On the transcendence of Fourier and other infinite series.

We investigate the transcendental nature of the sums

$$\sum_{n \in \mathbb{Z}} \frac{f(n)A(n)}{B(n)} \quad \text{and} \quad \sum_{n \in \mathbb{Z}} \frac{A(n)}{B(n)}$$

where $A(x), B(x)$ are polynomials with algebraic coefficients with $\deg A < \deg B$, $f$ is an algebraic valued periodic function, and the sum is over integers $n$ which are not zeros of $B(x)$. By relating these sums to the Fourier series of certain functions we are able to obtain transcendence results. In certain cases we relate these sums to a theorem of Nesterenko regarding the algebraic independence of $\pi$ and $e^{\pi \sqrt{D}}$ for positive integer $D$. (Received September 22, 2010)