A quantum state in an $n$-dimensional quantum system about which we have complete knowledge can be described by a vector in $n$-dimensional Hilbert space. However, quantum states about which we have less than complete knowledge are described by a density matrix, which is a positive semidefinite $n \times n$ matrix with trace 1. A quantum channel encapsulates the most general transformation that the laws of physics allow one to apply to a quantum state. There are several equivalent definitions of quantum channels, which we will explain in this lecture. Quantum channels are interesting mathematical objects which can be described solely using linear algebra, but about which there is still much to be discovered. We concentrate on the question of how much information can be transmitted over a quantum channel. The answer to this question for classical channels was given 1948 by Shannon’s famous capacity theorem. In the quantum case, several different capacities can be defined, these are closely related to measures of entanglement. We will present several recent discoveries about these capacities. (Received April 24, 2009)