

## PAPERS PRESENTED AT MEETINGS

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**THIS CALENDAR** lists meetings of the Society which have been approved by the Council at which papers may be presented. Programs of Annual Meetings appear in the *Notices* and on the AMS website; programs for sectional meetings appear on the AMS Web pages in the Meetings & Conferences section, and are electronically archived in the *Notices* section on the AMS website.

MEETING #	DATE	PLACE	ABSTRACT DEADLINE	ABSTRACT ISSUE
1126	March 10–12, 2017	Charleston, SC	January 17	Vol 38, No. 2
1127	April 1–2, 2017	Bloomington, IN	February 7	Vol 38, No. 2
1128	April 22–23, 2017	Pullman, WA	February 28	Vol 38, No. 2
1129	May 6–7, 2017	New York, NY	March 14	Vol 38, No. 2
1130	July 24–28, 2017	Montréal, Canada	March 31	NONE
1131	September 9–10, 2017	Denton, TX	July 18	Vol 38, No. 3
1132	September 16–17, 2017	Buffalo, NY	July 25	Vol 38, No. 3
1133	September 23–24, 2017	Orlando, FL	August 1	Vol 38, No. 4
1134	November 4–5, 2017	Riverside, CA	September 12	Vol 38, No. 4
1135	January 10–13, 2018	San Diego, CA	TBA	Vol 39, No. 1
1136	March 17–18, 2018	Columbus, OH	TBA	TBA
1137	April 14–15, 2018	Portland, OR	TBA	TBA
1138	April 14–15, 2018	Nashville, TN	TBA	TBA
1139	April 21–22, 2018	Boston, MA	March 6	TBA
1140	June 11–14, 2018	Shanghai, Peoples Rep of China	TBA	TBA

## ATLANTA, GA, January 4–7, 2017

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Abstracts of the 1125th Meeting.

### 00 ► *General*

1125-00-9      **Tobias Colding\***, Massachusetts Institute of Technology, Cambridge, MA 02139. *Arrival time.*

Modeling of a wide range of physical phenomena leads to tracking fronts moving with curvature-dependent speed. A particularly natural example is where the speed is the mean curvature. If the movement is monotone inwards, then the arrival time function is the time when the front arrives at a given point. It has long been known that this function satisfies a natural differential equation in a weak sense but one wonders what is the regularity. It turns out that one can completely answer this question. It is always twice differentiable and the second derivative is only continuous in very rigid situations that have a simple geometric description. The proof weaves together analysis and geometry. (Received May 22, 2016)

1125-00-22      **John Preskill\***, California Institute of Technology, Pasadena, CA 91125. *Quantum computing and the entanglement frontier.*

The quantum laws governing atoms and other tiny objects seem to defy common sense, and information encoded in quantum systems has weird properties that baffle our feeble human minds. John Preskill will explain why he loves quantum entanglement, the elusive feature making quantum information fundamentally different from information in the macroscopic world. By exploiting quantum entanglement, quantum computers should be able to solve otherwise intractable problems, with far-reaching applications to cryptology, materials, and fundamental physical science. Preskill is less weird than a quantum computer, and easier to understand. (Received May 23, 2016)

1125-00-105 **Aklilu Zeleke\*** ([zeleke@stt.msu.edu](mailto:zeleke@stt.msu.edu)), 919 E. Shaw Lane, Room W25, East Lansing, MI 48825. *Examples of Research by Undergraduates at the Summer Undergraduate Research Institute in Experimental Mathematics (SURIEM).*

Over the past seven years we involved over one hundred undergraduates from across the nation in mathematics research at the Summer Undergraduate Research Institute in Experimental Mathematics (SURIEM) at Michigan State University. Students worked in projects from a variety of mathematics disciplines including combinatorics, graph theory, discrete mathematics, probability, statistics and mathematical biology. In this talk we give examples of research problems that led to peer reviewed publications and conference presentations. We will also share problems for future investigations. (Received July 28, 2016)

1125-00-123 **Linda Brown Westrick\*** ([westrick@uconn.edu](mailto:westrick@uconn.edu)). *Computation and information in sofic shifts.*

Any two-dimensional sofic shift can be described as the set of infinite tilings from a fixed tileset, in which some of the distinctions between the tiles have subsequently been erased. Classically, there are tilesets whose infinite tilings perform arbitrary computations, so in a sofic shift these computations can be hidden, even as they control what is visible. By contrast, in an effectively closed shift, the restrictions on what can happen are enumerated by an algorithm that does not have to share physical space with the patterns it controls. The sofic shifts are a proper subclass of the effectively closed shifts, but the exact limitations on the sofic computations are not well understood. Towards one direction of this question, we construct classes of “computationally-intensive.. sofic shifts.. (Received August 02, 2016)

1125-00-263 **Frank Morgan\*** ([frank.morgan@williams.edu](mailto:frank.morgan@williams.edu)). *The Future of the AMS Notices.*

As the Notices of the American Mathematical Society develops an enhanced electronic presence while preserving an attractive hard copy, as Editor-in-Chief I describe some of our plans and seek your suggestions for making Notices more accessible, attractive, and valuable to our larger community. (Received August 21, 2016)

1125-00-328 **Maria Mannone\*** ([manno012@umn.edu](mailto:manno012@umn.edu)). *Theoretical Physics and Category Theory as Tools for Analysis of Musical Performance and Composition.*

Musical performance starts from an indication of movement (a curve) hidden in the score, which then is transformed by the musician into a physical gesture (another curve), connecting the symbolic reality of the score to the physical reality of acoustics. Composition from improvisation follows the inverse path, from physical to symbolic. Symbolic gestures can be ideally transformed into physical ones via a connecting surface, as a “world-sheet” in physics. This formalism can be applied to any musical instrument, including the voice. The relations between gestures on different musical instruments can be framed through category theory, allowing comparison within music itself, and between music and other fields. Experiments in which images and gestures in the visual arts have been transformed into music have their explanation in categorical terms, via gestural analogies and similarity. In fact, the mathematical definition of musical gestures, apart from explaining and modeling musical practice, may constitute a musical element itself in composition. We conclude the presentation with some examples of music from images, and an excerpt from an original orchestral piece, where gestural analogies and morphisms connect instruments and sounds. (Received August 26, 2016)

1125-00-618 **Meghan Stuart\*** ([mstuart3@vols.utk.edu](mailto:mstuart3@vols.utk.edu)), **Jorge Cisneros Paz**, **María José Pérez Pereda** and **Navdha Malhotra**. *Ballast Cleaning Scheduling Optimization.*

CSX Corporation is one of the nation’s leading railway transportation suppliers. A large part of CSX annual expenses results from track upkeep and repair work, including track-ballast cleaning. Having clean ballast prevents damage to ties, supports and holds the ties in place, facilitates water drainage, and reduces vegetation around railways. Ballast cleaners’ routes are normally scheduled manually due to frequent changes in the planned route with constraints such as geography, weather, types of track, cleaner speed, cleaner working hours, repair work on portions of track, and union furlough activity. In this presentation, we describe an algorithm to schedule optimized ballast-cleaner routes, completing all high priority jobs in minimum time. The algorithm is based on a modified version of a heuristic traveling salesman solution algorithm that takes as input a list of jobs with priorities, regions with curfew, and other relevant scheduling data, and outputs a near-optimal route for the following year. The algorithm is configurable and dynamic to change as conditions and constraints are altered. (Received September 08, 2016)

1125-00-929 **Shenglan Yuan\***, 31-10 Thomson Ave, Long Island City, NY 10009. *The Experience of Mentoring Undergraduate Researchers*. Preliminary report.

In this talk we'll describe the experience—and challenges—of bringing authentic mathematical research to undergraduate students at a community college. In particular we will address the importance of finding suitable topics for students. We'll highlight several applied math projects completed by students in such a program. These include mathematical modeling problems, optimal location problems, and origami applications. (Received September 13, 2016)

1125-00-1200 **Huan Lei\***, 2000 Stevens Dr, Apt 207, Richland, WA 99354, and **Nathan Baker** and **Xiantao Li**. *Quantifying quasi-equilibrium and non-equilibrium properties of biomolecule systems*.

Biomolecules exhibit conformation fluctuations near equilibrium states, inducing uncertainty in various biological properties near metastable states as well as transition between the states. We have developed a general method to quantify the uncertainty of target properties induced by conformation fluctuations. For local properties, to alleviate the high dimensionality of the conformation space, we propose a method to increase the sparsity by defining a set of collective variables within active subspace, which increases the accuracy of the surrogate model. For dynamic properties, we develop a data-driven method to evaluate the memory kernel of the energy-dissipation process based generalized Langevin Equation. The method is demonstrated on solvation properties and is generalizable to investigate uncertainty in numerous biomolecular properties. (Received September 15, 2016)

1125-00-1227 **Michael Weingart\*** ([weingart@math.rutgers.edu](mailto:weingart@math.rutgers.edu)). *Rutgers TA Training: Continuity and Change*.

Rutgers mathematics TAs complete a semester-long training course, typically in spring of the first year of graduate study, before receiving a teaching assignment. Components include recorded practice teaching, self-criticism of the recordings, one-on-one discussion of the recordings with an experienced faculty member, as well as training in running an active learning format in calculus, and in grading the write-ups of the more involved types of homework assigned in certain courses.

A major challenge in recent years has been to adjust our course formats and modes of teaching to the rapid change in faculty demographics (a cluster of retirements) and student enrollment patterns (a surge of interest in STEM subjects, including math courses beyond calculus). TA roles now exist which hadn't just a few years ago – such as providing regular course-wide support in an online setting for a large multi-lecture linear algebra course – and our training program must catch up to the new reality. Another challenge is to provide the training and professional development to non-tenure track faculty, whether full or part time. This talk will discuss some of the productive details of the training program as it has been and continues to be, as well as ideas for addressing new challenges. (Received September 15, 2016)

1125-00-1407 **Jeremy Kastine\*** ([jkastine@highlands.edu](mailto:jkastine@highlands.edu)). *Rhythmic and Melodic L-Canons*. Preliminary report.

Let  $A$  be a finite set of real numbers, and let  $F$  be a finite set of transformations of the form  $f_{c,d} = cx + d$  where  $c \neq 0$ . We will say that  $(A, F)$  is a rhythmic L-canon provided that  $|f_{c_1,d_1}(A) \cap f_{c_2,d_2}(A)| \leq 1$  for distinct  $f_{c_1,d_1}, f_{c_2,d_2} \in F$  and  $|f_{c_1,d_1}(a_1) - f_{c_2,d_2}(a_2)| \in \{0\} \cup [1, \infty)$  for all  $f_{c_1,d_1}, f_{c_2,d_2} \in F$  and  $a_1, a_2 \in A$ . Given fixed  $|A|$  and  $|F|$ , the rhythmic L-canons of the most musical and mathematical interest are those for which  $\{f(a) : a \in A, f \in F\}$  has a relatively small range. In this talk, I will describe a process for finding rhythmic L-canons which are locally and globally optimal in this sense. I will also demonstrate how to extend rhythmic L-canons to melodic L-canons, in which each part is a transformed version of a single melody and any two parts are in unison whenever they overlap. (Received September 16, 2016)

1125-00-1438 **Sandra Rucker\*** ([srucker@cau.edu](mailto:srucker@cau.edu)), Clark Atlanta University, Atlanta, GA 30314. *A Historical Perspective of Mathematics at Morris Brown College*.

We will discuss mathematics and mathematics related degree programs at Morris Brown College from a historical perspective. The current status of these programs also will be addressed. In addition, we will examine the scholarly contributions of its alumni to the Atlanta University Center and the larger academic community. (Received September 16, 2016)

1125-00-1595 **Elaheh Gorgin\*** ([elaheh.gorgin@minotstateu.edu](mailto:elaheh.gorgin@minotstateu.edu)). *Numerical Treatment of Ill-posed Linear Systems*. Preliminary report.

Inverse problems arise in many branches of science and engineering including statistics, geophysics, remote sensing, astronomy, physics, weather predictions, and many other fields. An equation  $Ax = b$  in which the

matrix  $A$  is generally very ill-conditioned and the data  $b$  is not exact, represents an ill-posed problem. This kind of problems are essentially under-determined due to the cluster of small singular values. The computed solution is potentially very sensitive to perturbations of the data. In such a case, it is essential to use numerical regularization theory to provide efficient and numerically stable methods that lead to useful stabilized solutions. Tikhonov regularization is one of the most popular regularization method for estimating the solutions of ill-posed problems and a significant drawback for this method is the need to choose the regularization parameter.

In this work, we summarize two known parameter choice techniques, the L-curve method and the multiplicative regularization method, and also we introduce a new parameter choice strategy. We go over the science and the theory behind these methods, compare their performances on fourteen different test problems for various noise levels, and discuss some of the applications. (Received September 18, 2016)

1125-00-1635     **Jon Kochavi\*** (jkochav1@swarthmore.edu), Department of Music and Dance, Swarthmore College, 500 College Ave., Swarthmore, PA 19081. *The Fibonacci sequence as metric suspension in Luigi Nono's "Il canto sospeso"*. Preliminary report.

In most Western and non-Western music, there is some degree of beat hierarchy, encapsulated in the metric time signature in the notated score. In his 1956 settings of defiant texts drawn from the final letters written by captured Italian resistance fighters awaiting execution, Luigi Nono eschews this approach, creating a seamless flow of rhythmically sustained and punctuated sounds. Despite the lack of metric regularity, however, the pitch durations (as well as pitches) are entirely compositionally controlled. This talk will explore the technique Nono uses to achieve this metric suspension in the second of these settings. Nono draws upon the Fibonacci sequence to determine pitch durations. Properties of the Fibonacci sequence modulo  $m$ , including the Pisano period and the distribution of residues of the sequence modulo  $m$ , will be discussed in relation to Nono's setting. The expressive effect of this compositional choice resonates both with the ideals of the *resistenza* and the post-war artistic responses to it. (Received September 18, 2016)

1125-00-1722     **Ju Zhou\*** (zhou@kutztown.edu) and **Yun Lu** (lu@kutztown.edu). *Assessing online teaching versus traditional face-to-face teaching across multiple sections of service mathematics courses*. Preliminary report.

In recent years, online teaching is more and more popular. But it is still controversial about whether mathematics can be successfully delivered through online teaching. This presentation is about a project designed to assess the effectiveness of online teaching versus the traditional face-to-face teaching across multiple sections of service mathematics courses. The traditional face-to-face teaching is largely lecture-based, while the online teaching strategy uses video, lecture notes, discussion board, etc. The Projector Directors will conduct the assessment in two sections (one online and one in regular class) of MAT 017 Introduction to Mathematics course. For each course, the data about student attitude towards mathematics, student attitude toward self-performance, and student mathematics efficacy about the course subject will be collected before the course began and near the end of the course. The Projector Directors will compare the two sets of data among one section to determine their improvement in the course subject, and compare the data between two sections to identify the effectiveness of online teaching versus the traditional face-to-face teaching. (Received September 19, 2016)

1125-00-1723     **Ilker Kocyyigit\*** (ilkerk@umich.edu). *Incorporating Sparsity Information in Inverse Problems*.

The existence of prior knowledge in inverse problems offers new challenges as well as new opportunities for bringing new ideas to inverse problems. The prior knowledge of sparsity of unknown localized targets might be used to improve various aspects of some inverse problems such as the ones arising from array imaging and Synthetic Aperture Radar (SAR). In this talk we discuss some of the sparsity promoting methods used in these inverse problems as well as the computational aspects of their construction and analysis. We discuss the stability and resolution of the reconstructed images and how they are related to the features of the source configuration and the measured data. Such features include number of measurement data sets and its richness, sparsity and well-separatedness properties of the support of the unknown configuration. We present numerical simulations to support the presented results. (Received September 19, 2016)

1125-00-1816     **Joachim Mueller-Theys\*** (mueller-theys@gmx.de). *A Mathematical Linguistics*. Preliminary report.

The fundamental principle of language is to take some  $s \in S$  to represent some  $o \in O$ . A set  $R$  of such *namings*  $\langle s, o \rangle$  is called a *representation* :iff all  $s \in S$  are non-vacuous and non-ambiguous with respect to  $R$ . It turns out that the representations can be identified with the functions  $\mu: S \rightarrow O$ , therefore called *meaning functions*.  $s, t$  are *equivalent* :iff  $\mu(s) = \mu(t)$ .

For such *languages*  $\mu: S \rightarrow O$  and  $\nu: D \rightarrow O$ ,  $\tau: S \rightarrow D$  is called a *translation* iff there is invariance of meaning, viz.  $\nu \circ \tau = \mu$ . If  $d = \tau(s)$ ,  $s$  is called *source* and  $d$  *destination*.

A mapping  $\kappa$  from some  $C$  to  $S$  is called an *encoding*, where  $\kappa(c)$  is  $c$  in *clear*.  $\kappa$  becomes a translation, not before  $\nu := \mu \circ \kappa$  bestows meaning on all *codes*.

Concepts are namings of sets, predicates of relations, elementary propositions of circumstances. Accordingly, we define meaning functions for predicate symbols and elementary sentences. (Received September 21, 2016)

1125-00-1872 **Sameeksha Khillan\*** (sameeksha.khillan@gmail.com), **Edgar Sucar** (edgarsucar@gmail.com), **Xiran Liu** (liu.xiran@wustl.edu) and **Zichao Li** (lizichao7@gmail.com). *Optimizing the Quality-Cost Trade-off of Human Annotation for labeling web-pages to train web page classifiers.*

Google gets human annotators to get training data for binary web page classifiers. Since this data is used as gold standard for training and evaluating classification models, its accuracy, based on the quality of human annotation, is of great importance to the classifier's performance. We want to develop and analyze strategies to mitigate human mistakes to achieve highest possible classifier performance respecting time and budget constraints. We simulate human annotated data based on previous records of rater accuracy and test these strategies. We develop two strategies: the first is an iterative multiple annotation strategy in which multiple humans annotate each web page with a particular distribution of raters (optimized for best data quality) assigned to each web page in advance. The second is based on prior probability of human error combined with confidence output by the classifier to clean evaluation data. We believe these strategies offer solutions to optimize quality-cost trade-off of human annotation. We observe that certain multiple rating orders and combinations lead to better data quality and also data quality is more relevant than data size for better performance, which indicates that a better strategy for rater combination is to spend more on better quality raters. (Received September 19, 2016)

1125-00-1951 **Girija Sarada Nair-Hart\*** (nairhaga@uc.edu). *Fostering concept modification through Light Board technology.*

Technology can promote concept modification by increasing the momentum to effectively complete the cycle of accommodation and equilibration of new concepts. To overcome misconceptions, a student may need to interact with alternate explanations and figures and diagrams multiple times. Light board videos are easy to construct and edit and the text or sketch stays visible while it is produced. Students appear to be more inclined to watch these high quality, colorful videos in which their instructor's face remains visible throughout the presentation. The increased attention and mind capture made possible by light board technology can thus expedite concept modification in learners. (Received September 19, 2016)

1125-00-1997 **Jason D Yust\*** (jason.yust@gmail.com), Boston University School of Music, 855 Commonwealth Ave., Boston, MA 02215. *Geometrical Realizations of Two- and Three-Dimensional Generalized Tonnetze.*

Some recent work on generalized Tonnetze has examined the topologies resulting from Richard Cohn's common-tone based formulation, while other work has reformulated the Tonnetz as a network of voice-leading relationships and investigated the resulting geometries. This paper adopts the original common-tone based formulation and takes a geometrical approach, showing that Tonnetze can always be realized in toroidal spaces, and that the resulting spaces always correspond to one of the possible Fourier phase spaces. We can therefore use the DFT to optimize the given Tonnetz to the space (or vice-versa). I interpret two-dimensional Tonnetze as simplicial decompositions of the 2-torus into regions associated with the representatives of a single trichord type. The natural generalization to three dimensions is therefore simplicial decompositions of the 3-torus. This means that a three-dimensional Tonnetze is, in the general case, a network of *three* tetrachord-types related by shared trichordal subsets. I list the possible tetrachordal Tonnetze for the  $Z_{12}$  case and propose a classification of them based on different kinds of degeneracies. (Received September 19, 2016)

1125-00-2018 **Victorie Gore\*** (gorev@southwestern.edu). *Extreme Precipitation: Changes in Rain Frequency from 1895-2015 in Central Texas.*

Analysis of precipitation data reveals trends in extreme rainfall events over the last century. The United States has seen an increase in yearly precipitation, especially regarding extreme daily precipitation events. We apply multiple methods to longitudinal data from eleven different locations within a 100 mile radius of downtown Austin. We find a precipitation threshold for each station for multiple time periods, apply a declustering process, and then fit a Generalized Pareto Distribution to the values above the threshold. The resulting period curves created for each site and time period provide an estimate of the upper extremes of the precipitation spectrum.

Our models reveal the extent of change in the trend of extreme rainfall events near Austin, TX. (Received September 19, 2016)

1125-00-2164 **Maitreyee Chandramohan Kulkarni\*** ([mkulka2@1su.edu](mailto:mkulka2@1su.edu)), Baton Rouge, LA 70808. *Quivers from Double Bruhat Cells of Kac-Moody Algebras.*

In this talk, I will describe Berenstein-Fomin-Zelevinsky cluster structures on Schubert cells of symmetrizable Kac-Moody algebras. Geiss-Leclerc-Schröer found an additive categorification of these cluster algebras via Frobenius categories constructed from representations of preprojective algebras. The talk will introduce the construction of quivers by building cylinders over graphs, orientability of its faces, and the construction of nondegenerate potentials for categorification of these algebras with frozen variables. (Received September 19, 2016)

1125-00-2380 **Floyd B Johnson\*** ([fbjohnso@mtu.edu](mailto:fbjohnso@mtu.edu)), 310 Hancock Street, Hancock, MI 54016, and **Jie Sun.** *Reduction of ETRU to NTRU.* Preliminary report.

The NTRU cryptosystem is a ring based public key cryptosystem developed in 1998 by Hoffstein et al. based over an integer modulus. The NTRU cryptosystem is faster than other popular encryption techniques such as RSA or DES and is believed to be resilient to quantum computing attacks, but has the drawback of possible decryption failure (See: Jarvis and Nevins, ETRU: NTRU over the Eisenstein integers(2015)). The ETRU cryptosystem is NTRU over the Eisenstein Integers with an Eisenstein Prime modulus of  $q$ , which are  $\mathbb{Z}[\omega]/\langle q \rangle$  where  $\omega$  is the third root of unity. Some attacks to find the private key include brute force, meet in the middle, and most strongly lattice methods. Traditionally ETRU is more resilient than NTRU for lattice methods and hence is considered to be stronger (Jarvis and Nevins). Presented here is the possibility of reducing the ETRU lattice to a smaller version of the NTRU lattice by constructing an explicit isomorphic mapping from  $\mathbb{Z}/\langle |q|^2 \rangle$  to  $\mathbb{Z}[\omega]/\langle q \rangle$  then reversing this process. This mapping has restrictions on which Eisenstein prime  $q$  can be used. The repercussions of this reduction including decryption failure rate, combinatorial security, and lattice attacks will then be discussed. (Received September 20, 2016)

1125-00-2493 **Richard J Plotkin\*** ([richardp@buffalo.edu](mailto:richardp@buffalo.edu)). *Quantifying functional accent.* Preliminary report.

Accentuation plays an essential role in our experience of musical time, progression, expression, and clarity. This paper builds upon a theory that broadly organizes three categories of accent: metric, grouping, and phenomenal. These varieties of accent are always at work in different ways, and dealing with the integration of these accent types is a daunting task. The goal here is to move forward a discussion of accent quantification by attributing functional roles to synergistic patterns. For instance, where alone neither a downbeat nor the beginning of a rhythmic group necessarily indicates an initiating function, together these two accents strongly project initiation. If that is so, then what sort of measurable accentual quantity and functionality does one attribute to a rhythmic pattern that begins on the second beat of a measure in common time? Does the metric accent of beat three demonstrate initiating function, or does the moment-by-moment misalignment of accent yield a group that starts in the middle of an accent-functional progression? These questions are especially pertinent to the analysis of song, bringing in scansion, prosody, and rhyme as linguistic indicators for the musical moments. (Received September 20, 2016)

1125-00-2546 **Klaus Hulek\*** ([hulek@math.uni-hannover.de](mailto:hulek@math.uni-hannover.de)), Institut für Algebraische Geometrie, Leibniz Universität Hannover, Welfengarten 1, 30167 Hannover, Germany. *zbMATH – Challenges and Perspectives.*

Zentralblatt MATH (zbMATH) is the world's most comprehensive and longest-running abstracting and reviewing service in pure and applied mathematics. In the rapidly changing world of mathematical information and research dissemination, which are the essential traditional and new purposes it should serve in the framework the future digital mathematical library? We outline recent developments and illustrate new perspectives like the connection of mathematical publications with software and research data, formula search, and the role of bibliometrics. (Received September 20, 2016)

1125-00-2587 **Tin Phan\*** ([tin.t.phan@asu.edu](mailto:tin.t.phan@asu.edu)), **Javier Baez** and **Yang Kuang.** *Mathematical Models for Prostate Cancer with Androgen Resistance under Intermittent Androgen Suppression Therapy.*

A standard treatment for advanced prostate cancer is the Androgen Deprivation Therapy (ADT). This accounts for the fact that tumor cells' growth is androgen-dependent, but the development of androgen-independent tumor cells usually takes place and renders the treatment ineffective after several years. Due to the reduction in the male hormone during treatment, undesirable effects cause loss in quality of life. Intermittent Androgen Suppression Therapy is the idea of alternating between on and off treatment period in accordance to the prostate specific

antigen level. This has been shown to give patients better life quality; however, it remains controversial whether it is superior to the continuous-ADT in term of prolonging the life of the patient. Among other issues, there is a rising need for predicting power of cancer progress to supply patients and physicians with the necessary information to decide on the best course of action. Numerous mathematical models have been developed to study prostate cancer. We review some of the major efforts in the last decade and put forward a novel approach that incorporates both PSA and androgen clinical data. We compare the models and find that incorporating the androgen in a more realistic way can increase the accuracy in prediction. (Received September 20, 2016)

1125-00-2593 **Olaf Teschke\*** ([olaf.teschke@fiz-karlsruhe.de](mailto:olaf.teschke@fiz-karlsruhe.de)), FIZ Karlsruhe, Franklinstraße 11, 10589 Berlin, Germany. *Status of the Global Digital Mathematics Library Initiative.* Preliminary report.

At the ICM 2014, the International Mathematical Union set up a working group to pursue the development of the Global Digital Mathematics Library, extending the vision of the World Digital Mathematical Library which was endorsed by the IMU in 2006. A special session at the JMM 2016 on Mathematical Information in the Digital Age of Science assembled a broad variety of projects and approaches toward this aim. We give a report of the current status of the initiative, ongoing developments, and upcoming perspectives. (Received September 20, 2016)

1125-00-2622 **Matthew D Bradley\*** ([bradley3@wit.edu](mailto:bradley3@wit.edu)). *An Infinitude of Proofs for the Infinitude of Primes.*

A prime number is an integer greater than 1 which is only integrally divisible by 1 and itself. It was first proven by Euclid in 300 BC that there are infinitely many of these prime numbers. Mathematicians from Greece to Japan have studied the mysterious set of prime numbers. Throughout the ages a multitude of additional proofs for the infinitude of primes have been discovered. These proofs elegantly incorporate many branches of mathematics, from point-set topology, to analysis and number theory. (Received September 20, 2016)

1125-00-2899 **Amy Buchmann**, [abuchman@tulane.edu](mailto:abuchman@tulane.edu), and **Candice Price\***. *What is the EDGE(Enhancing Diversity in Graduate Education) Program?*

To kick off our session, "Pure and Applied Talks by Women Math Warriors Presented by EDGE (Enhancing Diversity in Graduate Education)", we will introduce you to the EDGE program. We will give an overview of the program and its goals. (Received September 20, 2016)

1125-00-2994 **Shawn Firouzian\*** ([sfirouzi@ucsd.edu](mailto:sfirouzi@ucsd.edu)), 1 Miramar St, 929265, La Jolla, CA 92092, **Chris Rasmussen** ([crasmussen@mail.sdsu.edu](mailto:crasmussen@mail.sdsu.edu)), San Diego, CA 92120, **Richard Levine** ([rlevine@mail.sdsu.edu](mailto:rlevine@mail.sdsu.edu)), San Diego, CA 92120, and **Matt Anderson** ([manderson@mail.sdsu.edu](mailto:manderson@mail.sdsu.edu)), San Diego, CA 92120. *Adaptations of Learning Glass Technology in Undergraduate STEM Education.*

The Learning Glass is an innovative new instructional technology that holds considerable promise for engaging STEM class students and improving their learning outcomes. The Learning Glass screen acts as a transparent whiteboard. The instructor writes on a glass screen with LED illuminated edges. A camera on the opposite side of the glass records the video and horizontally flips the image (and hence the instructor is not required to write backwards). In this report, we share the results of two phase efficacy study between online calculus-based physics courses using Learning Glass technology, and large auditorium-style lecture-hall taught via document projector. All the classes were taught by the same instructor using identical content and materials. Our quasi-experimental design involved identical pre- and post-course assessments evaluating students' attitudes and their conceptual learning gains. In this presentation, we will share the results of both studies and compare them. The results so far are promising, with equivalent learning gains for all students, including minority and economically disadvantaged students. (Received September 21, 2016)

1125-00-3044 **Kate Heenan L Heenan\***, [klheen18@g.holycross.edu](mailto:klheen18@g.holycross.edu), and **David B Damiano**. *Topological data analysis of ballistic deposition models.*

In this project we apply methods from computational topology to analyze the void structure of simulations of ballistic deposition. In particular, we adapt and apply the concept of persistent homology dimension developed by MacPherson and Schweinhart. Further, we compare the persistent homology dimension to fractal measures of the void structure. Considering a large range of deposition sizes and different probabilistic rules for the depositions, we hope to capture and quantify the characteristics of the voids to come to an understanding of the complexity of the depositions. (Received September 20, 2016)

1125-00-3059 **Norman Carey\*** ([ncarey@gc.cuny.edu](mailto:ncarey@gc.cuny.edu)). *Christoffel Words, Rich Words, and Perfect Balance*. Preliminary report.

All Christoffel Words (which are the patterns found in well-formed scales) are palindromically rich words. Some, but not all, perfectly balanced scales are also rich. This paper will look at the distinction between rich and not-rich perfectly balanced constructions. The notion of palindromic defect describes by how much a word fails to be rich. I hypothesize that perfectly balanced words are either rich or exhibit a demonstrably low defect. (Received September 20, 2016)

1125-00-3080 **Clifton Callender\*** ([clifton.callender@fsu.edu](mailto:clifton.callender@fsu.edu)), 1937 Patsy Ann Court, Tallahassee, FL 32303. *Self-Similarity, Continuous Processes, and New Compositional Resources*.

I have previously presented research and compositions demonstrating that self-similar structures and continuous processes can give rise to novel musical possibilities, including canons with infinite solutions and infinite, aperiodic rhythmic tiling canons. This talk is a continuation and extension of these ideas to include alternate constructions of infinite melodic and rhythmic canons, irrational polyrhythms and hemiolas, and gradually morphing rhythms. (Received September 21, 2016)

1125-00-3081 **David John Baker\*** ([davidjohnbaker1@gmail.com](mailto:davidjohnbaker1@gmail.com)), 2800 July Street, Unit 39, Baton Rouge, LA 70808. *Measuring Distances of Musical Schemata*.

In order to understand the relationships between musical ideas, we must first define what it means for musical ideas to be similar. This could mean one of many things, including rhythmically similar, similar in contour, or containing similar intervals or pitches. While making such judgments lies at the nexus of music research, the notion of similarity and pattern discovery is often thought of as an abstract, intangible concept. Scholars have theorized that elements of the musical surface are able to be abstracted in order to create a schematic representation of essential musical features. These abstractions, once created, can then serve as a representation from which similarity measures of essential features can be modelled mathematically. This study examines how improvisatory patterns ("licks") might be analyzed in terms of similarity and distance metrics. Temporal pattern-matching, distance algorithms (COSITEC, SIAM, and others), and interval matching are all discussed and compared, with a discussion focusing on the results that might provide the most ecologically valid description of common jazz improvisational ideas. (Received September 21, 2016)

1125-00-3123 **James M Henle\*** ([jhenle@smith.edu](mailto:jhenle@smith.edu)). *The Affinity of Mathematics and Music is Cultural not Technical*.

Mathematics has a special relationship with music, stronger and more emotional than with the other arts. I argue that this can't be explained by the obvious mathematical characteristics of music but by looking at how the two affect and are affected by cultural currents. My evidence for this comes from considering mathematics as an art.

As an art, we can discern the same periods (Renaissance, Baroque, Classical, etc.) for mathematics as for the other arts. When we do, we notice that the timeline for mathematics lags behind the timelines of painting, architecture, and literature. The mathematical periods, however, mostly agree with those of music. (Received September 21, 2016)

1125-00-3145 **Garikai Campbell\***, Morehouse College, 830 Westview Drive SW, Atlanta, GA 30314. *The Changing Higher Education Landscape: One Mathematician Turned Administrator's View*.

This talk is about the changing higher education landscape. (Received September 21, 2016)

1125-00-3168 **Iian Smythe\*** ([ismythe@math.cornell.edu](mailto:ismythe@math.cornell.edu)), Cornell University. *Madness in vector spaces*. A collection of finite dimensional subspaces of a vector space is maximal almost disjoint, or mad, if distinct elements have finite-dimensional intersection and the collection is maximal with respect to this property. We outline and consider the basic questions regarding these families, such as their cardinality under set theoretic assumptions, the existence of P-indestructible mad families for notions of forcing P, and the existence of denable, i.e., analytic, such families. This last concern motivates connections with the author's local Ramsey theory for block sequences in vector spaces. (Received October 03, 2016)

1125-00-3169 **Sean Sovine\*** (sovine5@marshall.edu), Marshall University. *Weihrauch reducibility and finite dimensional subspaces.*

We study the Weihrauch problem of producing a finite dimensional or one dimensional subspace of a countable vector space, and related problems for producing finite dimensional subsets of a countable matroid. This extends Reverse Mathematics work of Downey, Hirschfeldt, Kach, Lempp, Mileti, and Montalbán (2007) and recent work of Hirst and Mummert (2016). We also study the Weihrauch problem of decomposing a countable vector space or countable matroid into one dimensional subspaces.

[1] Rodney G. Downey, Denis R. Hirschfeldt, Asher M. Kach, Steffen Lempp, Joseph R. Mileti, and Antonio Montalbán. Subspaces of computable vector spaces. *Journal of Algebra*, vol. 314 (2007), no. 2, pp. 888–894.

[2] Jerry Hirst and Carl Mummert. Reverse mathematics of matroids. *Lecture Notes In Computer Science*. To appear (2016).

(Received October 03, 2016)

1125-00-3170 **Timothy Trujillo\*** (trujillo@mines.edu), Colorado School of Mines. *Parametrizing by the Ellentuck space.*

We introduce a new construct that can be used to parametrize a topological Ramsey space by the collection of infinite subsets of natural numbers. We show that these parametrized spaces are topological Ramsey spaces. Then we use these spaces to prove some parametrized perfect set theorems. We conclude with a discussion of how to extend the results to the abstract setting and open questions related to applying the results to the Tukey theory of ultrafilters. (Received October 03, 2016)

1125-00-3178 **Seth Harris\*** (sharris2@drew.edu), Drew University. *On-line algorithms and reverse mathematics.*

Consider a two-player game (played by Alice and Bob) in which Alice asks a sequence  $\langle a_0, \dots, a_k \rangle$  and Bob responds with a sequence  $\langle b_0, \dots, b_k \rangle$  with no knowledge of Alice's future requests. A problem P is solvable by an *on-line algorithm* if Bob has a winning strategy in this game, where Bob wins the game if  $(\bar{a}, \bar{b})$  constitutes a solution to P.

Given a problem P, the corresponding *sequential problem* SeqP asserts the existence of an infinite sequence of solutions to P. For example, if P states “every finite graph without odd cycles is bipartite,” then SeqP is the statement: “for every sequence of finite graphs without odd cycles, there exists a sequence of bipartitions.”

We will show that the reverse-mathematical strength of SeqP is directly related to the on-line solvability of the non-sequential problem P, and we will exactly characterize which sequential problems are solvable in  $\text{RCA}_0$ ,  $\text{WKL}_0$ , or  $\text{ACA}_0$ . This is joint work with François Dorais, and generalizes James Schmerl's results specially for on-line graph colorings [2].

[1] F. G. Dorais and S. Harris, *Evasion, prediction, and on-line algorithms*, in preparation.

[2] J. Schmerl, *Reverse mathematics and graph coloring: eliminating diagonalization*, **Reverse Mathematics 2001, vol. 21 of Lecture Notes in Logic** (Stephen G. Simpson, editor), A K Peters, Ltd., Wellesley, MA, 2005, pp. 331–348.

(Received October 31, 2016)

## 01 ► History and biography

1125-01-64 **Shirley B. Gray\*** (sgray@calstatela.edu), Department of Mathematics, California State University, 5151 State University Drive, Los Angeles, CA 90032-8204. *Fusion Mathematics.*

The 1998 auction of the Palimpsest of Archimedes at Christie's, followed by collaborative work centered at the Walters Art Museum led to traveling museum exhibits, newspaper articles, television specials, and dozens of presentations. Mathematicians and other scholars attracted a new and significant audience. The singed, battered, faded, mildewed, damaged 10th century manuscript - the world's oldest copy of The Method of Archimedes - sold for \$2 million "under the hammer." In the true Archimedean experimental tradition, my team of students, staff and faculty decided to look not retrospectively at the content of propositions in The Method but rather in terms of 21st century mathematics and technology. "We believe we participated in every scholar's quest to have a Eureka moment - we found the Golden Ratio in our attempts to image the footprint of Archimedes." This discovery led to several publications and a Wikipedia article. But perhaps more importantly, our work in simultaneously embracing computer graphics, computer science, pure mathematics and 3-D printer modeling may reap huge benefits for future research in a new field that we call fusion mathematics.

Palimpsest Calculus Golden Ratio 3D Printer Superparabola Superconics (Received July 04, 2016)

1125-01-80 **Donald A. Sokol\*** (donsokol17@gmail.com), 11S047 Palisades Road, Burr Ridge,, IL 60527. *Plimpton 322: The Rosetta Stone of the Integer (Pythagorean) Triple.*

The Babylonian Clay Tablet (circa 1800-1700 B.C.) identified presently as Plimpton 322 in the museum at Columbia University represents the opportunity for a new look at the Pythagorean Theorem. The tablet has 15 lines of information related to the relationship,  $a^2+b^2=c^2$ . Line 11 contains the values  $c=75$  and  $b=45$  of a integer triple in  $a$ ,  $c$  and  $b$ . The value of "a", although missing, has been identified by numerous others as 60. These values are multiples of the prime integer triple 4, 5, 3 and the multiplier is 15. And 60, 75, 45 are also multiples of 1.0, 1.25, 0.75, and the multiplier is 60 (The triangular number for one is 1) . The result is  $a:60$  as  $Nt: 15$  and  $a=4Nt$ , where  $Nt$  is a triangular number, and "a" is the even value in an integer triple. The modifier for accommodating changes in  $x$  and  $y$  in an appropriate spread sheet mapping is  $y(x-1)/2$ ; so that  $a=4[y(y+1)/2+y(x-1)/2]=2(x+y)y$ . Also,  $c=a+x^2$  and  $b=c-2y^2$ . Square roots and negative numbers, avoided by both Babylonians and Greeks address scale and orientation. (Received July 12, 2016)

1125-01-110 **Mohammad K. Azarian\*** (azarian@evansville.edu), Department of Mathematics, University of Evansville, 1800 Lincoln Avenue, Evansville, IN 47722. *Mathematics Contributions of 'Abd al-'Alī al-Bīrjandī [Bīrjandī].*

Nizām al-Dīn 'Abd al-'Alī ibn Muhammad ibn Husain al-Bīrjandī (d. 934 A.H.L./1527-28), known as 'Abd al-'Alī Bīrjandī, was an acclaimed Iranian polymath. He was born in the second half of the 15th century in Bīrjand, the center city of Southern Khorāsān province in present day Iran, and was educated in Bīrjand and Herāt . It is believed that he was the youngest scientist working at Samarqand observatory before going to Isfahān in 1501. There, he was engaged in scientific work at the court of Shāh Ismā'il Safavid [Safavī] until 1524. There are up to 22 pieces of scholarly work attributed to Bīrjandī. Bīrjandī's best known works are in mathematics and astronomy. However, he also wrote treatises, commentaries, and books on astrology, logic, cosmology, agriculture, physics, geography, and religion. We will discuss his contributions to mathematics, including Sharh-i Za'ī-i Ulugh Beg ("Commentaries on Ulugh Beg's Astronomical Tables"), Sharh-i tahrīr al-Majistī ("Commentaries on Exposition of Almagest"), and Bīst bāb dar taqwīm [taqvīm]("Twenty Chapters on the Calendar"). (Received July 30, 2016)

1125-01-196 **Shigeru Masuda\*** (hj9s-msd@asahi-net.or.jp). *The Motion Equations and Theories of Heat by Fourier and Poisson.* Preliminary report.

We discuss historical development of classical heat theory from the viewpoint of mathematical physics, in particular, of Fourier and Poisson. After the arrival of concept of continuum, the new mathematics is put forth in pure mathematics but also in mathematical physics, and in the theory of heat communication, which is the then conventional object of heat study.

Poisson issues the papers on theory of heat : Poisson 1823 and the last book 1835 in rivalry to Fourier 1822, in which Poisson discusses the essential theories emphasizing his hypothesis of molecular emission and absorption of heat and which dues to the Newton's law, and basing on an impregnable belief of mathematical science.

