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Thomas Noll* (noll@cs.tu-berlin.de), Pfarrstr. 6, 04860 Torgau, Germany. *Well-formed Scales and Alteration: An Arithmetic Investigation into Music Notation.*

The paper elaborates upon theoretical cross links between E. Regener's mathematical approach to musical notation, N. Carey and D. Clampitt's theory of well-formed scales, E. Agmon's investigation into combined diatonic and chromatic tone relations, and J. Hooks theory of key signatures and enharmonic systems. It studies an extension of a linear automorphism of the cyclic group \mathbb{Z}_m to an isomorphism between two group extensions of this group by the group of integers \mathbb{Z} . Music-theoretically, the isomorphism extends the generic level of a well-formed scale to a level of chromatic alteration. The middle arrow α in the diagram

$$\begin{array}{ccccccccc} E_1: & 0 & \rightarrow & \mathbb{Z} & \rightarrow & \mathbb{Z} \times \{0, \dots, m-1\} & \rightarrow & \mathbb{Z}_m & \rightarrow & 0 \\ & & & \downarrow id & & \downarrow \alpha & & \downarrow \cdot n & & \\ E_{\bar{n}}: & 0 & \rightarrow & \mathbb{Z} & \rightarrow & \mathbb{Z} \times \{0, \dots, m-1\} & \rightarrow & \mathbb{Z}_m & \rightarrow & 0 \end{array}$$

controls the conversion from generation order to scalar order under the effect of chromatic alteration. The extension of the generic level by register behaves analogously. The combination of both extensions refines the concept of height-width duality from recent joint work with D. Clampitt. (Received September 22, 2010)