This talk will focus on how partially ordered sets help record topological structure, including mentioning some limitations in how much they can capture. I will briefly discuss my work on discrete Morse theory for order complexes of partially ordered sets and how this has been used e.g. to count by inclusion-exclusion. Then I'll turn things around and discuss more recent work on how topological structure of a stratified space can sometimes be gleaned from combinatorics of its closure poset combined with codimension one topology. This is used to show that certain stratified spaces arising from combinatorial representation theory are regular CW complexes homeomorphic to balls. Familiarity with this area will not be assumed. (Received September 19, 2010)