However, Fourier's equation is only the first half part of the general expression by Poisson and according to Poisson, Fourier's style is a particular case. We think, Poisson's method comes from the fluid dynamics and the wave theory in which he introduces an origin of the Navier-Stokes equations and wave equations. Heat theory produces more fruitful harvest than fluid dynamics in the mathematical history, for its easiness of linearity than the latter. (Received August 11, 2016)

1125-01-240 **David R Bellhouse\*** (bellhouse@stats.uwo.ca), Dept of Statistical and Actuarial Sciences, University of Western Ontario, London, Ontario N6A5B7, Canada. *Mathematicians as Consultants in Eighteenth-Century England.*

In the 1730s the French art critic, Jean-Bernard Le Blanc visited England and commented that compared to those on the Continent, English mathematicians received little if any financial support from the government. This continued to be the case throughout the eighteenth century. A few English mathematicians obtained positions at the universities. The rest typically earned their living through teaching in various schools and as private tutors. For example, in about 1750 James Dodson's salary for teaching at the Royal Mathematical School was 100 pounds per annum. Le Blanc estimated that in France someone like Abraham De Moivre would receive a pension of 1000 crowns per annum with minimal duties attached. Some of the English mathematicians supplemented their income by providing clients with valuations of life contingent contracts related to property. I explore the nature of this consulting activity and compare it to my own experience in statistical consulting over the past few years. (Received August 17, 2016)

1125-01-253 **Amy Ackerberg-Hastings\*** ([aackerbe@verizon.net](mailto:aackerbe@verizon.net)). *John Playfair's Approach to "the Practical Parts of the Mathematics"*. Preliminary report.

Best known for *Elements of Geometry* (1795) and *Illustrations of the Huttonian Theory of the Earth* (1802), University of Edinburgh mathematics and natural philosophy professor John Playfair (1748–1819) also wrote several dozen books, expository articles, and opinion pieces as individual publications or for *Philosophical Transactions of the Royal Society of London*, *Transactions of the Royal Society of Edinburgh*, and *Edinburgh Review*. Most of these works have been digitized and are readily available for study. In contrast, Playfair's 1793 *Prospectus of a Course of Lectures on Some of the Practical Parts of the Mathematics* has nearly disappeared, with as few as eight surviving copies. In this talk, I will consider what we can learn from this document, which consists of a nineteen-page list of potential topics. Despite its brevity, the *Prospectus* includes a number of themes and priorities that recurred throughout Playfair's writing and teaching. It also suggests how Playfair understood the role of mathematics with respect to the physical and natural sciences and reminds us of his influence as an educator and cosmopolitan intellectual. (Received August 18, 2016)

1125-01-254 **William Dunham\*** ([bdunham@brynawr.edu](mailto:bdunham@brynawr.edu)). *Did Euler Scoop Möbius?*

The Möbius function is a fixture of number theory. It is usually traced to an 1831 paper by August Ferdinand Möbius where, somewhat surprisingly, it arose in an analytic, rather than a number theoretic, setting. But perhaps more surprisingly, the function can be spotted in Leonhard Euler's classic text, *Introductio in analysin infinitorum*, from 1748.

In this talk, we examine Euler's clever reasoning that anticipated, by nearly a century, the work of Möbius (who, by the way, had both a front and a back). (Received August 19, 2016)

1125-01-279 **Johannes C. Familton\*** ([jfamilton@bmcc.cuny.edu](mailto:jfamilton@bmcc.cuny.edu)), Borough of Manhattan Community College, The City University of New York, 199 Chambers Street, New York, NY 10007. *The Joining of Quaternions with Grassmann algebras: William Kingdon Clifford*.

Most people who work with quaternions know the story of Hamilton's breakthrough on Broome bridge in Dublin, Ireland where he carved the famous formula  $i^2 = j^2 = k^2 = ijk = -1$ , but some may not know much about Grassmann (Hermann Günther Grassmann; April 15, 1809 – September 26, 1877) or Clifford (William Kingdon Clifford FRS; May 4, 1845 – March 3, 1879) and the history of their contributions to this subject.

Hamilton's approach was mainly connected to algebra, while Grassmann arrived at similar conclusions through geometry. Although Hamilton saw the adoption of quaternions in his lifetime, Grassmann was barely recognized during his. It wasn't until after he passed that other scientists and mathematicians began to recognize the depth of his work. One of these was Clifford. Although Clifford's life was relatively short due to tuberculosis his insight into how to connect Hamilton's algebra to Grassmann's geometry has endured.

In this talk Dr. Familton will give a brief discussion of Grassmann algebras, their history, and how Clifford connected the best of Grassmann's and Hamilton's work. (Received August 22, 2016)

1125-01-307 **Thomas Preveraud\*** ([thomaspreveraud@yahoo.com](mailto:thomaspreveraud@yahoo.com)), Lille, France. *The Early Teaching of Descriptive Geometry in the United States (1817-1915)*.

In the United States, descriptive geometry was a subject very few mathematicians, teachers or engineers knew about before 1820. Most of them were self-taught, as it was not introduced in any curriculum before 1817. This communication presents the first course of descriptive geometry ever taught in the United States by French polytechnician Claude Crozet, professor of civil engineering at West Point between 1817 and 1823, who introduced the subject in West Point curriculum in 1817. Descriptive geometry soon became a subject taught in colleges, especially in those that had already started to offer their students elective courses, or special engineer-training programs. Thus, descriptive geometry went gradually from a restrictive audience subject to a general-interest subject often shown as a sequel of the classical geometry course. Textbooks authors introduced then new élémentations of the method of projections in order to fit the changing readership and the changing place of the subject in the various curricula. After 1875, the practical role played by descriptive geometry remained crucial in emerging technical institutions and it found back there its original mission as a graphic art for the training of engineers. (Received August 25, 2016)

1125-01-308 **Christopher D. Hollings\*** ([christopher.hollings@maths.ox.ac.uk](mailto:christopher.hollings@maths.ox.ac.uk)), Mathematical Institute, University of Oxford, Woodstock Road, Oxford, OX2 6GG, United Kingdom. *A Tour of Group Generalisations of the 1920s and 1930s*.

The process of generalisation in mathematics sometimes gets a bad press owing to the presence in the literature of many an ill-motivated 'generalisation for generalisation's sake'. However, a number of different generalisations of the group concept emerged in the 1920s and 1930s, each designed to solve a particular problem. Although

not all of these newly defined objects went on to receive broader study, they are nevertheless good examples of well-motivated generalisation. I will give a survey of a selection of these, and point out some of their interesting interconnections. (Received August 25, 2016)

1125-01-318 **Tony Royle\*** ([tony.royle@open.ac.uk](mailto:tony.royle@open.ac.uk)). *The Perilous Practice of ‘Flying and Applying’*. Early aeronautical research in Britain was advanced by a decision to allow a number of the nation’s finest young mathematicians to train as pilots and conduct airborne experiments using full-scale aircraft. Given that many would subsequently perish in flying accidents, was the risk worth the reward? (Received August 26, 2016)

1125-01-375 **Daniel S Silver\*** ([silver@southalabama.edu](mailto:silver@southalabama.edu)), Dept. of Mathematics and Statistics, ILB 325, University of South Alabama, Mobile, AL 36688. *Mathematical Induction and Nature of British Miracles*.

The term “mathematical induction” is often attributed to Augustus De Morgan, who used it casually in an encyclopedia article of 1838. In fact, the term is older. We describe how mathematical induction inspired Charles Babbage as he conceived his calculating engines. (Received August 30, 2016)

1125-01-380 **Amy Shell-Gellasch\*** ([amy.shell-gellasch@montgomerycollege.edu](mailto:amy.shell-gellasch@montgomerycollege.edu)). *The Jullien Models of Descriptive Geometry*.

Descriptive Geometry was developed by Gaspard Monge in the 18th century and quickly became an important part of the education of engineers and architects, as well as mathematicians. This area of geometry uses projections to exhibit the properties of three-dimensional objects on the plane. French mathematician A. Jullien wrote a popular text book on descriptive geometry in the 19th century. As an aide to learning, he also developed a set of thirty physical models to exhibit how the projections are made. A set of Jullien Models is among the Smithsonian National Museum of American History’s collections. In this talk we will explore these beautiful teaching models. (Received August 30, 2016)

1125-01-444 **June Barrow-Green\*** ([june.barrow-green@open.ac.uk](mailto:june.barrow-green@open.ac.uk)), School of Mathematics & Statistics, The Open University, Walton Hall, Milton Keynes, MK7 6AA, United Kingdom. *“Knowledge gained by experience”: Olaus Henrici – engineer, geometer and maker of mathematical models*.

The (Danish-born) German mathematician Olaus Henrici (1840–1918) studied in Karlsruhe, Heidelberg and Berlin before making his career in London, first at University College and then, from 1884, at the newly formed Central Technical College where he established a Laboratory of Mechanics. Although Henrici’s original training was as an engineer, he became known as a promoter of projective geometry. In my talk I shall explore connections between these two sides of Henrici’s work. (Received September 02, 2016)

1125-01-538 **Fernando Q Gouvea\*** ([fggouvea@colby.edu](mailto:fggouvea@colby.edu)), Dept of Mathematics and Statistics, Colby College, 5836 Mayflower Hill, Waterville, ME 04901. *The Power of Power Series: Hensel and Analogy*. Preliminary report.

Hensel’s creation and use of the  $p$ -adic numbers was motivated to a large extent by the analogy between number fields and function fields. This analogy led him to new ideas and results but also to at least one disastrous mistake. We will look at both, leading us to think about the power and dangers of analogy in mathematics. (Received September 06, 2016)

1125-01-607 **Ezra Brown\*** ([ezbrown@math.vt.edu](mailto:ezbrown@math.vt.edu)), 225 Stanger St., mail code 0123, Mathematics Department, Virginia Tech, Blacksburg, VA 24061-0123. *An incomplete chronology of the (7, 3, 1) block design*. Preliminary report.

The  $(7, 3, 1)$  block design has appeared in many times and places, frequently disguised as something else. In this talk, we trace the chronology of  $(7, 3, 1)$  from its apparent beginnings in the mid-nineteenth century up to present time. We’ll talk about difference sets, tournaments, finite geometries, normed algebras, Hadamard matrices, error-correcting codes, Venn diagrams, and many other matters. And what is meant by “apparent” beginnings? Well, come and find out. (Received September 07, 2016)

1125-01-776 **Michael R. Raugh\*** ([michael.raugh@gmail.com](mailto:michael.raugh@gmail.com)). *How did Leibniz solve the catenary problem?* Preliminary report.

In Acta Eruditorum of June 1691 Leibniz presented the first solution to the catenary problem as a classical straightedge-and-compass construction. Johann Bernoulli also presented a solution obtained by correctly formulating a differential equation derived from considerations of static equilibrium. Leibniz did not explain his derivation, and there are some oddities. His curve cannot be a catenary because  $e$  is not constructible. But he names features that can only be true for a catenary; a line segment equal to a specified arc and the tangent

at a point. It is remarkable that his construction (with slight qualification) exhibits the catenary structured as a hyperbolic cosine, not then known by name or formulation. Another oddity: Bernoulli proves that Leibniz's construction was indeed a catenary! Did he not notice that it wasn't? For proof, he shows that the construction yields the same differential equation he had derived, but he began with a logarithmic curve correctly defined by a differential equation, thereby avoiding Leibniz's limitations of construction. I will present a simple solution obtained using methods that I believe were available to Leibniz. I don't claim he solved the problem in this way, but I do think that paths like this were open to him. (Received September 11, 2016)

1125-01-804 **Niccolo' Guicciardini\*** ([niccolo.guicciardini@unibg.it](mailto:niccolo.guicciardini@unibg.it)). *Publishing mathematics in 18C Geneva and Lausanne.*

During the eighteenth century, several towns located in what is known today as Suisse romande were extremely receptive towards scientific culture. I will focus on editorial enterprises taking place in Geneva and Lausanne that were important for the development of eighteenth century mathematics. (Received September 12, 2016)

1125-01-861 **Eric Barkan** ([ebarkan@jps.net](mailto:ebarkan@jps.net)), 98 Gazania Court, Novato, CA 94945, and **David Sklar\*** ([dsklar@sfsu.edu](mailto:dsklar@sfsu.edu)), Department of Mathematics, San Francisco State University, 1600 Holloway, San Francisco, CA 94132. *Riemann, Siegel, and a Translation of Siegel's 1932 paper "Uber Riemanns Nachlass zur analytischen Zahlentheorie"*. Preliminary report.

Riemann published one paper in number theory. In this 1859 paper he obtained an explicit formula for the prime counting function, revealed the deep connection between the distribution of primes and the zeros of the zeta function, and stated the Riemann hypothesis.

An unpublished asymptotic expansion of the zeta function was discovered in Riemann's private notes by Carl Ludwig Siegel, and published in his 1932 paper "Uber Riemann's Nachlass zur analytischen Zahlentheorie". This expansion, now called the Riemann-Siegel formula, remains our primary tool for numerical investigation of the zeta function. Siegel's paper showed that, in analytic number theory, Riemann was seventy years ahead of his time.

Hoping to learn what Siegel found in Riemann, what he added, and how he worked his way through Riemann's fragmented papers; we decided to look at Siegel's paper. Although this historic paper is widely cited we were unable to locate an English translation. Despite our limited knowledge of German, we have produced a translation of the paper as it appears in Siegel's collected works. We are currently working on an annotated version.

In this talk we discuss some of what we learned and didn't learn about the Riemann-Siegel collaboration. (Received September 20, 2016)

1125-01-1032 **Della Dumbaugh\*** ([ddumbaugh@richmond.edu](mailto:ddumbaugh@richmond.edu)), Department of Mathematics & Computer Science, University of Richmond, Richmond, VA 23173. *Solomon Lefschetz: The Man, The Mathematics.* Preliminary report.

Solomon Lefschetz played a critical role in the American mathematical community in the early twentieth century. He contributed significantly to algebraic topology, its applications to algebraic geometry, and the theory of non-linear ordinary differential equations. He not only exhibited academic excellence in mathematics, but he also demonstrated leadership as a faculty member at the University of Princeton and as President of the American Mathematical Society. He also edited the Annals of Mathematics and revised American engineering education. Even with all of his contributions to mathematics and the broader community, Lefschetz is often described as a man with rather unpleasant characteristics. Yet, Lefschetz alone wrote the letter that would bring Emil Artin and his young family to America to escape the situation in Nazi Germany. This talk offers a richer understanding of Lefschetz the man and his work in mathematics. (Received September 14, 2016)

1125-01-1043 **Ronald E. Mickens\*** ([rmockens@auc.edu](mailto:rmockens@auc.edu)), Clark Atlanta University, Atlanta, GA 30314. *Applied Mathematics Research at Atlanta University/Clark Atlanta University: 1980-2015.*

We give an overview of research related to applied mathematics at Atlanta University/Clark Atlanta University during the period of 1980- 2015. This interval was selected because of the lack of full documentation for earlier times and also because the speaker knows and/or knew all of the major "players" involved in such research efforts after 1981, the time when he arrived at Atlanta University, which then became Clark Atlanta University in 1989. In addition to brief bio-sketches of the researchers, we also provide concise summaries of their research topics. Our definition of applied mathematics is defined to include both theoretical and mathematical physics. (Received September 14, 2016)

1125-01-1134 **Laura E Turner\*** ([1turner@monmouth.edu](mailto:1turner@monmouth.edu)), Department of Mathematics, Monmouth University, West Long Branch, NJ 07764. *The Krieger-Nelson Prize Lectureship*.

The Krieger-Nelson Prize Lectureship honours outstanding research by women members of the Canadian mathematical community. First awarded in 1995, it is named after Cecilia Krieger (1894-1974), the first woman to earn a Ph.D. in mathematics from a Canadian university, and Evelyn Nelson (1943-1987), a prolific researcher in universal algebra. In this talk, we explore the origins and early history of this prize, from the contributions of its namesakes to the motivations behind the prize itself. (Received September 15, 2016)

1125-01-1220 **Karen V. H. Parshall\*** ([khp3k@virginia.edu](mailto:khp3k@virginia.edu)). *The Lay of the American Mathematical Landscape in the 1920s*. Preliminary report.

In the 1920s, American mathematicians returned to their pre-WWI research agendas with renewed vigor at the same time that they trained a next generation. This talk will survey the lay of the American mathematical landscape during that decade by locating mathematicians within their varied institutional settings, by gauging their appetite for mathematical research, and by taking, as a case in point, American geometrical research. (Received September 15, 2016)

1125-01-1410 **Erik R. Tou\*** ([etou@uw.edu](mailto:etou@uw.edu)), University of Washington – Tacoma, Interdisciplinary Arts & Sciences, 1900 Commerce St., Campus Box 358436, Tacoma, WA 98402. *Thousand-year-old geometry: al-Karaji's Treatise on the Extraction of Hidden Waters*. Preliminary report.

Abu Bakr ibn Muhammad ibn al-Husayn al-Karaji (c. 1000 CE) is remembered by mathematicians as an algebraist, and as one who helped make the subject less geometric and more abstract. However, this notion fails to capture the geometric (and more specifically, Euclidean) inheritance of mathematicians of al-Karaji's time and place (11th century Baghdad). Using selections from a forthcoming English translation of his *Treatise on the Extraction of Hidden Waters*, we look at some geometric constructions and proofs from this late-career work of al-Karaji. (Received September 16, 2016)

1125-01-1417 **Lawrence A. D'Antonio\*** ([ldant@ramapo.edu](mailto:ldant@ramapo.edu)), 505 Ramapo Valley Rd., Mahwah, NJ 07430. *Jean le Rond d'Alembert: l'enfant terrible of the French Enlightenment*. Preliminary report.

The year 2017 sees the tercentenary of the birth of Jean d'Alembert, a mathematician and philosopher of undoubted achievement, but also a frequent combatant in disputes and controversies. In this talk we will review his mathematical career, including those disputes. We will also examine his central role in the Enlightenment as a member of the Académie des Sciences and as co-editor of the *Encyclopédie*. (Received September 16, 2016)

1125-01-1476 **Robert E Bradley\*** ([bradley@adelphi.edu](mailto:bradley@adelphi.edu)), Adelphi University, Dept. of Mathematics and Computer Science, Garden City, NY 11530. *From L'Hôpital to Lagrange: Analysis Textbooks in the 18th Century*.

We examine broad trends in the evolution of analysis in the 18th century, as reflected in textbooks, particularly those of de l'Hôpital, Euler and Lagrange. (Received September 17, 2016)

1125-01-1492 **J. J. Tattersall\*** ([tat@providence.edu](mailto:tat@providence.edu)), Department of Mathematics, 1 Cunningham Square, Providence, RI 02918. *The Mathematical Department of the Educational Times and Journal of the College of Preceptors*.

A number of significant mathematical journals have included a section devoted to mathematical problems intended to challenge and educate their readers. None has had a more extensive list of contributions and world-wide readership than the monthly periodical *The Educational Times and Journal of the College of Preceptors*. Between 1848 and 1918, there were more than eighteen thousand contributions to the mathematical department from amateur and professional mathematicians. Beginning in 1864, the columns devoted to mathematical problems and solutions was republished in six-month installments as *Mathematical Questions and Solutions from the Educational Times*. We discuss the data collected on contributors and their contributions, and describe plans to put the information into an accessible database. (Received September 17, 2016)

1125-01-1766 **Ursula Martin\*** ([ursula.martin@cs.ox.ac.uk](mailto:ursula.martin@cs.ox.ac.uk)). *Writing the mathematical biography of Ada Lovelace*.

Ada, Countess of Lovelace (1815-1852) is famous for a paper published in 1843, which presented Charles Babbage's unbuilt general-purpose computer, the Analytical Engine. A detailed description of the machine was accompanied by a table, displaying how it might compute the Bernoulli numbers, often called "the first computer programme". Lovelace has been celebrated in a number of biographies which focus more on her famous

father (the poet Lord Byron), her relationship with her mother, and her tragically short life, than her mathematical and scientific contribution.

Yet the archives include accounts of her early mathematical education, and a remarkable correspondence course undertaken with Augustus De Morgan when she was in her mid-20s, in which she studies advanced mathematics, and discusses research frontiers of the day, such as quaternions and divergent series. They allow us to reassess the knowledge that enabled Lovelace to write so articulately about Babbage's engines, and to undo the misapprehensions of biographers who have variously dismissed her mathematics as "hieroglyphics", or misunderstood the context of mathematics or of women's education. It also raises broader question about how we misremember and misreport the achievements of female mathematicians. (Received September 19, 2016)

1125-01-1926     **Johnny L. Houston\*** (jlhouston602@gmail.com). *The Culture and History of Mathematics In The Atlanta University Center (AUC)*.

The Atlanta University Center (AUC) is the largest contiguous consortium of private African Americans institutions in higher education in the United States and the AUC has a rich legacy for producing large numbers of African American scholars and leaders in the fields of Science, Technology, Engineering, and Mathematics (STEM). Specifically, The AUC legacy in the field of mathematics is exemplary. In this presentation the author explains the unique culture and history of mathematics in AUC institutions. He identifies dozens of notable mathematics faculty who taught in AUC institutions and scores of notable mathematical scientists that AUC institutions have produced; some of these individuals are discussed in details. The author/presenter entered AUC as a student in 1960, where he earned two degrees in mathematics: a BA degree from Morehouse, in 1964, and an MS degree in mathematics from Clark Atlanta University in 1966. The presenter also served as Chair of the Mathematics Department at Clark Atlanta University for six (6) years and later as a Visiting Professor of Mathematics at Morehouse College. Also, he has given mathematical presentations at Spelman College/Morris Brown College. (Received September 19, 2016)

1125-01-1959     **Brittany Shields\*** (bshields@seas.upenn.edu), Bioengineering Department, 210 South 33rd Street, Suite 240 Skirkanich Hall, Philadelphia, PA 19104. *Scientific Diplomacy & Identity: Richard Courant in the 20th Century*.

This paper will explore the complicated, and sometimes conflicting, social roles of American mathematicians in the twentieth century. New York University's Courant Institute of Mathematical Sciences will be taken as a focal point in which to review the lives and careers of its founding mathematicians, including Richard Courant, Kurt O. Friedrichs and James J. Stoker. Their involvement in a multitude of social roles – from the founding of the institute in the 1930s, through contractual military work during World War II, and then diplomatic efforts in the Cold War era – will be considered. The paper asks how individuals, such as Courant, navigated the complicated terrain of twentieth century scientific diplomacy, balancing both national and international demands, in times of both war and peace. (Received September 19, 2016)

1125-01-2107     **Jordan Bell and V. Frederick Rickey\*** (fred.rickey@me.com), 11 Stately Oaks, Cornwall, NY 12518. *From "Vir Celeberrime" to "Hochedelgebohner Herr": From Euler to Goldbach*. Preliminary report.

For 35 years Euler and Goldbach corresponded about many facets of mathematics: algebra, analysis, geometry, and, especially, number theory. They discuss theorems, conjectures, the status of their attempted proofs, as well as books, travel, family, and their contemporaries. A new edition of the correspondence in Euler's *Opera Omnia*, IVA.IV, 2014, edited by Franz Lemmermeyer and Martin Mattmüller, contains English translations of the letters, as well as a great deal of information about eighteenth-century mathematics. We shall discuss some of the high points of this wonderful correspondence. (Received September 19, 2016)

1125-01-2433     **Colm Mulcahy\*** (colm@spelman.edu). *A century of mathematical excellence at Spelman College*. Preliminary report.

The first documented mathematics major at Spelman graduated in 1929, and she went on to do a masters in math at the University of Wisconsin in Madison. Since 1960, we've graduated about 700 math majors, and many of these achieved excellence in academia, industry, and government, with several serving in leadership roles in colleges and universities.

We'll survey some of these remarkable careers, as well as the history of Spelman's department of mathematics, whose teaching and mentoring laid the foundation which made all of this possible.

So far, over 55 of Spelman's grads have earned doctoral degrees in math or related fields, and this shows every sign of continuing at an increasing pace. By 2029 we expect that number to be much higher. (Received September 20, 2016)

1125-01-2437 **Kim Plofker\*** ([kim\\_plofker@alumni.brown.edu](mailto:kim_plofker@alumni.brown.edu)), Department of Mathematics, Union College, Schenectady, NY 12308. *An Indian version of al-Kāshī's method of iterative approximation of  $\sin 1^\circ$ .*

An ingenious technique attributed to the 15th-century astronomer-mathematician Jamshīd al-Kāshī for iteratively approximating the sine of  $1^\circ$  was adapted into a Sanskrit recension sometime in the early 18th century. We explore the adaptation process and some refinements in the Sanskrit version. (Received September 20, 2016)

1125-01-2767 **Bruce S. Burdick\***, Dept. of Mathematics, Roger Williams University, 1 Old Ferry Rd., Bristol, RI 02809. *Almanack and Almanaque: the Calendrical Periodical in Seventeenth Century North America.*

Originally, the primary content of an almanac was an ephemeris, a detailed calendar indicating the motions of the planets. Eclipses for the year, if any, needed to be predicted. Religious holidays and predictions of the weather were usually included. These items are still present in most modern almanacs but they no longer dominate.

In the early centuries of printing, the owner of a press had an incentive to find an almanac author because almanacs were expected to sell out their print run. An almanac author needed to be either a good calculator or a good plagiarist.

None of the printed almanacs from seventeenth century Mexico are known to have survived, but we have manuscript copies of many of them among the Inquisition records at the Archivo General in Mexico City. The majority of the almanacs printed in the English colonies in the same century have survived in their published form, although they are all quite rare.

We will look at the almanac traditions in both of these areas. By way of having a representative of each, we will focus on the careers and writings of Carlos de Sigüenza y Góngora, who lived and published in Mexico City, and John Tulley of Saybrook, Connecticut, who wrote for presses in Boston. (Received September 20, 2016)

1125-01-2821 **Duane Cooper\*** ([dcooper@morehouse.edu](mailto:dcooper@morehouse.edu)), Department of Mathematics, Morehouse College, 830 Westview Drive SW, Atlanta, GA 30314. *Morehouse Mathematics: Making a Difference from "Pop" through "Doc" to the Present.*

The Department of Mathematics at Morehouse College was honored to be recognized by the AMS in 2016 with its Mathematics Programs That Make a Difference award. In this presentation, we describe how our department's current efforts build on the rich legacy of past department leaders Claude B. "Pop" Dansby, Henry "Doc" Gore, and others who have taught generations of Black mathematics students and inspired many towards advanced degrees and careers in the mathematical sciences. (Received September 20, 2016)

1125-01-3109 **Joseph W. Dauben\*** ([jdauben@att.net](mailto:jdauben@att.net)), 635 West 42nd Street 41-D, New York, NY 10036. *Rationales for Mathematics and its Significance in Recently-Excavated Bamboo Texts from Ancient China.* Preliminary report.

This presentation will investigate the rationales given in prefaces to ancient mathematical texts about the reasons and rationales for doing mathematics, and what its significance was for masters and teachers alike according to recently excavated bamboo texts from ancient China. (Received September 21, 2016)

## 03 ► *Mathematical logic and foundations*

1125-03-42 **Maarten McKubre-Jordens\*** ([maarten.jordens@canterbury.ac.nz](mailto:maarten.jordens@canterbury.ac.nz)), University of Canterbury, Private Bag 4800, Christchurch, Canterbury 8140, New Zealand, and **Hannes Diener** and **Louis Warren**. *Classifying Material Implications over Minimal Logic.*

The so-called paradoxes of material implication and omniscience principles have motivated the development of much non-classical mathematics over the years. In this paper, we investigate some of these principles and classify them, over minimal logic. We provide proofs of equivalence and semantic models separating the principles where appropriate. A number of equivalent groups arise, all of which collapse with unrestricted use of double negation elimination. Interestingly, the principle *ex falso quodlibet* and several weaker principles, turn out to be distinct. Moreover, in the first-order analysis it turns out that Markov's principle of unbounded search is *not* minimally derivable from excluded middle alone. This separates the analysis clearly from both the classical and constructive interpretations which validate intuitionistic logic.

This research was in part supported by the New Zealand Marsden Fund, and the EC (FP7) project Correctness by Construction. (Received July 05, 2016)

1125-03-120 **Clinton Conley\*** (clintonc@andrew.cmu.edu). *Measurable chromatic numbers.*

The chromatic number of a graph, the least number of colors required to paint the vertices so that no two adjacent vertices share a color, can change drastically when various measurability constraints are placed on the coloring function. In this talk we survey several results over the past few years analyzing Borel and  $\mu$ -measurable (with respect to some fixed Borel probability measure  $\mu$ ) chromatic numbers, highlighting connections with ergodic theory. In particular, we pay special attention to graphs with hyperfinite/amenable connectedness relation: even in this special case the ability to discard a null set can have a big impact on how many colors are necessary. The talk will include joint work with Jackson, Kechris, Marks, Miller, Seward, and Tucker-Drob. (Received September 20, 2016)

1125-03-122 **Rahim Moosa\*** (rmoosa@uwaterloo.ca). *An application of model theory to noncommutative algebra.*

I will report on recent work, joint with Jason Bell and Omar León Sánchez, that uses the model theory of differentially closed fields to study the classical Dixmier-Moeglin equivalence problem for certain skew polynomial rings over commutative finitely generated algebras. (Received September 20, 2016)

1125-03-124 **Alexandra Shlapentokh\*** (shlapentokha@ecu.edu), Department of Mathematics, East Carolina University, Greenville, NC 27858. *On Existential Definability of C.E. Sets over Function Rings of Characteristic 0.* Preliminary report.

The connection between computably enumerable sets and existential (or Diophantine) definability in the language of rings goes back to the solution of Hilbert's Tenth Problem, when Davis, Putnam, Robinson and Matiyasevich showed that all c.e. subsets of natural numbers were existentially definable (over natural numbers). This result implied that Hilbert's Tenth Problem had no solution, i.e. there was no algorithm to determine whether an arbitrary equation in several variables with integer coefficients had integer solutions. Since the time of the solution of this problem, the question of definability of c.e. sets was explored over other domains, in particular over rings of functions of characteristic 0. I will discuss some old and new results concerning existential definability over these rings.

(Received August 22, 2016)

1125-03-127 **Henry P. Towsner\*** (htowsner@math.upenn.edu). *Hypergraph Regularity, Ultraproducts, and a Game Semantics.*

One approach to Szemerédi's graph regularity lemma, its generalization to hypergraphs, and its applications is to pass to infinitary limiting object - an ultraproduct - take advantage of measure-theoretic tools like the conditional expectation. Indeed, in a formal sense, graph regularity is equivalent to the existence of a particular case of the conditional expectation.

We describe how these infinitary arguments can be systematically reinterpreted as constructive, explicit, finitary arguments. More generally, we describe an alternative semantics for first-order logic in which statements about limiting objects like ultraproducts are reinterpreted as finite statements about winning strategies in a certain two-player game. (No prior knowledge about graph regularity or ultraproducts will be assumed.) (Received September 18, 2016)

1125-03-857 **Alfred R Dolich\*** (alfredo.dolich@kbcc.cuny.edu). *Expansions of Divisible Ordered Abelian Groups Without the Independence Property.*

We wish to consider structures of the form  $\mathcal{R} = \langle R, +, <, \dots \rangle$  which are expansions of divisible ordered Abelian groups and attempt to study under what conditions such structures or their theories may be deemed "well-behaved". Recall that the archetypal class of well-behaved structures of the form  $\mathcal{R}$  are the o-minimal structures, and for this class of structures we have a wealth of desirable properties. We look for other classes of well-behaved structures  $\mathcal{R}$  among those whose theory does not have the independence property (the so-called NIP theories) or some strengthening thereof. Essentially if  $Th(\mathcal{R})$  is o-minimal then any of these variants of not having the independence property hold for  $Th(\mathcal{R})$  and in this talk we survey to what extent assuming that  $Th(\mathcal{R})$  is NIP or some variant allows us to conclude that at least to a small extent  $Th(\mathcal{R})$  exhibits the good behavior witnessed by the o-minimal theories. (Received September 12, 2016)

1125-03-1048 **Eduardo Dueñez\*** (eduardo.duenez@utsa.edu), Mathematics Department, The University of Texas at San Antonio, 1 UTSA Circle, San Antonio, TX 78249-0664, and **José N Iovino.** *Uniformly metastable convergence in metric structures.*

We study Tao's finitary viewpoint of convergence in metric spaces, as captured by the notion of metastability with a uniform rate. We adopt the perspective of continuous model theory. We show that, in essence, uniform metastability is the only formulation of metric convergence that can be captured by a theory in continuous

first-order logic, a result we call the Uniform Metastability Principle. Philosophically, this principle amounts to the following meta-theorem: “If a classical statement about convergence in metric structures is refined to a statement about uniformly metastable convergence, then the validity of the original statement implies the validity of its uniformly metastable version.” As an instance of this phenomenon, we formulate an abstract version of Tao’s Metastable Dominated Convergence Theorem as a statement about axiomatizable classes of metric structures, and show that it is a direct consequence of the Uniform Metastability Principle.

(Research supported by NSF grant DMS-1500615.) (Received September 14, 2016)

1125-03-1259 **Maria Nogin\*** ([mnogin@csufresno.edu](mailto:mnogin@csufresno.edu)), 5245 N. Backer Ave. M/S PB 108, CSU Fresno, Fresno, CA 93740, and **Bing Xu**. *Modal logic axioms valid in quotient spaces of finite CW-complexes and other families of topological spaces.*

Considering the topological interpretations of the classical logic extended by a “box” operator interpreted as interior, we present extensions of S4 that are sound over some families of topological spaces, including particular point topological spaces, excluded point topological spaces, and quotient spaces of finite CW-complexes. (Received September 20, 2016)

1125-03-1339 **Michael C. Laskowski** ([mcl@math.umd.edu](mailto:mcl@math.umd.edu)), Department of Mathematics, University of Maryland College Park, College Park, MD 20742, and **Christopher S. Shaw\*** ([cshaw@colum.edu](mailto:cshaw@colum.edu)), Columbia College Chicago, 600 S. Michigan Ave, Chicago, IL 60605. *Skolem functions for a weakly o-minimal structure with a new convex predicate.*

For an o-minimal structure  $\mathcal{M}$  expanding a group and a new convex predicate  $U$ , we define  $T$ -resistance for the pair  $(\mathcal{M}, U)$ , a generalization of  $T$ -convexity. Using  $T$ -resistance and building on results of L. van den Dries and A. Lewenberg, we show that for a properly convex subset  $U$ , the theory of the expanded structure  $\mathcal{M}' = (\mathcal{M}, U)$  has definable Skolem functions precisely when  $\mathcal{M}'$  is valuatinal. In addition we present some insight about a particular algorithm for computing these functions when they are present. (Received September 16, 2016)

1125-03-1361 **David Milovich\*** ([ultrafilter@gmail.com](mailto:ultrafilter@gmail.com)), Dept. of Mathematics and Physics, 5201 University Blvd., Laredo, TX 78041. *Amalgamating many Boolean algebras.*

$\Delta$ -systems of overlapping Boolean algebras always extend to a common Boolean algebra, but non- $\Delta$ -systems may not: we might have  $x <_A y <_B z <_C x$ . Non- $\Delta$ -systems are unavoidable when constructing a Boolean algebra (or any structure) of size  $\geq \aleph_3$  as a directed union of countable structures. We prove a nontrivial sufficient condition for  $n$  overlapping Boolean algebras to have a common extension. Along the way, we prove an  $n$ -ary version of the Interpolation Theorem of propositional logic.

Using also the set-theoretic technique of long  $\omega_1$ -approximation sequences (also known as Davies sequences), we obtain a flexible method of constructing (in ZFC) arbitrarily large Boolean algebras as direct limits of countable Boolean algebras. Our main application is a Boolean algebra of size  $\aleph_n$  with the  $n$ -ary FN but not the  $(n+1)$ -ary FN where the  $n$ -ary FN is a higher-arity variant of the Freese-Nation property.

Our techniques also yield a new characterization of projective Boolean algebras that implies purely finitary facts about Vietoris hyperspace and symmetric power functors in the category of finite discrete spaces. (Received September 16, 2016)

1125-03-1537 **Simon Cho\*** ([seamcho@math.upenn.edu](mailto:seamcho@math.upenn.edu)). *On a variant of continuous logic.*

Continuous logic has found successful applications to e.g. ergodic theory in showing that certain convergence results hold uniformly in a precise sense, as described in the paper by J. Avigad and J. Iovino. This talk will describe a variant of continuous logic (which we call *geodesic logic*) which relaxes some of the continuity requirements of continuous logic and replaces them with ‘linear structures’ designed to mimic e.g. vector space structures of Banach spaces. Using this logic, we are able to apply the Avigad-Iovino approach to examples in the fixed point theory of Banach spaces involving functions which are a priori discontinuous, to obtain uniformity results in the same sense as above. (Received September 17, 2016)

1125-03-1667 **Katalin Bimbó\*** ([bimbo@ualberta.ca](mailto:bimbo@ualberta.ca)), 2–40 Assiniboia Hall, University of Alberta, Department of Philosophy, Edmonton, Alberta T6G2E7. *Multisets, a ternary relation and decidability.*

Multisets with finitely many elements (over a denumerable set) correspond to positive integers. Some logics such as the relevance logic  $R_+$  or classical linear logic can be formulated as sequent calculi using multisets of formulas. The decidability of classical propositional linear logic (LL) was proved by J. M. Dunn and myself in 2015 (by expanding the decidability result for *MELL*, the multiplicative–exponential fragment of LL proved by me in *Theoret. Comput. Sci.* **597** (2015), pp. 1–18.).

In this talk, I give a different proof of the decidability of  $LL$  using Kopylov's normalization of  $LL$  together with the correspondence between multisets and positive integers. (Received September 18, 2016)

1125-03-1673 **Maarten McKubre-Jordens\*** ([maarten.jordens@canterbury.ac.nz](mailto:maarten.jordens@canterbury.ac.nz)), University of Canterbury, Private Bag 4800, Christchurch, 8140, New Zealand, and **Erik Istre**. *A proof-theoretic solution to the set-theoretic paradoxes.*

Soon after its discovery by Frege and others, *Naïve Set Theory* (NST) was known to be paradoxical, giving rise to a wide variety of paradoxes, such as that of Russell. Several attempts to found NST using non-classical logics have so far obtained partial results. In this paper we provide a mathematically principled, motivated approach that makes sense of the set-theoretic paradoxes within the theory itself. By addressing the paradoxes structurally—that is, within the very notion of proof itself—we retain proving ability while increasing expressiveness permitted in the theory beyond what is possible in the classical conception. We use properties of proof normalization in ways suggested by Prawitz, Hallnäs, and Ekman to ensure our system is coherent, and discuss notions of implication within the formal system.

This research was in part supported by the New Zealand Marsden Fund, and the Marie Curie IRSES (FP7) project Correctness by Construction. (Received September 18, 2016)

1125-03-1839 **Francis Adams\*** ([fsadams@ufl.edu](mailto:fsadams@ufl.edu)). *Ideals of Compact Anticliques in Borel Graphs.*

Given a Borel graph  $G$  on a Polish space  $X$ , we can define the  $\sigma$ -ideal  $I_G$  generated by the compact  $G$ -anticliques. We investigate the cardinal characteristics  $\text{non}(I_G)$  and  $\text{cov}(I_G)$ , and in particular compare them to the bounding number  $\mathfrak{b}$ . I introduce a class of closed graphs for which it is consistent that  $\text{non}(I_G) < \mathfrak{b}$ . I will also develop some properties of this class of graphs and provide specific examples and non-examples. (Received September 19, 2016)

1125-03-1961 **Oscar Levin\*** ([oscar.levin@unco.edu](mailto:oscar.levin@unco.edu)), School of Mathematical Sciences, Campus Box 122, 501 20th Street, Greeley, CO 80639. *Graph labelings and computability theory.* Preliminary report.

A *graph labeling* assigns weights to vertices, edges, or both, subject to certain conditions. For example, a labeling of vertices is *graceful* or *harmonious* if the difference or sum, respectively, of labels on adjacent vertices is distinct for all edges, while a labeling is *edge-magic* if the sum of the labels on an edge and its incident vertices is constant for all edges. In this talk we will explore graph labelings in the context of computability theory, sharing a few initial findings and many directions for future research. (Received September 19, 2016)

1125-03-1963 **Stephen Flood** and **Matthew Jura\*** ([matthew.jura@manhattan.edu](mailto:matthew.jura@manhattan.edu)), 4513 Manhattan College Parkway, Riverdale, NY 10471, and **Oscar Levin** and **Tyler Markkanen**. *The reverse mathematics of a theorem of Steffens.* Preliminary report.

A *matching* of a graph  $G = (V, E)$  is a set  $M \subseteq E$  of pairwise disjoint edges. A *perfect matching* of a graph  $G$  is a matching  $M$  of  $G$  such that  $V(M) = V$ . In 1977, Steffens discovered a necessary and sufficient condition for a countable graph to possess a perfect matching, which he called “condition (A).” We say that a graph  $G$  satisfies condition (A) if for every matching  $M$  and for every vertex  $s \in V(G) \setminus V(M)$  there exists an  $M$ -augmenting path which starts at  $s$ . Steffens’ Theorem states that a countable graph has a perfect matching if and only if it satisfies condition (A). We classify the proof-theoretic strength of a number of principles related to Steffens’ Theorem in the context of reverse mathematics. (Received September 19, 2016)

1125-03-1968 **Jared Holshouser\*** ([jaredholshouser@my.unt.edu](mailto:jaredholshouser@my.unt.edu)), 1030 Dallas Drive, #1022, Denton, TX 76205. *Partition Properties for Non-Ordinal Sets Under the Axiom of Determinacy.* Preliminary report.

It is well established that the axiom of determinacy imbues normal cardinals with large cardinal properties. Recently it was shown that, in  $L(\mathbb{R})$ , all cardinals below  $\Theta$  are Jonsson and other cardinals are Rowbottom (weak Ramsey properties). It remained open, however, whether any non-ordinal sets are Jonsson. We have answered this question in the affirmative. For the fragment of  $L_\Theta(\mathbb{R})$  which is generated by cardinals and hyperfinite quotients of  $\mathbb{R}$  using finite unions and products, we have characterized when the Ramsey, Rowbottom, and Jónsson properties occur. (Received September 19, 2016)

1125-03-2232 **William C. Calhoun\*** ([wcalhoun@bloomu.edu](mailto:wcalhoun@bloomu.edu)), Department of Math. & Digital Sciences, 400 East Second Street, Bloomsburg, PA 17815. *Strongly nontrivial minimal Turing degrees.* Preliminary report.

