We introduce the $R$ cut-off covering spectrum and the cut-off covering spectrum of a metric space or Riemannian manifold. The spectra measure the sizes of localized holes in the space and are defined using covering spaces called $\delta$ covers and $R$ cut-off $\delta$ covers. They are investigated using $\delta$ homotopies which are homotopies via grids whose squares are mapped into balls of radius $\delta$.

On locally compact spaces, we prove that these new spectra are subsets of the closure of the length spectrum. We prove the $R$ cut-off covering spectrum is almost continuous with respect to the pointed Gromov-Hausdorff convergence of spaces and that the cut-off covering spectrum is also relatively well behaved. This is not true of the covering spectrum defined in our earlier work which was shown to be well behaved on compact spaces. We close by analyzing these spectra on Riemannian manifolds with lower bounds on their sectional and Ricci curvature and their limit spaces. (Received September 11, 2009)