Shift operators and stability in delayed dynamic equations.

In this paper, we use what we call (Adivar-Raffoul) shift operator so that general delay dynamic equations of the form

\[ x^\Delta(t) = a(t)x(t) + b(t)x(\delta_-(h,t))\delta^\Delta(h,t), \quad t \in [t_0, \infty)_T \]

can be analyzed with respect to stability and existence of solutions. By means of the shift operators we define a general delay function opening an avenue for the construction of Lyapunov functional on time scales. Thus, we use Lyapunov’s direct method to obtain inequalities that lead to stability and instability. Therefore, we extend and unify stability analysis of delay differential, delay difference, delay \( h \)-difference, and delay \( q \)-difference equations which are the most important particular cases of our delay dynamic equation.

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