The complexity of a set of natural numbers  $A$  can be measured by the growth rate of  $K(A \upharpoonright n)$ , where  $K$  is Kolmogorov complexity and  $A \upharpoonright n$  is  $A$  restricted to  $n$ . An order function is a nondecreasing unbounded

function. We define  $K_{order}$  to be the collection of sets  $A$  such that  $K(A \upharpoonright n)$  is bounded by  $p(K(n))$  for some order function  $p$ . We say that a set is strongly nontrivial if it is not a member of  $K_{order}$ . Using a modification of the Sack's minimal degree construction, we show there is a strongly nontrivial  $\Delta_2^0$  set of minimal Turing degree. (Received September 20, 2016)

1125-03-2344 **Monica M VanDieren\*** ([vandieren@rmu.edu](mailto:vandieren@rmu.edu)), Department of Mathematics, RMU, 6000 University Blvd, Moon Township, PA 15146. *Symmetry and Superstability in Abstract Elementary Classes.*

In this talk I will survey some of the recent developments in understanding superstability in abstract elementary classes. In particular, I will describe the concept of symmetry and how it has been used to answer several open questions and tie together research on good frames with tame classes. (Received September 20, 2016)

1125-03-2766 **Douglas Cenzer** ([cenzer@uf1.edu](mailto:cenzer@uf1.edu)), Department of Mathematics, University of Florida, P.O. Box 118105, Gainesville, FL 32611-8105, and **Diego A. Rojas\*** ([darz0545@uf1.edu](mailto:darz0545@uf1.edu)), 1132 SW 11th Ave, Gainesville, FL 32601-7841. *On the Computational Complexity and Randomness of Derivatives of Online Functions.* Preliminary report.

Recently, the study of algorithmic randomness has become increasingly significant in logic and theoretical computer science. Current research has defined a notion for random continuous functions, and the integrals of these functions have been investigated. However, the derivatives of these random continuous functions have not been explored or even defined. In this paper, we provide a notion for the derivative of continuous functions on  $2^{\mathbb{N}}$ . Furthermore, we discuss the implications of this derivative definition for online functions in computational complexity and randomness. Finally, we establish some connections between online functions and their derivatives on  $2^{\mathbb{N}}$  and functions on  $\mathbb{R}$  and their derivatives, where we can represent real-valued functions as functions acting on the dyadic representation of real numbers under an online function. (Received September 20, 2016)

## 05 ► Combinatorics

1125-05-8 **Raj Raina\*** ([rraina@stanford.edu](mailto:rraina@stanford.edu)), **Andrey Grinshpun** and **Rik Sengupta.** *Minimum Degrees of Minimal Ramsey Graphs for Almost-Cliques.*

For graphs  $F$  and  $H$ , we say  $F$  is *Ramsey for  $H$*  if every 2-coloring of the edges of  $F$  contains a monochromatic copy of  $H$ . The graph  $F$  is *Ramsey  $H$ -minimal* if  $F$  is Ramsey for  $H$  and there is no proper subgraph  $F'$  of  $F$  so that  $F'$  is Ramsey for  $H$ . Burr, Erdős, and Lovász defined  $s(H)$  to be the minimum degree of  $F$  over all Ramsey  $H$ -minimal graphs  $F$ . Define  $H_{t,d}$  to be a graph on  $t+1$  vertices consisting of a complete graph on  $t$  vertices and one additional vertex of degree  $d$ . We show that  $s(H_{t,d}) = d^2$  for all values  $1 < d \leq t$ .

We also make some further progress on some sparser graphs. Fox and Lin observed that  $s(H) \geq 2\delta(H) - 1$  for all graphs  $H$ , where  $\delta(H)$  is the minimum degree of  $H$ ; Szabó, Zumstein, and Zürcher investigated which graphs have this property and conjectured that all bipartite graphs  $H$  without isolated vertices satisfy  $s(H) = 2\delta(H) - 1$ . Fox, Grinshpun, Liebenau, Person, and Szabó further conjectured that all connected triangle-free graphs with at least two vertices satisfy this property. We show that  $d$ -regular 3-connected triangle-free graphs  $H$ , with one extra technical constraint, satisfy  $s(H) = 2\delta(H) - 1$ . (Received May 20, 2016)

1125-05-58 **Bo Lin** ([linbo@berkeley.edu](mailto:linbo@berkeley.edu)), Berkeley, CA, **Bernd Sturmfels** ([bernd@berkeley.edu](mailto:bernd@berkeley.edu)), Berkeley, CA, and **Xiaoxian Tang** and **Ruriko Yoshida\*** ([ruriko.yoshida@uky.edu](mailto:ruriko.yoshida@uky.edu)), Lexington, KY. *Convexity in treespaces.*

We study the geometry of metrics and convexity structures on the space of phylogenetic trees, here realized as the tropical linear space of all ultrametrics. The CAT(0)-metric of Billera-Holmes-Vogtman arises from the theory of orthant spaces. While its geodesics can be computed by the Owen-Provan algorithm, geodesic triangles are complicated and can have arbitrarily high dimension. Tropical convexity and the tropical metric are better behaved, as they exhibit properties that are desirable for geometric statistics. (Received June 30, 2016)

1125-05-74 **J. Hu, E. Shan, C. Wang, Shaohui Wang\*** ([shaohuiwang@yahoo.com](mailto:shaohuiwang@yahoo.com)) and **B. Wei.** *Total domination polynomials of graphs.* Preliminary report.

Given a graph  $G$ , a total dominating set  $D_t$  is a set that every vertex of  $G$  is adjacent to some vertices of  $D_t$  and let  $d_t(G, i)$  be the number of all total dominating sets with size  $i$ . The total domination polynomial, defined as  $D_t(G, x) = \sum_{i=1}^{|V(G)|} d_t(G, i)x^i$ , recently has been the focus of considerable extended research in the field of domination theory. In this paper, we obtain the vertex-reduction and edge-reduction formulas of total domination polynomials. As consequences, we give the total domination polynomials for paths and cycles.

Additionally, we determine the sharp upper bounds of total domination polynomials for trees and characterize the corresponding graphs with such bounds. Finally, we use the reduction-formulas to investigate the relations between vertex sets and total domination polynomials in  $G$ . (Received July 11, 2016)

1125-05-134 **Ruth Lopez\***, Department of Mathematics and Statistics, 1250 Bellflower Blvd, California State University, Long Beach, Long Beach, CA 90840, and **Jacob Worrell**, Dept. of Psychological and Brain Sciences, 1101 E. 10th Street, Indiana University - Bloomington, Bloomington, IN 47405-7007. *Towards a Characterization of Graphs with Distinct Betweenness Centralities.*

The betweenness centrality of a vertex  $v$  is the ratio of the number of shortest paths between two other vertices  $u$  and  $w$  which contain  $v$  to the total number of shortest paths between  $u$  and  $w$ . We consider the problem of characterizing all graphs with distinct betweenness centralities. We begin by solving the problem for all graphs with less than or equal to seven vertices. Next, we investigate graph properties such as density and minimality. Finally, we determine sufficient conditions for graphs with distinct betweenness centralities to be extended to infinite families of graphs of the same type. (Received August 03, 2016)

1125-05-135 **Jenny Kaufmann\***, Department of Mathematics, Fine Hall, Washington Road, Princeton University, Princeton, NJ 08544-1000, and **Henry Wickus**, Dept. of Mathematics and Computer Science, 2755 Station Avenue, DeSales University, Center Valley, PA 18034. *On Some Edge Folkman Numbers, Large and Small.*

"How large a structure do we need to force a certain substructure to appear in any of its colorings?" This question is the essence of Ramsey theory. Edge Folkman numbers  $F_e(G_1, G_2; k)$ , a generalization of more commonly studied Ramsey numbers, are defined as the smallest order of any  $K_k$ -free graph  $F$  such that any red-blue coloring of the edges of  $F$  contains either a red  $G_1$  or a blue  $G_2$ . We discuss findings on edge Folkman numbers involving graphs  $J_s = K_s - e$ . We prove the general results  $F_e(J_3, K_n; n+1) = 2n-1$ ,  $F_e(J_3, J_n; n) = 2n-1$ , and  $F_e(J_3, J_n; n+1) = 2n-3$ . We also present results on  $F_e(J_4; J_4; k)$  for all cases other than  $k=4$ , with exact values given for  $k > 6$  and bounds given for  $k$  equal to 5 or 6. In particular we present the upper bound  $F_e(J_4; J_4; 5) \leq 1297$ , using computational methods modified from the study of classical Folkman numbers. We also describe our attacks on  $F_e(3, 3; 4)$ , the smallest unsolved problem in the theory of Folkman numbers and the problem which originally motivated our research. (Received August 03, 2016)

1125-05-188 **Kristofer Siy\*** ([kristofer.siy@tufts.edu](mailto:kristofer.siy@tufts.edu)), Department of Mathematics, 503 Boston Avenue, Tufts University, Medford, MA 02155, and **Heather Weaver**, Mathematics, Applied Mathematics, and Stats, 2049 Martin Luther King Jr. Drive, Case Western Reserve University, Cleveland, OH 44106-7057. *Minimum Saturation Numbers of Double Stars.*

A graph  $G$  is said to be  $F$ -saturated if  $G$  contains no subgraph isomorphic to  $F$ , but for every edge in the complement of  $G$ ,  $G + e$  contains a subgraph isomorphic to  $F$ . For an  $F$ -saturated graph  $G$ , the average saturation number  $\text{avsat}(G, F)$  is the average number of copies of  $F$  created over adding all possible edges in the complement of  $G$ , and the minimum and maximum saturation numbers  $\text{minsat}(n, F)$  and  $\text{maxsat}(n, F)$  are the minimum and maximum values of the parameter  $\text{avsat}(G, F)$  over all graphs  $G$  with  $n$  vertices.

We will examine certain specific cases of trees, but also provide results of a general nature including the family of double stars.

A double star  $S_{m,k}$  is a tree of diameter 3, namely, the graph generated by taking a  $K_2$  and adding  $m+k$  vertices,  $m$  of which are connected to one endpoint of the  $K_2$  and the other  $k$  of which are connected to the other endpoint of the  $K_2$ . Berman et al. showed in 2015 that the only trees with an infinite number of uniquely  $T$ -saturated graphs are precisely the balanced double stars, namely, all double stars for which  $m=k$ . We generalise this result and determine an asymptotically sharp value for  $\text{minsat}(n, S_{m,k})$ . (Received August 11, 2016)

1125-05-205 **Shelby P Cox\*** ([shelbycox@umass.edu](mailto:shelbycox@umass.edu)) and **Amal Mattoo** ([amal.mattoo@gmail.com](mailto:amal.mattoo@gmail.com)). *Euler characteristic of Hilbert schemes via colored Young diagrams.*

We are interested in finding a generating function for the Euler characteristic of the Hilbert scheme of points in the (singular) variety  $\mathbb{C}^2/\mathbb{Z}_n$ , and the Hilbert scheme of points in the (smooth) orbifold  $[\mathbb{C}^2/\mathbb{Z}_n]$ . For the former case, the problem reduces to finding a generating function for all 0-generated Young diagrams which contain a certain number of 0 colored squares i.e. those in correspondence with ideals generated by monomials trivially acted upon by the group action. We found a theorem which greatly simplifies the problem, and in some cases it reduces the problem into already solved cases.

For the orbifold case, the problem reduces to finding a generating function for all Young diagrams with a given coloring. We developed a procedure which results in the desired generating function, as well as closed

form generating functions for some cases. We also explored the method of vertex operator algebras, which had previously been applied to prove MacMahon's formula counting plane partitions.

[Mentor: Amin Gholampour, University of Maryland College Park] (Received August 11, 2016)

1125-05-217 **Joseph M DiMuro\*** (joseph.dimuro@biola.edu). *The Warden's Game - Greedy and Necklace Constructions of Universal Cycles*. Preliminary report.

Let  $\mathbf{T}(n, k)$  be the set of strings over  $\{1, 2, \dots, k\}$  of length  $n$ . The lexicographically minimal universal cycle for  $\mathbf{T}(n, k)$  (also known as a de Bruijn cycle) can be constructed via a greedy algorithm; at each step, the smallest possible symbol is appended to the cycle, while ensuring that no substring of length  $n$  is repeated. This universal cycle can also be constructed by taking all necklaces in  $\mathbf{T}(n, k)$  in lexicographic order, and appending their aperiodic prefixes.

Let  $S \subseteq \mathbf{T}(n, k)$  be a subset that is closed under rotations. Aaron Williams asked for necessary and sufficient conditions on  $S$  so that a universal cycle for  $S$  can be generated via the greedy algorithm or the necklace algorithm. With the help of a dice-turning game, we will answer both questions. (Received September 20, 2016)

1125-05-225 **G. Eric Moorhouse** (moorhous@uwyo.edu), **Shuying Sun\*** (shuying@udel.edu) and **Jason Williford** (jwillif1@uwyo.edu). *The Eigenvalues of the Graphs  $D(4, q)$* . Preliminary report.

The graphs  $D(k, q)$  give the best known bounds on extremal problems with forbidden even cycles, and are denser than the well-known graphs of Lubotzky Phillips, Sarnak and Margulis. Despite this, little about the spectrum and expansion properties of these graphs is known. In this paper we find the spectrum for  $D(4, q)$ , the smallest open case. For each prime power  $q$ , the graph  $D(4, q)$  is  $q$ -regular graph on  $2q^4$  vertices, all of whose eigenvalues other than  $\pm q$  are bounded by  $2\sqrt{q}$ . Accordingly, these graphs are good expanders, in fact very close to Ramanujan. (Received August 19, 2016)

1125-05-238 **Aleksandr Kodess\*** (kodess@uri.edu), University of Rhode Island, 5 Lippitt Road, Lippitt Hall, room 200, Kingston, RI 02881. *Algebraic Digraphs*.

Let  $q$  be a prime power,  $\mathbb{F}_q$  denote the finite field of  $q$  elements. Let  $f_i: \mathbb{F}_q^2 \rightarrow \mathbb{F}_q$  be arbitrary functions,  $1 \leq i \leq l$ . The digraph  $D = D(q; \mathbf{f})$ , where  $\mathbf{f} = (f_1, \dots, f_l): \mathbb{F}_q^2 \rightarrow \mathbb{F}_q^l$ , is defined as follows. The vertex set  $V$  of  $D$  is  $\mathbb{F}_q^{l+1}$ . There is an arc from  $(x_1, \dots, x_{i+1}) \in V$  to  $(y_1, \dots, y_{i+1}) \in V$  if and only if  $x_i + y_i = f_{i-1}(x_1, y_1)$  for all  $i$ ,  $2 \leq i \leq l+1$ .

When  $l = 1$  and  $f = f_1$  can be represented by the polynomial  $X_1^m X_2^n$ , the digraph  $D = D(q; m, n)$  is called a *monomial* digraph. The digraphs  $D(q; \mathbf{f})$  and  $D(q; m, n)$  are directed analogues of a well studied class of algebraically defined undirected bipartite graphs  $B\Gamma(q; f)$  having many applications, most noticeably in extremal graph theory supplying a lower bound of the best magnitude on some  $\text{ex}(\nu, C_{2l})$ .

We present a number of results on the strong connectivity of the general algebraic digraph  $D(q; \mathbf{f})$  and the diameters of monomial digraphs. We also discuss the isomorphism problem for all monomial digraphs for a fixed  $q$ . (Received August 16, 2016)

1125-05-261 **Annie Raymond\*** (raymonda@uw.edu), **James Saunderson**, **Mohit Singh** and **Rekha R Thomas**. *Symmetry and Turan Sums of Squares*.

Given a graph  $H$ , the Turan graph problem asks to find the maximum number of edges in a  $n$ -vertex graph that does not contain any subgraph isomorphic to  $H$ . In recent years, Razborov's flag algebra methods have been applied to Turan hypergraph problems with great success. We show that these techniques are equivalent to symmetry-reduction methods for sum of squares representations of invariant polynomials. This connection gives an alternate computational framework for Turan problems with the potential to go further. Our results expose the rich combinatorics coming from the representation theory of the symmetric group present in both methods. This is joint work with James Saunderson, Mohit Singh, and Rekha R. Thomas. (Received August 20, 2016)

1125-05-283 **Miaomiao Han\*** (mahan@mix.wvu.edu), 654 Protzman Street, Apt 1, Morgantown, WV 265.5, and **You Lu** and **Rong Luo**. *Neighbor sum distinguishing total coloring of  $d$ -degenerate graphs*. Preliminary report.

A proper total  $k$ -coloring  $\phi$  of a graph  $G$  is a mapping from  $V(G) \cup E(G)$  to  $\{1, 2, \dots, k\}$  such that no adjacent or incident elements in  $V(G) \cup E(G)$  receive the same color. Let  $m_\phi(v)$  denote the sum of colors on the edges incident with vertex  $v$  and the color on vertex  $v$ . A proper total  $k$ -coloring of  $G$  is called neighbor sum distinguishing if  $m_\phi(u) \neq m_\phi(v)$  for each edge  $uv \in E(G)$ . Let  $\chi_\Sigma^t(G)$  be the neighbor sum distinguishing total chromatic number of a graph  $G$ . Piłśniak and Woźniak conjectured that for any graph  $G$ ,  $\chi_\Sigma^t(G) \leq \Delta(G) + 3$ . In this

paper, we present  $\chi_{\Sigma}^t(G) \leq \max\{\Delta(G) + \lfloor \frac{3\text{col}(G)}{2} \rfloor - 1, 3\text{col}(G) - 2\}$ , where  $\text{col}(G)$  is the coloring number of  $G$ . In particular, for a 2-degenerate graph  $G$ , we determine the exact values of  $\chi_{\Sigma}^t(G)$  if  $\Delta(G) \geq 6$  and show that  $\chi_{\Sigma}^t(G) \leq 7$  if  $4 \leq \Delta(G) \leq 5$ . (Received August 23, 2016)

1125-05-300 **Miaomiao Han, Xinmin Hou, Hong-Jian Lai and Jiaao Li\*** (joli@mix.wvu.edu), Department of Mathematics, West Virginia University, Morgantown, WV 26506-6310. A *Ramsey-Type theorem on Modulo Orientations*.

A mod  $(2p+1)$ -orientation  $D$  is an orientation of  $G$  such that  $d_D^+(v) - d_D^-(v) \equiv 0 \pmod{2p+1}$  for any vertex  $v \in V(G)$ . Jaeger conjectured that every  $4p$ -edge-connected graph has a mod  $(2p+1)$ -orientation. For  $p=1$ , it is the Tutte's 3-Flow Conjecture. The  $p=2$  case, if true, would imply Tutte's 5-Flow Conjecture. The Ramsey theorem states that, when  $|V(G)|$  is sufficient large, either  $G$  or its complement  $G^c$  contains a complete graph  $K_n$  as a subgraph. We show a Ramsey-Type theorem on modulo orientations that if  $G$  is a graph with  $|V(G)| \geq N(p) = 1152p^4$  and  $\min\{\delta(G), \delta(G^c)\} \geq 4p$ , then either  $G$  or  $G^c$  has a mod  $(2p+1)$ -orientation. (Received August 24, 2016)

1125-05-303 **Federico Ardila\***, San Francisco State University, **Hanner Bastidas**, Universidad del Valle, **Cesar Ceballos**, University of Vienna, and **John Guo**, San Francisco State University. *The configuration space of a robotic arm in a tunnel*.

We study the motion of a robotic arm inside a rectangular tunnel. We prove that the configuration space of all possible positions of the robot is a CAT(0) cubical complex. This allows us to use techniques from geometric group theory to find the optimal way of moving the arm from one position to another. We also compute the diameter of the configuration space, that is, the longest distance between two positions of the robot. The key ingredient in the proofs is the bijection between CAT(0) cubical complexes and "posets with inconsistent pairs". The talk will assume no previous knowledge of the subject and will be accessible to undergraduates. (Received August 24, 2016)

1125-05-316 **Yue Cai\*** (ycai@math.tamu.edu), Department of Mathematics, MS 3368, Texas A&M University, College Station, TX 77843, and **Margaret Readdy** (margaret.readdy@uky.edu). *A poset approach to the  $q$ -Stirling numbers*.

In this talk, we show the classical  $q$ -Stirling numbers of the second kind can be expressed more compactly as a pair of statistics on a subset of restricted growth words. The resulting expressions are polynomials in  $q$  and  $1+q$ . We extend this enumerative result via a decomposition of the Stirling poset of the second kind. This poset supports an algebraic complex and a basis for integer homology is determined. Time permitting, a parallel enumerative, poset theoretic and homological study for the  $q$ -Stirling numbers of the first kind will also be discussed. (Received August 25, 2016)

1125-05-319 **Po-Shen Loh, Michael Tait and Craig Timmons\*** (craig.timmons@csus.edu). *Induced Turán numbers*.

Let  $s \geq t \geq 2$  be integers and  $H$  be a non-bipartite graph. It is easy to see that the maximum number of edges in an  $n$ -vertex graph with no induced copy of  $K_{s,t}$  is  $\binom{n}{2}$  because  $K_n$  has no induced  $K_{s,t}$ . However, if we ask for the maximum number of edges in an  $n$ -vertex graph with no copy of  $H$  and no induced copy of  $K_{s,t}$ , then we can no longer use the complete graph. A  $(\chi(H)-1)$ -partite Turán graph, which will be  $H$ -free, cannot be used for a lower bound either because a sufficiently large Turán graph will contain induced copies of  $K_{s,t}$ . We will present some bounds on the number of edges in an  $n$ -vertex graph with no copy of  $H$  and no induced copy of  $K_{s,t}$ , as well as some related results involving clique counts. This is joint work with Po-Shen Loh and Mike Tait. (Received August 26, 2016)

1125-05-356 **Ryan R. Martin, Brendon Stanton and Shanise Walker\*** (shanise1@iastate.edu), 443 Carver Hall, 411 Morrill Rd, Ames, IA 50011. *A new lower bound for a vertex-identifying code in general graphs*.

Let  $N[v]$  denote the closed neighborhood of a vertex  $v$ . For a finite graph  $G$ , a vertex-identifying code in  $G$  is a subset  $C \subset V(G)$ , with the property that  $N[u] \cap C \neq N[v] \cap C$ , for all distinct  $u, v \in V(G)$  and  $N[v] \cap C \neq \emptyset$ , for all  $v \in V(G)$ . Karpovsky, Chakrabarty, and Levitin proved that for a graph  $G$  on  $n$  vertices such that  $|N[v]| < \beta$  for all  $v \in V(G)$  and  $C$  a vertex-identifying code,  $|C| \geq \max\left\{\lceil \log_2(n+1) \rceil, \left\lceil \frac{2n}{\beta+1} \right\rceil\right\}$ . We improve upon the lower bounds of Karpovsky, et al. when  $e\sqrt{n} < \beta < \frac{n}{2} - \sqrt{\frac{n \ln 2}{2}}$ . (Received August 29, 2016)

1125-05-357 **Matthew J Willis\*** (mjwillis@wesleyan.edu), 38 Ward St., Middletown, CT 06457.  
*Parabolic generalizations of the Catalan numbers.*

These results originated from the study of flagged Schur functions, which are polynomials produced by summing the weights of certain sets of semistandard Young tableaux. These sets are determined by  $n$ -tuples  $\phi$  with  $1 \leq \phi_1 \leq \dots \leq \phi_n \leq n$  called flags which serve as upper row bounds for the tableaux. The set of all flags is the distributive lattice denoted  $L(n, n)$  by Stanley. The set determined by each flag is a principal ideal in the poset of Young tableaux under the entrywise comparison ordering. When the shape of the tableaux is strict, i.e. all column lengths are present, the distinct flagged Schur functions are enumerated by the Catalan numbers. What happens if the shape is not strict? The missing column lengths determine a parabolic subgroup of the symmetric group. To answer the question, we sharpen some earlier results of Postnikov and Stanley that related flagged Schur functions to key polynomials (type A Demazure characters), which are polynomials determined by a permutation. This leads to new sub-posets of  $n$ -tuples with some interesting properties and applications, in particular counts that are analogous to the Catalan numbers for each of the  $2^{n-1} - 1$  parabolic subgroups. These results are joint with Robert Proctor. (Received August 29, 2016)

1125-05-381 **Robert Davis** (davisr@math.msu.edu), Department of Mathematics, Michigan State University, East Lansing, MI 48824-1027, and **Bruce E Sagan\*** (sagan@math.msu.edu), Department of Mathematics, Michigan State University, East Lansing, MI 48824-1027.  
*Pattern-avoiding polytopes and Bruhat orders I.*

The Birkhoff polytope,  $B_n$ , is the convex hull in  $\mathbb{R}^{n \times n}$  of the permutations matrices  $M_\sigma$  as  $\sigma$  runs over the symmetric group  $\mathfrak{S}_n$ . Given a set of permutations  $\Pi$ , let  $\text{Av}_n(\Pi)$  denote the set of all  $\sigma \in \mathfrak{S}_n$  which avoid the permutations in  $\Pi$ . There is a corresponding polytope  $B_n(\Pi)$  which is the convex hull of all  $M_\sigma$  for  $\sigma \in \text{Av}_n(\Pi)$ . For certain  $\Pi$  these polytopes exhibit very interesting behavior. In particular, we consider  $B_n(132, 312)$  and  $\tilde{B}_n(123)$  where the tilde indicates that we only take the alternating permutations in  $\text{Av}_n(123)$ . Restricting weak Bruhat order to these two permutation classes give lower order ideals in Young's lattice for certain shifted and left-justified shapes, respectively. By analyzing EL-shellings of these posets, we obtain information about the corresponding polytopes. (Received August 30, 2016)

1125-05-382 **Robert Davis\*** (davisr@math.msu.edu), Department of Mathematics, Michigan State University, 619 Red Cedar Rd., East Lansing, MI 48824, and **Bruce E. Sagan**.  
*Pattern-avoiding polytopes and Bruhat orders II.*

We continue studying  $B_n(132, 312)$  and  $\tilde{B}_n(123)$ , showing that the EL-shellings of the corresponding posets induce highly-structured triangulations of the polytopes. To do so, we examine associated toric ideals, which are algebraic objects that capture relations among the lattice points in the polytopes. In particular, we identify the reduced Gröbner bases of these ideals. A correspondence due to Sturmfels then shows that the order complexes of these posets form triangulations of the polytopes which are regular, unimodular, and flag. Moreover, the theory of  $(P, \omega)$ -partitions implies that the Ehrhart  $h^*$ -vectors of the polytopes are palindromic, which provides a strong condition on certain semigroup algebras generated by the polytopes. Together, these properties imply that the polytopes have unimodal  $h^*$ -vectors, as well as easily-computable normalized volumes. If time permits, some natural open problems will be discussed. (Received August 30, 2016)

1125-05-420 **Ronald J. Gould\*** (rg@mathcs.emory.edu), Dept. Math and Computer Science, Emory University, Atlanta, GA 30322. *On Saturation Spectrum.*

Given graphs  $G$  and  $H$ , we say that  $G$  is  $H$ -saturated if  $G$  does not contain a copy of  $H$  as a subgraph, but the addition of any edge  $e \notin E(G)$  produces at least one copy of  $H$  in  $G \cup e$ . Given a positive integer  $n$ , the saturation number,  $\text{sat}(n, H)$ , is the minimum number of edges in an  $H$ -saturated graph on  $n$  vertices. Of course, the well studied extremal number,  $\text{ext}(n, H)$  is the maximum number of edges in an  $H$  saturated graph on  $n$  vertices. One question is now obvious: For what values of  $m$ ,  $\text{sat}(n, H) \leq m \leq \text{ext}(n, H)$  does there exist an  $H$ -saturated graph of order  $n$  with  $m$  edges? The set of all such values is called the saturation spectrum of  $H$ . In this talk we will explore this question for several families of graphs. (Received September 01, 2016)

1125-05-478 **Thomas McConville\*** (thomasmcconvillea@gmail.com). *A poset of reduced words for the longest element.*

A Coxeter group is a group with a particular kind of presentation. A remarkable property of such a group is that any two minimum length expressions (reduced words) for the same element are connected by a sequence of elementary moves. We consider turning the graph of reduced words of the longest element into the Hasse diagram of a poset. This can be done by orienting the graph to "point away from" a particular reduced word  $r_0$ . However, not all choices of  $r_0$  are of equal value. We show that for a particular choice of  $r_0$  in types A

and  $B$ , the poset is bounded and its proper part is homotopy equivalent to a sphere. Our proof uses Rambau's Suspension Lemma. Furthermore, we conjecture that every interval is either contractible or homotopy equivalent to a sphere. (Received September 03, 2016)

1125-05-481 **Colin R. Defant\*** ([cdefant@ufl.edu](mailto:cdefant@ufl.edu)). *Anti-Power Prefixes of the Thue-Morse Word.*

Recently, Fici, Restivo, Silva, and Zamboni defined a  $k$ -anti-power to be a word of the form  $w_1 w_2 \cdots w_k$ , where  $w_1, w_2, \dots, w_k$  are distinct words of the same length. They defined  $AP(x, k)$  to be the set of all positive integers  $m$  such that the prefix of length  $km$  of the word  $x$  is a  $k$ -anti-power. Let  $\mathbf{t}$  denote the Thue-Morse word, and let  $\mathcal{F}(k) = AP(\mathbf{t}, k) \cap (2\mathbb{Z}^+ - 1)$ . We show that  $(2\mathbb{Z}^+ - 1) \setminus \mathcal{F}(k)$  is finite whenever  $k \geq 3$ . For  $k \geq 3$ ,  $\gamma(k) = \min \mathcal{F}(k)$  and  $\Gamma(k) = \max((2\mathbb{Z}^+ - 1) \setminus \mathcal{F}(k))$  are well-defined odd positive integers. Fici et al. speculated that  $\gamma(k)$  grows linearly in  $k$ . We prove that this is indeed the case by showing that  $1/2 \leq \liminf_{k \rightarrow \infty} (\gamma(k)/k) \leq 9/10$  and  $1 \leq \limsup_{k \rightarrow \infty} (\gamma(k)/k) \leq 3/2$ . In addition, we prove that  $\liminf_{k \rightarrow \infty} (\Gamma(k)/k) = 3/2$  and  $\limsup_{k \rightarrow \infty} (\Gamma(k)/k) = 3$ . (Received September 03, 2016)

1125-05-504 **Robert Aldred** and **Michael D. Plummer\*** ([michael.d.plummer@vanderbilt.edu](mailto:michael.d.plummer@vanderbilt.edu)),  
Vanderbilt University, Nashville, TN 37240. *Matching extension in prism graphs.*

If  $G$  is any graph, the *prism graph* of  $G$ , denoted  $P(G)$ , is the cartesian product of  $G$  with a single edge, or equivalently, the graph obtained by taking two copies of  $G$ , say  $G_1$  and  $G_2$ , with the same vertex labelings and joining each vertex of  $G_1$  to the vertex of  $G_2$  having the same label by an edge. A connected graph  $G$  has property  $E(m, n)$  (or more briefly " $G$  is  $E(m, n)$ ") if for every pair of disjoint matchings  $M$  and  $N$  in  $G$  with  $|M| = m$  and  $|N| = n$  respectively, there is a perfect matching  $F$  in  $G$  such that  $M \subseteq F$  and  $N \cap F = \emptyset$ . A graph which has the  $E(m, 0)$  property is also said to be  $m$ -*extendable*. In this paper, we begin the study of the  $E(m, n)$  properties of the prism graph  $P(G)$  when  $G$  is an arbitrary graph as well as the more special situations when, in addition,  $G$  is bipartite or bicritical. (Received September 04, 2016)

1125-05-578 **Mary Flagg\*** ([flaggm@stthom.edu](mailto:flaggm@stthom.edu)), Department of Mathematics, University of St. Thomas, 3800 Montrose, Houston, 77096, and **Daniela Ferrero**, **Katherine F. Benson**, **Veronika Furst**, **Leslie Hogben** and **Violeta Vasilevska**. *Nordhaus-Gaddum problems for the power domination number of a simple graph.*

Power domination is a coloring game played on a simple graph which was originally motivated by the challenge of determining where to place phase measurement units (PMUs) to efficiently monitor an electric power grid. The power domination number of a graph corresponds to the smallest number of PMUs needed to monitor the whole grid represented by that graph. As with other graph parameters, Nordhaus-Gaddum problems for power domination involve finding (sharp) lower and upper bounds for the sum or product of the power domination number of a graph and that of its complement in terms of the order of the graph. The focus of our research has been Nordhaus-Gaddum bounds for graphs in which both the graph and its complement are connected. Weakening the connected graph requirement somewhat, we will exhibit sharp lower and upper bounds for Nordhaus-Gaddum sums and products for power domination for trees and for graphs in which each connected component has at least three vertices. (Received September 06, 2016)

1125-05-632 **Alexander Diaz-Lopez**, **Lucas Everham** and **Pamela E Harris\***  
([peh2@williams.edu](mailto:peh2@williams.edu)), Bronfman #204, 18 Hoskey Street, Williamstown, MA 01267, and **Erik Insko**, **Vincent Marcantonio** and **Mohamed Omar**. *Peak Sets of Graphs.*

If  $G$  is a connected graph with  $n$  vertices denoted  $v_0, \dots, v_{n-1}$ , then a permutation of length  $n$  corresponds to a labeling (or  $n$ -coloring) of the vertices of  $G$ . We say that a permutation  $\pi$  has a peak at the vertex  $v_i$  on  $G$  if the label of  $v_i$  is greater than all of the labels of  $v_i$ 's neighboring vertices, with the caveat that we do not allow peaks at vertices of degree 1 or 0, as these are more like cliffs than peaks. The  $G$ -peak set of a permutation  $\pi$  is defined to be the set  $P_G(\pi) = \{i \in [n] : \pi \text{ has a peak at the vertex } v_i\}$ , where  $[n] = \{1, 2, 3, \dots, n\}$ . Given a subset  $S \subseteq V(G)$  we denote the set of all permutations with  $G$ -peak set  $S$  by  $\mathcal{P}_S(G) = \{\pi \in \mathfrak{S}_n \mid P_G(\pi) = S\}$ . We note that the peaks sets  $P_S(n)$  originally studied by Billey, Burdzy, and Sagan corresponded to studying peak sets on the path graph  $P_n$ , i.e.,  $P_S(n) = \mathcal{P}_S(G)$  where  $G = P_n$ . In this talk, we present a recursive formula for enumerating  $|\mathcal{P}_S(G)|$  and provide closed formulas for the number of permutations with a given peak set for a collection of interesting families of graphs. (Received September 08, 2016)

1125-05-648 **Satyan L Devadoss\*** ([devadoss@sandiego.edu](mailto:devadoss@sandiego.edu)) and **Mia Smith**. *Colorful Graph Associahedra*. Preliminary report.

Given a graph  $G$ , there exists a simple convex polytope called the graph associahedron whose face poset is based on the connected subgraphs of  $G$ . Motivated by ideas in algebraic topology and computational geometry, we

define the colorful graph associahedron based on an assignment of a color parameter. We show it to be a simple abstract polytope, provide its construction based on the classical permutohedron and prove various combinatorial and topological properties. (Received September 08, 2016)

1125-05-661 **Emily Gunawan\*** ([egunawan@umn.edu](mailto:egunawan@umn.edu)), 800 West College Avenue, Saint Peter, MN 56082, and **Gregg Musiker** and **Hannah Vogel**. *Cluster algebraic interpretation of infinite friezes*. Preliminary report.

Originally studied by Conway and Coxeter, friezes appeared in various recreational mathematics publications in the 1970s. We construct a periodic infinite frieze using a class of peripheral elements of a cluster algebra of type D or affine A. We discover new symmetries and formulas relating the entries of this frieze and so-called bracelets. We also present a correspondence between Broline, Crowe and Isaacs's classical matching tuples (which is a generalization of Conway and Coxeter's work) and various recent combinatorial interpretations of elements of cluster algebras from surfaces. (Received September 15, 2016)

1125-05-665 **David Anderson\*** ([anderson.2804@osu.edu](mailto:anderson.2804@osu.edu)). *Diagrams and essential sets for signed permutations*.

The essential set of a permutation, defined via its Rothe diagram by Fulton in 1992, gives a minimal list of rank conditions cutting out the corresponding Schubert variety in the flag manifold. I will describe an analogous notion for signed permutations, giving minimal conditions for Schubert varieties in flag varieties for other classical groups. This is related to a poset-theoretic construction of Reiner, Woo, and Yong, and thus gives a diagrammatic method for computing the latter. (Received September 08, 2016)

1125-05-674 **Thotsaporn Thanatipanonda\*** ([thotsaporn@gmail.com](mailto:thotsaporn@gmail.com)), Bangkok, Thailand. *On the Minimum Number of Monochromatic Generalized Schur Triples*.

In 1996, Ronald Graham asked the question about the minimum number of monochromatic triples  $(x, y, z)$  satisfies equation  $x + y = z$  of any 2-coloring of the interval  $[1, n]$ . The answer was confirmed by many people to be  $\frac{n^2}{22} + \mathcal{O}(n)$ . Recently Wong and myself showed that the minimum numbers of monochromatic triples of the form  $\{x, y, x + ay\}$ ,  $a \geq 2$  are  $\frac{n^2}{2a(a^2 + 2a + 3)} + \mathcal{O}(n)$ . We will also mention about the conjectures of other equations. (Received September 09, 2016)

1125-05-718 **James A Sellers\*** ([sellersj@psu.edu](mailto:sellersj@psu.edu)), Department of Mathematics, Penn State University, 104 McAllister Building, University Park, PA 16802. *A Combinatorial Proof of a Relationship Between Maximal  $(2k - 1, 2k + 1)$ -cores and  $(2k - 1, 2k, 2k + 1)$ -cores*.

Integer partitions which are simultaneously  $t$ -cores for distinct values of  $t$  have attracted significant interest in recent years. When  $s$  and  $t$  are relatively prime, Olsson and Stanton have determined the size of the maximal  $(s, t)$ -core  $\kappa_{s,t}$ . When  $k \geq 2$ , a conjecture of Amdeberhan on the maximal  $(2k - 1, 2k, 2k + 1)$ -core  $\kappa_{2k-1, 2k, 2k+1}$  has also recently been verified by numerous authors.

In this work, we analyze the relationship between maximal  $(2k - 1, 2k + 1)$ -cores and maximal  $(2k - 1, 2k, 2k + 1)$ -cores. In previous work, Nath noted that, for all  $k \geq 1$ ,

$$|\kappa_{2k-1, 2k+1}| = 4|\kappa_{2k-1, 2k, 2k+1}|$$

and requested a combinatorial interpretation of this unexpected identity. Here, using the theory of abaci, partition dissection, and elementary results relating triangular numbers and squares, we provide such a combinatorial proof. This is joint work with Rishi Nath. (Received September 09, 2016)

1125-05-726 **Ashvin Anand Swaminathan\*** ([aaswaminathan@college.harvard.edu](mailto:aaswaminathan@college.harvard.edu)), 388 Eliot Mail Center, 101 Dunster Street, Cambridge, MA 02138, and **Noam D Elkies** ([elkies@math.harvard.edu](mailto:elkies@math.harvard.edu)), 1 Oxford Street, Cambridge, MA 02138. *Permutations that Destroy Arithmetic Progressions in Elementary  $p$ -groups*.

Given an abelian group  $G$ , it is natural to ask whether there exists a permutation  $\pi$  of  $G$  that “destroys” all nontrivial 3-term arithmetic progressions (APs), in the sense that  $\pi(b) - \pi(a) \neq \pi(c) - \pi(b)$  for every ordered triple  $(a, b, c) \in G^3$  satisfying  $b - a = c - b \neq 0$ . This question was resolved for infinite groups  $G$  by Hegarty, who showed that there exists an AP-destroying permutation of  $G$  if and only if  $G/\Omega_2(G)$  has the same cardinality as  $G$ , where  $\Omega_2(G)$  denotes the subgroup of all elements in  $G$  whose order divides 2. In the case when  $G$  is finite, however, only partial results have been obtained thus far. Hegarty has conjectured that an AP-destroying permutation of  $G$  exists if  $G = \mathbb{Z}/n\mathbb{Z}$  for all  $n \neq 2, 3, 5, 7$ , and together with Martinsson, he has proven the conjecture for all  $n > 1.4 \times 10^{14}$ . In this paper, we show that if  $p$  is a prime and  $k$  is a positive integer, then there is an AP-destroying permutation of the elementary  $p$ -group  $(\mathbb{Z}/p\mathbb{Z})^k$  if and only if  $p$  is odd and  $(p, k) \notin \{(3, 1), (5, 1), (7, 1)\}$ . (Received September 09, 2016)

1125-05-742 **Vincent Coll, Jonelle Hook, Colton Magnant, Karen McCreedy\***  
(karenmccready@kings.edu) and **Kathleen Ryan**. *The Proper Diameter of a Graph*.  
Preliminary report.

In fiber-optic communication, where carrier waves for each incoming and outgoing signal to a node must be operating at a different frequency in order to avoid interference, finding a path over which a signal can travel without interference can be thought of as a problem of edge-colored graphs. An edge-colored graph is properly connected if there exists a properly colored path between every pair of vertices in the graph, where a path is properly colored if consecutive edges have distinct colors. In such a graph, we define the proper diameter of the graph to be the maximum length of a shortest properly colored path between any two vertices in the graph. We consider various families of graphs to compare the diameter of the graph to possible proper diameters for 2-colorings. (Received September 10, 2016)

1125-05-744 **Adam Berliner, Nate Dean, Jonelle Hook, Alison Marr, aBa Mbirika\***  
(mbirika@uwec.edu) and **Cayla McBee**. *Coprime and prime labelings of ladder graphs  
and complete bipartite graphs*.

