Continuous Dependence Results for Ill-posed Evolution Problems in a Banach Space.

We prove Hölder-continuous dependence results for the difference between certain ill-posed and well-posed evolution problems in a Banach space $X$. We consider the ill-posed evolution problem

$$\frac{du(t)}{dt} = A(t, D)u(t) \quad 0 \leq t < T$$

$$u(0) = \chi,$$

where $iD$ is the generator of a bounded strongly continuous group on $X$, and $A(t, D) = \sum_{j=1}^{k} a_j(t)D^j$ with $a_j \in C([0, T] : \mathbb{C})$ for each $1 \leq j \leq k$. We determine families $\{f(t, D)\}_{t \in [0, T]}$ of operators in $X$ such that the problem

$$\frac{dv(t)}{dt} = f(t, D)v(t) \quad 0 \leq t < T$$

$$v(0) = \chi$$

is well-posed and such that solutions of the well-posed problem (2) approximate known solutions of the original ill-posed problem (1). We use $C$-regularized evolution systems to obtain our approximation which establishes continuous dependence on modeling for the problems under consideration. Namely, assuming $u(t)$ and $v(t)$ are solutions of (1) and (2) respectively, we prove that

$$\|u(t) - v(t)\|_1 \leq C_1\beta^{1-\frac{\tau}{\gamma}}M^{\frac{\tau}{\gamma}}$$

for a suitable norm $\| \cdot \|_1$, where $0 < \beta < 1$, and $C_1$ and $M$ are constants independent of $\beta$. (Received September 21, 2009)