Suppose $M$ is a $W^*$-algebra, that $E$ is a $W^*$-correspondence over $M$, and that $\mathcal{T}_+(E)$ is the tensor algebra of $E$. If $\sigma$ is a normal representation of $M$ on a Hilbert space $H$, then there is a $W^*$-correspondence over $\sigma(M)'$, denoted $E^\sigma$ and called the $\sigma$-dual of $E$, such that elements of $\mathcal{T}_+(E)$ can be represented as $B(H)$-valued functions defined on the closed unit ball, $\overline{D}(E^\sigma)$. The functions are continuous on $\overline{D}(E^\sigma)$ and analytic on the open unit ball, $D(E^\sigma)$, as $B(H)$-valued functions, but they have additional structure that we shall describe. We shall discuss the properties of these functions in several concrete settings, showing connections with the theory of rings of generic matrices, and we shall present a number of unsolved problems that we have found interesting. (Received September 10, 2010)