Michael N Bleicher* (bleicher@math.wisc.edu), Prof. Michael Bleicher, Mathematics - Clark Atlanta University, 223 James P. Brawley Dr. SW, Atlanta, GA 30314, and J. Marshall Osborn (osborn@math.wisc.edu), Prof. J. Marshall Osborn, Mathematics - University of Wisconsin, Van Vleck Hall, Madison, WI 53706. Isoperimetric Networks With Five Regions In The Euclidean Plane. Preliminary report.
Isoperimetric problems were considered by Xenodorus in classical Greek times. The problem of surrounding regions of given areas as efficiently as possible by is an important practical and theoretic problem. Implicit in the classical statement, regions were connected and there were just the required number of regions. Recently, Frank Morgan posed a more general problem by relaxing the conditions that the regions be connected and that no additional regions (dead spaces) be surrounded. It is intuitively clear that the relaxation should not change the solution to the problem of surrounding $n$ regions of given area with the shortest length of arcs, but the proof has been elusive. In the corresponding problem with natural boundary given, it is possible to surround more area by allowing disconnected regions. Also, if the natural boundary has infinite length, a solution may not exist. We show that if, for all finite numbers of connected regions the optimal solution, which must exist, has no dead spaces, then for any solution of the relaxed problem in which the number of components of the complement of the arc network is finite must also lack dead spaces. Further we show that for an optimal network surrounding at most five connected regions, a dead space will not exist. (Received October 01, 2000)