A coprime labeling of a simple graph of order  $n$  is a labeling in which adjacent vertices are given relatively prime labels, and a graph is prime if the labels used can be taken to be the first  $n$  positive integers. In this talk, we consider when ladder graphs are prime and when the corresponding labeling may be done in a cyclic manner around the vertices of the ladder. Furthermore, we discuss coprime labelings for complete bipartite graphs. This is joint work begun at REUF 4 in 2012 with collaborators Adam Berliner, Nate Dean, Jonelle Hook, Alison Marr, and Cayla McBee. (Received September 10, 2016)

1125-05-745 **aBa Mbirika\*** (mbirika@uwec.edu). *Lattice point visibility on straight and curved lines of sight*.

An integer lattice point  $(x, y)$  is visible from the origin along a straight line of sight if and only if the greatest common divisor of  $x$  and  $y$  equals one—equivalently, if  $x$  and  $y$  are relatively prime. It is known that approximately 40 percent of the integer lattice is hidden from view from the origin along these straight lines of sight. We give an elementary number theory proof of this fact. Remarkably, arbitrarily large  $n \times n$  squares of hidden points can be found in this 40 percent of the integer lattice. We use the Chinese Remainder Theorem to give a proof of this fact. A natural question to ask is what changes if we have curved lines of sights from the origin? In particular, we focus our attention on lines of sight given by power functions of the form  $f(x) = ax^b$  where  $a \in \mathbb{Q}$  and  $b \in \mathbb{N}$ . This is joint work with my undergraduate research students at the University of Wisconsin-Eau Claire. (Received September 10, 2016)

1125-05-791 **Shyam S Narayanan\*** (shyam.s.narayanan@gmail.com), 8209 W 143rd Ter, Overland Park, KS 66223, and **Evan Chen**. *The 26 Wilf-equivalence classes of length five quasi-consecutive patterns*.

We present two families of Wilf-equivalences for consecutive and quasi-consecutive vincular patterns. These give new proofs of the classification of consecutive patterns of length 4 and 5. We then prove additional equivalences to explicitly classify all quasi-consecutive patterns of length 5 into 26 Wilf-equivalence classes. (Received September 11, 2016)

1125-05-792 **Adam T O'Neal\*** (oneala@marshall.edu), Huntington, WV, and **Michael Schroeder**, WV. *On sets of cardinality 2 of nondecreasing diameter*.

Our problem comes from Ramsey theory. For positive integers  $m, r$ , and  $t$ , we say that a coloring of  $n$  integers,  $[n]$ , with  $r$  colors is  $(m, r, t)$ -permissible if there exist  $t$  monochromatic subsets  $B_1, B_2, \dots, B_t$  such that

- (a)  $|B_1| = |B_2| = \dots = |B_t| = m$ ,
- (b)  $\max(B_i) < \min(B_{i+1})$  for  $1 \leq i \leq t - 1$ , and
- (c)  $\max(B_{i+1}) - \min(B_{i+1}) \geq \max(B_i) - \min(B_i)$  for  $1 \leq i \leq t - 1$ ;  
that is, the diameters of the subsets are nondecreasing.

Let  $f(m, r, t)$  be the smallest integer  $n$  such that any coloring of  $[n]$  is  $(m, r, t)$ -permissible. In this presentation, we fix  $m = r = 2$  and show that  $5t - 5 < f(2, 2, t) \leq 5t - 2$ . We conjecture that  $f(2, 2, t) = 5t - 4$  and prove the conjecture in certain cases. We conclude by investigating colorings with more than two colors. (Received September 15, 2016)

1125-05-815 **Shih-Wei Chao\*** (shih-wei.chao@ung.edu), **Neil Calkin**, **Catherine Davison**, **Krista Kelly** and **Ana Andreea Stoica**. *Counting kings and a connection with Catalan numbers*. Preliminary report.

Let  $f_{m,n}$  be the number of placements of non-attacking kings on a  $m \times n$  chessboard. We will analyze the generating function  $f_{m,n}(x)$  using formal power series to (1) find the connection between Catalan numbers and Fibonacci numbers and (2) give the formula for  $[x^k]f_{m,n}(x)$  for certain  $k$ ,  $m$ , and  $n$ . We will finish the talk by introducing some open questions including but not limited to matrices and 3-D chessboard. (Received September 12, 2016)

1125-05-816 **Dustin Hedmark\***, dustin.hedmark@uky.edu. *Homology of Filters in the Partition Lattice*.

Starting with the computation of the Möbius function of the even partition lattice by Sylvester in 1976, there has been much interest in understanding the topology and representation theory of filters in the partition lattice. In this talk I will speak on current work with Richard Ehrenborg where we compute the homology groups, as well as the  $\mathfrak{S}_{n-1}$  action on these homology groups, for arbitrary filters in the partition lattice  $\Pi_n$  using Mayer Vietoris Sequences. We will spend most of our time looking at examples of computations of homologies in the partition lattice, notably a derivation of Wach's well known results on the  $d$ -divisible partition lattice (Received September 12, 2016)

1125-05-818 **Liz Lane-Harvard\***, elaneharvard@uco.edu. *Strongly Regular Graphs from Arcs in Non-Desarguesian Planes*. Preliminary report.

Utilizing ovals in Desarguesian projective planes, Tits, in 1968, was able to construct generalized quadrangles. In 1969 and 1971, utilizing hyperovals in Desarguesian projective planes, Ahrens-Szekeres and Hall were able to construct generalized quadrangles. Similarly, in 1985, Payne constructed generalized quadrangles from  $q$ -arcs in Desarguesian projective planes. The concurrency graph of a generalized quadrangle is a strongly regular graph; hence, the above generalized quadrangles are associated with strongly regular graphs. We have removed the hypothesis that the plane must be Desarguesian, and we construct strongly regular graphs with the same parameters as the concurrency graphs of the generalized quadrangles arising from ovals, hyperovals, and  $q$ -arcs. (Received September 12, 2016)

1125-05-839 **S-Y Choi** and **Jonathan Needleman\*** (needlejs@lemoyne.edu). *Central-Peripheral Appendage Numbers of Uniform Central Graphs*.

Uniform central graphs (UCGs) were introduced in the late 1990s. They are graphs in which the eccentric vertices of the central vertices are independent of the central vertex chosen. These eccentric vertices then form a "centered" periphery.

Appendage numbers of a graph  $G$  are the fewest number of vertices that need to be appended to  $G$  in order to create a new supergraph  $H$  with a specified property.

In this talk we will describe appendage numbers for any pairs of graphs  $C$  and  $P$  where the supergraph is a UCG with center  $C$  and centered periphery  $P$ . In doing so, we show the appendage numbers are much more dependent on the structure of  $P$  than of  $C$ . This extends the work of Gu who found appendage numbers for centers  $C$ , independent of choice of  $P$ . (Received September 12, 2016)

1125-05-843 **Hao Huang** and **Yi Zhao\*** (yzhao6@gsu.edu). *Degree version of the Erdős-Ko-Rado Theorem*.

We use an algebraic method to prove a degree version of the celebrated Erdős-Ko-Rado theorem: given  $n > 2k$ , every intersecting  $k$ -uniform hypergraph  $H$  on  $n$  vertices contains a vertex that lies on at most  $\binom{n-2}{k-2}$  edges. (Received September 12, 2016)

1125-05-846 **Joanna A. Ellis-Monaghan\*** (jellis-monaghan@smcvt.edu), Saint Michael's College, One Winooski Park, Colchester, VT 05439. *New Problems in Graph Theory from Self-Assembly*.

Newly emerging technologies in self-assembly, both at the nanoscale with for example DNA origami, and the macroscale with for example robotic assembly, are now generating fascinating and challenging new design problems for which graph theory is a natural tool. Because this is a new area, many aspects require 'first principle' approaches—hard, creative thinking and serious problem formulation work, but a fairly manageable ramp-up—and hence it is particularly suitable for undergraduate involvement. Furthermore, the work is highly interdisciplinary, with research teams frequently including professional collaborators and students from mathematics, computer science, physics, biology, and even English and art.

We will present some of these new applications in self-assembly and describe some of the graph-theoretical design strategy problems arising from them. We will describe how we achieve substantive and pragmatically useful results with students who may join the group with very little mathematical background. We will also share our experience in identifying, establishing, and funding such a project, and how it may be used for recruitment of mathematics majors and minors. (Received September 12, 2016)

1125-05-854 **Nina V Zubrilina\*** ([nina57@stanford.edu](mailto:nina57@stanford.edu)). *Dimension and edge dimension: random graphs and counterexamples.*

Let  $G(V, E)$  be a connected simple undirected graph. The distance between an edge  $e = v_1v_2$  and a vertex  $v$  is defined as  $d(e, v) = \min\{d(v_1, v), d(v_2, v)\}$ . A set  $S \subset V$  generates  $E$  if for any  $e_1 \neq e_2 \in E$  there exists  $s \in S$  such that  $d(e_1, s) \neq d(e_2, s)$ . The cardinality of the smallest generating set of  $E$  is called the edge metric dimension of  $G$  and denoted  $edim(G)$ . We investigate various properties of  $edim(G)$ . We determine  $edim$  of the random graph  $G(n, p)$  for constant  $p \in (0, 1)$ . We also classify the graphs for which  $edim(G) = n - 1$  and show that  $\frac{edim(G)}{dim(G)}$  isn't bounded from above (here  $dim(G)$  is the standard metric dimension of  $G$ ). Lastly, we compute  $edim(G \square P_n)$  and  $edim(G + K_1)$ . (Received September 12, 2016)

1125-05-896 **Alireza Abdollahi, Russ Woodroffe\*** ([rwoodroffe@math.msstate.edu](mailto:rwoodroffe@math.msstate.edu)) and **Gjergji Zaimi**. *The Frankl Conjecture for subgroup lattices.*

A conjecture attributed to Frankl (from 1979) asks whether every nontrivial finite lattice  $L$  has an join-irreducible element  $x$  such that at most half the elements of  $L$  lie above  $x$ . In joint work with Alireza Abdollahi and Gjergji Zaimi, we have verified this conjecture for all subgroup lattices of finite groups, as well as a broad family of additional lattices. (Received September 13, 2016)

1125-05-908 **Chinenye Ofodile\*** ([chinenye.ofodile@asurams.edu](mailto:chinenye.ofodile@asurams.edu)), Albany State University, 504 College Dr., Albany, GA 31705. *The Enumeration of 132-Pattern Occurrences in Dumont Permutations of Length Four.*

In this undergraduate research project, we enumerate Dumont permutations of length four containing pattern 132. Patterns are order-isomorphism classes of permutations (or, more generally, of strings over a totally ordered alphabet). Strings are order-isomorphic if and only if pairs of elements in the same positions satisfy the same pairwise comparisons. Dumont permutations are classes of permutations that satisfy certain restrictions based on parity of positions or values of elements. This work parallels the work of Burstein, Elizalde and Mansour, who enumerated Dumont permutations that avoided certain three or four letter patterns, and the work of Noonan and Zeilberger, who enumerated all permutations with one or two occurrences of some patterns. (Received September 13, 2016)

1125-05-920 **Jessica Fuller\*** ([jessica.fuller@emory.edu](mailto:jessica.fuller@emory.edu)) and **Ronald J. Gould**. *Saturation and Constructing  $(K_t - e)$ -saturated graphs.* Preliminary report.

Given a graph  $H$ , we say a graph  $G$  is  $H$ -saturated if  $G$  does not contain  $H$  as a subgraph and the addition of any edge  $e' \notin E(G)$  results in  $H$  as a subgraph. The question of the minimum number of edges of an  $H$ -saturated graph on  $n$  vertices, known as the *saturation number*, and the question of the maximum number of edges possible of an  $H$ -saturated graph, known as the Turán number, has been addressed for many different types of graphs. We are interested in the existence of  $H$ -saturated graphs for each edge count between the saturation number and the Turán number. We prove that  $(K_4 - e)$ -saturated graphs do not exist for small values of  $|E(G)|$  and construct  $(K_4 - e)$ -saturated graphs with  $|E(G)|$  in the interval  $[2n - 4, \lfloor \frac{n}{2} \rfloor \lceil \frac{n}{2} \rceil - n + 6]$ . We then extend the  $(K_4 - e)$ -saturated graphs to  $(K_t - e)$ -saturated graphs. (Received September 13, 2016)

1125-05-955 **Cayla D. McBee\*** ([cmcbree@providence.edu](mailto:cmcbree@providence.edu)). *Prime Labelings of Hypercube Graphs.* Preliminary report.

A simple graph on  $n$  vertices has a prime labeling if and only if there exists a labeling of the vertices by the numbers  $1, 2, 3, \dots, n$  such that any two adjacent vertices have labels that are relatively prime. There are many problems related to prime labeling that are well suited for collaborations with undergraduate students. This talk looks at the problem of determining the values of  $k$  for which the hypercube graph,  $Q_k$ , is prime and presents some recent work with students. (Received September 13, 2016)

1125-05-971 **Ali Kemal Uncu\*** ([akuncu@uf1.edu](mailto:akuncu@uf1.edu)), 704 SW 16th Ave, 306, Gainesville, FL 32601. *Weighted Partition Results Inspired by Nathan Fine's False Theta Identities.*

We utilize results of Nathan Fine to discover new partition identities involving weights. These relations connect Göllnitz–Gordon type partitions and partitions with distinct odd parts, partitions into distinct parts and ordinary partitions, respectively. Some of these weights involve new partition statistics. One example of such statistics

is the number of different odd parts of a partition larger than or equal to a given value. We later will mention another weighted partition identity in the same spirit emerging from Ramanujan's work as the time permits. (Received September 13, 2016)

1125-05-1007 **Katsuhiko Ota\*** (ohta@math.keio.ac.jp), Department of Mathematics, Keio University, 3-14-1, Hiyoshi, Kohoku-ku, Yokohama, 223-8522, Japan. *Small theta subgraphs in sparse graphs.*

A *theta graph* is a graph consisting of three internally disjoint paths with common end vertices. By considering a BFS tree in a graph, it is not difficult to prove that if  $G$  is a graph of order  $n$  with minimum degree 3, then  $G$  contains a theta subgraph of order at most  $6 \log_2 n$ . Note that the minimum degree condition is sharp, and there exists a graph of order  $n$  with average degree 3 which does not contain a theta subgraph of order  $o(n)$ .

In this talk, we consider slightly weaker conditions, which ensure the existence of small theta subgraphs.

(1) Let  $\alpha > 0$  and let  $G$  be a graph of order  $n$  with average degree at least  $3 + \alpha$ . Then,  $G$  contains a theta subgraph of order at most  $(\frac{9}{\alpha} + 3) \log_2 n$ .

(2) Let  $\beta > 0$  and let  $G$  be a graph of order  $n$  without isolated vertices. For  $d \in \{1, 2\}$ , let  $n_d$  denote the number of vertices of degree  $d$  in  $G$ . If  $4n_1 + 3n_2 \leq (1 - \beta)n$ , then  $G$  contains a theta subgraph of order at most  $(\frac{6}{\beta} + 1)(6 \log_2 n + 1)$ .

These results enable us to prove that every large enough graph with minimum degree at least  $2k + 1$  contains  $k$  vertex-disjoint isomorphic theta subgraphs.

This is a joint work with Y. Egawa, S. Fujita and T. Sakuma. (Received September 14, 2016)

1125-05-1017 **Darren A Narayan\***, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623. *Puzzle type undergraduate projects in combinatorics and graph theory.*

We present a series of puzzle type research problems from graph theory and combinatorics that are ideal for undergraduate research. The problems are variants of successful undergraduate research projects. The problems require little background and are suitable for students who have taken a course in discrete mathematics. (Received September 14, 2016)

1125-05-1019 **Hoang Dau**, CSL, Urbana, IL 61801, and **Olgica Milenkovic\*** (milenkov@illinois.edu), Urbana, IL 61801. *Boolean Intersection Graphs.*

We propose a new latent Boolean feature model for complex networks that captures different types of node interactions and network communities. The model is based on a new concept in graph theory, termed the Boolean intersection representation of a graph, which generalizes the notion of an intersection representation. We mostly focus on one form of Boolean intersection, termed cointersection, and describe how to use this representation to deduce node feature sets and their communities. We derive several general bounds on the minimum number of features used in cointersection representations and discuss graph families for which exact cointersection characterizations are possible. Our results also include algorithms for finding optimal and approximate cointersection representations of a graph. (Received September 14, 2016)

1125-05-1029 **Yezhou Wu**, Zhejiang University, Hangzhou, Zhejiang 310027, Peoples Rep of China, and **Dong Ye\*** (dong.ye@mtsu.edu), Department of Mathematical Sciences, Middle Tennessee State University, Murfreesboro, TN 37132. *Circuit Covers of Cubic Signed Graphs.*

A signed graph is a graph  $G$  associated with a mapping  $\sigma : E(G) \rightarrow \{-1, +1\}$ , denoted by  $(G, \sigma)$ . A cycle of  $(G, \sigma)$  is a connected 2-regular subgraph. A cycle  $C$  is *positive* if it has an even number of negative edges, and negative otherwise. A *circuit* of a signed graph  $(G, \sigma)$  is a positive cycle or a barbell consisting of two edge-disjoint negative cycles joined by a path. A circuit cover of  $(G, \sigma)$  is a family of circuits covering all edges of  $(G, \sigma)$ . A shortest circuit cover of  $(G, \sigma)$  is a circuit cover with the shortest length which is denoted by  $\text{scc}(G, \sigma)$ . Bouchet proved that a signed graph with a circuit cover if and only if it is flow-admissible (i.e., has a nowhere-zero integer flow). We show that every 2-edge-connected cubic signed graph has  $\text{scc}(G, \sigma) \leq 26|E(G)|/9$  if it is flow-admissible. (Received September 14, 2016)

1125-05-1042 **Luis Montejano\*** (luismontej@gmail), Av. San Isidro No 303, Juriquilla, Queretaro., 76230 Queretaro, Mexico. *Rotors in triangles and tetrahedra.* Preliminary report.

*Rotors in triangles and tetrahedra.* Abstract We say that a convex body  $K$  in euclidean  $n$ -space is a rotor of a polytope  $P$  if for each rigid movement  $R$  there exist a translation  $t$  so that  $P$  is circumscribed about  $t(R(K))$ .

It is well known that if  $K$  is a convex plane figure which is a rotor in the polygon  $P$ , then every support line of  $K$  intersects its boundary in exactly one point, and if  $K$  intersect each side of  $P$  at the points  $A_1, \dots, A_n$ , then the normals of  $K$  at these points are concurrent.

In this paper we shall prove that if  $P$  is a triangle, then there is a barycentric formula that describe the curvature of the boundary  $K$  at the points  $A_1, A_2, A_3$ . We prove also that if  $K$  is a three dimensional convex body which is a rotor in a tetrahedron  $T$ , and if  $K$  intersect each face of  $T$  at the points  $x_1, x_2, x_3, x_4$ , then the normals lines of  $K$  at  $x_1, x_2, x_3, x_4$  generically belong to a one ruling of a quadric surface. (Received September 14, 2016)

1125-05-1052 **Steven Klee\***, Seattle University Department of Mathematics, 901 12th Avenue, Seattle, WA 98122. *Prime labelings of graphs with the Gaussian integers.*

A classic conjecture in graph theory asks whether it is always possible to label the vertices of a tree on  $n$  vertices using the numbers  $1, 2, \dots, n$  so that the labels on adjacent vertices are relatively prime. During the summer of 2015, some of my REU students studied an extension of this problem to the Gaussian integers, which are the complex numbers whose real and imaginary parts are both integers. In this talk, I will outline what we know, what we would like to know, and some interesting complications that arise when extending this problem to the Gaussian integers. (Received September 14, 2016)

1125-05-1066 **Benjamin J. Hamlin\*** (bh7394@stu.armstrong.edu), **Joshua K. Lambert** and **Mark R. Budden**. *Enumeration of Triangles in a  $2^t$ th Residue Graph.*

For a fixed prime  $p \equiv 1 \pmod{16}$ , we shall count the number of triangles in a 16th residue graph. The aforementioned number requires the enumeration of consecutive 16th residues modulo  $p$ . We focus our efforts to enumerating the triangles in the  $2^t$ th residue graphs. (Received September 14, 2016)

1125-05-1073 **Matthew Yancey\*** (mpyance@super.org). *Hyperbolicity and Congestion.*

A promising new direction of network analysis is to understand when a network is hyperbolic (as defined by Gromov), and what such a label would imply. We examine the proposed implication that when a transportation network is hyperbolic, with few exceptions, it will experience congestion. In this scenario, *congestion* in a probabilistic model for a network means that there exists a vertex  $w$  and an  $\epsilon > 0$  such that for a randomly uniformly chosen pair of vertices  $a, b$  and a random uniformly chosen geodesic  $P$  from  $a$  to  $b$ , we have that  $w \in P$  with probability at least  $\epsilon$ . While several examples of exceptional networks are known, there is no understanding of what the exceptions are. We will present several theorems whose individual conclusions are that the network has congestion, and whose assumptions are that the network is hyperbolic, along with modest additional assumptions. (Received September 14, 2016)

1125-05-1080 **Steve Kirkland\*** (stephen.kirkland@umanitoba.ca). *On the Characteristic Set, Centroid, and Centre for a Tree.* Preliminary report.

Let  $T$  be a tree on vertices  $1, \dots, n$ , and let  $L$  be the corresponding Laplacian matrix. The algebraic connectivity of  $T$  is the smallest positive eigenvalue of  $L$ ; a corresponding eigenvector is known as a Fiedler vector. Given a Fiedler vector  $v$ , either a) there is a unique vertex  $i$  such that  $v_i = 0$  and  $i$  is adjacent to a vertex  $j$  with  $v_j$  nonzero, or b) there is a unique pair of adjacent vertices  $k, l$  such that  $v_k v_l < 0$ . The characteristic set for  $T$  is  $\{i\}$  in case a), and is  $\{k, l\}$  in case b). The characteristic set can be viewed as a ‘middle’ of the tree.

A tree has both a unique centroid and a unique centre, and each can also be viewed as a ‘middle’ of the tree. In view of that observation, it is natural to wonder how far the characteristic set for a tree can be from its centroid and centre, respectively. In this talk we identify families of trees that maximise the distance from the characteristic set to the centroid and centre, respectively. We also determine the asymptotics for the maximum distance (taken over all trees on  $n$  vertices) between the characteristic set and the centroid, and between the characteristic set and the centre. (Received September 14, 2016)

1125-05-1101 **Emily Meehan\*** (emeehan@ncsu.edu). *Baxter posets.*

In this talk, we define a family of combinatorial objects, which we call Baxter posets. We prove that the Baxter numbers count Baxter posets by demonstrating that Baxter posets are the adjacency posets of diagonal rectangulations. Several known families of Baxter objects are closely related to Catalan combinatorics, and we motivate the definition of Baxter posets by summarizing some of these relationships. Given a diagonal rectangulation, we will describe the cover relations in the associated Baxter poset. If time allows, we will also describe a method for obtaining the Baxter permutation associated to a Baxter poset. (Received September 14, 2016)

1125-05-1113 **Katherine Perry\*** ([kep0024@auburn.edu](mailto:kep0024@auburn.edu)), 221 Parker Hall, Auburn, AL 36849, and **Chris Rodger, Hung-Lin Fu** and **Yuan-Hsun Lo**. *Rainbow Spanning Trees in Edge-Colored Complete Graphs*.

A spanning tree of a properly edge-colored complete graph,  $K_n$ , is rainbow provided that each of its edges receives a distinct color. In 1996, Brualdi and Hollingsworth conjectured that if  $K_{2m}$  is properly  $(2m - 1)$ -edge-colored, then the edges of  $K_{2m}$  can be partitioned into  $m$  rainbow spanning trees except when  $m = 2$ . The existence of  $\lfloor m/(500 \log(2m)) \rfloor$  mutually edge-disjoint rainbow spanning trees in the case where  $m \geq 500,000$  was recently proved using probabilistic techniques. By means of an explicit, constructive approach, we construct  $\lfloor \sqrt{6m + 9}/3 \rfloor$  mutually edge-disjoint rainbow spanning trees for any positive value of  $m$ . Not only are the rainbow trees produced, but also some structure of each rainbow spanning tree is determined in the process. This improves upon best constructive result to date in the literature which produces exactly three rainbow trees. It also improves upon the probabilistic result for all  $m$ , at most  $5.7 \times 10^7$ .

Keywords: edge-coloring, complete graph, rainbow spanning tree (Received September 14, 2016)

1125-05-1117 **Adam Giambrone\*** ([adam.giambrone@uconn.edu](mailto:adam.giambrone@uconn.edu)). *Sigma-Adequate Link Diagrams and the Tutte Polynomial*. Preliminary report.

A well-known bijection between checkerboard-colored link diagrams  $D$  and edge-signed planar graphs  $G$  (called *Tait graphs*) has led to a number of connections between the fields of knot theory and graph theory. As an example, Thistlethwaite used this bijection to characterize A-adequate link diagrams  $D$  both in terms of the edge-restricted and edge-contracted Tait graphs  $G|E_+$  and  $G/E_+$ , respectively, and in terms of the nonvanishing behavior of a product  $\chi_{G|E_+}(0, t) \cdot \chi_{G/E_+}(t, 0)$  of two Tutte polynomials. In this talk, we extend Thistlethwaite's results to the wider class of sigma-adequate link diagrams. Furthermore, we show that the Tutte polynomial  $\chi_G(t, t)$  of the Tait graph  $G$  can be written as a sum of products of two Tutte polynomials, where the sum is over the sigma-adequate states of  $D$ . Using this state sum, we show that the number of sigma-adequate states of  $D$  is bounded above by the number of spanning trees in  $G$  and we give a method to find all of the sigma-adequate states of  $D$ . (Received September 14, 2016)

1125-05-1131 **Alberto Ravagnani\***, [alberto.ravagnani@unine.ch](mailto:alberto.ravagnani@unine.ch). *Equidistant Subspace Codes*.

A subspace code is a collection of vector spaces of the same dimension over a finite field, with the property that each two spaces intersect in low dimension. Subspace codes were introduced in 2008 by Koetter and Kschischang for error correction in random linear network coding.

Equidistant codes are subspace codes in which each two spaces intersect the same dimension. They were proposed by Etzion and Raviv for distributed storage applications.

We provide an almost complete classification of equidistant codes of large cardinality, showing that for most parameters they are either sunflowers, or the orthogonals of sunflowers. As an application, we prove that optimal equidistant codes have a very simple structure for most choices of the parameters.

We then show how to construct equidistant codes of asymptotically optimal cardinality for all parameters, and how to decode them efficiently.

The new results in the talk are joint work with Elisa Gorla. (Received September 15, 2016)

1125-05-1142 **George E. Andrews\*** ([gea1@psu.edu](mailto:gea1@psu.edu)), 306 McAllister Bldg., Mathematics Department, Pennsylvania State University, University Park, PA 16802. *4-Shadows in q-Series, the Kimberling Index, and Garden of Eden Partitions*.

This talk is devoted to discussing the implications of a very elementary technique for proving mod 4 congruences in the theory of partitions. It starts with a tribute to the late Hans Raj Gupta and leads in unexpected ways to partitions investigated by Clark Kimberling, to Bulgarian Solitaire, and to Garden of Eden partitions. (Received September 15, 2016)

1125-05-1144 **James J Madden**, Baton Rouge, LA, and **Trevor McGuire\*** ([tmcguire@iwu.edu](mailto:tmcguire@iwu.edu)), Bloomington, IL. *Infinite Neighbor Complexes*.

Scarf introduced the idea of the complex of neighbors in the context of integer programming in 1981. This work was given an algebraic application by Bayer, Peeva and Sturmfels in the late 1990s; that application was a remarkable connection between the neighbor complex and free resolutions of monomial ideals in  $k[x_1, \dots, x_n]$ . This work was later extended by Bayer and Sturmfels to include a similar technique for resolving lattice ideals.

Underpinning both of these applications was the requirement that the set of neighbors be *generic*. However, if the set was not generic, in both applications, there is a technique of *deformations*, in which the ideal in question is replaced with an ideal in the group algebra  $k[\mathbb{R}^n]$ . This application shows the usefulness of considering subsets of  $\mathbb{R}^n$  as a base set for Scarf's construction.

In this presentation, we will look at the original tools that Scarf developed for subsets of  $\mathbb{N}^n$ , and apply them to subsets of  $\mathbb{R}^n$ . The main result is to provide the technicalities and special considerations needed to use infinite neighbor complexes to index resolutions. (Received September 15, 2016)

1125-05-1150 **Matt Baker\*** ([mbaker@math.gatech.edu](mailto:mbaker@math.gatech.edu)). *Hodge Theory in Combinatorics*.

In 2015, Karim Adiprasito, June Huh, and Eric Katz announced a proof of a 50-year old conjecture of Heron, Rota, and Welsh asserting that the coefficients of the characteristic polynomial of a matroid form a log-concave sequence. The proof of this conjecture establishes far more: the authors prove combinatorial analogues of Poincare duality, the hard Lefschetz theorem, and the Hodge-Riemann relations for the Chow ring of an arbitrary matroid. This work opens up new horizons in both algebraic geometry and combinatorics, applying deep algebro-geometric intuition to a combinatorial problem with no direct link to algebraic geometry. (Received September 15, 2016)

1125-05-1196 **Emily Marshall\*** ([emarshall@lsu.edu](mailto:emarshall@lsu.edu)) and **Guoli Ding**. *Minors in 3-connected planar non-Hamiltonian graphs*. Preliminary report.

Hamilton cycles in graphs can be hard to find, and there is a large body of research that outlines sufficient conditions for the existence of such a cycle. For graphs on the plane, Whitney proved that all 4-connected triangulations are Hamiltonian and this result was later strengthened by Tutte to all 4-connected planar graphs. This talk approaches the problem from the other direction by looking instead at non-Hamiltonian graphs. It is known that not all 3-connected planar graphs are Hamiltonian and the Herschel graph is the smallest example. We focus on 3-connected planar non-Hamiltonian graphs and report on progress towards proving the existence of a Herschel minor in all such graphs with an initial restriction to triangulations. (Received September 15, 2016)

1125-05-1212 **Sergi Elizalde** and **Peter R. W. McNamara\*** ([peter.mcnamara@bucknell.edu](mailto:peter.mcnamara@bucknell.edu)). *The structure of the consecutive pattern poset*.

The consecutive pattern poset is the infinite partially ordered set of all permutations, where  $\sigma \leq \tau$  if  $\tau$  has a subsequence of adjacent entries in the same relative order as the entries of  $\sigma$ . We study the structure of the intervals in this poset from topological, poset-theoretic, and enumerative perspectives. Among other results, we classify the intervals of the following types: disconnected; shellable; rank-unimodal; strongly Sperner. (Received September 15, 2016)

1125-05-1245 **Mark Ellingham\*** ([mark.ellingham@vanderbilt.edu](mailto:mark.ellingham@vanderbilt.edu)). *A combinatorial condition for nets of triangular polyhedra*.

A very old and still-unsolved question asks whether a convex polyhedron can be cut along a subset of its edges and then unfolded into a non-self-overlapping planar shape, which is called a *net* of the polyhedron. The net can be regarded as an outerplanar graph, and such a graph must have certain properties. In particular, if the outer cycle of the net has  $2n$  vertices, every vertex of the net must have degree at most  $n + 1$ . We show that in the case of maximal outerplanar graphs this necessary condition is also sufficient in a combinatorial sense. In other words, any maximal outerplanar graph with outer cycle of length  $2n$  and with every vertex of degree at most  $n + 1$  can have its outer cycle glued together by identifying pairs of edges so that the result is a polyhedral (3-connected and planar) graph.

This is joint work with too many people to name individually, but including Theo Douvropoulos, David Richter and a large group from the 2013 BIRS Workshop on Topological and Geometric Graph Theory. (Received September 15, 2016)

1125-05-1299 **Liam Solus\*** ([liam.solus@gmail.com](mailto:liam.solus@gmail.com)), KTH Royal Institute of Technology. *Combinatorial optimization in Markov equivalence of DAG models*.

A directed acyclic graphical model is a family of multivariate probability distributions that satisfy a set of conditional independence relations encoded in a directed acyclic graph (DAG). Multiple DAGs can encode the same set of conditional independence relations, and such DAGs are called Markov equivalent. From the perspective of complexity of model selection it is desirable to understand the number and size of these equivalence classes. We will see how this important question from model selection can be recast into the language of combinatorial optimization. In doing so, a pair of polynomial generating functions arise to describe the problem. We study this important statistical question by studying the complexity of computing the associated combinatorial statistics encoded by these polynomials. This is joint work with Adityanarayanan Radhakrishnan (MIT) and Caroline Uhler (MIT). (Received September 16, 2016)

1125-05-1300 **Liam Solus\*** ([liam.solus@gmail.com](mailto:liam.solus@gmail.com)), KTH Royal Institute of Technology. *On consistency guarantees for DAG model selection.*

A directed acyclic graphical model is a multivariate statistical model in which the nodes of a directed acyclic graph (DAG) represent random variables and the edges encode a set of CI relations. Given a set of observed CI relations  $\mathcal{C}$ , an important goal in statistics is to identify a DAG model that best encodes the relations  $\mathcal{C}$  with respect to some scoring criterion. In recent work, Mohammadi, Uhler, Wang, and Yu show that if  $\mathcal{C}$  is a DAG gaussoid then it can be represented by a polytope called a DAG associahedron. The authors then proposed a greedy algorithm for DAG model selection that walks along the edges of the DAG associahedron. In this talk, we will discuss consistency guarantees for this algorithm, considering well-studied assumptions such as faithfulness, adjacency faithfulness, and the SMR assumption. We will see that the algorithm is provably consistent under the faithfulness assumption by relating it to the popular Greedy Equivalence Search algorithm (GES). We find that each edge traversal of the DAG associahedron encodes multiple iterations of Chickering's Apply-Edge Operation algorithm, the underlying mechanism in GES. This talk is based on joint work with Caroline Uhler (MIT) and Lenka Matejovicova (IST Austria). (Received September 20, 2016)

1125-05-1318 **Elizabeth Drellich\***, [edrelli1@swarthmore.edu](mailto:edrelli1@swarthmore.edu). *GKM theory for non-GKM spaces.*

For certain families of algebraic varieties, including flag varieties, GKM theory provides tools to calculate the torus equivariant cohomology explicitly. These results descend to the ordinary cohomology and are a powerful tool in Schubert calculus. Recently some of the power of this tool has been used to give explicit Schubert calculus-style combinatorial formulas for spaces that are not GKM, like  $p$ -compact spaces, Hessenberg varieties, and in particular the Peterson variety. This talk will present some of those spaces and show how aspects of GKM theory can be used on a larger collection of spaces. (Received September 16, 2016)

1125-05-1353 **Nathan Bowler, Johannes Carmesin, Shadisadat Ghaderi\*** ([shghaderi@mix.wvu.edu](mailto:shghaderi@mix.wvu.edu)) and **Jerzy Wojciechowski**. *The Almost Intersection Property for Pairs of Matroids on Common Groundset.*

The theory of finite matroids was introduced by Whitney to capture and generalize the concept of linear independence in vector spaces. This theory was later generalized to infinite sets in a series of papers and recently it attracts a substantial amount of attention. The most important open problem is the Infinite Matroid Intersection conjecture proposed in 1990 by Nash-Williams. It says that each pair of matroids on the same ground set has the Intersection Property and is a suitable restatement in the infinite case of the well-known finite matroid intersection theorem of Edmonds. Some progress was made towards proving this conjecture, but it is still open even for finitary matroids on a countable ground set. We introduce the Almost Intersection Property, which is in essence, the Intersection Property modulo finite sets. We prove that the Almost Intersection Property implies the Intersection Property and present some corollaries of that result. (Received September 16, 2016)

1125-05-1365 **Ryan N Alweiss\*** ([ryeguy10@gmail.com](mailto:ryeguy10@gmail.com)), 3 Ames Street, Cambridge, MA 02142. *Ramsey Numbers of Odd Cycles Versus Larger Even Wheels.*

The generalized Ramsey number  $R(G_1, G_2)$  is the smallest positive integer  $N$  such that any red-blue coloring of the edges of the complete graph  $K_N$  either contains a red copy of  $G_1$  or a blue copy of  $G_2$ . Let  $C_m$  denote a cycle of length  $m$  and  $W_n$  denote a wheel with  $n + 1$  vertices. In 2014, Zhang, Zhang and Chen determined many of the Ramsey numbers  $R(C_{2k+1}, W_n)$  of odd cycles versus larger wheels, leaving open the case where  $n = 2j$  is even and  $k < j < 3k/2$ . They conjectured that for these values of  $j$  and  $k$ ,  $R(C_{2k+1}, W_{2j}) = 4j + 1$ . In 2015, Sanhueza-Matamala confirmed this conjecture asymptotically, showing that  $R(C_{2k+1}, W_{2j}) \leq 4j + 334$ . In this paper, we prove the conjecture of Zhang, Zhang and Chen for almost all of the remaining cases. (Received September 16, 2016)

1125-05-1366 **Jennifer Vandenbussche\*** ([jvandenb@kennesaw.edu](mailto:jvandenb@kennesaw.edu)), **Victor Larsen** and **Erik Westlund**. *Recent results on extending Hall precolorings of graphs.*

Hall's condition for list coloring is a generalization of Hall's condition for the existence of systems of distinct representatives, and it is an obvious necessary condition for the existence of a list coloring of  $V(G)$  from a list assignment  $L$ . In this talk, we present recent results regarding precolorings of graphs whose associated list assignments satisfy Hall's condition (Hall precolorings). We discuss the number of additional colors needed to extend a Hall precoloring of  $G$  to a proper coloring of  $G$ . We also answer a question of Bobga et al. regarding the relationship between the ability to extend Hall  $m$ -precolorings of  $G$  and the ability to extend Hall  $(m + k)$ -precolorings of  $G$ . (Received September 16, 2016)

1125-05-1375 **Kirsti Wash\*** ([kirsti.wash@trincoll.edu](mailto:kirsti.wash@trincoll.edu)). *The packing chromatic number of subdivisions of subcubic graphs.*

The packing chromatic number  $\chi_\rho(G)$  of a graph  $G$  is the smallest integer  $k$  such that the vertex set of  $G$  can be partitioned into sets  $X_1, \dots, X_k$  where  $X_i$  is an  $i$ -packing for each  $i \in \{1, \dots, k\}$ . Gastineau and Togni recently conjectured that  $\chi_\rho(S(G)) \leq 5$  where  $S(G)$  is the subdivision of any subcubic graph. We show that the conjecture is indeed true for all generalized prisms of a cycle other than the Petersen graph. We also give a more general class of 2-connected subcubic graphs for which the conjecture holds and discuss the obstacles of proving the conjecture in general. (Received September 16, 2016)

1125-05-1414 **William J Keith\*** ([wjkeith@mtu.edu](mailto:wjkeith@mtu.edu)), Fisher 316, 1400 Townsend Drive, Houghton, MI 49931. *Parity similarity formulas for partitions and multipartitions.*

We produce several formulas relating the parity of the partition function in arithmetic progressions to parities of partition and multipartition functions. With  $p_t(n)$  the number of  $t$ -multipartitions of  $n$ , these formulas are of the type

$$q \sum_{n=0}^{\infty} p_t(an + b)q^n \equiv \frac{1}{(q; q)_\infty^{at}} + \frac{1}{(q^a; q^a)_\infty^t} \pmod{2}$$

as well as similar formulas with more terms. We use combinatorial reasoning and recent work of Radu on Ramanujan-Kolberg type identities to prove our results. The list of identities obtained so far is strongly suggestive of an infinite family, and several corollaries can be drawn that may be useful in investigations of the parity of the partition function more generally.

