
It has been known for at least 2500 years that mathematics and music are directly related. This article explains and extends ideas originating with Euler involving labeling parts of graphs with notes in such a way that other parts of the graphs correspond in a natural way to chords. The principal focus of this research is the notion of diatonic labelings of cubic graphs, that is, labeling the edges with pitch classes in such a way that vertices are incident with edges labeled with the pitch classes of a triad in a given diatonic scale. The pitch classes are represented in a natural way with elements of $\mathbb{Z}_{12}$, the integers modulo twelve.

Several classes of cubic graphs are investigated and shown to be diatonic. Among the graphs considered are Platonic Solids, cylinders, and Generalized Petersen Graphs. It is shown that there are diatonic cubic graphs on $n$ vertices for even $n \geq 14$. Also it is shown that there are cubic graphs on $n$ vertices that do not have diatonic labelings for all even $n \geq 4$. The question of forbidden subgraphs is investigated, and a forbidden subgraph for diatonic graphs, or “clash”, is demonstrated. (Received September 06, 2010)