We investigate relations between cubic graphs and Riemann surfaces that are constructed from a random choice of a graph and orientation. Our goal is to describe the global geometry of such a "typical" Riemann Surface. This model of constructing surfaces from graphs enables us to study properties like the Cheeger constant, systole length, and the size of embedded balls in large genus surfaces by examining random cubic graphs. (Received September 11, 2009)