This is joint work with Fabrizio Zanello and Samuel Judge. (Received September 16, 2016)

1125-05-1434 **Sammy Y. Luo\*** ([sammyluo@mit.edu](mailto:sammyluo@mit.edu)), 3 Ames St, H309, Cambridge, MA 02142. *Short Zero-Sum Sequences Over Abelian  $p$ -Groups of Large Exponent.*

Let  $G$  be a finite abelian group with exponent  $n$ . Let  $\eta(G)$  denote the smallest integer  $\ell$  such that every sequence over  $G$  of length at least  $\ell$  has a zero-sum subsequence of length at most  $n$ . We determine the precise value of  $\eta(G)$  when  $G$  is a  $p$ -group whose Davenport constant is at most  $2n - 1$ . This confirms one of the equalities in a conjecture by Schmid and Zhuang from 2010. (Received September 16, 2016)

1125-05-1458 **Akira Saito\*** ([asaito@chs.nihon-u.ac.jp](mailto:asaito@chs.nihon-u.ac.jp)), Nihon University, Japan, and **Colton Magnant**, Georgia Southern University. *Forbidden subgraphs in edge-colored graphs.*

For a graph  $G$ , a function  $c: E(G) \rightarrow \{1, 2, \dots\}$  is called an *edge-coloring*, and the pair  $(G, c)$  is called an edge-colored graph. An edge-colored graph  $(G, c)$  is said to be *rainbow* if  $c(e) \neq c(f)$  for every pair of distinct edges  $e$  and  $f$  of  $G$ . For a connected graph  $H$ ,  $(G, c)$  is said to be *rainbow  $H$ -free* if  $G$  does not contain a subgraph  $G'$  which is isomorphic to  $H$  and  $(G', c|_{E(G')})$  is rainbow. For a graph  $H_1$  and its connected subgraph  $H_2$ , every rainbow  $H_2$ -free graph is trivially rainbow  $H_1$ -free. In this talk, we consider the opposite phenomenon and investigate the conditions which make a rainbow  $H_1$ -free graph rainbow  $H_2$ -free. (Received September 16, 2016)

1125-05-1472 **Shoichi Tsuchiya\*** ([wco.liew6.te@gmail.com](mailto:wco.liew6.te@gmail.com)), 2-1-1 Higashimita, Tama-ku, Kawasaki-shi, Kanagawa 214-8580, Japan. *On fullerene graphs.*

*Fullerenes* are cubic carbon molecules in which the atoms are arranged on a sphere in pentagons and hexagons. *Fullerene graphs* are 3-connected cubic plane graphs with pentagonal and hexagonal faces. Such graphs are suitable models for fullerenes: carbon atoms are represented by vertices of the graph, whereas the edges represent bonds between adjacent atoms. It is known that fullerene graphs satisfy many properties. For example, every fullerene graph  $G$  is 2-extendable, contains at least  $2^{\frac{|G|-380}{61}}$  perfect matchings, and so on. On the other hand, there are open problems on fullerene graphs. For example, deciding whether a fullerene graph has Hamilton cycle or not is still open. In this talk, we introduce new results on fullerene graphs. (Received September 17, 2016)

1125-05-1478 **Kathleen Ryan\***, Department of Mathematics & Computer Science, 2755 Station Avenue, Center Valley, PA 18034, and **Vincent Coll, Jonelle Hook, Colton Magnant** and **Karen McCready**. *Forget Yellow: Follow the Properly Colored Brick Road Instead.*

An edge-colored graph is *properly connected* if there exists a properly colored path between every pair of vertices. In such a graph, we re-consider the notions of distance and diameter by requiring that only properly colored paths are traversable. (Sorry, Dorothy.) So in a properly connected edge-colored graph  $G$ , we say that the *proper distance* between two vertices is the length of the shortest properly colored path between them, and we define the *proper diameter* of  $G$  to be the maximum proper distance between any pair of vertices in  $G$ . Depending

on the coloring of  $G$ , the difference between the diameter and the proper diameter of  $G$  varies, and we explore this difference for various graph families such as cycles, fans, complete bipartite graphs, and grids. (Received September 17, 2016)

1125-05-1493 **Alexander Berkovich\*** (alexberk@ufl.edu), Department of Mathematics, University of Florida, 358 Little Hall, Gainesville, FL 32611. *New Weighted Partition Identities, the Smallest Part of Partition and all that.*

I explain how to use the q-binomial theorem, the q-Gauss sum, and the transformation of Jackson to discover and prove many new weighted partition identities. These identities involve unrestricted partitions, over-partitions and partitions with distinct even parts. Smallest part of the partitions plays an important role in this analysis. This talk is based on my joint work with Ali Uncu. (Received September 17, 2016)

1125-05-1502 **J Tim Dwyer\***, Dartmouth College, Dept. of Mathematics, 27 N. Main Street, Hanover, NH 03755. *Wilf-equivalences of non-overlapping permutations, the cluster method, and linear extensions of posets.* Preliminary report.

We will discuss applications of the cluster method of Goulden & Jackson to counting occurrences of a consecutive pattern  $\pi$  in permutations  $\sigma$ , that is substrings of  $\sigma$  that are order isomorphic to  $\pi$ . The cluster method has been used previously to find many examples of Wilf-equivalences of patterns as well as a very general sufficient condition. We use the relationship between the cluster method and linear extensions of posets to prove a necessary condition for two non-overlapping permutations to be Wilf-equivalent. (Received September 17, 2016)

1125-05-1522 **John Polhill\*** (jpolhill@bloomu.edu), Dept. of Mathematical and Digital Sciences, Bloomsburg University of PA, 400 East Second Street, Bloomsburg, PA 17815. *Families of Strongly Regular Graphs and Two-Weight Codes from Partial Difference Sets.* Preliminary report.

A partial difference set is a subset of a finite group with an additional difference property. The author of this paper has in the past given several product constructions of infinite families of these sets. It so happens that partial difference sets correspond both to certain strongly regular graphs as well as projective two-weight codes. The purpose of this talk is to explore the partial difference set constructions in the contexts of strongly regular graphs and two-weight codes. We will focus on a result that is not yet published and is in groups of order  $3^k$  so that the resulting infinite family of codes is ternary. The family of codes includes the optimal ternary code of length 10, dimension 4, and nonzero weights 6 and 9. (Received September 17, 2016)

1125-05-1538 **Xiangfei Ni\***, nxf@zjnu.cn, and **Aihua Li**, lia@mail.montclair.edu. *Interlace Polynomials of Paths Attached to a Star Head.*

Graph polynomials have been used to describe interior structural properties of the underground graphs and have applications in other fields such as biology. In this paper, we study interlace polynomials of a special type of graph consisting of a path of size  $n$  and a star head of size  $m$ . We denote such a graph as  $T(n,m)$ . The selection of such graphs is motivated by distribution networks of products. Both iterative and explicit formula for the interlace polynomial of  $T(m,n)$  are given. Several values of the polynomial are obtained which reflect properties of  $T(m,n)$ . The result is applied to solve a matrix theory problem. (Received September 17, 2016)

1125-05-1574 **Robert W. Peck\*** (rpeck@lsu.edu), 102 School of Music, Louisiana State University, Baton Rouge, LA 70803. *Difference sets in mathematical music theory.* Preliminary report.

We investigate music-theoretical instances of difference sets in Generalized Interval Systems. Collections of pitches with flat interval distributions (or nearly flat distributions) are of particular interest in music theory, such as the all-interval tetrachords  $(0,1,4,6)$  and  $(0,1,3,7)$  in the modular space of twelve pitch classes. Generalized Interval Systems apply certain aspects of pitch intervals to other group structures. We draw on examples from cyclic, non-cyclic abelian, and non-abelian groups of order  $\leq 57$ . The flat-interval sets in these systems belong to three categories:  $(v,k,1)$  planar difference sets,  $(v,k,\lambda)$  non-planar difference sets, and  $(v,k,\lambda,t)$  almost difference sets. (Received September 20, 2016)

1125-05-1579 **Isaac Benjamin Michael\*** (isaac\_michael@baylor.edu), 207 North Calvert Street, Franklin, TX 77856, and **Mark Sepanski**. *Net Regular Signed Trees.*

The notion of a signed graph was first introduced by Harary. A signed graph consists of a graph and a labeling of the edges with  $\pm 1$ . A signed graph is called net regular if the sum of the signs of every edge incident to each vertex is constant. Graphs that admit a signing making them net regular are called net regularizable.

In this paper, net regular signed trees are studied, including general properties, conditions for a finite tree to be net regularizable, and an algorithm that computes the initial terms of the generating function for the number of net regularizable trees. (Received September 18, 2016)

1125-05-1615 **Anders S. Buch\*** (asbuch@math.rutgers.edu). *Puzzles for 3-step flag varieties*. Preliminary report.

A conjecture of Knutson from 1999 states that the cohomological Schubert structure constants of all partial flag varieties of Lie type A can be expressed as the number of puzzles that can be created from a list of puzzle pieces. This conjecture has been proved for Grassmannians and 2-step flag varieties, but is incorrect for 3-step flag varieties and beyond. However, in the case of 3-step flag varieties one obtains a formula that withstands computer testing by adding four extra puzzle pieces to Knutson's list. I will speak about partial progress towards proving this conjecture. (Received September 18, 2016)

1125-05-1664 **Sun Kim\*** (sunkim2@illinois.edu). *Bressoud's Conjecture*.

In 1980, D. M. Bressoud obtained an analytic generalization of the Rogers-Ramanujan-Gordon identities. He then tried to establish a combinatorial interpretation of his identity, which specializes to many well-known Rogers-Ramanujan type identities. He proved that a certain partition identity follows from his identity in a very restrictive case and conjectured that the partition identity holds true in general. In this talk, we discuss Bressoud's conjecture for the general case. (Received September 18, 2016)

1125-05-1672 **Eric Stucky\*** (stuck127@umn.edu), **Garner Cochran**, **Andrew W. Herring**, **Ranjan Rohatgi** and **Corbin Groothuis**. *Polynomial Chebyshev Quotients, Combinatorially*. Preliminary report.

For any graph  $G$  we may construct an associated polynomial called the matching polynomial, which is a variant on a generating function for matchings of  $G$ . When  $G$  is a cycle or path graph with  $n$  vertices, the resulting polynomials are essentially the Chebyshev polynomials  $T_n(x)$  and  $U_n(x)$  respectively. It is known that the only divisibility relations among the  $U_n$  have the form  $\frac{U_{mn-1}}{U_{n-1}} = U_{m-1} \circ T_n$ ; we interpret this equality combinatorially. In particular we show the right-hand side is an object with combinatorial meaning, called the  $d$ -matching polynomial by Hall, Pruder and Sawin (2015). (Received September 18, 2016)

1125-05-1679 **Lisa Warshauer Lowrance\*** (llowranc@skidmore.edu), **James Oxley**, **Charles Semple** and **Dominic Welsh**. *Properties of almost all matroids*.

We discuss several results about the asymptotic behavior of matroids. Specifically, almost all matroids are simple and cosimple and, indeed, are 3-connected. This verifies a strengthening of a conjecture of Mayhew, Newman, Welsh, and Whittle. We also discuss several quantitative results of almost all matroids. (Received September 18, 2016)

1125-05-1720 **Sheila Sundaram\*** (shsund@comcast.net). *On conjugacy classes of  $S_n$  containing all irreducibles*. Preliminary report.

We show that for the conjugation action of the symmetric group  $S_n$ , when  $n = 6$  or  $n \geq 8$ , all  $S_n$ -irreducibles appear as constituents of a single conjugacy class, namely, one indexed by a partition  $\lambda$  of  $n$  with at least two parts, whose parts are all distinct and taken from the set of odd primes and 1. We investigate a sequence of representations  $W_n$  defined as a multiplicity-free sum of every  $S_n$ -irreducible except two: namely, those indexed by  $(n-1, 1)$  and  $(2, 1^{n-2})$ . We describe precisely when the induced outer tensor products of these representations contain all irreducibles. (Received September 19, 2016)

1125-05-1735 **Saeed Akbari** (s\_akbari@sharif.edu), Tehran, Iran, **Amir Hossein Ghodrati** (ghodrati\_ah@mehrs.sharif.ir), Tehran, Iran, and **Shahriar Shahriari\*** (sshahriari@pomona.edu), 610 N College Ave, Claremont, CA 91711. *Nowhere-zero bases for the nullspace of the incidence matrices of graphs*.

A nowhere-zero  $k$ -flow for a directed or undirected graph is a vector in the nullspace of the incidence matrix of the graph such that all the entries of the vector are from the set  $\{\pm 1, \pm 2, \dots, \pm(k-1)\}$ . We consider the problem of finding a *basis* consisting of nowhere-zero  $k$ -flows for the nullspace of the incidence matrix. For a variety of graphs—including the complete graphs—we find such bases with  $k = 2$  or  $k = 3$ . We conjecture that all directed graphs with no cut-edge have such a basis with  $k = 5$ . If true, this would strengthen Tutte's celebrated conjecture on nowhere-zero 5-flows. (Received September 19, 2016)

1125-05-1744 **Alexander Engström\*** ([alexander.engstrom@aalto.fi](mailto:alexander.engstrom@aalto.fi)). *Quantum Annealing for Combinatorial Algebraic Geometry*. Preliminary report.

I will discuss some examples of theoretical considerations encountered when trying to implement computational algorithms in combinatorial algebraic geometry on an existing type of quantum computer. (Received September 19, 2016)

1125-05-1762 **Everett Sullivan\*** ([everett.n.sullivan.gr@dartmouth.edu](mailto:everett.n.sullivan.gr@dartmouth.edu)). *Linear chord diagrams with long chords*.

Linear chord diagrams, also known as matchings, are pairings of integers into sets of size two. This important combinatorial object arises in the fields of permutation patterns, knot theory, genomic rearrangements, and others. When restricting to the subset of chord diagrams with a prescribed minimum length interesting geometric sequences appear. We prove the existence of these geometric sequences bijectively. (Received September 19, 2016)

1125-05-1780 **Arran Hamm** and **Kristen Melton\*** ([meltonk6@mailbox.winthrop.edu](mailto:meltonk6@mailbox.winthrop.edu)). *A Lower Bound on the Hadwiger Number of a Random Subgraph of the Kneser Graph*.

Hadwiger's Conjecture is one of the most famous open problems in graph theory; it states that  $h(G) \geq \chi(G)$  for any graph  $G$  (where  $h(G)$  is the hadwiger number of  $G$ ). A theorem of Kostochka gives a lower bound on  $h(G)$  in terms of the average degree of  $G$ . This talk will be focused on giving a lower bound on  $h(G)$  where  $G$  is a (binomial) random subgraph of a Kneser graph. (Recall: A Kneser graph with parameters  $n$  and  $k$ , denoted  $KG(n, k)$ , has the set of  $k$ -subsets of  $\{1, \dots, n\}$  as its vertex set where two  $k$ -sets are adjacent if and only if they are disjoint.) So  $G$  is given by keeping each edge of  $KG(n, k)$  independently with probability  $p$ . For certain values of  $n$ ,  $k$ , and  $p$  we improve upon the bound given in Kostochka's theorem. (Received September 19, 2016)

1125-05-1787 **Baogang Xu** ([baogxu@njnu.edu.cn](mailto:baogxu@njnu.edu.cn)), School of Mathematical Sciences, Nanjing Normal University, 1 Wenyuan Road, Nanjing, Jiangsu 210023, Peoples Rep of China, and **Xiaoya Zha\*** ([xzha@mtsu.edu](mailto:xzha@mtsu.edu)), Department of Mathematical Sciences, Middle Tennessee State University, Murfreesboro, TN 37132. *Partition graphs with non-negative Euler characteristics*. Preliminary report.

In this talk, we show that every non-planar graph with non-negative Euler characteristic (i.e., projective planar, toroidal, or Klein bottle embeddable) can be edge partitioned into a planar graph and an outer-planar graph. (Received September 19, 2016)

1125-05-1790 **Katie Haymaker\*** ([kathryn.haymaker@villanova.edu](mailto:kathryn.haymaker@villanova.edu)) and **Sally Robertson**. *Counting colorful tilings of rectangular arrays*. Preliminary report.

How many ways are there to tile a rectangular board with painted squares and dominoes, when there are  $a$  available colors for the squares and  $b$  available colors for the dominoes? There is no closed-form expression for the number of tilings of an  $m \times n$  board with dominoes and squares, but the problem has been extensively studied in the area of mathematical physics, where the pieces are called monomers and dimers. In this talk we will give a recursive formula for the number of colorful tilings of a  $2 \times n$  board with squares, dominoes, and trominoes. Time permitting, we will also sketch a general method for calculating the number of colorful tilings of an  $m \times n$  board. (Received September 19, 2016)

1125-05-1809 **Martin Rolek\*** ([mrolek@knights.ucf.edu](mailto:mrolek@knights.ucf.edu)) and **Zi-Xia Song**. *Coloring graphs with forbidden minors*.

Hadwiger's conjecture from 1943 states that for every integer  $t \geq 1$ , every graph either can be  $t$ -colored or has a subgraph that can be contracted to the complete graph on  $t + 1$  vertices. Proving that graphs with no  $K_7$  minor are 6-colorable is the first case of Hadwiger's conjecture that is still open. It is not known yet whether graphs with no  $K_7$  minor are 7-colorable. Using a Kempe-chain argument along with the fact that an induced path on three vertices is dominating in a graph with independence number two, we first give a very short and computer-free proof of a recent result of Albar and Gonçalves and generalize it to the next step by showing that every graph with no  $K_t$  minor is  $(2t - 6)$ -colorable, where  $t \in \{7, 8, 9\}$ . We then prove that graphs with no  $K_8^-$  minor are 9-colorable and graphs with no  $K_8^=$  minor are 8-colorable. Finally we prove that if Mader's bound for the extremal function for  $K_p$  minors is true for all  $p \geq 10$ , then every graph with no  $K_p$  minor is  $(2t - 6)$ -colorable. We believe that the Kempe-chain method we have developed in this paper is of independent interest. (Received September 19, 2016)

1125-05-1861 **Kristin Heysse\*** ([keheysse@iastate.edu](mailto:keheysse@iastate.edu)), 396 Carver Hall, Ames, IA 50011.  
*Constructions for distance cospectral graphs.*

The distance matrix of a connected graph is the symmetric matrix with columns and rows indexed by the vertices and entries that are the pairwise distances between the corresponding vertices. We give a construction for graphs which differ in their edge counts yet are cospectral with respect to the distance matrix. Further, we identify a subgraph switching behavior which constructs additional distance cospectral graphs. The proofs for both constructions rely on a perturbation of (most of) the distance eigenvectors of one graph to yield the distance eigenvectors of the other. (Received September 19, 2016)

1125-05-1864 **Franklin H. J. Kenter\*** ([kenter@usna.edu](mailto:kenter@usna.edu)), Annapolis, MD 21401. *Spectral Approaches to Graph Coloring And Its Variations.*

Hoffman was the first to give a lower bound on the chromatic number of a graph,  $\chi(G)$ , and the eigenvalues of the adjacency matrix. He showed that  $\chi(G) \geq 1 - \frac{\lambda_{\max}}{\lambda_{\min}}$  where  $\lambda_{\max}$  is the maximum eigenvalue and  $\lambda_{\min}$  is the minimum eigenvalue.

In this talk, we investigate several variations or extensions of this equality. We will discuss frugal coloring (graph coloring with additional restrictions), hypergraph coloring, and even finding large bipartite subgraphs. (Received September 19, 2016)

1125-05-1878 **Christin Bibby\*** ([cbibby2@uwo.ca](mailto:cbibby2@uwo.ca)), **Graham Denham** and **Eva Maria Feichtner.** *A Leray model for the Orlik-Solomon algebra.*

We give a sequence of combinatorial models (CDGAs) for the Orlik-Solomon algebra of a matroid, arising from a sequence of combinatorial blowups. In the case that the matroid is complex realizable, each is a model in the sense of rational homotopy theory, arising from the Leray spectral sequence associated to the inclusion of the projective complement into a certain projective variety. (Received September 19, 2016)

1125-05-1895 **Guantao Chen\*** ([gchen@gsu.edu](mailto:gchen@gsu.edu)), Department of Mathematics and Statistics, Georgia State University, Atlanta, GA 30303. *Cycles in critical edge coloring graphs with large maximum degrees.* Preliminary report.

Vizing conjectured that every  $k$ -critical edge coloring graphs with  $k \geq 3$  contains a 2-factor. By imposing some conditions on the lower bound of maximum degree, we verify the conjecture. Moreover, we show that such graphs have a hamiltonian cycles. These are joint work with Xiaodong Chen, Songling Shan, and Yue Zhao, respectively. (Received September 19, 2016)

1125-05-1914 **Portia Anderson\*** ([panderson@smith.edu](mailto:panderson@smith.edu)), Department of Mathematics and Statistics, Smith College, Northampton, MA 01063, **Mariel Jones**, Department of Mathematics and Statistics, Smith College, Northampton, MA 01063, **Minjun Li**, Department of Mathematics and Statistics, Smith College, Northampton, MA 01063, **Caitlin O'Neill**, Department of Mathematics and Statistics, Smith College, Northampton, MA 01063, **Cleophelia Roberts**, Department of Mathematics and Statistics, Smith College, Northampton, MA 01063, **Carolyn Stephen**, Department of Mathematics and Statistics, Smith College, Northampton, MA 01063, and **Claudia He Yun**, Department of Mathematics and Statistics, Smith College, Northampton, MA 01063. *Generalized splines on infinite graphs.*

Given an edge-labeled graph, a generalized spline is a labeling of the vertices so that the difference between adjacent vertex labels is a multiple of the corresponding edge label. This definition generalizes the notion of splines from classical analysis and applied mathematics.

We discuss several new results on generalized splines, including how to construct bases for splines for certain infinite graphs. Surprisingly, this answers the question of how to construct the equivariant cohomology of a geometric space called an affine Springer fiber. Time permitting, we may also discuss results about bases for other spline spaces related to questions in approximation theory. (Received September 19, 2016)

1125-05-1930 **Linda Eroh** ([eroh@uwosh.edu](mailto:eroh@uwosh.edu)), **Henry Escuadro\*** ([escuadro@juniata.edu](mailto:escuadro@juniata.edu)), **Raluca Gera** ([gera@nps.edu](mailto:gera@nps.edu)), **Samuel Prahlow** ([samuel.prahlow@valpo.edu](mailto:samuel.prahlow@valpo.edu)) and **Karl Schmitt** ([karl.schmitt@valpo.edu](mailto:karl.schmitt@valpo.edu)). *A Graph Theoretical Analysis of the Number of Edges in  $k$ -Dense Graphs.*

For each edge  $uv \in E(G)$ , the **edge multiplicity** of  $uv$  in  $G$  is given by

$$m_G(uv) = |N_G(u) \cap N_G(v)|.$$

For an integer  $k$  with  $k \geq 2$ , a  **$k$ -dense community** of a graph  $G$ , denoted by  $DC_k(G)$ , is a maximal connected subgraph of  $G$  induced by the vertex set

$$V_{DC_k(G)} = \{v \in V(G) : \exists u \in V(G) \text{ such that } uv \in E(G) \text{ and } m_{DC_k(G)}(uv) \geq k - 2\}.$$

In this study, we characterize which graphs are  $k$ -dense but not  $(k+1)$ -dense for some values of  $k$  and study the minimum and maximum number of edges such graphs can have.

Keywords:  $k$ -dense subnetworks (or  $k$ -dense subgraph),  $k$ -dense community,  $k$ -dense graph,  $k$ -core,  $k$ -core subnetwork (Received September 19, 2016)

1125-05-1996 **Benjamin J. Braun** and **Wesley K. Hough\*** ([wesley.hough@uky.edu](mailto:wesley.hough@uky.edu)), Department of Mathematics, University of Kentucky, 715 Patterson Office Tower, Lexington, KY 40506. *Matching and Independence Complexes Related to Small Grids.*

In this talk, we briefly survey tools from discrete Morse theory useful for analyzing the independence complex of graphs. More specifically, we develop particular matching tree algorithms applied to certain grid graphs that obey cell-counting recursions connecting back to interesting combinatorial sequences. (Received September 19, 2016)

1125-05-1999 **Sarah Loeb\*** ([sloeb2@illinois.edu](mailto:sloeb2@illinois.edu)) and **Douglas B. West.** *Circular Separation Dimension.*

Given a linear ordering  $\sigma$  of  $V(G)$ , a pair of non-incident edges is *separated* by  $\sigma$  if both vertices of one edge precede both vertices of the other. The *separation dimension*  $\pi(G)$  of a graph  $G$  is the minimum size of a set of vertex orderings needed to separate every pair of non-incident edges of  $G$ . Given a circular ordering  $\sigma$  of  $V(G)$ , a pair of non-incident edges is *separated* by  $\sigma$  if the vertices of the two edges do not alternate. The *circular separation dimension*  $\pi^\circ(G)$  of a graph  $G$  is the minimum size of a set of circular vertex orderings needed to separate every pair of non-incident edges of  $G$ .

We show that  $\pi^\circ(G) = 1$  if and only if  $G$  is an outerplanar graph. While  $\pi(G)$  is unbounded for bipartite graphs, we prove  $\pi^\circ(G) \leq 2$  for every bipartite graph. Finally, we prove that  $\pi^\circ$  is unbounded by showing that  $\pi^\circ(K_n) \geq \log_2 \log_3(n-1)$ . (Received September 19, 2016)

1125-05-2048 **Hongliang Lu**, Xi'an, Shaanxi, Peoples Rep of China, **Yan Wang**, Atlanta, GA 30332, and **Xingxing Yu\*** ([yu@math.gatech.edu](mailto:yu@math.gatech.edu)), Atlanta, GA 30332. *On almost perfect matchings in  $k$ -partite  $k$ -graphs.*

The minimum co-degree threshold for a perfect matching in a  $k$ -graph with  $n$  vertices was determined by Rödl, Ruciński and Szemerédi for the case when  $n \equiv 0 \pmod{k}$ . Recently, Han resolved the remaining cases, establishing a conjecture of Rödl, Ruciński and Szemerédi. In this paper, we determine the minimum co-degree threshold for almost perfect matchings, answering a question of Rödl and Ruciński. (Received September 19, 2016)

1125-05-2069 **Neil Lyall**, **Ákos Magyar** and **Hans Parshall\*** ([hans@math.uga.edu](mailto:hans@math.uga.edu)). *Spherical configurations over finite fields.*

In their 1973 paper, Erdős, Graham, Montgomery, Rothschild, Spencer, and Straus proved that every Euclidean Ramsey set is contained in some sphere, and Graham conjectures that every finite spherical set is indeed Ramsey. This conjecture remains open (and contested) even in the case of a generic four point subset of a circle. We provide evidence for Graham's conjecture by proving something stronger in the finite field setting: for any  $\alpha \in (0, 1)$ , every  $A \subseteq \mathbf{F}_q^{10}$  with  $|A| \geq \alpha q^{10}$  contains an isometric copy of every four point spherical set spanning two dimensions, provided  $q$  is taken sufficiently large with respect to  $\alpha$ . For  $d \geq 2k + 6$ , comparable results are obtained in  $\mathbf{F}_q^d$  for arbitrary  $(k+2)$ -point spherical configurations spanning  $k$  dimensions. (Received September 19, 2016)

1125-05-2087 **Yelena Mandelshtam\*** ([yelena13@stanford.edu](mailto:yelena13@stanford.edu)). *On graphs representable by pattern-avoiding words.*

In this paper we study graphs defined by pattern-avoiding words. Word-representable graphs have been studied extensively following their introduction in 2000 and are the subject of a book published by Kitaev in 2015. Recently there has been interest in studying graphs represented by pattern-avoiding words. In particular, in 2016, Gao, Kitaev, and Zhang investigated 132-representable graphs, that is, word-representable graphs that can be represented by a word which avoids the pattern 132. They proved that all 132-representable graphs are circle graphs and provided examples and properties of 132-representable graphs. They posed several questions, some of which we answer in this paper.

One of our main results is that not all circle graphs are 132-representable, thus proving that 132-representable graphs are a proper subset of circle graphs, a question that was left open in the paper by Gao et al. We show that 123-representable graphs are also a proper subset of circle graphs, and are different from 132-representable graphs. We also study graphs represented by pattern-avoiding 2-uniform words, that is, words in which every letter appears exactly twice. (Received September 19, 2016)

1125-05-2094 **sarah-marie belcastro\*** ([smbelcas@toroidalsnark.net](mailto:smbelcas@toroidalsnark.net)) and **Ruth Haas**. *Grünbaum colorings extended to non-facial 3-cycles*. Preliminary report.

A *Grünbaum coloring* of a triangulation of a surface assigns 3 colors to the edges such that every facial triangle receives all three colors. It is known that all planar triangulations and many classes of toroidal, projective-planar, and Klein-bottle triangulations have Grünbaum colorings.

We consider first the question of when a triangulation with a Grünbaum coloring can be edge-colored with three colors such that the non-facial cycles also receive all three colors. We show that for the sphere, every triangulation has this type of generalized Grünbaum coloring, and that for every other topological surface there exist triangulations with such a generalized Grünbaum coloring and triangulations that have Grünbaum colorings but that cannot have a generalized Grünbaum coloring.

We also consider the question of how many colors are needed to assign colors to edges of a triangulation of a surface such that every 3-cycle (both facial and non-facial) receives three distinct colors. (Received September 19, 2016)

1125-05-2113 **Heather D Jordan\*** ([hjordan@albion.edu](mailto:hjordan@albion.edu)) and **Genevieve Newkirk**. *4-Cycle Decompositions of Complete 3-Uniform Hypergraphs*. Preliminary report.

A 3-uniform complete hypergraph of order  $n$  has vertex set  $\{1, 2, \dots, n\}$  and, as its edge set, the set of all possible subsets of size 3. A 4-cycle  $v_1, e_1, v_2, e_2, v_3, e_3, v_4, e_4, v_1$  in this hypergraph, where each  $v_i$  is a vertex and each  $e_i$  is an edge, has the property that  $v_i, v_{i+1} \in e_i$  for  $i = 1, 2, 3$  and  $v_4, v_1 \in e_4$ , also known as a Berge cycle. A decomposition of a hypergraph is a partition of its edge set into edge-disjoint subsets. In this talk, we investigate decompositions of the 3-uniform complete hypergraph of order  $n$  into 4-cycles. (Received September 19, 2016)

1125-05-2170 **M. N. Ellingham** ([mark.ellingham@vanderbilt.edu](mailto:mark.ellingham@vanderbilt.edu)), **Songling Shan\*** ([songling.shan@vanderbilt.edu](mailto:songling.shan@vanderbilt.edu)), **Dong Ye** ([dong.ye@mtsu.edu](mailto:dong.ye@mtsu.edu)) and **Xiaoya Zha** ([xiaoya.zha@mtsu.edu](mailto:xiaoya.zha@mtsu.edu)). *Toughness condition for  $k$ -trees in  $K_4$ -minor-free graphs*.

Let  $k$  be a positive integer. A  $k$ -tree is a tree with maximum degree at most  $k$ , and a  $k$ -walk is a closed walk with each vertex repeated at most  $k$  times. A  $k$ -walk can be obtained from a  $k$ -tree by visiting each edge twice. Jackson and Wormald in 1990 conjectured that any  $\frac{1}{k-1}$ -tough graph contains a spanning  $k$ -walk for  $k \geq 2$ . This conjecture is widely open even for planar graphs. We confirm this conjecture for  $K_4$ -minor-free graphs, an important subclass of planar graphs, by showing that any  $\frac{1}{k-1}$ -tough  $K_4$ -minor-free graph contains a spanning  $k$ -tree for any integer  $k \geq 2$ . (Received September 19, 2016)

1125-05-2175 **Brendan W Sullivan\*** ([sullivanb@emmanuel.edu](mailto:sullivanb@emmanuel.edu)), 400 The Fenway, Boston, MA 02115, and **Nikolas Townsend** and **Mikayla Werzanski**. *The  $3 \times 3$  Rook's graph is the unique smallest graph with lazy cop number 3*.

In the pursuit-evasion game *cops and robbers*, a team of cops and a robber occupy vertices of a connected graph and alternately move along its edges. The minimum number of cops required to catch the robber is called the *cop number* (denoted  $c$ ). Previous work by Beveridge et al. has shown that the Petersen graph is the unique smallest graph which requires three cops: it's the only graph on 10 vertices with  $c = 3$ , and all graphs on 9 or fewer vertices have  $c \leq 2$ . (This result was previously found by Baird & Bonato via computational search.)

In the variant *lazy cops and robbers*, the cops may only choose one member of their squad to make a move when it's their turn. Analogously to Beveridge's result, we have found the  $3 \times 3$  Rook's graph ( $R_3 = K_3 \square K_3$  with 9 vertices) is the unique smallest graph with  $c_L = 3$ . We will share a self-contained proof of this fact. In addition, we will share computational results for graphs on 10 or more vertices, hunting for distinct structures that necessitate 3 lazy cops. Finally, we will share progress made towards the general conjecture that  $R_n$  is the unique smallest graph with  $c_L = n$ . (Received September 19, 2016)

1125-05-2198 **John Sinkovic\*** ([john.sinkovic@uwaterloo.ca](mailto:john.sinkovic@uwaterloo.ca)). *Graphs for which inertia bounds are not tight*.

Given a simple graph  $G$ , a weight matrix  $W$  of  $G$  is a real symmetric matrix such that  $w_{ij} = 0$  whenever  $ij$  is not an edge of  $G$ . The inertia of  $W$  is an ordered triple  $(n_+, n_-, n_0)$  consisting of the number of positive, negative,

and zero eigenvalues (multiplicity of zero). The inertia of a weight matrix can be used to give an upper bound on the independence number,  $\alpha(G)$ . The following is attributed to D.M. Cvetković.

$$\alpha(G) \leq n_0 + \min\{n_+, n_-\}$$

for any weight matrix  $W$ .

Recently, C. Elphick and P. Wocjan proposed a lower bound for the chromatic number  $\chi(G)$ .

$$1 + \max\left\{\frac{n_+}{n_-}, \frac{n_-}{n_+}\right\} \leq \chi(G)$$

It is interesting and oft times difficult to find graphs for which a bound is not tight. We give some examples of when the inequalities above are strict. (Received September 19, 2016)

1125-05-2212 **Kristen Wetzler\*** (kwetz11@lsu.edu). *Graphs whose only odd cycles are small.*

It is well known that a graph is bipartite if and only if all of its cycles are even. Maffray proved in 1992 that a 2-connected simple graph whose only odd cycles are triangles is bipartite unless it is  $K_4$ , or the graph obtained from  $K_{2,n}$  by adding an edge joining the vertices in the 2-vertex class. In this talk, we will generalize this result first by increasing the length of the largest odd cycle and then by moving to binary matroids. (Received September 20, 2016)

1125-05-2276 **Emily J Olson\*** (ognacevi@msu.edu) and **Bruce Sagan**. *Progress on the 1/3 – 2/3 Conjecture.* Preliminary report.

In a partially ordered set  $P$ , let a pair of elements  $(x, y)$  be called  $\alpha$ -balanced if the proportion of linear extensions that has  $x$  before  $y$  is between  $\alpha$  and  $1 - \alpha$ . The 1/3 – 2/3 Conjecture states that every finite poset which is not a chain has some 1/3-balanced pair. While the conjecture remains unsolved, we present progress in certain cases which include products of two chains and certain other distributive lattices. (Received September 20, 2016)

1125-05-2279 **Alexander Halperin** (adhalperin@salisbury.edu), 1101 Camden Ave., Salisbury, MD 21801, and **Adam Jump\*** (ajump2@gulls.salisbury.edu), 356 Hickory Point Rd., Pocomoke City, MD 21851. *A Sharp Upper Bound on the  $k$ -Color Connection Number of a Graph.* Preliminary report.

How much security is needed to optimally harden a network against cyberattack? How many types of cargo need exist between a freight carrier and its destination in order to guarantee it has the most possible supplies? These problems can be phrased in terms of rainbow Ramsey theory: how many edge colors does a graph need to contain a path with  $k$  distinct colors between every pair of vertices? Equivalently, what is the  $k$ -color connection number of a graph? Denoting the  $k$ -color connection number of  $G$  as  $cc_k(G)$ , Coll et al. conjectured  $cc_k(G) \leq 2k - 1$  and, for a sharp condition, observed that  $cc_k(C_{2k-1}) = 2k - 1$ . We prove  $cc_k(G) \leq 2k - 1$  by analyzing the  $k$ -color connection number of chorded cycles. Additionally, we display our Mathematica package that determines the  $k$ -color connection number of chorded cycles. (Received September 21, 2016)

1125-05-2301 **Mark Bly\*** (blymark83@gmail.com). *Two  $q$ -countings related to  $q$ -multinomial coefficients.* Preliminary report.

Let  $V$  be an  $n$ -dimensional vector space over a finite field, and let  $\Delta$  be the set of sequential arrangements of  $n$ -element multisets of the form  $\{1^{n_1}, 2^{n_2}, \dots, k^{n_k}\}$ . In the spirit of D.Knuth's (1971) providing a  $q$ -counting of integer partitions relating to  $q$ -binomial coefficients, we provide two  $q$ -countings of  $\Delta$  relating to  $q$ -multinomial coefficients: (1) via a map from the set of chains of subspaces of  $V$ ; (2) via a map from the set of ordered direct sum decompositions of  $V$ . (Received September 20, 2016)

1125-05-2304 **Sarfraz Ahmad\*** (sarfraz11@gmail.com), M. A. Jinah Campus, Raiwand Road, Off Defence, Road, Lahore, Punjab 54700, Pakistan. *On Different Topological Indices and polynomials of the Boron Zigzag nanotube  $BNT[p, q]$ .*

In mathematical chemistry, a topological index is computed using structure of the molecular graph and is a numerical parameter which describe its topology. In this article we give a complete description of different types of the Zagreb indices of the zigzag boron triangular nanotubes  $BNT[p, q]$ . Using combinatorial techniques we also provide explicit formulas of the  $GA$  index, the Randic index and the atom-bond connectivity index of the nanotubes  $BNT[p, q]$ .

The Omega polynomial and its four counting polynomials have great significance in the study of QSPR/QSTR/QSAR. These are also useful to demonstrate topological indices by virtue of quasi-orthogonal cuts of the edge strips in the polycyclic graphs. In the end we give a complete description of the Omega and the Sadhana polynomial of the family of zigzag boron nanotube  $BNT[p, q]$  and provide its mathematical proof. We also give explicit

formulae for the PI and the Theta polynomial of zigzag boron nanotubes  $BNT[p, q]$ . (Received September 20, 2016)

1125-05-2324 **Sarah D Brauner\***, Department of Mathematics and Statistics, 44 College Lane, Northampton, MA 01063, **Marissa E Miller**, Department of Mathematics and Statistics, 44 College Lane, Northampton, MA 01063, **Leslie K Nordstrom**, Department of Mathematics and Statistics, 44 College Lane, Northampton, MA 01063, and **Jamie A Oliva**, Department of Mathematics and Statistics, 44 College Lane, Northampton, MA 01063. *Further results on a generalization of Lie(k).*

The representation of the symmetric group  $S_k$  on the homogeneous component of the free Lie algebra is known as  $Lie(k)$ . We consider instead the analogous representation when the Lie algebra is replaced with a LAnKe, a vector space equipped with an antisymmetric commutator of  $n$ , rather than 2, elements of the vector space, together with a generalized Jacobi identity. There are recent results on this new topic by T. Friedmann, P. Hanlon, R. Stanley, and M. Wachs, and we discuss several additional results. (Received September 20, 2016)

1125-05-2353 **Guanghui Wang\*** (ghwang@sdu.edu.cn), Jinan, Shandong 250100. *Rainbow matchings in graphs.*

A rainbow matching in an edge colored graphs is a matching such that all edges have distinct colors. Rainbow matchings are closely related to latin squares, orthogonal matchings. In this talk, we focus on the existence of rainbow matchings in properly edge colored graphs. (Received September 20, 2016)

1125-05-2360 **Grigoriy Blekherman, Rainer Sinn\*** (sinn@math.gatech.edu) and **Mauricio Velasco**. *Positive semidefinite matrix completion and algebraic geometry*. Preliminary report.

We sketch how the positive semidefinite matrix completion problem is related to questions in algebraic geometry and commutative algebra. The connection is made via geometry of spectrahedra. We will see recent results from joint work with Greg Blekherman and Mauricio Velasco. (Received September 20, 2016)

1125-05-2361 **Huilan Li, Jennifer Morse and Patrick Shields\*** (prs49@drexel.edu). *Structure constants for K-theory of Grassmannians, revisited.*

The problem of computing products of Schubert classes in the cohomology ring can be formulated as the problem of expanding skew Schur polynomials into the basis of ordinary Schur polynomials. In contrast, the problem of computing the structure constants of the Grothendieck ring of a Grassmannian variety with respect to its basis of Schubert structure sheaves is not equivalent to expanding skew stable Grothendieck polynomials into the basis of ordinary stable Grothendiecks. Instead, we show that the appropriate  $K$ -theoretic analogy is through the expansion of skew reverse plane partitions into the basis of polynomials which are Hopf-dual to stable Grothendieck polynomials. We combinatorially prove this expansion is determined by Yamanouchi set-valued tableaux. A by-product of our results is a dual approach proof for Buch's  $K$ -theoretic Littlewood-Richardson rule for the product of stable Grothendieck polynomials. (Received September 20, 2016)

1125-05-2364 **Michael J. Joseph\*** (michael.j.joseph@uconn.edu). *On Orbits of Toggling Actions on Independent Sets of a Path Graph and the Homomesy Phenomenon*. Preliminary report.

We consider systems consisting of a finite set  $S$  of objects, and an invertible map that partitions  $S$  into orbits. For many such systems one can find natural statistics on  $S$  that have the same average across any orbit; we call such statistics "homomesic". This phenomenon occurs unexpectedly frequently, and often homomesic statistics can be used to answer seemingly unrelated questions about the orbits. In this talk we consider an example of this in detail, for which the invertible map is a product of toggling involutions on independent sets of a path graph. (Received September 20, 2016)

1125-05-2371 **Anne C Sinko\*** (asinko@csbsju.edu), 37 S. College Ave., St. Joseph, MN 56374. *The upper and lower independence-partition number on bipartite graphs.*

Colored-independence is a storage/scheduling problem which, in addition to the standard restriction involving pairs of elements that cannot be placed together, considers sets of elements that must be placed together. A set  $S$  is a colored-independent set if, for each color class  $V_i$ ,  $S \cap V_i = V_i$  or  $S \cap V_i = \emptyset$ .  $\beta(G; S)$  is the maximum cardinality of a colored-independent set. The independence-partition number,  $\beta_{PRT}(G)$ , is then defined to be the maximum cardinality over all  $\beta(G; S)$ . The lower independence-partition number,  $i_{PRT}(G)$ , is defined to be the maximum cardinality over all  $i(G; S)$ , where  $i(G; S)$  is the minimum cardinality of a maximal colored-independent set. This talk will examine results of  $\beta_{PRT}(G)$  and  $i_{PRT}(G)$  on bipartite graphs. Particular attention will be given to the characterization of bipartite graphs that achieve  $i_{PRT}(T) = |V_1|$  where  $V_1$  is the smaller of the bipartition sets of graph  $G$ . (Received September 20, 2016)

1125-05-2396 **Jonathan Bloom** and **Carl Hammarsten\*** ([hammarsc@lafayette.edu](mailto:hammarsc@lafayette.edu)). *An algorithm toward finding solutions to the general multi-constraint cable-trench problem.*

Given a weighted graph, a classic problem in graph theory involves finding spanning trees whose edge weights minimize certain constraints. Two particularly well-known versions of this problem are the shortest-path tree (SPT) and the minimal spanning tree (MST). In both of these cases, there exist efficient algorithms to construct solutions - Dijkstra's algorithm for the former and Prim's algorithm for the latter.

The cable-trench problem may be thought of as an interpolation between the SPT and the MST. Namely, the constraint we minimize is a weighted sum of the constraints for the two previous cases.

