Bernardo M Abrego* (abrego@math.rutgers.edu), Department of Mathematics, Hill Center, Rutgers University, 110 Frelinghuysen Road, Piscataway, NJ 08854, and Gyorgy Elekes and Silvia Fernandez (sfernand@math.rutgers.edu), Department of Mathematics, Hill Center, Rutgers University, 110 Frelinghuysen Road, Piscataway, NJ 08854. Finite point sets with many similar copies of a given pattern. Preliminary report.
Consider a $k$-element subset $P$ of the plane. It is known that the maximum number of sets similar to $P$ that can be found among $n$ points in the plane is $\Theta\left(n^{2}\right)$ if and only if the cross ratio of any quadruplet of points in $P$ is algebraic. In this talk we study the structure of the extremal $n$-sets $A$ which have $c n^{2}$ similar copies of $P$. As our main result we prove the existence of large lattice-like structures in such sets $A$. In particular we prove that, for $n$ large enough, $A$ must contain $m$ points in a line forming an arithmetic progression, and moreover, when $P$ is not cocyclic or collinear, $A$ contains $m \times m$ lattices. We show that this result is best possible when $P$ is cocyclic or collinear by constructing $n$-element sets $A$ with $c_{P} n^{2}$ copies of $P$ and without $k \times k$ lattice subsets. Finally, we look at the case when $P$ is an equilateral triangle. We give non-trivial bounds for the maximum number of equilateral triangles determined by $n$ points without arithmetic progressions of size $\lambda(n)$. (Received October 02, 2000)

