Dhanapati Adhikari* (dadhika@math.okstate.edu), 401 Mathematical Sciences, Stillwater, OK 74078, and Chongsheng Cao and Jiahong Wu. Global regularity results for the 2D Boussinesq equations with vertical dissipation. Preliminary report.

This paper furthers the study of Adhikari, Cao and Wu [Journal of Differential Equations 249 (2010) No. 5 1078-1088] on the global regularity issue concerning the 2D Boussinesq equations with vertical dissipation and vertical thermal diffusion. The norm of the vertical velocity $v$ in the Lebesgue space $L^q$ with $2 \leq q < \infty$ is shown to be bounded by $C_1 q$ for $C_1$ independent of $q$. This bound significantly improves the previous exponential bound in $q$. In addition, we prove that, if $v$ satisfies $\int_0^T \sup_{q \geq 2} \frac{\|v(\cdot,t)\|_{L^q}^2}{q} \, dt < \infty$, then the associated solution of the 2D Boussinesq equations preserve its smoothness on $[0,T]$. In particular, $\|v\|_{L^q} \leq C_2 \sqrt{q}$ implies global regularity. (Received September 22, 2010)