We present an algorithm which aims to find a solution to the cable-trench problem via modifying a candidate solution tree by studying local costs associated to each edge and determining where improvements can be made. We present empirical evidence that our algorithm solves the problem for many graphs and a large variety of edge weights. We will also show that our algorithm naturally addresses the situation where the edge weights used for the SPT and the those used for the MST need not be directly related (general cable-trench) and our algorithm is easily modifiable to address the situation when more than two constraints are combined (multi-constraint cable-trench). (Received September 20, 2016)

1125-05-2400 **Kyungyong Lee**, **Li Li** and **Nicholas A. Loehr\*** ([nloehr@vt.edu](mailto:nloehr@vt.edu)), Virginia Tech  
Department of Mathematics, 460 McBryde Hall, 225 Stanger Street, Blacksburg, VA 24060.  
*Chain decompositions for  $q, t$ -Catalan numbers.*

The  $q, t$ -Catalan numbers  $C_n(q, t)$  are polynomials in  $q$  and  $t$  that reduce to the ordinary Catalan numbers when  $q = t = 1$ . These polynomials have important connections to representation theory, algebraic geometry, and symmetric functions. Haglund and Haiman discovered combinatorial formulas for  $C_n(q, t)$  as weighted sums of Dyck paths (or equivalently, integer partitions contained in a staircase shape). This talk investigates the joint symmetry property  $C_n(q, t) = C_n(t, q)$ . We conjecture some structural decompositions of Dyck objects into "mutually opposite" subcollections that lead to a bijective explanation of joint symmetry in certain cases. A key new idea is the construction of infinite chains of partitions that are independent of  $n$  but induce the joint symmetry for all  $n$  simultaneously. Using these methods, we can prove combinatorially that for  $0 \leq k \leq 9$  and all  $n$ , the terms in  $C_n(q, t)$  of total degree  $\binom{n}{2} - k$  have the required symmetry property. (Received September 20, 2016)

1125-05-2419 **Lindsay A Erickson\*** ([lindsay.erickson@augie.edu](mailto:lindsay.erickson@augie.edu)), 2001 S Summit Avenue, FSC  
#383, Sioux Falls, SD 57197. *Gamification of combinatorial research via Nim on graphs.*

Although there is an increasingly sizable body of knowledge on how converting traditional pedagogy into game play improves educational outcomes, little work has been done examining how games can be used for research outcomes, particularly in mathematics. What has been done in the area of digital topology shows great promise for this novel line of research. We examine how game data can be used to positively impact research into the solution of Nim on graphs by developing and programming a human vs. computer version of Nim on the  $K_{3,3}$ , enabling web play, collecting and filtering game data, and analyzing data to find a solution to the game. In the process of solving the  $K_{3,3}$ , the solution to other classes of graphs, including the general solution to Nim on the  $K_{2,n}$ , were found. (Received September 20, 2016)

1125-05-2422 **Lindsey-Kay Lauderdale\*** ([llauderdale@uttyler.edu](mailto:llauderdale@uttyler.edu)), 3900 University Blvd., Tyler,  
TX 75799, and **Christina Graves** and **Stephen Graves**. *Vertex Minimal Graphs with Dihedral Symmetry.*

Let  $D_{2n}$  denote the dihedral group of order  $2n$ , where  $n$  is an integer greater than 3. In this talk we build upon the findings of Haggard and McCarthy who, for certain values of  $n$ , each produced a vertex minimal graph whose automorphism group is isomorphic to  $D_{2n}$ . Specifically, Haggard considered the situation where  $\frac{n}{2}$  or  $n$  is a power of a prime number and McCarthy investigated the case when  $n$  is not divisible by 2, 3 nor 5. Here we construct a vertex minimal graph whose automorphism group is isomorphic to  $D_{2n}$  where  $n$  is not divisible by 4. These results provide a new geometric interpretation of the dihedral group. (Received September 20, 2016)

1125-05-2452 **Andrew Meier\*** ([ameier@nebrwesleyan.edu](mailto:ameier@nebrwesleyan.edu)), **Austin Mohr** ([amohr@nebrwesleyan.edu](mailto:amohr@nebrwesleyan.edu))  
and **Thomas Schuler** ([tschule2@nebrwesleyan.edu](mailto:tschule2@nebrwesleyan.edu)). *Pruning techniques for SUBGRAPH ISOMORPHISM using matchings and vertex cuts.* Preliminary report.

Given two graphs, called the host and the pattern, SUBGRAPH ISOMORPHISM is the problem of determining whether the host contains a subgraph that is isomorphic to the pattern. We describe new techniques involving the theory of graph matchings and vertex cuts that appear to greatly reduce the search space involved in SUBGRAPH ISOMORPHISM. Matchings are used to avoid the exploration of branches in the depth-first search tree in which

a large subset of the pattern vertices must be mapped to a smaller subset of the host vertices. Vertex cuts are used to prioritize vertex assignments in such a way that the host is quickly broken into connected components, which allows one to proceed recursively on each of the (much simpler) components. Our observations about cut vertices are most effective in the special case of SPANNING SUBGRAPH ISOMORPHISM, wherein the host and pattern are of equal order. We conclude with empirical data for several instances of SPANNING SUBGRAPH ISOMORPHISM, all of which suggest that the consideration of matchings and vertex cuts yields a great reduction in runtime. (Received September 20, 2016)

1125-05-2481 **Isaiah Harney\*** (isaiah.harney@uky.edu) and **Heide Gluesing-Luerssen** (heide.gl@uky.edu). *On Robust Colorings of the Hamming-Distance Graph.*

The Hamming-Distance Graph,  $H_q(n, d)$ , is defined as the graph with vertex set  $\mathbb{Z}_q^n$  where two vertices are adjacent if their Hamming distance is at least  $d$ . Cliques of  $H_q(n, d)$  correspond to  $q$ -ary block codes of length  $n$  and Hamming distance at least  $d$ . In a paper by Rouayheb et al., the authors use graph theoretical techniques applied to  $H_q(n, d)$  to produce new proofs for many classical bounds on error-correcting codes. Moreover, they determine the chromatic number of this graph for many parameters.

A result by Greenwell/Lovász exists which states that  $\chi(H_q(n, n)) = q$  for all  $n$  and that all minimal colorings of  $H_q(n, n)$  are coordinate colorings, meaning the color of every vertex is assigned as its value in a fixed coordinate. For  $d < n$ , we can color  $H_q(n, d)$  in a similar way using  $n - d + 1$  coordinates. Moreover, the results from Rouayheb et al. show that these coordinate colorings are minimal colorings for all parameters for which the authors were able to determine the chromatic number of the graph. However, it is easily demonstrated that these coordinate colorings are not the only minimal colorings when  $d < n$ .

In this talk, we will present an analog of the result of Greenwell/Lovász which holds for the case  $q = 2$  and  $d = n - 1$ . (Received September 20, 2016)

1125-05-2500 **Katie Anders\*** (kanders@uttyler.edu) and **Kassie Archer**. *Unimodal rooted forests.*

We say that an unordered rooted labeled forest avoids the pattern  $\pi \in S_n$  if the sequence obtained from the labels along the path from the root to any vertex does not contain a subsequence that is in the same relative order as  $\pi$ . We enumerate the class of forests that avoid  $\{312, 213\}$ , which are the unimodal forests, via a bijection with the set of ordered cycle decompositions of permutations in  $S_n$ . (Received September 20, 2016)

1125-05-2506 **Brent Gorbutt\***, bgorbutt@gmu.edu. *On the equivariant  $K$ -theory of type A Peterson varieties.*

We reprove the Monk formula for the  $S^1$ -equivariant cohomology of type A Peterson varieties proved by Harada-Tymoczko using a formula developed by Goldin-Knutson. We then explore the  $S^1$ -equivariant  $K$ -theory of type A Peterson varieties. (Received September 20, 2016)

1125-05-2533 **Elizabeth Niese\*** (niese@marshall.edu), Department of Mathematics, Marshall University, One John Marshall Drive, Huntington, WV 25755. *A Remmel-Whitney style rule for products of quasisymmetric and symmetric Schur functions.* Preliminary report.

Remmel and Whitney provided an algorithmic method to compute the Littlewood-Richardson coefficients appearing when the product of two Schur functions is expanded in the Schur basis. The quasisymmetric Schur functions,  $QS_\alpha$ , introduced by Haglund, Luoto, Mason, and van Willigenburg provide a basis for the ring of quasisymmetric functions. While products of two quasisymmetric Schur functions do not necessarily expand positively as a sum of quasisymmetric Schur functions, the product of a quasisymmetric Schur function and (symmetric) Schur function does. There is a combinatorial description of the coefficients of this expansion in terms of “Littlewood-Richardson composition tableaux” satisfying certain requirements. In this talk we present an alternate way to obtain these coefficients by algorithmically constructing a set of composition tableaux whose shapes correspond to the coefficients appearing in the product of  $QS_\alpha \cdot s_\lambda$ . (Received September 20, 2016)

1125-05-2577 **Kassie Archer\*** (karcher@uttyler.edu) and **Lindsey-Kay Lauderdale**. *Pattern-avoiding cycles.* Preliminary report.

A permutation  $\pi \in S_n$  avoids the pattern  $\sigma \in S_k$  if there is no subsequence of  $\pi$  in the same relative order as  $\sigma$ . In this talk, we discuss pattern-avoiding permutations which are composed of a single  $n$ -cycle. (Received September 20, 2016)

1125-05-2579 **Stephen M. Gagola III\*** (gagolasm@miamioh.edu), Department of Mathematics, Miami University, Oxford, OH 45056. *Multiplicative properties of partitions of integers.*

Here we give a combinatorial proof of an inequality that was first proven by Christine Bessenrodt and Ken Ono. Bessenrodt and Ono proved that the number of partitions of  $n$ , say  $p(n)$ , satisfies  $p(a)p(b) > p(a+b)$  for  $a, b > 1$

and  $a + b > 9$  by using a result of Lehmer and asked whether a combinatorial proof exists. Here we prove the inequality combinatorially and show that the proof can also be extended to prove the analogous inequality for  $k$ -regular partitions with  $k \geq 2$ . For  $2 \leq k \leq 6$ , these inequalities were first proven to hold for  $k$ -regular partitions by Olivia Beckwith and Christine Bessenrodt using similar methods to the  $p(n)$  case. (Received September 20, 2016)

1125-05-2585 **Bruno Benedetti\*** ([bruno@math.miami.edu](mailto:bruno@math.miami.edu)), Department of Mathematics, 1365 Memorial Drive, Coral Gables, FL 33146. *Local constructions of manifolds*. Preliminary report.

As kids we learned a trick to make paper dices. First we drew a tree of squares; from that, we started to recursively glue together two incident boundary edges; we stopped when all boundary edges had been matched. This idea applies to all dimensions. For example, in dimension three, let us call "Mogami" all triangulated 3-manifolds that can be obtained from a "tree of tetrahedra" (i.e. a triangulation of the 3-ball whose dual graph is a tree), by recursively gluing together two incident boundary triangles. Are all triangulations of the 3-ball Mogami? Or are there some that cannot be obtained this way? (The name is after the physicist who introduced this model in discrete quantum gravity, for asymptotic enumeration purposes; so if not all 3-balls are Mogami, it would be interesting to estimate how many are not...) (Received September 20, 2016)

1125-05-2589 **Joshua D Laison\*** ([jlaison@willamette.edu](mailto:jlaison@willamette.edu)), **Erin M McNicholas** and **Nicole S Seaders**. *Base Size Sets and Determining Sets*.

The determining number or fixing number of a graph  $\Gamma$  is the smallest size of a subset of vertices  $S$  of  $\Gamma$  such that any automorphism of  $\Gamma$  that stabilizes  $S$  stabilizes all of  $\Gamma$ . The determining set  $d(G)$  of a finite group  $G$  is the set of all determining numbers of all finite graphs for which  $G$  is the automorphism group.

We can think of the determining set of  $G$  as a parameter determined by the faithful actions of  $G$  on vertex sets of graphs. In this talk we compare this set to the base size set  $b(G)$ , the same parameter extended to all faithful actions of  $G$  on finite sets, where the action is no longer determined by preserving edges of the graph. We find groups  $G$  for which these parameters are different. (Received September 20, 2016)

1125-05-2649 **Timothy Alland\*** ([tim.alland@okstate.edu](mailto:tim.alland@okstate.edu)), 4817 River View Dr, Fort Worth, TX 76132, and **Edward Richmond**. *A Pattern Avoidance Criteria for Parabolic Fiber Bundle Structures of Schubert Varieties*. Preliminary report.

We prove that a Schubert variety has a fiber bundle structure if and only if the corresponding permutation avoids the split patterns  $3|12$  and  $23|1$  in a certain way. To do this, we find the necessary conditions for a permutation to have a Billey-Postnikov (BP) decomposition. Continuing, we prove that a Schubert variety has a complete parabolic bundle structure if and only if the corresponding permutation avoids patterns  $3412$ ,  $52341$ , and  $635241$ . This combinatorial condition gives a direct way to determine if a Schubert variety has a fiber bundle structure as compared to doing so via the Schubert variety's intersection conditions. (Received September 20, 2016)

1125-05-2661 **Pakawut Jiradilok\***, Mathematics Department, Harvard University, 1 Oxford Street, Cambridge, MA 02138. *Reconstructing Partitions from their Multisets of  $k$ -Minors*.

For non-negative integers  $n$  and  $k$  with  $n \geq k$ , a  $k$ -minor of a partition  $\lambda = [\lambda_1, \lambda_2, \dots]$  of  $n$  is a partition  $\mu = [\mu_1, \mu_2, \dots]$  of  $n - k$  such that  $\mu_i \leq \lambda_i$  for all  $i$ . The multiset  $\widehat{M}_k(\lambda)$  of  $k$ -minors of  $\lambda$  is defined as the multiset of  $k$ -minors  $\mu$  with multiplicity of  $\mu$  equal to the number of standard Young tableaux of skew shape  $\lambda/\mu$ . We show that there exists a function  $G(n)$  such that the partitions of  $n$  can be reconstructed from their multisets of  $k$ -minors if and only if  $k \leq G(n)$ . Furthermore, we prove that  $\lim_{n \rightarrow \infty} G(n)/n = 1$  with  $n - G(n) = O(n/\log n)$ . For a minor  $\mu$  of the partition  $\lambda$ , we study the excitation factor  $E_\mu(\lambda)$  which appears as a crucial part in Naruse's Skew-Shape Hook Length Formula. We observe that certain excitation factors of  $\lambda$  can be expressed as a  $\mathbb{Q}[k]$ -linear combination of the elementary symmetric polynomials of the hook lengths in the first row of  $\lambda$  where  $k = \lambda_1$  is the number of cells in the first row of  $\lambda$ . (Received September 20, 2016)

1125-05-2671 **Daniela Ferrero\*** ([dferrero@txstate.edu](mailto:dferrero@txstate.edu)), Department of Mathematics, San Marcos, TX 78666, and **Leslie Hogben**, **Franklin H. J. Kenter** and **Michael Young**. *The relationship between  $k$ -forcing and  $k$ -power domination*.

Let  $G = (V, E)$  be a graph and  $k$  a positive integer. For a set  $S \subseteq V$ , recursively define a family of sets,  $S^{(i,k)}$ ,  $i \geq 0$  by  $S^{(0,k)} = S$ ,  $S^{(1,k)} = N[S]$ , and for each  $i \geq 1$ ,  $S^{(i+1,k)} = S^{(i,k)} \cup \{w : \exists v \in S^{i,k} \text{ such that } |N(v) \setminus S^{i,k}| \leq k \text{ and } w \in N(v) \setminus S^{i,k}\}$ . The set  $S$  is a  $k$ -power dominating set of a graph  $G$  if there is an integer  $\ell$  such that  $S^{(\ell,k)} = V$  and the minimum integer  $\ell$  such that  $S^{(\ell,k)} = V$  is the  $k$ -power propagation time for  $S$  in  $G$ .

Analogously, associate with  $S$  another family of sets,  $B^{(i,k)}$ ,  $i \geq 0$  defined by  $B = B^{(0,k)}$  and for each  $i \geq 0$ :  $B^{(i+1,k)} = \{w : \exists v \in B^{(i,k)}, \text{ such that } |N(v) \setminus B^{(i,k)}| \leq k \text{ and } w \in N(v) \setminus B^{(i,k)}\}$ . The set  $S$  is a  $k$ -forcing

set of  $G$  if there is an integer  $\ell$  such that  $S^{(\ell,k)} = V$  and the minimum integer  $\ell$  such that  $S^{(\ell,k)} = V$  is the *k-propagation time* for  $S$  in  $G$ .

We show how methods and techniques used to study  $k$ -power domination transfer to the study of  $k$ -forcing and vice versa. (Received September 20, 2016)

1125-05-2785 **Dewey Taylor\*** (dttaylor2@vcu.edu). *Undergraduate research projects in graph theory.* Graph theory is an area of mathematics rich with open and solvable research problems that require only minimal background knowledge. Even freshmen and sophomores can often begin working on open questions without having a formal course in graph theory. In particular, this is true for the theory of associative graph products. This talk will provide examples of past successful undergraduate research projects as well as highlight several open problems involving graph products. (Received September 20, 2016)

1125-05-2788 **Mark Kempton\*** (mkempton@cmsa.fas.harvard.edu). *Quantum state transfer on graphs with potential.* Preliminary report.

Given a graph, we can describe the evolution of the quantum state of a particle moving along the edges of the graph. We say the perfect state transfer occurs if, given a particle starting at a vertex, there is a time at which the particle is at another single vertex with probability one. Considerable research has been done recently studying perfect state transfer on graphs, and it seems to be a rather rare phenomenon, and constructing examples is quite difficult. We study the effects of adding a potential to the vertices of the graph—that is, a function that assigns an amount of energy to each vertex of the graph. We show, in particular, that there are examples of graphs where perfect state transfer does not occur, but where adding a potential makes it possible. We also study the case of paths. Our techniques involve studying how the spectrum of the adjacency matrix of a graph is affected by adding a diagonal matrix. (Received September 20, 2016)

1125-05-2851 **Stephen J Young\*** (stephen.young@pnnl.gov), **Tobias Hagge**, **Patrick Mackey**, **Kathleen Nowak** and **Jennifer Webster.** *Not all communities are far apart: Incorporating community distance into community detection.* Preliminary report.

Recently Mendel and Naor, Dumitriu and Radcliffe, and Radcliffe and Williamson have generalized the idea of associated eigenvalues and eigenvectors to a graph to a non-linear (non-Euclidean) setting. We combine their ideas with the well studied techniques of spectral community detection to develop a new non-linear means of community detection. These techniques naturally incorporate distance between communities and can give insight into the relationship between various communities present in a network. (Received September 20, 2016)

1125-05-2855 **Ricardo Diaz**, **Quang-Nhat Le\*** (qnhatle@math.brown.edu) and **Sinai Robins.** *Counting lattice points with solid-angle weights in irrational polytopes.*

Counting the number of lattice points inside a polytope is a classical enumerative problem with far-reaching applications in number theory, algebraic geometry, combinatorics, and more. This lattice-point count and its variants fall into the realm of the Ehrhart theory of lattice-point enumerating functions of polytopes. One of the most well-known variants is Macdonald's solid-angle sum which counts lattice points in a polytope, with solid-angle weights.

Previously, the Ehrhart theory was studied mostly in the case of integer dilates of rational polytopes. In a joint work with Ricardo Diaz and Sinai Robins, we use the techniques of Fourier analysis to study the solid-angle sums in the general case of real dilates of real polytopes. We also obtain a closed form for the codimension-1 coefficient of the solid-angle sum; the leading coefficient is trivially the volume of the given polytope. (Received September 20, 2016)

1125-05-2872 **Nathan H Reff\*** (nreff@brockport.edu), The College at Brockport, State University of New York, Brockport, NY 14420. *Oriented Gain Graphs, Line Graphs and Eigenvalues.*

A theory of orientation on gain graphs (voltage graphs) is developed to generalize the notion of orientation on graphs and signed graphs. Using this orientation scheme, the line graph of a gain graph is studied. For a particular family of gain graphs with complex units, matrix properties are established. As with graphs and signed graphs, there is a relationship between the incidence matrix of a complex unit gain graph and the adjacency matrix of the line graph. (Received September 20, 2016)

1125-05-2900 **Taylor McMillan\*** (mcmi9872@bears.unco.edu) and **Oscar Levin** (oscar.levin@unco.edu). *Computing Odd Graceful Labelings in Noncomputable Graphs.*

A *graceful labeling* of a graph  $G = (V, E)$  is an injection  $\lambda : V \rightarrow \{0, 1, 2, \dots, |E|\}$  such that the induced edge label,  $|\lambda(v_i) - \lambda(v_j)|$  for  $\{v_i, v_j\} \in E$ , is distinct for all edges. This definition can be extended to countably infinite graphs. We explore the effective content of a result of Chawathe and Krishna about *odd* graceful labelings

(the induced labels must be distinct odd numbers) of bipartite graphs, working in the context of computability theory. We will show that there is a connected, locally finite, bipartite, computably enumerable graph that has an odd graceful labeling, but no such computable labeling. (Received September 20, 2016)

1125-05-2924 **Arthur F Diep-Nguyen\*** (diepnguy@bc.edu), 2915 Somerset Place, San Marino, CA 91108, and **Dylan King**. *Strategies for Weak Cop Number on Tilings of the Plane*. Preliminary report.

Traditionally, Cops and Robbers is a pursuit game played on a finite, connected graph.  $k$  cops and a single robber are placed on the vertices of a graph and take turns moving to adjacent vertices. If a cop occupies the same vertex as the robber, then the cops win; if the robber perpetually avoids the cops, then he wins. Much of the literature focuses on determining the cop number of a graph; the minimum number of cops needed to always capture the robber. Our research focuses on the infinite analogue to the traditional Cops and Robbers game, which involves infinite graphs and the weak cop number: the minimum number of cops to prevent the robber from returning to the same vertex infinitely often. In particular, we attempt to determine the weak cop number of infinite, locally-finite, connected, planar graphs using a combination of strategies: two of them from others' work and one developed by us. (Received September 20, 2016)

1125-05-2963 **Louis Deaett\*** (louis.deaett@quinnipiac.edu) and **Alexander Hutman** (alexander.hutman@quinnipiac.edu). *Faster computation of zero forcing parameters*. Preliminary report.

Given a finite graph with some of its vertices "filled," we can iteratively apply a simple rule: When a filled vertex has exactly one unfilled neighbor, that neighbor becomes filled. The *zero forcing number* of a graph is the size of a smallest set of vertices that, when initially filled, results in this process ending with all vertices filled. This number is (in a provable sense) hard to compute in general. We nevertheless discuss a strategy that allows its value to be computed more efficiently than via a brute-force search. In particular, we employ standard shortest-path algorithms in a sort of "metagraph" derived from the original. We also explore possible optimizations suggested by this strategy. (Received September 20, 2016)

1125-05-2972 **Vindya Bhat\*** (vbhat@cims.nyu.edu), 251 Mercer Street Office, Warren Weaver Hall Office 722, Courant Institute of Mathematical Sciences, New York, NY 10012, and **Jaroslav Nešetřil**, **Christian Reiher** and **Vojtěch Rödl**. *A Ramsey class for Steiner systems*.

We construct a Ramsey class whose objects are Steiner systems. In contrast to the situation with general  $r$ -uniform hypergraphs, it turns out that simply putting linear orders on their sets of vertices is not enough for this purpose: one also has to strengthen the notion of subsystems used from "induced subsystems" to something we call "strongly induced subsystems".

Moreover we study the Ramsey properties of other classes of Steiner systems obtained from this class by either forgetting the order or by working with the usual notion of subsystems. This leads to a perhaps surprising induced Ramsey theorem in which designs get colored. (Received September 20, 2016)

1125-05-2986 **Julie Anne Bowman** (s707410@sbuniv.edu) and **Nicholas Lindell\*** (n.lindell@uga.edu). *Games on Graphs: Seepage!* Preliminary report.

The game of Seepage, first described by Clarke, et al. in 2009, is played by two players, Sludge,  $S$ , and Green,  $G$ , on a directed acyclic graph with a single source and several sinks.  $S$  and  $G$  alternately claim vertices of the graph, which subsequently cannot be claimed by the opponent. Sludge begins by claiming, or 'contaminating', the source. Afterwards, in sequence,  $G$  can claim, or 'protect', any vertex on the graph, while  $S$  can contaminate any vertex adjacent to an already contaminated vertex.  $S$  is said to win if any sink is contaminated; otherwise,  $G$  wins. The generalized version of this game allows  $G$  to claim multiple vertices each turn. The *green number* of a graph  $H$ ,  $gr(H)$ , is defined to be the minimum  $k$  such that  $G$  can guarantee victory with at most  $k$  moves on each turn. Graphs are called *green-win* if  $gr(H) = 1$ , *sludge-win* if  $gr(H) > 1$  and  $k$ -*green-win* if  $gr(H) = k$ . In their paper, Clarke, et al. characterized green-win and  $k$ -green-win rooted trees  $T$ , providing a polynomial time algorithm for determining if  $gr(T) = k$ . We introduce a more generalized algorithm that determines if  $gr(H) = k$  for any directed acyclic graph, as well as methods to reduce the number of vertices and edges of a graph without changing the green number. (Received September 20, 2016)

1125-05-2996 **JiYoon Jung\*** (jungj@marshall.edu), **Suhyung An** and **Sangwook Kim**. *Enumeration of  $k$ -Fuss-Catalan paths and  $(k, r)$ -Fuss-Schröder paths*. Preliminary report.

In this paper we provide three results involving  $k$ -Fuss-Catalan paths and  $(k, r)$ -Fuss-Schröder paths. First, we enumerate the number of  $k$ -Fuss-Catalan paths of type  $\lambda$ . J. H. Przytycki and A. S. Sikora studied  $k$ -Fuss-Catalan paths of length  $n$ , and we extend the study to  $k$ -Fuss-Catalan paths with type  $\lambda$  and  $m$  connected

components. By taking the sum over  $m$  we get the number of  $k$ -Fuss-Catalan of type  $\lambda$ . Second, we enumerate the number of  $(k, r)$ -Fuss-Schröder paths of type  $\lambda$ . Y. Park and S. Kim studied Schröder paths with type  $\lambda$  and  $m$  connected components. Generalizing the results to  $(k, r)$ -Fuss-Schröder paths we give a combinatorial interpretation for the number of small  $(k, r)$ -Fuss-Schröder paths of type  $\lambda$  by using Chung-Feller style. We also give explicit formula for the number of large  $(k, r)$ -Fuss-Schröder paths of type  $\lambda$  with  $d$  diagonal steps touching the line  $y = kx$ , and a description for the number of all large  $(k, r)$ -Fuss-Schröder paths of type  $\lambda$ . Finally, we find two sets of sparse noncrossing partitions of  $[2(k+1)n+1]$  which are in bijection with the set of all small (respectively, large)  $(k, r)$ -Fuss-Schröder paths of type  $\lambda$ . (Received September 20, 2016)

1125-05-3005 **Charles D. Burnette\*** (cdb72@drexel.edu), Drexel University, Department of Mathematics, 3141 Chestnut Street, Philadelphia, PA 19104, and **Eric Schmutz** (eric.jonathan.schmutz@drexel.edu), Drexel University, Department of Mathematics, 3141 Chestnut Street, Philadelphia, PA 19104. *Involution Factorizations of Random Permutations Chosen from the Ewens Distribution*.

Given a permutation  $\sigma$  of  $[n]$ , let  $\mathbf{N}_n(\sigma)$  denote the number of ways to write  $\sigma$  as a product of two involutions of  $[n]$ . If we endow  $S_n$  with the Ewens measure, then the random variables  $\mathbf{N}_n$  are asymptotically lognormal. The proof is based upon the observation that, for most permutations  $\sigma$ ,  $\mathbf{N}_n(\sigma)$  is well-approximated by  $\mathbf{B}_n(\sigma)$ , the product of the cycle lengths of  $\sigma$ . Asymptotic lognormality of  $\mathbf{N}_n$  can therefore be deduced from Erdős and Turán's theorem that  $\mathbf{B}_n$  is itself asymptotically lognormal. (Received September 20, 2016)

1125-05-3006 **Conner Mattes\***, 1500 Illinois St., Golden, CO 80401, and **Zachary Chaney**, 1500 Illinois St., Golden, CO. *Triangular Ramsey Numbers*. Preliminary report.

We describe the combinatorial game of Mines and use it to introduce the notion of a triangular Ramsey number. We give exact values for some of the triangular Ramsey numbers and provide a lower bound using a modified version of the probabilistic method pioneered by Erdős. (Received September 20, 2016)

1125-05-3027 **Michelle L. Wachs\*** (wachs@math.miami.edu). *Homology of Partition Posets*. Preliminary report.

We give a survey of results obtained over the years, on (co)homology of various posets related to the partition lattice, focussing on bases for (co)homology and representations of the symmetric group on (co)homology. Examples include the partition lattice itself, the 1 mod  $k$  partition poset, and the recently studied weighted partition poset. (Received September 20, 2016)

1125-05-3030 **Gexin Yu** and **C. Santana Afton\*** (csafton@email.wm.edu), CSU 0433, 110 Sadler Center, Williamsburg, VA 23185. *Routing Permutations on Hypercube Graphs*. Preliminary report.

Let  $G$  be a graph, and let  $p_v$  be a pebble placed upon the vertex  $v$ . We define  $\pi$  as a permutation on the vertices of  $G$ , where we denote  $p_{\pi(v)}$  to be the destination for  $p_v$ . Our goal is to move each pebble to its destination. For each step, we select a disjoint set of edges and swap the pebbles that lie on each edge. The minimum number of steps to achieve  $\pi$  is denoted  $rt(G, \pi)$ , the routing number for  $\pi$ . The maximum routing number for any  $\pi$  is denoted as  $rt(G)$ . We will focus primarily on  $rt(Q_n)$ , where  $Q_n$  is the  $n$ -dimensional hypercube.

Recently, Li, Lu, and Yang showed that  $n+1 \leq rt(Q_n) \leq 2n-2$ , proving the conjecture of the lower bound proposed by Alon, Chung, and Graham. They also determined  $rt(Q_3)$  computationally. In this talk we will demonstrate an alternative proof to the lower bound, and an algorithm to find  $rt(Q_3)$  with negligible computation. (Received September 20, 2016)

1125-05-3106 **Amir Jafari\*** (ajafari@sharif.ir) and **Sharareh Alipour**. *On Chromatic Number of Generalized Kneser Graphs*. Preliminary report.

For integers  $n, k$  and  $i$ , the generalized Kneser graph  $K(n, k, i)$ , is a graph whose vertices are subsets of size  $k$  of  $\{1, 2, \dots, n\}$  and two vertices  $F$  and  $F'$  are connected if and only if their intersection has less than  $i$  elements. In this paper we study the chromatic number of this graph. Some new bounds and properties for this chromatic number are derived. (Received September 21, 2016)

## 06 ► Order, lattices, ordered algebraic structures

1125-06-1302 **Franck Jedrzejewski\*** ([franckjed@gmail.com](mailto:franckjed@gmail.com)), French Atomic Energy Commission (CEA), INSTN, 91191 Saclay, France. *Associahedra, combinatorial block designs and related structures.*

The purpose of this talk is to review some representations of low dimension structures such as associahedra (Stasheff polytope), permutohedra, and Tamari lattices and to study their adequacy to represent combinatorial objects. In particular, we look at some combinatorial block designs such that the number of blocks is a Catalan number. We investigate the relationship with the symmetric group, planar binary trees and the Loday's dendriform algebra. (Received September 16, 2016)

1125-06-1575 **Jason R. Elsinger\*** ([jelsinger@shc.edu](mailto:jelsinger@shc.edu)). *Quantum dimensions and fusion products for irreducible modules of orbifold lattice vertex algebras under an isometry of order two: a step toward the general case.*

Every isometry  $\sigma$  of a positive-definite even lattice  $Q$  can be lifted to an automorphism of the lattice vertex algebra  $V_Q$ . An important problem in vertex algebra theory and conformal field theory is to classify the representations of the  $\sigma$ -invariant subalgebra  $V_Q^\sigma$  of  $V_Q$ , known as an orbifold. It is a long-standing conjecture that all irreducible  $V_Q^\sigma$ -modules are obtained by restriction from twisted or untwisted  $V_Q$ -modules. Under certain assumptions, this conjecture has been proved recently in a series of papers by M. Miyamoto. In the case when  $\sigma$  is an isometry of  $Q$  of order two, we have classified and constructed all irreducible modules of the orbifold vertex algebra  $V_Q^\sigma$  and identified them as submodules of twisted or untwisted  $V_Q$ -modules. Here we calculate their quantum dimensions and fusion rules, and investigate the general order case. The example where  $Q$  is a direct sum of two copies of the root lattice  $A_2$  and  $\sigma$  is the permutation automorphism is presented in detail. (Received September 18, 2016)

1125-06-2738 **Christina Eubanks Turner**, Loyola Marymount University, and **Aihua Li\***, [lia@mail.montclair.edu](mailto:lia@mail.montclair.edu). *Graphical Properties of the Partially Ordered Set Derived from  $\text{Spec}(\mathbb{Z}[x])$ .*

Consider  $\text{GZ} = \text{Spec}(\mathbb{Z}[x])$  of nonzero prime ideals of  $\mathbb{Z}[x]$  as a partially ordered set by inclusion. We further view  $\text{GZ}$  as an infinite bipartite graph with the prime ideals as the vertices and the inclusion relations as the edges. In this paper, we investigate fundamental graph theoretic properties of  $\text{GZ}$ . In particular, we describe the diameter, circumference, girth, radius, eccentricity, global and local connectivity, and cliques of  $\text{GZ}$ . The complement of  $\text{GZ}$  is investigated as well. (Received September 20, 2016)

1125-06-2973 **Harry Altman\*** ([haltman@umich.edu](mailto:haltman@umich.edu)). *The Surreal Exponential and Order-Reversing Maps between Well-Ordered Set.* Preliminary report.

In this talk we will discuss, for given ordinals  $\alpha$  and  $\beta$ , how one may compute the maximum extending ordinal of the poset of all order-reversing functions from  $\alpha$  to  $\beta$  – a feat that has not actually been performed for that many other well partial orders. We focus on the case where we restrict to functions which are eventually zero, and observe that by choosing  $\alpha$  to be  $\omega$ , one may provide an order-theoretic interpretation for the surreal exponential as applied to limit ordinals. (Received September 20, 2016)

## 08 ► General algebraic systems

1125-08-1634 **Jeffrey O Wand\*** ([wand@gonzaga.edu](mailto:wand@gonzaga.edu)). *Constructing Demazure Flags and the Combinatorics Involved.*

In this talk we will look into the combinatorics motivated by studying the representation theory of Lie algebras. More specifically, we will be looking at a family of modules for the current algebra  $\mathfrak{sl}_n[t] = \mathfrak{sl}_n \otimes \mathbb{C}[t]$ , where  $\mathfrak{sl}_n$  is the space of complex  $n \times n$  matrices whose trace is zero and  $\mathbb{C}[t]$  is the space of polynomials with complex coefficients. The family of interest is the Demazure modules. The level  $\ell$  Demazure module is a cyclic module for  $\mathfrak{sl}_n[t]$  that is generated by a highest weight vector with certain defining relations. Our goal is to construct an explicit level 2 Demazure filtration of the level 1 Demazure module, something that was proven to exist by Naoi in 2011. In constructing our level 2 Demazure filtration interesting combinatorics arise. (Received September 18, 2016)

## 11 ► Number theory

1125-11-12 **Richard Taylor\***, Institute for Advanced Study, Princeton, NJ 08540. *Galois groups and locally symmetric spaces.*

Langlands proposed an extraordinary correspondence between representations of Galois groups and automorphic forms, which has deep, and completely unexpected, implications for the study of both objects. The simplest special case is Gauss' law of quadratic reciprocity. In the so called 'regular, self-dual' case much progress has been made in the roughly 40 years since Langlands made these conjectures. In this talk I will discuss recent progress in regular, but non-self-dual case. In this case the automorphic forms in question can be realized as cohomology classes for arithmetic locally symmetric spaces, i.e. quotients of symmetric spaces by discrete groups. Thus instead of the Langlands correspondence being a relationship between algebra and analysis, it can be thought of as a relationship between algebra and topology. This realization of the Langlands correspondence is in many ways more concrete. It also admits to generalizations not envisioned by Langlands, for instance relating mod  $p$  Galois representations with mod  $p$  cohomology classes. In this talk I will describe the expected Langlands correspondence in the setting of locally symmetric spaces. I will try both to present the general picture and to give numerical examples. I will also describe recent theorems of Lan, Harris, Thorne and myself on the Langlands correspondence in this setting and startling progress of Peter Scholze in the mod  $p$  case. I will not attempt to describe the proofs.

(Received May 22, 2016)

1125-11-54 **Ricardo Conceicao\***, 300 North Washington Street, Glatfelter Hall, Campus Box 402, Gettysburg, PA 17325, and **Rodrigo Gondim** and **Miguel Rodriguez**. *On a Frobenius problem for polynomials.*

We extend the famous diophantine Frobenius problem to the setting of polynomials over a field  $k$ . Similar to the classical problem, we show that the  $n = 2$  case of the Frobenius problem for polynomials is easy to solve. In addition, we translate a few results from the Frobenius problem over  $\mathbb{Z}$  to  $k[t]$ . When  $k$  is a finite field, we discuss some striking contrasts between the classical and the polynomial case, and mention a few ideas for future research. (Received June 27, 2016)

1125-11-55 **David Zureick-Brown\*** ([dzb@mathcs.emory.edu](mailto:dzb@mathcs.emory.edu)), 400 Dowman Drive, Atlanta, GA 30322, and **Eric Katz** and **Joe Rabinoff**. *Tropical geometry and uniformity of rational points.*

I will discuss recent progress on the uniformity conjecture – the existence of a universal bound on the number of rational points on curves of a fixed genus – and explain new ideas from tropical and non-archimedean analytic geometry which lead to a partial proof of the uniformity conjecture. (Received June 27, 2016)

1125-11-62 **Ernest Hunter Brooks\*** ([ernest.brooks@epfl.ch](mailto:ernest.brooks@epfl.ch)), EPFL SB Mathgeom GRJet, MA B3 494 (Bâtiment MA), 1015 Lausanne, Switzerland. *Unnatural Hecke operators: The local arithmetic of special cycles on Unitary Shimura Varieties.*

Special cycles on unitary Shimura varieties occurring in the context of the Gan–Gross–Prasad conjectures, which arise from considering embeddings of unitary groups, are defined over abelian extensions of imaginary quadratic fields. A necessary ingredient to develop an Iwasawa theory of special cycles is a “vertical distribution relation,” i.e., a relation between the natural Galois and Hecke actions on these cycles over the anticyclotomic  $\mathbb{Z}_p$ -extension. We establish these relations for the case of 1-cycles on a 3-fold using local methods, by reducing the problem to the combinatorics of some bizarre operators on Bruhat-Tits buildings.

This is joint work with Réda Boumasmoud and Dimitar Jetchev. (Received July 03, 2016)

1125-11-82 **John Greene\*** ([jgreene@d.umn.edu](mailto:jgreene@d.umn.edu)), Department of Mathematics and Statistics, 1117 University Drive, University of Minnesota Duluth, Duluth, MN 55812, and **Jesse Schmieg**, Department of Mathematics and Statistics, University of Minnesota Duluth, Duluth, MN 55812. *Conjugates and reduced surds for non simple continued fractions.* Preliminary report.

A quadratic surd,  $x$ , is called *reduced* if  $x > 1$  and  $-1 < \bar{x} < 0$ . It is well known that a simple continued fraction is purely periodic if and only if it is a reduced quadratic surd. Maxwell Anselm and Steven Weintraub investigated the generalization of a simple continued fraction where the “numerator” 1 was replaced by an arbitrary positive integer  $z$ . In this case, a quadratic surd only had a purely periodic expansion when  $x > z$  and  $-1 < \bar{x} < 0$ , and they called such an  $x$   $z$ -reduced. Here, we replace the “numerator” with an arbitrary real number  $z \geq 1$ , usually a rational number. In this case, it is possible for a rational number  $x$  to have a purely periodic expansion, even

though  $\bar{x}$  no longer has an obvious meaning. We give a definition for the conjugate and discuss its strengths and weaknesses. (Received July 13, 2016)

1125-11-87 **Gerry Myerson\*** ([gerry.myerson@mq.edu.au](mailto:gerry.myerson@mq.edu.au)), Mathematics, Macquarie University, NSW 2109, Australia.  $\sqrt{2}$ , *a musical exploration*.

An application of the work of Jacob Jacobs and Sholom Secunda. (Received July 17, 2016)

1125-11-116 **Barry R Smith\*** ([barsmith@lvc.edu](mailto:barsmith@lvc.edu)), 101 N. College Ave, Annville, PA 17003. *Indefinite forms, continued fractions, and binary necklaces*. Preliminary report.

We will give a construction that uses a continued fraction expansion to attach a binary necklace to each class of indefinite binary quadratic forms. We give three applications of the necklace invariant. (1) We show that two types of necklace symmetry translate into natural statements about the situation of the corresponding forms in the class group. (2) We describe how moving down the famous Markoff tree of forms corresponds to concatenation of the corresponding strings. (3) We classify forms of the types  $ax^2 + (ab+2)xy + by^2$  and  $ax^2 + (2ab+1)xy + by^2$  has having orders 1, 2, 3, 4, or  $\geq 5$  in the class group. In particular, a form has order 4 if and only if a continued fraction built from  $a$  and  $b$  has a very specific pattern of partial quotients. (Received August 01, 2016)

1125-11-118 **Kylie Hess** ([hessko@rose-hulman.edu](mailto:hessko@rose-hulman.edu)), **Jeremy Rouse** ([rouseja@wfu.edu](mailto:rouseja@wfu.edu)), **Emily Stamm\*** ([emstamm@vassar.edu](mailto:emstamm@vassar.edu)) and **Terrin Warren** ([terrinwarren@gmail.com](mailto:terrinwarren@gmail.com)). *When is  $a^n + 1$  the sum of two squares?* Preliminary report.

