The lemniscate constant $L$ is defined as half the perimeter of the lemniscate curve $r^2 = \cos(2\theta)$. This number shares some features of the number $\pi$. Recently Stedall [2] showed that the famous continued fraction for $\pi$ of Lord Brouncker is only one of an infinite sequence of continued fractions that he had discovered and that seem to have been forgotten in recent times. This author wrote about them in [1]. In this paper we find another infinite sequence of continued fractions very closely related to Lord Brouncker’s, only these converge to a rational function of $L^2/\pi$. Infinite products resembling the famous product of Wallis for $\pi$ are also found for these rational functions of $L^2/\pi$ as well as for $L$ itself. [1] Osler, Thomas J., Lord Brouncker’s forgotten sequence of continued fractions for $\pi$. International Journal of Mathematical Education in Science and Technology, Volume 41, Issue 1 January 2010 , pages 105 - 110. [2] Stedall, Jacqueline A., Catching Proteus: The Collaborations of Wallis and Brouncker. I. Squaring the Circle, Notes and Records of the Royal Society of London, Vol. 54, No. 3, (Sep., 2000), pp. 293 -316 (Received September 20, 2010)