In this paper, we investigate a particular Diophantine equation, \( X^2 + Y^3 = 6912Z^2 \), and a set of solutions to the equation, which are derived from some polynomials in \( \mathbb{Z}[x, y] \). We focus on three polynomials \( X = f(x, y) \), \( Y = g(x, y) \) and \( Z = h(x, y) \) that satisfy the Diophantine equation and the greatest common divisors for the integer values of the polynomials. These polynomials are relatively prime in \( \mathbb{Q}[x, y] \). However, for a fixed integer pair \( x_0, y_0 \), the integer values \( f(x_0, y_0) \), \( g(x_0, y_0) \) and \( h(x_0, y_0) \) are not necessarily relatively prime in \( \mathbb{Z}[x, y] \). We investigate the greatest common divisors (GCDs) between these three polynomial values for specific integer pairs \( x_0 \) and \( y_0 \). We focus on the cases where \( y = 1 \) and \( y = 2 \). For these cases, we give complete classifications on the distribution of the GCDs. We use the Gröbner Bases technique as an aid in investigating the GCDs for \( f, g, h \) in \( \mathbb{Z}[x, y] \). We then generalize the results from the cases \( y = 1 \) and \( y = 2 \) to obtain similar properties for the GCDs of \( f, g, h \) for all \( x \) and \( y \) in \( \mathbb{Z}[x, y] \). (Received September 17, 2010)