The geometry of projective structures on surfaces was studied classically by many people, with important contributions having been made by S. Lie, R. Liouville, A. Tresse, and J. Levine, among others. In particular, Liouville, in 1887, was the first to write down tensorial invariants and apply them to problems, such as characterizing projective structures that come from metrics. Cartan showed how to attach a projective connection to a projective structure and discussed how to generate differential invariants, though he did not find a method to generate all of the differential scalar invariants. In fact, a considerable amount of parabolic invariant theory machinery is still needed to do this.

In this talk, I will review the basics of projective structures on surfaces, including Liouville’s tensor and Cartan’s projective connection. I will discuss Liouville’s method of generating higher order differential invariants through projective differential operators and interpret this in the context of Cartan’s approach. Finally, I will apply this machinery to the classical problem, recently solved by Dujnaski, Eastwood, and myself, of characterizing those projective structures that arise from non-degenerate quadratic forms on surfaces. (Received June 23, 2009)