

1056-91-2025

Craig A. Tovey* (cat@gatech.edu), School of ISyE, Georgia Tech, Atlanta, GA 30332-0205, and
Dylan Shepardon. *A Smallest Tournament Not Realizable by $\frac{2}{3}$ -Majority Voting, and Some
Open Problems in Spatial Voting Theory.*

Define the *predictability* number $\alpha(T)$ of a tournament T to be the largest supermajority threshold $\frac{1}{2} < \alpha \leq 1$ for which T could represent the pairwise voting outcomes from some population of voter preference orders. We establish that the predictability number always exists and is rational. Only acyclic tournaments have predictability 1; the Condorcet voting paradox tournament has predictability $\frac{2}{3}$; Gilboa has found a tournament on 54 alternatives (i.e. vertices) that has predictability less than $\frac{2}{3}$, and has asked whether a smaller such tournament exists. We exhibit an 8-vertex tournament that has predictability $\frac{13}{20}$, and prove that it is the smallest tournament with predictability $< \frac{2}{3}$. Our methodology is to formulate the problem as a finite set of two-person zero-sum games, employ the minimax duality and linear programming basic solution theorems, and solve using rational arithmetic.

We conclude by advertising several open problems of computational geometry, combinatorics, and probability from spatial voting theory. (Received September 22, 2009)