The third lecture will be devoted to applications of expanders to geometry. The various equivalent definitions of expanders in the 1st lecture hinted toward seeing the expanding property as “an isoperimetric inequality”. For a compact Riemannian manifold $M$, its fundamental group $\pi_1(M)$ has property $(\tau)$ (namely, its finite quotients Cayley graphs form a family of expanders) iff the finite sheeted covers of $M$ satisfy a uniform isoperimetric inequality. This is a key observation (going back to Brooks) which enables to tackle some of the hardest geometric problems using expanders.

The main applications are toward Thurston conjecture on the non-vanishing of the first Betti number of finite sheeted covers of compact hyperbolic manifolds. It also tackles the conjecture that every compact hyperbolic 3-manifold has a finite sheeted cover which is Haken. After the work of Perelman, this is probably the most important open problem in the geometry of 3-manifolds.

If time permits we will also talk about the Baum-Connes conjecture. (A separate talk on this will be given by Paul Baum in the special session associated with these colloquium talks). (Received September 16, 2010)