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Neil Hindman* (nhindman@aol.com). *Monochromatic sums equal to products in \mathbb{N} .*

Csikvári, Gyarmati, and Sárközy asked whether, whenever the set \mathbb{N} of positive integers is finitely colored, there must exist monochromatic a, b, c , and d such that $a + b = cd$ and $a \neq b$. We provide an affirmative answer, establishing the following much stronger statement, (where FS and FP refer to “finite sums” and “finite products” respectively).

Theorem. *Let $m, r \in \mathbb{N}$ with $m > 1$ and let $\mathbb{N} = \bigcup_{k=1}^r A_k$. There exist $k \in \{1, 2, \dots, r\}$, $d \in \mathbb{N}$, and sequences $\langle x_t \rangle_{t=1}^m$ and $\langle y_t \rangle_{t=1}^m$ such that*

(1) $\langle x_t \rangle_{t=1}^m$ has distinct finite sums;

(2) $\langle y_t \rangle_{t=1}^m$ has distinct finite products;

(3) $\sum_{t=1}^m x_t = \prod_{t=1}^m y_t = d$;

(4) $FS(\langle x_t \rangle_{t=1}^m) \cup FP(\langle y_t \rangle_{t=1}^m) \subseteq A_k$; and

(5) $FS(\langle x_t \rangle_{t=1}^m) \cap FP(\langle y_t \rangle_{t=1}^m) = \{d\}$.

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