Let $\ell_j : \mathbb{R}^d \to \mathbb{R}^{d_j}$ be surjective linear transformations, let $P : \mathbb{R}^d \to \mathbb{R}$ be a real-valued polynomial, let $B$ be a ball in $\mathbb{R}^d$. The associated sublevel sets are

$$E_\varepsilon(P,g_1,\ldots,g_n) = \{ y \in B : |P(y) - \sum_{j=1}^n g_j(\ell_j(y))| < \varepsilon \},$$

where $g_j : \mathbb{R}^{d_j} \to \mathbb{R}$ are arbitrary measurable functions. We study upper measure bounds of the form

$$|E_\varepsilon(P,g_1,\ldots,g_n)| \leq \rho(\varepsilon)$$

which are uniform over all measurable functions $g_j$, with $\rho(\varepsilon) \to 0$ as $\varepsilon \to 0$. Such bounds would be implied by conjectured multilinear oscillatory integral inequalities. We prove the sublevel set bounds under the natural nondegeneracy hypothesis on $P$, supplemented by an auxiliary rationality hypothesis. The analysis involves an alternative notion called finitely witnessed nondegeneracy, and relies on a variant of Szemerédi’s theorem due to Furstenberg and Katznelson. (Received September 22, 2009)