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(sshahriari@pomona.edu), Department of Mathematics, Pomona College, Claremont, CA 91711, and Christopher Towse* (ctowse@scrippscollege.edu), Department of Mathematics, Scripps College, Claremont, CA 91711. Partitioning the Boolean lattice into chains of large minimum size, Part I.
Let $\mathbf{2}^{[n]}$ denote the Boolean lattice of order $n$, that is, the poset of subsets of $\{1, \ldots, n\}$ ordered by inclusion. Recall that $\mathbf{2}^{[n]}$ may be partitioned into what we call the canonical symmetric chain decomposition (due to de Bruijn, Tengbergen, and Kruyswijk), or CSCD. Motivated by a question of Füredi, we show that there exists a function $d(n) \sim \frac{1}{2} \sqrt{n}$ such that for any $n \geq 0, \mathbf{2}^{[n]}$ may be partitioned into $\binom{n}{\lfloor n / 2\rfloor}$ chains of size at least $d(n)$. (For comparison, a positive answer to Füredi's question would imply that the same result holds for some $d(n) \sim \sqrt{\pi / 2} \sqrt{n}$.) More precisely, we first show that for $0 \leq j \leq n$, the union of the lowest $j+1$ elements from each of the chains in the CSCD of $\mathbf{2}^{[n]}$ forms a poset $\mathbf{T}_{j}(n)$ with the normalized matching property and log-concave rank numbers. We then use our results on $\mathbf{T}_{j}(n)$ to show that the nodes in the CSCD chains of size less than $2 d(n)$ may be repartitioned into chains of large minimum size, as desired. (Received September 26, 2000)

