Potential-Based Strategies for Breaker for Maker-Breaker Tic-Tac-Toe on the Integer Lattice with Numerous Directions.

We consider a Tic-Tac-Toe game played on the $d$-dimensional integer lattice. The game that we investigate is a Maker-Breaker version of Tic-Tac-Toe. In a Maker-Breaker game, the first player, Maker, only tries to occupy a winning line and the second player, Breaker, only tries to stop Maker from occupying a winning line. We consider the bounded number of directions game, in which we designate a finite set of direction-vectors $S \subset \mathbb{Z}^d$ which determines the set of winning lines. We show, by using the Erdős-Selfridge theorem and a modification of a theorem by Beck about games played on almost-disjoint hypergraphs, that for the special case when the coordinates of each direction-vector are bounded, i.e., when $S \subset \{\vec{v} : \|\vec{v}\|_{\infty} \leq k\}$, Breaker can win this game if the length of each winning line is on the order of $d^2 \lg(dk)$ and $d^2 \lg(k)$, respectively. (Received September 22, 2009)