On Even and Odd Variation-Diminishing Convolution Transforms.

A theorem of J.J. Sylvester gives an upper bound for the number of zeros, and hence sign changes, of a polynomial that can be written as a linear combination of shifted monomials having a common degree. If this common degree is even(odd), the upper bound is the smallest even(odd) number greater or equal to the number of sign changes in the sequence of coefficients of the linear transformation. This is an example of an even(odd) variation-diminishing transformation. Sylvester’s Theorem can be extended to convolution on the real line with monomial kernels, and these transforms are also even(odd) variation-diminishing. We provide here a characterization of all kernels $\Lambda$ for which $f \rightarrow \Lambda \ast f$ is even or odd variation-diminishing. (Received September 22, 2009)