Reconstruction of graphs from metric balls of their vertices.

Given a graph $G$, the metric ball of radius $r$ about a vertex $v$ is $B_r(v) = \{ w \in V(G) : d(v, w) \leq r \}$. We prove a conjecture of Levenshtein, that if $G$ has girth at least $2r + 3$ and no terminal vertices then we can reconstruct $G$ from the function $B_r$. This is best possible since a cycle on $2r + 2$ vertices cannot be reconstructed in this way. The previous best known result was for graphs with girth at least $2r + 2\lceil (r - 1)/4 \rceil + 1$ and no terminal vertices. (Received September 21, 2010)