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**Thomas Warren Scanlon\*** ([scanlon@math.berkeley.edu](mailto:scanlon@math.berkeley.edu)), UC Berkeley, Department of Mathematics, Evans Hall, Berkeley, CA 94720-3840. *Counting special points: logic, Diophantine geometry and transcendence theory.*

I shall describe Jonathan Pila's recent unconditional proof of the André-Oort conjecture for powers of the  $j$ -line. While the theorem itself is beautiful, it is the proof which is really the most spectacular part of Pila's contribution in that he employs ideas from disparate areas and mathematical logic, in the form of the study of definability in o-minimal expansions of the real field is the central actor.

By an o-minimal expansion of the real field  $(\mathbb{R}, +, \cdot, 0, 1, \leq, \dots)$  we mean that we consider the real numbers as a first-order structure with at least the usual field operations and the usual order relation but possibly with other distinguished functions and relations so that every definable subset of the line is a finite union of points and intervals. Refining decomposition and parametrization theorems for sets definable in o-minimal structures, Pila and Wilkie produced upper bounds for the numbers of rational points lying on definable sets.

Using results of Peterzil and Starchenko on the definability of the  $j$ -function, Pila leverages the upper bounds on the number of rational points in definable sets against lower bounds on the size of the Galois orbits of CM points to prove his theorem. (Received September 22, 2010)