Using Fermat's two squares theorem and properties of cyclotomic polynomials, we prove assertions about when numbers of the form  $a^n + 1$  can be expressed as the sum of two integer squares. We prove that  $a^n + 1$  is the sum of two squares for all  $n \in \mathbb{N}$  if and only if  $a$  is a perfect square. We also prove that for  $a \equiv 0, 1, 2 \pmod{4}$ , if  $a^n + 1$  is the sum of two squares, then  $a^\delta + 1$  is the sum of two squares for all  $\delta|n$ ,  $\delta > 1$ . Using Aurifeuillian factorization, we show that if  $a$  is a prime and  $a \equiv 1 \pmod{4}$ , then there are either zero or infinitely many odd  $n$  such that  $a^n + 1$  is the sum of two squares. When  $a \equiv 3 \pmod{4}$ , we define  $m$  to be the least positive integer such that  $\frac{a+1}{m}$  is the sum of two squares, and prove that if  $a^n + 1$  is the sum of two squares for any odd integer  $n$ , then  $a^m + 1$  and  $\frac{n}{m}$  are both sums of two squares. (Received August 05, 2016)

1125-11-125 **Madeleine Barowsky\*** ([mbarowsk@wellesley.edu](mailto:mbarowsk@wellesley.edu)), **William Damron**, **Andres Mejia**, **Frederick Saia**, **Nolan Schock** and **Katherine Thompson**. *Classically Integral Quadratic Forms Excepting at Most Two Values*.

The study of universal quadratic forms has long been of interest to number theorists. Quadratic forms which are "almost universal," failing to represent some finite set of numbers, are less explored, though Halmos (1938) published a list of diagonal quaternary forms that except just one value. We seek to determine for which  $\{m, n\}$  there exists a quadratic form excepting precisely those values. Generalizing the techniques developed by Bhargava and Hanke (2005) for universal quadratic forms, we use escalator lattices, modular forms, and computational methods to answer this question for quaternary forms. We develop new strategies to handle forms of higher dimensions, yielding an enumeration of and proofs for the 73 possible pairs that a classically integral, positive definite quadratic form may except. (Received August 02, 2016)

1125-11-163 **David Hansen\***, Columbia University, Department of Mathematics, 2990 Broadway, New York, NY 10027. *Geometry and cohomology of local Shimura varieties*.

I'll describe some new results on the geometry and cohomology of local Shimura varieties and (more generally) moduli spaces of mixed-characteristic local shtukas as defined by Scholze. This is partially joint work with Jared Weinstein. (Received August 06, 2016)

1125-11-166 **Amy Feaver**, **Anna Haensch\*** ([haenscha@duq.edu](mailto:haenscha@duq.edu)), **Jingbo Liu** and **Gabi Nebe**. *Kneser-Hecke operators for codes over finite chain rings*.

There is a well known correspondence between lattices and codes. Via this classical construction, the weight enumerator for codes corresponds to the theta series for lattices, where one counts the number of codewords by composition, and the other counts the number of vectors in a lattice of a certain length. In this talk, we will explore how some of the attendant machinery of theta series are born out in this correspondence. In particular, we will consider the Kneser-Hecke operator, a code theoretic analogue of the classical Hecke operator. (Received August 06, 2016)

1125-11-235 **Annie S. Chen\*** ([annieboulder@gmail.com](mailto:annieboulder@gmail.com)), **T. Alden Gassert** ([thomas.alden.gassert@gmail.com](mailto:thomas.alden.gassert@gmail.com)) and **Katherine E. Stange** ([kstange@math.colorado.edu](mailto:kstange@math.colorado.edu)). *Index Divisibility in Dynamical Sequences and Cyclic Orbits Modulo  $p$* . Preliminary report.

Let  $\phi(x) = x^d + c$  be an integral polynomial of degree at least 2, and consider the sequence  $(\phi^n(0))_{n=0}^\infty$ , which is the orbit of 0 under iteration by  $\phi$ . Let  $D_{d,c}$  denote the set of positive integers  $n$  for which  $n \mid \phi^n(0)$ . We give a characterization of  $D_{d,c}$  in terms of a directed graph and describe a number of its properties, including its cardinality and the primes contained therein. In particular, we study the question of which primes  $p$  have the property that the orbit of 0 is a single  $p$ -cycle modulo  $p$ . We show that the set of such primes is finite when  $d$  is even, and conjecture that it is infinite when  $d$  is odd. (Received August 16, 2016)

1125-11-258 **Yunqing Tang\*** ([yqtang@math.harvard.edu](mailto:yqtang@math.harvard.edu)). *Cycles in the de Rham cohomology of abelian varieties over number fields*.

In his 1982 paper, Ogus defined a class of cycles in the de Rham cohomology of smooth proper varieties over number fields. In the case of abelian varieties, this class includes all the Hodge cycles by the work of Deligne, Ogus and Blasius. Ogus predicted that all such cycles are Hodge. In this talk, I will first introduce Ogus' conjecture as a crystalline analogue of Mumford–Tate conjecture and explain how a theorem of Bost on algebraic foliation is related. After this, I will discuss the proof of Ogus' conjecture for some families of abelian varieties under the assumption that the cycles lie in the Betti cohomology with real coefficients. (Received August 19, 2016)

1125-11-265 **Lubjana Beshaj\*** ([beshaj@math.utexas.edu](mailto:beshaj@math.utexas.edu)), Austin, TX, and **Tony Shaska** ([shaska@oakland.edu](mailto:shaska@oakland.edu)), Rochester, MI. *The number of binary forms of bounded moduli height*. Preliminary report.

We introduce the minimal absolute height and the moduli height for integral binary forms and show that for any binary form of degree  $d \geq 3$ , the moduli height is less than or equal to  $c$  times the minimal absolute height, where  $c$  is some constant depending on  $d$ . We present some computational results on the number of sextics (up to equivalence) with bounded moduli height and some conjectures about such number when the moduli height grows to infinity. (Received August 21, 2016)

1125-11-268 **Jonathan M. Gerhard\*** ([gerha2jm@dukes.jmu.edu](mailto:gerha2jm@dukes.jmu.edu)). *An exact product formula for abelian threefolds*. Preliminary report.

Let  $f$  be the characteristic polynomial of Frobenius of an abelian variety of dimension 3 over a finite field; we use  $f$  to relate three seemingly disjoint objects. First, we consider the factorizations of primes in  $\text{Split}(f)$ , a degree 6 number field  $K$ . Second, we use a parameterization of Shinoda (1980) to describe certain conjugacy classes of the matrix group  $\text{GSp}_6(\mathbb{F}_q)$ . Our main result (following Gekeler (2003) and Achter and Williams (2015)) is a product formula relating the class number of  $K$  to the relative densities of conjugacy classes of  $\text{GSp}_6(\mathbb{F}_q)$ . Finally, we give a (conjectural) application of our formula to the size of isogeny classes of abelian threefolds. (Received August 21, 2016)

1125-11-273 **Alicia Marino\*** ([amarino@wesleyan.edu](mailto:amarino@wesleyan.edu)) and **Wai Kiu Chan** ([wkchan@wesleyan.edu](mailto:wkchan@wesleyan.edu)). *Strictly  $k$ -regular quadratic forms*. Preliminary report.

An integral quadratic form is said to be strictly  $k$ -regular if it primitively represents all quadratic forms of  $k$  variables that are primitively represented by its genus. We show that, for  $k > 1$ , there are finitely many inequivalent positive definite primitive integral quadratic forms of  $k+4$  variables that are strictly  $k$ -regular. Our result extends a recent finiteness result of Earnest-Kim-Meyer (2014) on strictly regular quadratic forms of 4 variables. (Received August 22, 2016)

1125-11-284 **Adrian Barquero-Sanchez**, **Lindsay Cadwallader**, **Olivia Cannon**, **Tyler Genao\*** ([tgenao2013@fau.edu](mailto:tgenao2013@fau.edu)) and **Riad Masri**. *Faltings heights of CM elliptic curves and special gamma values*.

In a 1984 Séminaire Bourbaki article, Deligne evaluates the Faltings height of an elliptic curve with complex multiplication in terms of Euler's Gamma function at rational numbers, under the assumption that the endomorphism ring of the curve is a maximal imaginary quadratic order. In this talk, we describe our results in generalizing this to when the endomorphism ring is not necessarily maximal. (Received August 23, 2016)

1125-11-305 **Alyson Deines, Jenny G. Fuselier, Ling Long, Holly Swisher and Fang-Ting Tu\*** (tu@math.lsu.edu), 303 Locket Hall, Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803. *Generalized Legendre Curves and Quaternionic Multiplication.*

We are going to construct abelian surfaces with quaternionic multiplication from certain generalized Legendre curves

$$y^N = x^i(1-x)^j(1-\lambda x)^j, \quad \lambda \in \mathbb{C}, N, i, j, k \in \mathbb{N}.$$

For a given generalized Legendre curve with parameter  $\lambda$ , we denote  $J_\lambda^{new}$  the primitive part of its Jacobian variety. In this talk, we will give a criterion for  $\text{End}(J_\lambda^{new})$  containing quaternion algebra when  $N = 3, 4, 6$ . (Received August 25, 2016)

1125-11-310 **Thomas Brazelton** (tbrazel1@jhu.edu), **Joshua Harrington** (joshua.harrington@cedarcrest.edu), **Siddarth Kannan\*** (siddarth.kannan@pomona.edu) and **Matthew Litman** (my15470@psu.edu). *On Consecutive Primitive  $n$ th Roots of Unity Modulo  $q$ .*

Given  $n \in \mathbb{N}$ , we study the conditions under which a finite field of prime order  $q$  will have adjacent elements of multiplicative order  $n$ . In particular, we analyze the resultant of the cyclotomic polynomial  $\Phi_n(x)$  with  $\Phi_n(x+1)$ , and exhibit Lucas and Mersenne divisors of this quantity. For  $n \neq 1, 2, 3, 6$ , we prove the existence of a prime  $q_n$  for which there is an element  $\alpha \in \mathbb{Z}_{q_n}$  where  $\alpha$  and  $\alpha + 1$  both have multiplicative order  $n$ . Additionally, we use algebraic norms to set analytic upper bounds on the size and quantity of these primes. (Received August 25, 2016)

1125-11-349 **Ana Caraiani\*** (caraiani@math.uni-bonn.de), Venusbergweg 36, 53115 Bonn, Germany, and **Peter Scholze**. *On torsion in the cohomology of unitary Shimura varieties.*

I will discuss joint work with Peter Scholze on torsion in the cohomology of certain unitary Shimura varieties. The range of degrees that torsion can occur in is restricted by a genericity condition on the corresponding Galois representation. I will discuss how to prove this kind of result using perfectoid Shimura varieties and the geometry of the Hodge-Tate period morphism. (Received August 29, 2016)

1125-11-367 **Emma Previato\*** (ep@bu.edu), Department of Mathematics and Statistics, Boston University, Boston, MA 02215-2411. *Integrality of the Kleinian sigma function and Rankin-Cohen brackets on modular forms.*

Number-theoretic aspects of algebraic curves can be approached by the use of higher-genus Kleinian sigma functions, a generalization of the genus-one Weierstrass sigma function, the recent subject of much work in computational algebraic geometry and mathematical physics. This talk will review current work by Y. Ōnishi showing the integrality of sigma functions associated to smooth plane telescopic curves, which admit a Weierstrass model. With this in hand, the sigma function becomes a modular form under the group  $\text{Sp}(2g, \mathbb{Z})$ ,  $g$  being the genus of the curve, that can be reduced modulo any odd prime. The focus of the talk is then the study of the Rankin-Cohen brackets on the ring of modular forms, devised by D. Zagier to provide it with a graded differential structure. We present results on differential properties of modular forms naturally associated to algebraic curves, as well as problems on their reduction to positive characteristic. Our case study is Klein's plane quartic, the modular curve  $X(7)$ ; since it is not of plane telescopic type, we also present joint results (with J. Kameda and S. Matsutani, *Internat. J. Math.* **24**) on the extension of sigma to a class of affine-space curves, embedded via a non-symmetric Weierstrass semigroup at a point. (Received August 30, 2016)

1125-11-374 **Keerthi Madapusi Pera\*** (keerthi@math.uchicago.edu), 5734 S University Ave, Chicago, IL 60637. *Special subvarieties of Shimura varieties.* Preliminary report.

We will talk about certain rigid subvarieties of Shimura varieties defined using Scholze's Hodge-Tate period map (Received August 30, 2016)

1125-11-377 **Zhiwei Yun\*** (zhiweiyun@gmail.com), 10 Hillhouse Ave, New Haven, CT 06511, and **Wei Zhang** (wzhang@math.columbia.edu), 2990 Broadway, New York, NY 10027. *Intersection numbers and higher derivatives of  $L$ -functions for function fields.*

We prove a higher derivative analogue of the Waldspurger formula and the Gross-Zagier formula in the function field setting. Our formula relates the self-intersection number of certain cycles on the moduli of Shtukas for  $\text{GL}(2)$  to higher derivatives of automorphic  $L$ -functions for  $\text{GL}(2)$ . (Received August 30, 2016)

1125-11-384 **Romyar T. Sharifi\***, Department of Mathematics, UCLA, Los Angeles, CA 90095-1555. *Reciprocity maps with restricted ramification.*

In this talk, we will compare two maps that naturally arise in study of the cohomology of number fields with ramification restricted to a finite set  $S$  of primes. Using this comparison, we will relate the cokernel of one of these maps, which we call an  $S$ -reciprocity map, to dual Selmer groups of residual representations for newforms that satisfy congruences with Eisenstein series modulo a prime in  $S$ . (Received August 30, 2016)

1125-11-388 **Reinier Broker\*** ([reinierbroker@gmail.com](mailto:reinierbroker@gmail.com)), CCR, 805 Bunn Drive, Princeton, NJ 08540, and **Allan Keeton**. *Lower bounds for Hilbert class polynomials.*

Hilbert class polynomials (minimal polynomials of CM elliptic curves) have various algorithmic applications. There are various algorithms that compute them in time proportional to proven upper bounds on their sizes. Based on examples, it is widely believed that the proven upper bounds are close to the truth. In this talk we address whether we can rigorously prove any lower bounds on their sizes. As we will see, the answer depends on what we are willing to assume. (Received August 30, 2016)

1125-11-411 **George A Boxer\*** ([george.a.boxer@gmail.com](mailto:george.a.boxer@gmail.com)). *On the Serre weights of certain  $GS\mathbb{P}_4$  valued Galois representations.*

Given an irreducible, odd, mod  $p$  Galois representation  $\bar{\rho}: G_{\mathbb{Q}} \rightarrow \mathrm{GSp}_4(\overline{\mathbb{F}}_p)$ , a generalization of Serre's famous modularity conjecture predicts that  $\bar{\rho}$  is the mod  $p$  Galois representation associated to a Siegel modular form of genus 2. Then one can try to understand, as Serre did for elliptic modular forms, what the possible weights are for Siegel modular forms giving rise to  $\bar{\rho}$ . In its modern formulation, this problem is known as the "weight part of Serre's conjecture." We will explain a result about the Serre weights of a certain class of  $\bar{\rho}$ 's and give an application. This is joint work with Frank Calegari and Toby Gee. (Received September 01, 2016)

1125-11-435 **Irene Bouw, Jenny Cooley, Elisa Lorenzo-Garcia, Kristin Lauter and Michelle Manes\*** ([mmanes@math.hawaii.edu](mailto:mmanes@math.hawaii.edu)), Department of Mathematics, 2565 McCarthy Mall, Keller 401A, Honolulu, HI 96813, and **Rachel Newton and Ekin Ozman**. *Bad reduction of genus 3 curves with Complex Multiplication.*

Let  $C$  be a smooth, absolutely irreducible genus 3 curve over a number field  $M$ . Suppose that the Jacobian of  $C$  has complex multiplication by a sextic CM-field  $K$ . Suppose further that  $K$  contains no imaginary quadratic subfield. We give a bound on the primes  $\mathfrak{p}$  of  $M$  such that the stable reduction of  $C$  at  $\mathfrak{p}$  contains three irreducible components of genus 1. (Received September 04, 2016)

1125-11-438 **Joseph A Vandehey\*** ([vandehey.1@osu.edu](mailto:vandehey.1@osu.edu)), 100 Math Tower, 231 West 18th Avenue, Columbus, OH 43210. *Differencing methods for Korobov-type exponential sums.*

Korobov was very interested in exponential sums of the type

$$\sum_{n=0}^{N-1} e^{\frac{2\pi i ab^n}{q}}$$

for integers  $a, b, q$  with  $b$  and  $q$  coprime. This sum is closely related to the base- $b$  expansion of rational numbers. Many powerful techniques for estimating exponential sums involve differencing methods that are not helpful for these Korobov-type sums due to the exponential  $b^n$ . We introduce a new differencing method that can improve estimations in very large ranges of these sums. (Received September 01, 2016)

1125-11-454 **Steve Lester\*** ([sjlester@gmail.com](mailto:sjlester@gmail.com)) and **Zeev Rudnick**. *Small scale equidistribution of eigenfunctions on the torus.*

I will describe some recent results on the distribution of the  $L^2$ -mass of eigenfunctions of the Laplacian on the torus  $\mathbb{T}^d/2\pi\mathbb{R}^d$ . A special case of a result of Marklof and Rudnick implies that the  $L^2$ -mass of almost all such eigenfunctions equidistributes with respect to Lebesgue measure for  $d = 2$ . I will discuss results on the scales at which the  $L^2$ -mass equidistributes as well as mention some limitations on equidistribution, and relate these questions to arithmetic problems such as representing integers as sums of squares and the distribution of lattice points. This is joint work with Zeév Rudnick. (Received September 02, 2016)

1125-11-467 **Thomas A Hulse\*** ([tahulse@colby.edu](mailto:tahulse@colby.edu)), Colby College, Department of Mathematics and Statistics, 5830 Mayflower Hill, Waterville, ME 049018858, and **Chan Ieong Kuan, David Lowry-Duda and Alexander Walker**. *Multiple Dirichlet Series and Average Orders.*

Inspired by Gauss's Circle Problem and Dirichlet's Divisor Problem, here we consider the conjectured asymptotic behavior of partial sums of Fourier coefficients of holomorphic cusp forms. We are able to obtain new results by means of meromorphically continuing Dirichlet series whose coefficients are derived from these partial sums,

which we do by decomposing these objects into shifted multiple Dirichlet series and taking spectral expansions. (Received September 02, 2016)

1125-11-475 **Edray Herber Goins\*** ([egoins@purdue.edu](mailto:egoins@purdue.edu)), Mathematical Sciences Building, 150 North University Street, West Lafayette, IN 47907-2067. *Toroidal Belyi Pairs, Toroidal Graphs, and their Monodromy Groups.*

A Belyi map  $\beta : \mathbb{P}^1(\mathbb{C}) \rightarrow \mathbb{P}^1(\mathbb{C})$  is a rational function with at most three critical values; we may assume these values are  $\{0, 1, \infty\}$ . A Dessin d'Enfant is a planar bipartite graph obtained by considering the preimage of a path between two of these critical values, usually taken to be the line segment from 0 to 1. Such graphs can be drawn on the sphere by composing with stereographic projection:  $\beta^{-1}([0, 1]) \subseteq \mathbb{P}^1(\mathbb{C}) \simeq S^2(\mathbb{R})$ . Replacing  $\mathbb{P}^1$  with an elliptic curve  $E$ , there is a similar definition of a Belyi map  $\beta : E(\mathbb{C}) \rightarrow \mathbb{P}^1(\mathbb{C})$ . Since  $E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$  is a torus, we call  $(E, \beta)$  a toroidal Belyi pair. The corresponding Dessin d'Enfant can be drawn on the torus by composing with an elliptic logarithm:  $\beta^{-1}([0, 1]) \subseteq E(\mathbb{C}) \simeq \mathbb{T}^2(\mathbb{R})$ .

This project seeks to create a database of such Belyi pairs, their corresponding Dessins d'Enfant, and their monodromy groups. This work is part of PRiME (Purdue Research in Mathematics Experience) with Gabriel Ngwe, Caitlin Leinkaemper, Dionel Jaime, Ivan Gonzalez, and Baiming Qiao. (Received September 02, 2016)

1125-11-499 **Paul Savala\*** ([psavala@whittier.edu](mailto:psavala@whittier.edu)). *Computing the Laplace Eigenvalue and Level of Maass Cusp Forms.*

Let  $f$  be a primitive Maass cusp form for a congruence subgroup  $\Gamma_0(D) \subset \mathrm{SL}(2, \mathbb{Z})$  and  $\lambda_f(n)$  its  $n$ -th Fourier coefficient. In this talk we discuss a recent paper by the author which shows that with knowledge of only finitely many  $\lambda_f(n)$  one can often solve for the level  $D$ , and in some cases, estimate the Laplace eigenvalue to arbitrarily high precision. This is done by analyzing the resonance and rapid decay of smoothly weighted sums of  $\lambda_f(n)e(\alpha n^\beta)$  for  $X \leq n \leq 2X$  and any choice of  $\alpha \in \mathbb{R}$ , and  $\beta > 0$ . The methods include the Voronoi summation formula, asymptotic expansions of Bessel functions, weighted stationary phase, and computational software. These algorithms manifest the belief that the resonance and rapid decay nature uniquely characterizes the underlying cusp form. They also demonstrate that the Fourier coefficients of a cusp form contain all arithmetic information of the form. (Received September 04, 2016)

1125-11-506 **Boris Adamczewski** and **Jason P Bell\***, Department of Pure Mathematics, University of Waterloo, 200 University Ave. W., Waterloo, ON N2L3G1, Canada, and **Eric Delaygue**. *Algebraic independence of  $G$ -functions and Lucas congruences.*

We consider complex power series  $F(x)$  satisfying a linear homogeneous differential equation with polynomial coefficients that have the property that their coefficients lie in a finitely generated ring. For such a power series it is possible to reduce modulo certain primes (maximal ideals) and get a power series  $F_p(x)$  with coefficients in a finite field. We consider those that have the property that the coefficients of their  $F_p(x)$  satisfy a so-called Lucas style recurrence (that is,  $a_{pn+j} = a_n a_j$ , where  $a_i$  denotes the coefficient of  $x^i$  in  $F_p$ ). We show that many power series arising in number theory and combinatorics have this form and we show that one can often say interesting things about transcendence and special values. (This is joint work with Boris Adamczewski and Éric Delaygue.) (Received September 04, 2016)

1125-11-527 **Thotsaporn Aek Thanatipanonda** and **Doron Zeilberger\*** ([doronzeil@gmail.com](mailto:doronzeil@gmail.com)), 110 Frelinghuysen Rd, Department of Mathematics, Hill Center, Busch Campus, Piscataway, NJ 08854. *Debunking Richard Guy's Law of Small Numbers.*

Richard K. Guy (born, Sept. 30, 1916), who has recently celebrated his 100-th birthday, famously formulated the "Strong Law of Small Numbers" and gave lots of "cautionary tales" of pairs of sequences that agree for quite a few terms, only to disagree later on. These examples are often used by purists to uphold the current (misguided!) dogma in mathematics, that empirical proofs, based on checking many special cases, are not to be trusted, and that only fully rigorous proofs are safe.

We will argue that in many cases empirical proofs are very trustworthy, and will explain why some "cautionary tales" (including so-called Pisot sequences) should not intimidate us empiricists. (Received September 05, 2016)

1125-11-531 **Ira M Gessel\*** ([gessel@brandeis.edu](mailto:gessel@brandeis.edu)). *Exponential generating function mod  $p$ .* Preliminary report.

We find congruences to a prime modulus  $p$  for sequences defined by exponential generating functions by studying the ring of exponential generating functions modulo  $p$ . The algebraic structure of this ring is quite simple, but most of our results follow from properties, such as chain rules, for the derivations  $D^i$  where  $D$  is the derivative.

We find that many sequences defined by exponential generating functions are periodic modulo  $p$ , and in fact the set of such sequences is closed under addition, multiplication, composition, and compositional inversion

of their exponential generating functions. We also find that solutions of functional equations such as  $f(x) = x + f(e^x - x - 1)$  and  $2g(x) = x + g(e^x - 1)$  have coefficient sequences that are not periodic modulo  $p$ . (Received September 05, 2016)

1125-11-536 **Laure Flapan** and **Jaclyn Ann Lang\*** ([lang@math.univ-paris13.fr](mailto:lang@math.univ-paris13.fr)), Département de Mathématiques, LAGA, Institute Galilée, Université Paris 13, 99 av. J.-B. Clément, 93430 Villetaneuse, France. *Modularity for Schreieder's varieties*. Preliminary report.

Schreieder constructed a family of varieties from quotients of products of certain hyperelliptic curves. We show that certain subquotients of their cohomology are modular in the sense that the L-function matches that of a particular Hecke character. (Received September 06, 2016)

1125-11-551 **Benjamin Linowitz**, **D. B. McReynolds**, **Paul Pollack** and **Lola Thompson\*** ([101a.thompson@oberlin.edu](mailto:101a.thompson@oberlin.edu)). *Counting quaternion algebras*.

In this talk, we discuss how classical techniques from multiplicative number theory can be used to count quaternion algebras over number fields subject to various constraints. Because of the correspondence between maximal subfields of quaternion algebras and geodesics on arithmetic hyperbolic manifolds, these counts have interesting applications to the field of spectral geometry. (Received September 06, 2016)

1125-11-577 **Andrew V Sutherland\*** ([drew@math.mit.edu](mailto:drew@math.mit.edu)), Department of Mathematics, Massachusetts Institute of Technology, 77 Massachusetts Ave., Cambridge, MA 02139. *Computing L-series of genus 3 curves*. Preliminary report.

The efficient computation of L-series of low genus curves raises two closely related algorithmic problems that have applications to cryptography: counting points on curves over finite fields and performing group operations in their Jacobians. These have been extensively studied in genus 1 and 2, but genus 3 raises several new challenges. In particular, one must consider curves that are not hyperelliptic, and even for hyperelliptic curves, many of the algorithms that work well in genus 1 and 2 do not easily generalize to practical algorithms in genus 3.

I will discuss recent progress in both the hyperelliptic and non-hyperelliptic cases. (Received September 06, 2016)

1125-11-587 **Abdul Hassen\*** ([hassen@rowan.edu](mailto:hassen@rowan.edu)), 201 Mullica Hill Rd, Glassboro, NJ 08080, and **Christopher Ernest**. *Hyperbolic Euler Numbers and Polynomials*. Preliminary report.

Euler numbers are defined by

$$\frac{1}{\cosh(x)} = \frac{2e^x}{e^{2x} + 1} = \sum_{n=0}^{\infty} E_n \frac{x^n}{n!}.$$

We define a generalization of Euler numbers and polynomials by the generating functions

$$G_N(x) := \frac{2e^x}{e^{2x} + T_{N-1}(x)} = \sum_{n=0}^{\infty} E_{N,n} \frac{x^n}{n!},$$

and

$$G_N(x, z) := \frac{2e^{x(z+1)}}{e^{2x} + T_{N-1}(x)} = \sum_{n=0}^{\infty} E_{N,n}(z) \frac{x^n}{n!}$$

where

$$T_m(x) = \sum_{k=0}^m \frac{x^k}{k!}.$$

We refer these numbers and polynomials as *hyperbolic Euler numbers and polynomials of order  $N$* . Note that  $E_{1,n} = E_n$  and  $E_{1,n}(z) = E_n(z)$  are the classical Euler numbers and polynomials, respectively. In this talk we will focus on  $N = 2$  and consider some divisibility properties and give an explicit formula for these numbers. We will also prove similar result for the polynomials. For example, we will show that

$$E_{2,n} = 1 - \sum_{k=0}^{n-2} \binom{n}{k} 2^{n-k-1} E_{2,k}, \quad -2nE_{2,n-1}$$

and

$$E_{2,n} = n \sum_{k=0}^{n-1} \binom{n-1}{k} (-1)^{n-k} E_{2,k} - \sum_{k=0}^{n-1} \left( \frac{1 + (-1)^{n-k}}{2} \right) \binom{n}{k} E_{2,k}.$$

(Received September 07, 2016)

1125-11-641 **Valentijn Karemaker\*** (vkarem@math.upenn.edu), David Rittenhouse Labs, 209 South 33rd Street, Philadelphia, PA 19104-6395, and **Sara Arias-de-Reyna, Cécile Armana, Marusia Rebolledo, Lara Thomas and Núria Vila**. *The inverse Galois problem for symplectic groups*.

The inverse Galois problem asks whether any finite group occurs as a Galois group. Given any prime number  $\ell$ , we will construct a three-dimensional abelian variety  $A/\mathbf{Q}$  such that the Galois representation attached to its  $\ell$ -torsion realises the symplectic group  $\mathrm{GSp}(6, \mathbf{F}_\ell)$  as a Galois group. This solves the inverse Galois problem for an infinite family of groups. The abelian variety will be the Jacobian variety of a curve whose behaviour at two distinct primes  $p$  and  $q$  satisfies certain congruence conditions. (Received September 08, 2016)

1125-11-660 **Valerio De Angelis\*** (vdeangel@xula.edu), Mathematics Department, Xavier University of Louisiana, 1, Drexel Drive, New Orleans, LA 70125. *The Stern diatomic sequence  $a(n)$  as a function of the gaps between 1's in the binary expansion of  $n$* .

Let  $a(n)$  be the Stern's diatomic sequence, and let  $x_1, \dots, x_r$  be the distances between successive 1's in the binary expansion of the (odd) positive integer  $n$ . We show that  $a(n)$  is obtained by evaluating some generalized Chebyshev polynomials when the variables are given the values  $x_1 + 1, \dots, x_r + 1$ . Using this representation we derive an arithmetic property of  $a(n)$ , as well as a determinant expression for it. (Received September 08, 2016)

1125-11-666 **Moa Apagodu\*** (mapagodu@vcu.edu), 1015 Floyd Av, Richmond, VA 23238, and **Doron Zeilberger**. *Using the "Freshman's Dream" to Prove Combinatorial Congruences*.

Recently, William Y.C. Chen, Qing-Hu Hou, and Doron Zeilberger developed an algorithm for finding and proving congruence identities (modulo primes) of indefinite sums of many combinatorial sequences, namely those (like the Catalan and Motzkin sequences) that are expressible in terms of constant terms of powers of Laurent polynomials. We first give a leisurely exposition of their approach, and then extend it in two directions. The Laurent polynomials may be of several variables, and instead of single sums we have multiple sums. In fact we even combine these two generalizations. We conclude with some super-challenges. (Received September 08, 2016)

1125-11-670 **Guillermo Mantilla-Soler\*** (g.mantilla691@uniandes.edu.co), Bogotá, Colombia. *New results on arithmetic equivalence using a Galois representation analog of Tate's isogeny theorem*. Preliminary report.

Let  $K$  be a number field and let  $\zeta_K(s) = \sum_{n=0}^{\infty} \frac{a_n(K)}{n^s}$  be its Dedekind zeta function. Motivated by Tate's isogeny theorem we show that  $\zeta_K(s)$  is completely determined by  $a_\ell(K)$  for  $\ell$  prime. This new characterization of arithmetic equivalence is a priori weaker than previously known ones, but in a Galois theoretical sense, we believe, is more natural. (Received September 09, 2016)

1125-11-705 **Ozlem Ejder\***, 3620 S. Vermont Avenue KAP108, Los Angeles, CA 90089. *Monodromy, Fermat Curves and Modular Symbols*.

This work is motivated by the question of computing the monodromy of a specific fibration where a generic fiber is isomorphic to Fermat curve of degree  $n$ . We use the realization of Fermat curves in terms of modular curves and compute their modular symbols and cuspidal modular symbols, and hence compute the generators of the first homology group. And finally, we describe the monodromy for this specific fibration. (Received September 09, 2016)

1125-11-707 **Tessa Cotron\*** (tessa.cotron@emory.edu), 605 Asbury Circle, Box 122042, Atlanta, GA 30322, **Robert Dicks** (rdicks@emory.edu), 605 Asbury Circle, Atlanta, GA 30322, and **Sarah Fleming** (smf1@williams.edu), Paresky Center, Williams College, Williamstown, MA 01267. *Asymptotics and Congruences For Partition Functions which Arise from Finitary Permutation Groups*.

In a recent paper, Bacher and de la Harpe study the conjugacy growth series of finitary permutation groups. In the course of studying the coefficients of a series related to the finitary alternating group, they introduce generalized partition functions  $p(n)_e$ . The group theory in their work motivates the study of the asymptotics for these functions. Moreover, Bacher and de la Harpe conjecture over 200 congruences for these functions which are analogous to the Ramanujan congruences for the unrestricted partition function  $p(n)$ . We obtain asymptotic formulas for all of the  $p(n)_e$ , and prove their conjectured congruences. Their work also motivates an investigation into congruence relationships between the finitary symmetric group and the finitary alternating group. Using the Ramanujan congruences for the partition function  $p(n)$  and Atkin's generalization to the  $k$ -colored partition

function  $p_k(n)$ , we prove the existence of congruence relations between these two series modulo arbitrary powers of 5 and 7, which we systematically describe. Furthermore, we prove that such relationships exist modulo powers of all primes  $\ell \geq 5$ . (Received September 09, 2016)

1125-11-712 **Francesca Balestrieri, Jennifer Berg, Michelle Manes, Jennifer Park and Bianca Viray\*** ([bviray@uw.edu](mailto:bviray@uw.edu)). *Obstructions to the Hasse principle on Enriques surfaces.*

In 1970, Manin showed that the Brauer group can obstruct the existence of rational points, even when there exist points everywhere locally. Later, Skorobogatov defined a refinement of this Brauer-Manin obstruction, called the étale-Brauer obstruction. We show that this refined obstruction is necessary to understand failures of the Hasse principle on Enriques surfaces, thereby completing the case of Kodaira dimension 0 surfaces. (Received September 09, 2016)

1125-11-716 **James A Sellers\*** ([sellersj@psu.edu](mailto:sellersj@psu.edu)), Department of Mathematics, Penn State University, 104 McAllister Building, University Park, PA 16802. *Arithmetic Properties of  $m$ -ary Partitions Without Gaps.*

Motivated by recent work of Bessenrodt, Olsson, and Sellers on unique path partitions, we consider partitions of an integer  $n$  wherein the parts are all powers of a fixed integer  $m \geq 2$  and there are no “gaps” in the parts; that is, if  $m^i$  is the largest part in a given partition, then  $m^j$  also appears as a part in the partition for each  $0 \leq j < i$ . We will discuss a number of arithmetic properties satisfied by the unrestricted  $m$ -ary partition function, dating back to the 1960s. We then transition to recent work of Andrews, Brietzke, Rødseth, and Sellers who provided a proof of an infinite family of congruences modulo powers of  $m$  which are satisfied by the function which enumerates  $m$ -ary partitions with no gaps. (Received September 09, 2016)

1125-11-727 **Csanád Bertók\*** ([bertok.csanad@science.unideb.hu](mailto:bertok.csanad@science.unideb.hu)), Institute of Mathematics, Hajdú-Bihar megye, P.O. Box 400, Debrecen, 4002, Hungary, and **Attila Pethő** and **Michael E. Pohst**. *On multidimensional Diophantine approximation of algebraic numbers.*

In this talk we develop algorithms for solving the dual problems of approximating linear forms and of simultaneous approximation in number fields  $F$ . Using earlier ideas for computing independent units by Buchmann, Pethő and later Pohst we construct sequences of suitable modules in  $F$  and special elements  $\beta$  contained in them. The most important ingredient in our methods is the application of the *LLL*-reduction procedure to the bases of those modules. For *LLL*-reduced bases we derive improved bounds on the sizes of the basis elements. From those bounds it is quite straightforward to show that the sequence of coefficient vectors  $(x_1, \dots, x_n)$  of the presentation of  $\beta$  in the module basis becomes periodic. We can show that the approximations which we obtain are close to being optimal. Moreover, it is periodic on bases of real number fields. Thus our algorithm can be considered as a generalization, within the framework of number fields, of the continued fraction algorithm. (Received September 10, 2016)

1125-11-728 **Armin Straub\*** ([straub@southalabama.edu](mailto:straub@southalabama.edu)). *Core partitions into distinct parts and an analog of Euler’s theorem.*

A special case of an elegant result due to Anderson proves that the number of  $(s, s + 1)$ -core partitions is finite and is given by the Catalan number  $C_s$ . Amdeberhan recently conjectured that the number of  $(s, s + 1)$ -core partitions into distinct parts equals the Fibonacci number  $F_{s+1}$ . We prove this conjecture by enumerating, more generally,  $(s, ds - 1)$ -core partitions into distinct parts.

As a by-product of our results, we obtain a bijection between partitions into distinct parts and partitions into odd parts, which preserves the perimeter (that is, the largest part plus the number of parts minus 1). This simple but curious analog of Euler’s theorem appears to be missing from the literature on partitions.

Finally, we intend to discuss some more recent developments. (Received September 10, 2016)

1125-11-747 **Yara Elias, Ekin Ozman, Kristin E. Lauter and Katherine E Stange\*** ([kstange@math.colorado.edu](mailto:kstange@math.colorado.edu)). *Ring-LWE for the number theorist.*

The talk will give an overview for number theorists of the Ring-Learning-With-Errors problem, a number theoretical hard problem proposed for post-quantum cryptography. I will review joint work on this problem that came about as part of Women in Numbers 3. (Received September 10, 2016)

1125-11-748 **Hao Chen, Kristin E Lauter and Katherine E Stange\*** ([kstange@math.colorado.edu](mailto:kstange@math.colorado.edu)). *Security and attacks on Ring-Learning-with-Errors.*

The Ring-Learning-with-Errors problem is a hard problem based on ideal lattices proposed for post-quantum cryptography. I will give an overview of joint work investigating the security of the problem from the perspective of attacks based on the ring structure. (Received September 10, 2016)

1125-11-756 **Karl Dilcher\*** ([dilcher@mathstat.dal.ca](mailto:dilcher@mathstat.dal.ca)), Department of Mathematics & Statistics, Dalhousie University, Halifax, NS B3H 4R2, Canada, and **Sinai Robins**. *Zeros and irreducibility of gcd-polynomials.*

We study the family of self-inversive polynomials of degree  $n$ , whose  $j$ th coefficient is  $\gcd(n, j)^k$ , for each fixed integer  $k \geq 1$ . We prove that these polynomials have all of their roots on the unit circle, with uniform angular distribution. In the process we prove some new results on Jordan's totient function. We also prove that these polynomials are irreducible, apart from an obvious linear factor, whenever  $n$  is a power of a prime, and conjecture that this holds for all  $n$ . (Received September 10, 2016)

1125-11-770 **Karl Dilcher\*** ([dilcher@mathstat.dal.ca](mailto:dilcher@mathstat.dal.ca)), Department of Mathematics & Statistics, Dalhousie University, Halifax, N.S. B3H 4R2, Canada, and **Christophe Vignat**. *On the polynomial part of a restricted partition function.*

We prove an explicit formula for the polynomial part of a restricted partition function, also known as the first Sylvester wave. This is achieved by way of some identities for higher-order Bernoulli polynomials, one of which is analogous to Raabe's well-known multiplication formula for the ordinary Bernoulli polynomials. As a consequence of our main result we obtain an asymptotic expression of the first Sylvester wave as the coefficients of the restricted partition grow arbitrarily large. (Received September 11, 2016)

1125-11-783 **Marie José Bertin, Alice Garbagnai, Ruthi Hortsch\*** ([ruthi@beammath.org](mailto:ruthi@beammath.org)), **Odile Lecacheux, Makiko Mase, Cecilia Salgado and Ursula Whitcher**. *Classification of Elliptic Fibrations of a Singular K3 Surface.*

We classify, up to automorphism, the elliptic fibrations on the singular K3 surface  $X$  associated with the Laurent polynomial

$$x + \frac{1}{x} + y + \frac{1}{y} + z + \frac{1}{z} + \frac{x}{y} + \frac{y}{x} + \frac{y}{z} + \frac{z}{y} + \frac{z}{x} + \frac{x}{z},$$

the transcendental lattice of which is isometric to  $\langle 6 \rangle \oplus \langle 2 \rangle$ .

