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**Julia F. Knight\*** ([knight.1@nd.edu](mailto:knight.1@nd.edu)), 255 Hurley Hall, Mathematics Department, University of Notre Dame, Notre Dame, IN 46556-5641, and **Karen Lange**. *Structures associated with real closed fields.*

Let  $R$  be a countable real closed field. A *value group* for  $R$  is a subgroup  $G$  of  $(R^+, \cdot)$  with just one representative for each equivalence class under the Archimedean valuation. A *residue field* for  $R$  is a maximal Archimedean subfield  $k$ . An *integer part* is a discrete ordered subring  $I$  such that for all  $x \in R$ , there exists  $i \in I$  with  $i \leq x < i + 1$ .

We showed that  $R$  has a value group that is  $\Delta_2^0(R)$ , and there is a residue field that is  $\Pi_2^0(R)$ . Both results are sharp. By a result of Mourgues and Ressayre,  $R$  has an integer part. Using their procedure, we obtain an integer part that is  $\Delta_{\omega}^0(R)$ . For all we know, there is a simpler procedure, yielding an integer part that is  $\Delta_2^0(R)$ . There is a maximal discrete ordered subring  $I$  that is  $\Delta_2^0(R)$ . Moniri and Marker gave examples showing that this need not be an integer part— $I$  does not extend to a  $Z$ -ring. There is a  $\Delta_2^0(R)$  ring  $I \subseteq R$  such that  $I$  is a maximal  $Z$ -ring. With Paola D’Aquino, we showed that this need not be an integer part for  $R$ . We showed that there is a computable real closed field with no  $n$ -c.e. integer part for any  $n$ . (Received September 13, 2010)