The $F$-signature is a fundamental measure of the singularities of a local ring $R$ of characteristic $p > 0$. Explicitly, it measures the number of copies of $R$ that a direct sum decomposition of $R^{1/p^e}$ has, as $e$ increases. It is closely related to the notion of $F$-regularity.

In this talk, we discuss the generalization of the $F$-signature to a pair $(R, \Delta)$ where $\Delta$ a some $\mathbb{Q}$-divisor. This generalization is natural since concepts like $F$-regularity have long since been generalized to this context. Pairs are useful because, given a morphism of rings $R \to S$, they allow one to study the singularities of $R$ by studying the singularities of a certain pair on $S$ (which may be easier). This technique also allows us to answer an open question of Aberbach and Enescu related to the $F$-signature and the splitting prime. (Received September 20, 2010)