The Maximum-weight Stable Matching Problem.

Given a preference system $(G, \prec)$ and an integral weight function defined on the edge set of $G$ (not necessarily bipartite), the maximum-weight stable matching problem is to find a stable matching of $(G, \prec)$ with maximum total weight. In this talk we consider this $NP$-hard problem using linear programming and polyhedral approaches. We show that the Rothblum system for defining the fractional stable matching polytope of $(G, \prec)$ is totally dual integral if and only if this polytope is integral if and only if $(G, \prec)$ contains no so-called semistable partitions with odd cycles. We also present a combinatorial polynomial-time algorithm for the maximum-weight stable matching problem and its dual on any preference system containing no semistable partitions with odd cycles. Our results generalize Király and Pap’s theorem on the maximum-weight stable-marriage problem and rely heavily on their work. (Received September 17, 2010)