In the paper, we give each elliptic fibration by Dynkin diagrams characterizing its reducible fibers, and the rank and torsion of its Mordell-Weil group. We will review this and explain Nishiyama's method, which was used to obtain this classification. (Received September 11, 2016)

1125-11-803 **Ian Petrow\*** ([ian.petrow@epfl.ch](mailto:ian.petrow@epfl.ch)), Section des Mathématiques, Bâtiment MA, Station 8, 1004 Lausanne, Switzerland. *Cubic Moments of  $L$ -functions and the Petersson formula for newforms.*

Subconvex bounds for central values of  $L$ -functions are well-known to have many important arithmetic applications. In this talk I will present joint work with M.P. Young in which we use a cubic moment to prove a Weyl-type subconvexity bound for the central  $L$ -values of quadratic twists of a newform of square-free level, trivial nebentypus, and arbitrary even weight. This generalizes work of Conrey and Iwaniec. I will discuss a new more general Petersson formula for newforms of square-free level, which we developed as a tool for these estimates. (Received September 12, 2016)

1125-11-824 **Jeffrey Hatley\*** ([hatleyj@union.edu](mailto:hatleyj@union.edu)) and **Antonio Lei**. *Arithmetic properties of signed Selmer groups at non-ordinary primes.*

We extend many results (due to Greenberg-Vatsal, Emerton-Pollack-Weston, and Pollack-Weston) on Selmer groups for  $p$ -ordinary elliptic curves and modular forms to the non-ordinary setting. More precisely, we study the signed Selmer groups defined using the machinery of Wach modules over  $\mathbf{Z}_p$ -cyclotomic extensions. The main ingredient is the definition of residual and non-primitive Selmer groups at non-ordinary primes, which allow us to show that if two non-ordinary modular forms of even weight  $k \geq 2$  are congruent mod  $p$ , then their Selmer groups also enjoy a congruence property mod  $p$ . (Received September 12, 2016)

1125-11-825 **Alexandra M Florea\***, [amusat@stanford.edu](mailto:amusat@stanford.edu). *The 4th moment of quadratic Dirichlet  $L$ -functions in function fields.*

We focus on the fourth moment of quadratic Dirichlet  $L$ -functions at the critical point in function fields. We will explain how to obtain an asymptotic formula with some of the secondary main terms. (Received September 12, 2016)

1125-11-833 **Brandt Kronholm\***, University of Texas Rio Grande Valley, Department of Mathematics, Edinburg, TX 78539. *Infinite Families of Congruences for the Coefficients of Gaussian Polynomials.*

Divisibility properties for the unrestricted partition function  $p(n)$  are well known. Similar congruences for  $p(n, m)$ , partitions of  $n$  into at most  $m$  parts, have been established. Let  $p(n, m, N)$  denote the number of partitions of  $n$  into at most  $m$  parts, no part larger than  $N$ . These are the coefficients of Gaussian Polynomials. The purpose of this talk is to state and prove theorems regarding infinite families of congruences for  $p(n, m, N)$ . (Received September 12, 2016)

1125-11-841 **Ted Chinburg, Brett Hemenway, Nadia Heninger and Zachary Scherr\*** (zls002@bucknell.edu). *Capacity Theory and Optimality of Coppersmith's Theorem.*

Coppersmith's method is an approach to finding small integral solutions to polynomial congruences. Given a monic polynomial  $f(x) \in \mathbb{Z}[x]$  of degree  $d > 1$  and a positive integer  $N$ , Coppersmith devised a polynomial time method for finding all integers  $r$  for which

$$f(r) \equiv 0 \pmod{N}$$

and  $|r| < N^{1/d}$ . In this talk we will show a connection between Coppersmith's method and adelic capacity theory, as developed by Cantor and Rumely. We will be able to use results from capacity theory to prove that the  $N^{1/d}$  is sharp in Coppersmith's method. Time permitting, we will also show why proposed modifications to Coppersmith's algorithm still cannot break this barrier for  $N$  of cryptographic interest. This is joint work with Ted Chinburg, Brett Hemenway and Nadia Heninger. (Received September 12, 2016)

1125-11-845 **Ayla R. Gafni\*** (agafni@ur.rochester.edu). *Partitions into values of a polynomial.* Preliminary report.

Let  $f(x)$  be an integer-valued polynomial, and let  $p_f(n)$  denote the number of partitions of  $n$  into values of  $f$ . That is, the number of ways to write

$$n = f(a_1) + f(a_2) + \cdots + f(a_m),$$

where  $m \geq 1$  and  $a_i \geq a_{i+1}$  for each  $i$ . We will examine this partition function over various classes of polynomials, and provide an asymptotic formula for  $p_f(n)$  given appropriate conditions on  $f$ . (Received September 19, 2016)

1125-11-847 **Tristan Freiberg\*** (tfreiberg@uwaterloo.ca), **Pär Kurlberg** and **Lior Rosenzweig**. *Poisson spacings between sums of two squares and spectral correlations for the square billiard.*

We investigate the level spacing distribution for the quantum spectrum of the square billiard. Extending work of Connors–Keating and Smilansky, we formulate an analogue of the Hardy–Littlewood prime  $k$ -tuple conjecture for sums of two squares, and show that it implies that the spectral gaps, after removing degeneracies, are Poisson distributed. We also give numerical evidence for the conjecture and its implications. (Received September 12, 2016)

1125-11-864 **Brad Rodgers\*** (rbrad@umich.edu), 530 Church St., Ann Arbor, MI 48109. *Sums of arithmetic functions over short intervals.*

The distribution of sums over short intervals of certain arithmetic functions, such as  $\Lambda(n)$  and  $d_k(n)$ , is closely connected to random matrix phenomena in number theory. In this talk we will attempt to explain this connection along with some surprising behavior exhibited by these sums. We discuss both conjectures over the integers and rigorous work in a function field setting that has sometimes motivated new conjectures. We will also discuss a decomposition of arithmetic functions that makes some arithmetic sense of the behavior these sums exhibit. A part of this talk is joint work with Jon Keating, Edva Roditty-Gershon, and Zeev Rudnick. (Received September 12, 2016)

1125-11-879 **Ashvin A Swaminathan\*** (aaswaminathan@college.harvard.edu), 388 Eliot Mail Center, 101 Dunster Street, Cambridge, MA 02138, **Aaron Landesman** (aaronlandesman@stanford.edu), 450 Serra Mall, Stanford, CA 94305, **James Tao** (jamestao@college.harvard.edu), 434 Winthrop Mail Center, 32 Mill Street, Cambridge, MA 02138, and **Yujie Xu** (yujiex@caltech.edu), 1200 East California Boulevard, Pasadena, CA 91125. *Surjectivity of Galois Representations in Rational Families of Abelian Varieties.*

We show that for any family of abelian varieties over a rational base with big geometric monodromy, those members that have adelic Galois representation with image as large as possible form a density-1 subset. Our results can be applied to a number of interesting families of abelian varieties, such as rational families dominating

the moduli of Jacobians of hyperelliptic curves, trigonal curves, or plane curves. As a consequence, we prove that for any dimension  $g \geq 3$ , there are infinitely many abelian varieties over  $\mathbb{Q}$  with adelic Galois representation having image equal to all of  $\mathrm{GSp}_{2g}(\hat{\mathbb{Z}})$ . (Received September 12, 2016)

1125-11-880 **Robert Schneider\*** ([robert.schneider@emory.edu](mailto:robert.schneider@emory.edu)), Dept. of Mathematics and Computer Science, Emory University, Atlanta, GA 30322. *Jacobi's triple product, mock theta functions, and the  $q$ -bracket of Bloch-Okounkov.*

In Ramanujan's final letter to Hardy, he wrote of a strange new class of infinite series he called "mock theta functions". It turns out all of Ramanujan's mock theta functions are essentially specializations of a so-called universal mock theta function  $g_3(z, q)$  of Gordon-McIntosh. Here we show that  $g_3$  arises naturally from the reciprocal of the classical Jacobi triple product—and is intimately tied to rank generating functions for unimodal sequences, which are connected to mock modular and quantum modular forms—through the action of an operator from partition theory, the  $q$ -bracket of Bloch-Okounkov, that has recently been studied by Zagier and other authors due to connections to quasimodular and  $p$ -adic modular phenomena. (Received September 12, 2016)

1125-11-889 **Elena C Covill\*** ([ec20covi@siena.edu](mailto:ec20covi@siena.edu)). *On the subgroup generated by solutions of Pell's equation.*

Equivalence classes of solutions of the Diophantine equation  $a^2 + mb^2 = c^2$  form an infinitely generated abelian group  $G_m$  under the operation induced by complex multiplication, where  $m$  is a fixed square-free positive integer. Solutions of Pell's equation  $x^2 - my^2 = 1$  generate a subgroup  $P_m$  of  $G_m$ . We prove that  $G_m/P_m$  has infinite rank for infinitely many values of  $m$ . (Received September 12, 2016)

1125-11-900 **Ernest Hunter Brooks\*** ([ernest.brooks@epfl.ch](mailto:ernest.brooks@epfl.ch)), EPFL SB MATHGEOM GR-JET, MA B3 494 (Bâtiment MA), Station 8, 1015 Lausanne, Vaud, Switzerland, and **Dimitar P. Jetchev** and **Benjamin Pierre Charles Wesolowski**. *Isogeny Graphs of Ordinary Abelian Varieties.*

Fix a prime number  $\ell$ . Graphs of isogenies of  $\ell$ -power degree are well-understood for elliptic curves, but not for higher-dimensional abelian varieties. We study the case of ordinary abelian varieties over a finite field of characteristic  $p \neq \ell$ , with a particular focus on principally polarized surfaces. We analyse graphs of so-called  $\ell$ -isogenies, showing that they are (almost) volcanoes. In the case of abelian surfaces, we can further describe graphs of isogenies whose kernels are maximal isotropic in the  $\ell$ -torsion. Among applications to cryptography is a "surfacing" algorithm in genus two, which computes an isogeny from an arbitrary ordinary abelian surface to one whose endomorphism ring is the maximal order in its endomorphism algebra. This is joint work with Dimitar Jetchev and Benjamin Wesolowski. (Received September 13, 2016)

1125-11-903 **David Jao\*** ([djao@uwaterloo.ca](mailto:djao@uwaterloo.ca)), 200 University Ave. W, Waterloo, Ontario N2L3G1, Canada. *Post-quantum public-key cryptography based on isogenies between supersingular elliptic curves.*

According to our current knowledge of quantum mechanics, computers based on quantum phenomena can potentially solve certain problems much more quickly than is possible on any classical computer, including most of the mathematical problems upon which current public-key cryptosystems are based. In response, researchers have developed *post-quantum cryptosystems* — alternative cryptosystems based on new mathematical problems which are hard to solve even on a quantum computer. Mainstream post-quantum cryptosystems can be categorized into several broad families: lattice-based, code-based, hash-based, and schemes based on multivariate polynomials. A fifth family of cryptosystems, based on isogenies between supersingular elliptic curves, offers a promising alternative to these schemes. Compared to other schemes, isogeny-based cryptosystems are unique in the following ways: they achieve the smallest possible public key size; they are based on number-theoretic complexity assumptions; implementations can leverage existing elliptic curve cryptography libraries; and the security level has a simple linear relationship to the key size. In this presentation we survey existing constructions of isogeny-based cryptosystems and present recent results on key sizes, performance, and security. (Received September 13, 2016)

1125-11-925 **Jesse Thorner** and **Asif Zaman\*** ([asif.ali.zaman@gmail.com](mailto:asif.ali.zaman@gmail.com)). *Brun-Titchmarsh, Chebotarev, and Lang-Trotter.*

The classical Brun-Titchmarsh theorem provides an upper bound for the number of primes in an arithmetic progression in a far wider range than that afforded by the Prime Number Theorem for arithmetic progressions. This feature makes it a fundamental tool in many arguments. The Chebotarev density theorem, on the other hand, has few alternatives to adequately estimate the number of prime ideals with a prescribed splitting behaviour in a Galois extension of number fields. The first such result with appropriate field uniformity is due

to Lagarias-Montgomery-Odlyzko in their influential 1979 paper and, as far as we are aware, there have not been any advances on this type of general bound. Unfortunately, the valid range of their estimate is inadequate for many applications. In fact, it does not specialize to the classical Brun-Titchmarsh for arithmetic progressions.

We will report on a new generalization of Brun-Titchmarsh associated to the Chebotarev density theorem which improves on the prior work of Lagarias-Montgomery-Odlyzko. Our result has consequences for counting primes represented by certain binary quadratic forms and refining the best known unconditional bounds towards the Lang-Trotter conjectures for elliptic curves. This is joint work with Jesse Thorner. (Received September 13, 2016)

1125-11-933 **Sneha Chaubey, Elena Fuchs, Robert Hines\*** ([robert.hines@colorado.edu](mailto:robert.hines@colorado.edu)) and **Katherine Stange**. *Super-Apollonian continued fractions*. Preliminary report.

We consider a pair of dynamical systems on the complex plane inspired by the action of the super-Apollonian group, realized here as reflections in the sides of a right-angled ideal hyperbolic octahedron. These systems are “reflective” versions of Asmus Schmidt’s continued fraction algorithms over  $\mathbb{Q}(i)$ . We study aspects of the dynamics including the natural extension, invariant measure, action on rational points, and restriction to the real line. (Received September 13, 2016)

1125-11-945 **Martin Burke\*** ([martin31113@gmail.com](mailto:martin31113@gmail.com)). *A Short Proof of Fermat’s Last Theorem,  $x < z$  and  $y < z$*

$x, y$  and  $z$  are integers  $> 0$  and  $n > 2$ . For  $x^2 + y^2 = z^2$ , if  $x = z$ , or  $y = z$ , or  $x$  and  $y = z$ , then  $x^2 + y^2 > z^2$ . So  $x < z$  and  $y < z$ .

Considering  $x^3 + y^3 = z^3$ ,  $x^2x + y^2y = z^2z$ , the individual terms  $x, y$  and  $z$  act like constants that multiply the  $x^2, y^2$ , and  $z^2$  terms. For example  $3^2 + 4^2 = 5^2$  and  $3^2 * 2 + 4^2 * 2 = 5^2 * 2$ .

However  $x < z$  and  $y < z$ , and multiplication by the individual  $x, y$  and  $z$  terms causes an inequality in  $x^3 + y^3 = z^3$ . So  $x^3 + y^3 \neq z^3$ .

Similarly  $x^4 + y^4 = z^4$ ,  $x^2x^2 + y^2y^2 \neq z^2z^2$ . Considering  $x^2x^{n-2} + y^2y^{n-2} = z^2z^{n-2}$ ,  $x^{n-2}$ ,  $y^{n-2}$ , and  $z^{n-2}$  multiply  $x^2, y^2$ , and  $z^2$ .

However  $x < z$  and  $y < z$ . Therefore  $x^{n-2} < z^{n-2}$  and  $y^{n-2} < z^{n-2}$ .

Multiplication by the individual  $x^{n-2}, y^{n-2}$ , and  $z^{n-2}$  terms causes an inequality in  $x^n + y^n = z^n$ .

$x^n + y^n \neq z^n$  QED.

The proof for  $x^2 + y^2 <> z^2$  will also be presented. (Received September 13, 2016)

1125-11-951 **Cassie Williams\*** ([w111i5c1@jmu.edu](mailto:w111i5c1@jmu.edu)), James Madison University, Harrisonburg, VA. *Numerical secondary terms for a conjecture of Cohen and Lenstra*. Preliminary report.

In 1984, Cohen and Lenstra published their classic paper describing a heuristic to explain the observed frequency with which finite abelian groups occur as the class group of a quadratic number field and applied their theoretical framework to make several conjectures about such class groups. Thirty years of improvements in computing and algorithms have made it easy to obtain large data sets against which to test the Cohen-Lenstra conjectures, and discrepancies between their asymptotic predictions and reality have been noted by several authors. Analytic approaches to determining secondary terms only work for some of the conjectures, and so we instead turn to a numerical approach. We used Sage to perform a numerical investigation of the discrepancy between one of the Cohen-Lenstra conjectures for real quadratic fields and the actual data. We will share our results, including numerical secondary terms for various small primes, the error in our new estimates, and some interesting patterns. (This is joint work with undergraduate research student Codie Lewis.) (Received September 16, 2016)

1125-11-953 **Sandie Han, Ariane M Masuda, Satyanand Singh and Johann Thiel\***, 300 Jay St., Brooklyn, NY 11201. *The growth of coefficients in certain PLFT  $(u, v)$ -Calkin-Wilf trees*. Preliminary report.

A positive linear fractional transformation (PLFT) is a function of the form  $f(z) = \frac{az+b}{cz+d}$  where  $a, b, c$ , and  $d$  are nonnegative integer coefficients with determinant  $ad - bc \neq 0$ . Nathanson defined a PLFT  $(u, v)$ -Calkin-Wilf tree, with  $u, v$  positive integers, as an infinite rooted binary tree where every vertex is labelled by a PLFT using a simple set of rules. If a vertex is labelled by the PLFT  $f(z)$ , then the left child of the vertex is labelled by  $L_u(f(z))$  and the right child is labelled by  $R_v(f(z))$  where  $L_u(z) = \frac{z}{uz+1}$  and  $R_v(z) = z + v$ . In this talk we study the size of the coefficients of PLFTs appearing in certain PLFT  $(u, v)$ -Calkin-Wilf trees. This is joint work with Sandie Han, Ariane M. Masuda, and Satyanand Singh (Received September 13, 2016)

1125-11-965 **Ted Chinburg\*** ([ted@math.upenn.edu](mailto:ted@math.upenn.edu)), Dept. of Math, U. Penn., 209 S. 33rd Street, Philadelphia, PA 19104, **Brett Hemenway**, Dept. of Computer Science, 3330 Walnut St., University of Pennsylvania, Philadelphia, PA 19104, **Nadia Heninger**, Department of Computer Science, 3330 Walnut St., University of Pennsylvania, Philadelphia, PA 19104, and **Zachary Scherr**, Bucknell University, Department of Mathematics, 380 Olin Science Building, Lewisburg, PA 17837. *Capacity theory and Coppersmith's algorithm for integral points*. Preliminary report.

In 1996, Coppersmith described polynomial time algorithms for finding (i) small solutions to one variable polynomial congruences, and (ii) small integral solutions to polynomial equations in two variables. I will describe how capacity theory can be used to quantify how far one can extend Coppersmith's method of treating problem (ii). This has applications to finding an unknown divisor  $d$  of a given large integer  $N$  given a sufficiently close approximation to  $d$ . (Received September 13, 2016)

1125-11-968 **Elena Fuchs\*** ([efuchs@math.ucdavis.edu](mailto:efuchs@math.ucdavis.edu)). *Local to global principles in integral circle packings*.

One of the most spectacular results on arithmetic of Apollonian circle packings is the "almost" local to global principle for curvatures in any given integral Apollonian packing as described by Bourgain-Kontorovich in 2014. The methods in their work, inspired originally by an observation of Sarnak's in his letter to Lagarias on Apollonian circle packings, apply to a much larger class of circle packings. In this talk, we clarify what "almost" local to global means, and describe what the larger class is, as well as what aspects of the packings in this class seem necessary in order to conclude an "almost" local to global result using the analytic tools from work of Bourgain-Kontorovich. This is joint work with Stange and Zhang. (Received September 13, 2016)

1125-11-986 **Jesse Thorner\*** ([jthorner@stanford.edu](mailto:jthorner@stanford.edu)). *Averages of twisted  $GL(2)$   $L$ -functions*.

Let  $K$  be a number field, and let  $\pi$  be a cuspidal automorphic representation of  $GL(2)$  over  $K$ . Rohrlich proved that for any fixed complex number  $s_0$ , there exist infinitely many ray class characters  $\chi$  of  $K$  for which  $L$ -function  $L(s_0, \pi \otimes \chi, K) \neq 0$ . Using mean value theorems for  $L(s, \pi \otimes \chi, K)$  arising from the theory of multiple Dirichlet series and large sieve inequalities for ray class characters due to Blomer, Goldmakher, and Louvel, we make Rohrlich's result quantitative when  $L(s, \pi, K)$  satisfies the generalized Ramanujan conjecture. We also obtain non-vanishing results for  $L'(1/2, \pi, K)$ , and we nontrivially bound the average analytic rank of  $L(s, \pi, K)$  in families of quadratic twists. (Received September 13, 2016)

1125-11-990 **Sneha Chaubey\*** ([chaubey2@illinois.edu](mailto:chaubey2@illinois.edu)), **Amita Malik**, **Nicolas Robles** and **Alexandru Zaharescu**. *Zeros of normalized combinations of the Riemann Xi function on the critical line*.

We consider functions of the form  $F_{\mathcal{E}, a, T}(s) = \sum_{j=0}^M \frac{c_j (-1)^j}{L^{2j}} \xi^{(a+2j)}(s)$ , with  $L = \log \frac{T}{2\pi}$  and  $c_j$  real constants satisfying certain constraint. We show that as  $T \rightarrow \infty$ , the proportion of zeros of  $F_{\mathcal{E}, a, T}(s)$  on the critical line  $\text{Re}(s) = 1/2$  tends to 1, at a rate depending on  $a$  but not on the choice of the  $c_j$ 's. (Received September 13, 2016)

1125-11-992 **Elena C. Covill**, **Mohammad Javaheri** and **Nikolai A. Krylov\*** ([nkrylov@siena.edu](mailto:nkrylov@siena.edu)), Siena College, Department of Mathematics, 515 Loudon Road, Loudonville, NY 12211. *On the subgroup generated by solutions of Pell's equation and elements of order 2 in the corresponding quotient group*.

Equivalence classes of solutions of the Diophantine equation  $a^2 + mb^2 = c^2$  form an infinitely generated abelian group  $G_m$  under the operation induced by complex multiplication, where  $m$  is a fixed square-free positive integer. Solutions of Pell's equation  $x^2 - my^2 = 1$  generate a subgroup  $P_m$  of  $G_m$ . I will show how the sequence of decreasing convergents of the continued fraction expansion of  $\sqrt{2}$  generates elements of order 2 in the quotient groups  $G_m/P_m$  for certain  $m$ . To do that I will use a homomorphism  $f_m : G_m \rightarrow Cl(\mathbb{Q}[\sqrt{-m}])$  into the ideal class group of the imaginary quadratic field  $\mathbb{Q}[\sqrt{-m}]$ , and show that  $P_m \subseteq \ker(f_m)$ , when the ring of integers of the real quadratic field  $\mathbb{Q}[\sqrt{m}]$  has units of norm -1. (Received September 13, 2016)

1125-11-1010 **Atul Dixit\*** ([adixit@iitgn.ac.in](mailto:adixit@iitgn.ac.in)), 5/340, Department of Mathematics, Indian Institute of Technology Gandhinagar, Palaj, Gandhinagar, Gujarat 382355, India, and **Koustav Banerjee** ([banerjeeekoustav98@gmail.com](mailto:banerjeeekoustav98@gmail.com)), Department of Mathematics, Ramkrishna Mission Vivekananda University, PO Belur Math, Howrah, W. Bengal 711202, India. *New representations for  $\sigma(q)$  via reciprocity theorems*.

In 1986 George E. Andrews proved two results involving "sums of tails" on page 14 of Ramanujan's Lost Notebook using a reciprocity theorem of Ramanujan. These identities can be thought of as representations for the function

$\sigma(q) = \sum_{n=0}^{\infty} \frac{q^{n(n+1)/2}}{(-q)_n}$ . In this talk, we give two new representations for Ramanujan's function  $\sigma(q)$  derived using more general reciprocity theorems of Soon-Yi Kang, and Andrews. The advantage of these representations is that they involve free complex parameters - one in the first representation, and two in the second. This is joint work with Koustav Banerjee. (Received September 16, 2016)

1125-11-1035 **Andrew V. Sills\*** (asills@georgiasouthern.edu), asills@georgiasouthern.edu.  
*Rogers–Ramanujan type partition identities.* Preliminary report.

The partition theoretic Rogers–Ramanujan identities assert that for  $a = 0, 1$  and any  $n$ , the number of partitions of  $n$  into parts greater than  $a$  that mutually differ by at least 2 equals the number of partitions of  $n$  into parts congruent to  $\pm(a+1) \pmod{5}$ . A *Rogers–Ramanujan type partition identity* asserts the equality, for all  $n$ , of two classes of restricted partitions where in one class, the parts are restricted to certain arithmetic progressions with a fixed modulus, and in the other class parts must satisfy some (possibly quite complicated) difference conditions and initial conditions. Many examples of RR type partition identities, including many infinite families, are now known. In the 1940's Derek Lehmer and Henry Alder proved the nonexistence of certain *a priori* plausible families of RR type partition identities. Despite numerous advances over the past half-century by Andrews, Gordon, and others, an overarching theory of why certain identities exist and why others are impossible, remains elusive. In this talk, I will share some observations of what I believe to be previously unnoticed features of known RR type partition identities in the hope that this may move us a step closer to a general understanding of such identities. (Received September 20, 2016)

1125-11-1047 **Bhargav Narayanan** and **Stephen D. Miller\***, 110 Frelinghuysen Rd, Piscataway, NJ 08854, and **Ramarathnam Venkatesan**. *Some remarks on Coppersmith's method and small-exponent RSA.* Preliminary report.

The talk will concern joint work with Ramarathnam Venkatesan and Bhargav Narayanan on small exponent RSA and variants of Boneh-Durfee's method. (Received September 14, 2016)

1125-11-1055 **Sebastian I Troncoso\*** (troncosomath@gmail.com), Department of Mathematics, 619 Red Cedar Road, C212 Wells Hall, East Lansing, MI 48824. *Bound for preperiodic points for maps with good reduction.*

Let  $K$  be a number field and let  $\phi$  in  $K(z)$  be a rational function of degree  $d \geq 2$ . Let  $S$  be the places of bad reduction for  $\phi$  (including the archimedean places). Let  $Per(\phi, K)$ ,  $PrePer(\phi, K)$ , and  $Tail(\phi, K)$  be the set of  $K$ -rational periodic, preperiodic, and purely preperiodic points of  $\phi$ , respectively. This work presents two main results. The first result gives a bound for  $|PrePer(\phi, K)|$  in terms of the number of places of bad reduction  $|S|$  and the degree  $d$  of the rational function  $\phi$ . This bound significantly improves a previous bound given by J. Canci and L. Paladino 2014. For the second result, assuming that  $|Per(\phi, K)| \geq 4$  (resp.  $|Tail(\phi, K)| \geq 3$ ), we prove bounds for  $|Tail(\phi, K)|$  (resp.  $|Per(\phi, K)|$ ) that depend only on the number of places of bad reduction  $|S|$  (and not on the degree  $d$ ). We show that the hypotheses of this result are sharp, giving counterexamples to any possible result of this form when  $|Per(\phi, K)| < 4$  (resp.  $|Tail(\phi, K)| < 3$ ). (Received September 14, 2016)

1125-11-1108 **Ana Caraiani**, **Ellen Eischen** and **Jessica Fintzen\*** (fintzen@umich.edu), Department of Mathematics, University of Michigan, 530 Church St, Ann Arbor, MI 48109, and **Elena Mantovan** and **Ila Varma**. *p-adic q-expansion principle and families of automorphic forms on unitary groups of arbitrary signature.*

We discuss a variant of the q-expansion principle (called the Serre-Tate expansion principle) for p-adic automorphic forms on unitary groups of arbitrary signature. We outline how this can be used to produce p-adic families of automorphic forms on unitary groups, which has applications to the construction of p-adic L-functions. This is done via an explicit description of the action of certain differential operators on the Serre-Tate expansion. (Received September 14, 2016)

1125-11-1111 **Nathan G McNew\*** (nmcnew@towson.edu), 8000 York Road, Towson, MD 21252.  
*Random multiplicative walks on the residues modulo n.*

Consider a multiplicative random walk on the set,  $\mathbb{Z}/n\mathbb{Z}$ , of residues modulo  $n$ , where at each step one chooses a residue uniformly at random, and multiplies the current state by it. This is an absorbing random walk with a single absorbing state,  $0 \pmod{n}$ . We are interested in the expected time to absorption, which we denote by  $a(n)$ . We give several ways to compute  $a(n)$  and find that it is closely related to  $P_1(n)$ , the largest prime divisor of  $n$ . Both functions have the same average order asymptotically and the difference between the functions is  $o(1)$  as  $n$  tends to infinity on a set with positive density. Finally we find that in an average sense, a better

approximation for  $a(n)$  is  $a(n) \approx P_1(n) + \frac{\pi}{4}P_2(n)$  where  $P_2(n)$  is the second largest divisor of  $n$ . (Received September 14, 2016)

1125-11-1114 **Edray H Goins** (egoins@math.purdue.edu) and **Alejandra Alvarado\*** (aalvarado2@eiu.edu). *Arithmetic Progressions on Conic Sections.*

The set  $\{1, 25, 49\}$  is a 3-term collection of integers which forms an arithmetic progression of perfect squares. We view the set  $\{(1, 1), (5, 25), (7, 49)\}$  as a 3-term collection of rational points on the parabola  $y = x^2$  whose  $y$ -coordinates form an arithmetic progression. In this exposition, we provide a generalization to 3-term arithmetic progressions on arbitrary conic sections  $\mathcal{C}$  with respect to a linear rational map  $\ell : \mathcal{C} \rightarrow \mathbb{P}^1$ . We explain how this construction is related to rational points on the universal elliptic curve  $Y^2 + 4XY + 4kY = X^3 + kX^2$  classifying those curves possessing a rational 4-torsion point. (Received September 14, 2016)

1125-11-1115 **Amita Malik\*** (amalik10@illinois.edu), University of Illinois at Urbana-Champaign, and **Armin Straub**. *Sporadic Apéry-like numbers modulo primes.*

At the ICM in 1978, R. Apéry presented a proof of the irrationality of  $\zeta(3)$ . In this proof, he introduced a sequence of integers, now known as Apéry sequence. Apéry-like numbers are special integer sequences, studied by Beukers and Zagier, which are modeled after Apéry numbers. Among their remarkable properties are connections with modular forms, Calabi-Yau differential equations, and a number of  $p$ -adic properties, some of which remain conjectural. A result of Gessel shows that Apéry's sequence satisfies Lucas congruences. We prove corresponding congruences for all sporadic Apéry-like sequences. While, in some cases, we are able to employ approaches due to McIntosh, Samol-van Straten and Rowland-Yassawi to establish these congruences, there are few others for which we require a finer analysis. As an application, we investigate modulo which numbers these sequences are periodic. In particular, we show that the Almkvist-Zudilin numbers are periodic modulo 8, a special property which they share with the Apéry numbers. This is joint work with Armin Straub. (Received September 14, 2016)

1125-11-1118 **Nickolas Andersen\*** (nandersen@math.ucla.edu). *The mock theta conjectures.*

The mock theta conjectures are two families of identities from Ramanujan's Lost Notebook involving the fifth order mock theta functions. These identities were first proven by Hickerson (Inventiones 1988) using Hecke-type  $q$ -series identities discovered by Andrews. We prove an equality between two vector-valued harmonic Maass forms of weight  $1/2$  which encodes these identities, thus providing a simple, conceptual proof of the mock theta conjectures. (Received September 14, 2016)

1125-11-1129 **Kristin Estella Lauter\*** (klauter@microsoft.com). *Supersingular Isogeny Graphs and Quantum Arithmetic.*

The National Institute of Standards and Technology (NIST) will be running an international competition over the next few years to select a new system for Post-Quantum Cryptography (PQC). One of the possible candidates is based on the hardness of finding isogenies between supersingular elliptic curves. This hard problem was first proposed by Charles-Goren-Lauter in 2006 as the basis for a new cryptographic hash function construction. The isogeny graph of supersingular elliptic curves can be interpreted in terms of Brandt matrices representing Hecke operators acting on spaces of modular forms. The idea behind the cryptographic applications is to use the hardness of finding paths in these Ramanujan graphs (or inverting random walks) as a way to construct a one-way function. The hard problem is then to find paths in the graph, given the starting and ending point. In this talk, we will discuss an algorithm for path-finding in the related Ramanujan graphs constructed by Lubotzky-Phillips-Sarnak, and highlight a surprising connection with quantum arithmetic. (Received September 15, 2016)

1125-11-1130 **Wladimir Pribitkin\*** (wladimir.pribitkin@csi.cuny.edu). *Simple Proof of the Partition Function Formula.*

We present a simple proof of Rademacher's celebrated exact formula for the number of unrestricted partitions of a positive integer. (Received September 15, 2016)

1125-11-1139 **Sharon M Frechette\*** (sfrechet@holycross.edu), **Julia Gordon** and **Lance Robson**. *Orbital Integrals and Shalika Germs for  $\mathfrak{sl}_n$  and  $\mathfrak{sp}_{2n}$ .* Preliminary report.

Shalika germs were introduced as a tool for studying orbital integrals, objects that play a large role in harmonic analysis on  $p$ -adic groups. The Shalika germ expansion expresses regular semisimple orbital integrals in terms of nilpotent ones, in a neighborhood of the origin. Exact values of Shalika germs elude computation, except for those of a few Lie algebras of small rank. We prove that Shalika germs on  $\mathfrak{sl}_n$  and  $\mathfrak{sp}_{2n}$  belong to a class of motivic functions defined by Cluckers and Loeser by means of a first-order language of logic (Denef-Pas language). The proof involves Nevins' combinatorial matching between two parametrizations of nilpotent orbits:

a parametrization involving partitions, and DeBacker's parametrization arising from the Bruhat-Tits building. As a result, we establish bounds on the Shalika germs that are uniform in  $p$ . This is joint work with Julia Gordon and Lance Robson. (Received September 15, 2016)

1125-11-1140 **George E. Andrews\*** ([gea1@psu.edu](mailto:gea1@psu.edu)), 306 McAllister Bldg., Mathematics Department, Pennsylvania State University, University Park, PA 16802. *Sequences in partitions*. Preliminary report.

Several years ago, the generating function for partitions with short sequences was presented (G.E. Andrews, Partitions with short sequences and mock theta functions, Proc. Nat. Acad. Sci., 102(13),(2005), 4666-4671). Recently, in joint work with E. Deutsch (A note on the method of Erdos and the Stanley-Elder partition theorems, INTEGERS, 16, A24(2016)), we proved a very general theorem which, in a special case, counted the total number of sequences of length  $k$  in the partitions of  $n$  (a result originally found by A. Knopfmacher and A. Munagi, Ramanujan J., 18(2009),239-255). In this talk, we follow up on the possible ways of merging these studies of sequences in partitions (Received September 15, 2016)

1125-11-1156 **Kannan Soundararajan\*** ([ksound@stanford.edu](mailto:ksound@stanford.edu)), Department of Mathematics, Stanford University, Stanford, CA 94305. *Tao's work on the Erdos discrepancy problem*.

The Erdos discrepancy problem asks whether every assignment of signs to the natural numbers must have large imbalances among the multiples of some integer. In 2015 Tao found a remarkable proof that there must be such irregularities. The key is a logarithmic version of the Chowla and Elliott conjectures from multiplicative number theory, which Tao established building upon another breakthrough of Matomaki and Radziwill on multiplicative functions in short intervals. My goal will be to describe some of the story and ideas behind this proof. (Received September 15, 2016)

1125-11-1193 **John R. Doyle\*** ([john.doyle@rochester.edu](mailto:john.doyle@rochester.edu)), Department of Mathematics, University of Rochester, Rochester, NY 14627. *Dynamical modular curves for quadratic polynomial maps*. Preliminary report.

Given a polynomial  $\phi$  of degree  $d \geq 2$  defined over a number field  $K$ , the set of  $K$ -rational preperiodic points for  $\phi$  comes naturally equipped with the structure of a directed graph  $G(\phi, K)$ . Given an abstract directed graph  $G$ , one might ask whether there exists a polynomial  $\phi \in K[z]$  such that  $G(\phi, K)$  contains a subgraph isomorphic to  $G$ . We will discuss the case of quadratic polynomials, where the appropriate moduli space for this problem is an algebraic curve — a dynamical analogue of the classical modular curves for torsion points on elliptic curves. We will briefly describe these dynamical modular curves and some of their properties, and we will discuss various applications, including results toward a dynamical version of Merel's uniform boundedness theorem. (Received September 15, 2016)

1125-11-1224 **Antonino Leonardi\*** ([a.leonardi@gmail.com](mailto:a.leonardi@gmail.com)), Via Aosta, 17, 20155, Milan, Italy. *Simple applications of continued fractions and an elementary result on Heron's algorithm (with a generalization to  $n$ -adic numbers)*.

The talk will start with some applications of continued fractions in order to obtain geometrical illusions. After this, it will deal with the pythagorean problem of the right-angled isosceles triangles finding all solutions to the simple diophantine equation  $l^2 + (l + 1)^2 = d^2$ , which will give a "pseudo-pythagoric" triangle. The author will also prove a theorem (main result) which relates continued fractions with Heron's algorithm, giving some examples. A generalization of the theorem in the  $n$ -adic completions is also being analyzed by the author and will possibly be considered during the talk. (Received September 15, 2016)

1125-11-1229 **R. Davis, R. Pries, V. Stojanoska and K. Wickelgren\*** ([kwickelgren3@math.gatech.edu](mailto:kwickelgren3@math.gatech.edu)), School of Math, Georgia Institute of Technology, 686 Cherry Street, Atlanta, GA 30308. *Galois action on Fermat curves*.

Consider the Fermat curve  $x^p + y^p = 1$  where  $p$  is an odd prime. Let  $K = \mathbb{Q}(\zeta_p)$  be the cyclotomic field. We extend work of Anderson about the action of the absolute Galois group  $G_K$  on a relative homology group of the Fermat curve. Anderson proved that the action factors through  $Q = \text{Gal}(L/K)$  where  $L$  is the splitting field of  $1 - (1 - x^p)^p$ . For  $p$  satisfying Vandiver's conjecture, we find an explicit formula for the action of  $q \in Q$  on the relative homology. This is joint work by R. Davis, R. Pries, V. Stojanoska, and K. Wickelgren. (Received September 15, 2016)

1125-11-1253 **Rachel Davis\*** ([rachel.davis@wisc.edu](mailto:rachel.davis@wisc.edu)), **Rachel Pries**, **Vesna Stojanoska** and **Kirsten Wickelgren**. *Fermat curves and Heisenberg extensions*.

Using the explicit formula for the Galois action on Fermat curves from the first talk of this pair, we determine the maps between several Galois cohomology groups which arise in connection with obstructions for rational points on the generalized Jacobian. Heisenberg extensions play a key role in the outcome. (Received September 15, 2016)

1125-11-1273 **Bianca A Thompson\*** ([bthompson@g.hmc.edu](mailto:bthompson@g.hmc.edu)), 250 N College Park Drive #P38, Upland, CA 91786. *Lower bounds on the canonical heights associated to some rational maps*. Preliminary report.

In prior work by Ingram, he showed that the lower bound of the canonical height of the morphism  $f(z) = z^d + c$  can be obtained for  $f$  evaluated at a wandering point. Building on his techniques, we can find a lower bound on the canonical height of some rational maps. (Received September 15, 2016)

1125-11-1306 **Karl Dilcher\*** ([dilcher@mathstat.dal.ca](mailto:dilcher@mathstat.dal.ca)), Department of Mathematics & Statistics, Dalhousie University, Halifax, N.S. B3H 4R2, Canada, and **Larry Ericksen**. *Continued fractions and Stern polynomials*.

By using specific subsequences of two different types of generalized Stern polynomials, we obtain several related classes of finite and infinite continued fractions involving  $z^{t^j}$  in their partial numerators, where  $z$  is a complex variable and  $t \geq 2$  an integer. The talk concludes with some additional related results. (Received September 16, 2016)

1125-11-1309 **Lenny Jones** ([lkjone@ship.edu](mailto:lkjone@ship.edu)) and **Tristan Phillips\*** ([tp7924@ship.edu](mailto:tp7924@ship.edu)), Department of Mathematics, Shippensburg University, Shippensburg, PA 17257. *Arithmetic Progressions of Polygonal Numbers with Polygonal Common Difference*. Preliminary report.

In 1965, Sierpinski showed that there are infinitely many triples of triangular numbers in arithmetic progression for which the common difference is itself a triangular number. More recently, Ide and Jones showed that there are no 3-term arithmetic progressions of squares for which the common difference is a square. They also showed that no 3-term arithmetic progressions of the triangular numbers have a square common difference. In this talk, we generalize these ideas to other polygonal numbers. (Received September 16, 2016)

1125-11-1323 **Adriana Salerno\*** ([asalerno@bates.edu](mailto:asalerno@bates.edu)), **Damaris Schindler** and **Amanda Tucker**. *Symmetries of Rational Functions Arising in Ecalle's Study of Multiple Zeta Values*.

In Ecalle's theory of multiple zeta values he makes frequent use of certain properties that express symmetries of rational functions in several variables. We focus on the properties of push-invariance, circ-neutrality, and alternality. Ecalle states and uses several implications about the relations between these symmetries. In this talk we will introduce these concepts and prove two results: first, that push-invariance and circ-neutrality imply the first alternality relation, but not the more general alternality relations, and second, that alternality does, indeed, imply circ-neutrality. (Received September 16, 2016)

1125-11-1391 **Alexander Borselli\*** ([apb211@lehigh.edu](mailto:apb211@lehigh.edu)), Christmas Saucon Hall, 14 E. Packer Ave., Bethlehem, PA 18015.  *$\rho$ -Minimal Transitive Permutation Groups and Ranks of CM Types in Degree 24*.

The rank of an Abelian variety  $A$  of type  $(K, \Phi)$ ,  $t(A) = t(\Phi)$ , is the rank of the free  $\mathbb{Z}$ -module  $M$  spanned by the  $G$ -orbit of  $\Phi$ , where  $G = \text{Gal}(K^C/\mathbb{Q})$ , inside the  $\mathbb{Z}$ -module spanned by the  $2n$  embeddings of  $K$  into  $\mathbb{C}$ . If  $t(A) = n + 1$ ,  $A$  is said to be nondegenerate. If  $t(A) = t(\Phi) < n + 1$  and  $A$  is simple, then  $A$  is said to be degenerate. The groups,  $G$ , are transitive permutation groups of degree  $2n$  with even order center, of which there are 19, 126 in degree  $2n = 24$ . To grasp this problem we look at  $\rho$ -minimal transitive permutation groups and minimally transitive permutation groups with even order center. A  $\rho$ -minimal group,  $G$ , of degree  $2n$  is a group with a central order 2 element,  $\rho$ , such that  $G = \langle \rho \times M \rangle$ , where  $M$  is a minimally transitive permutation group of degree  $2n$ , and  $G$  has no proper transitive subgroup with a central order 2 element. In this talk I will discuss their construction in degree 24 and give the ranks of types for these groups and for minimal groups with even order centers. (Received September 16, 2016)

1125-11-1395 **Joshua Harrington** and **Lenny Jones\*** ([lkjone@ship.edu](mailto:lkjone@ship.edu)). *Gaps Between Elements of the Same Order in  $\mathbb{F}_p$* . Preliminary report.

Let  $N \geq 3$  be an integer, let  $p \equiv 1 \pmod{N}$  be a prime and let

$$a = \min \{ a_2 - a_1, a_3 - a_2, \dots, a_{\phi(N)} - a_{\phi(N)-1} \},$$

where

$$a_1 < a_2 < \cdots < a_{\phi(N)}$$

are the elements of order  $N$  in the finite field  $\mathbb{F}_p = \{0, 1, \dots, p-1\}$ . Recently, Brazelton, Harrington, Kannan and Litman investigated the set of primes  $p$  such that  $a = 1$ . In this talk, we discuss extending their investigation to the situation when  $a \geq 2$ . (Received September 16, 2016)

1125-11-1419 **Byungchan Kim\*** (bkim4@seoultech.ac.kr), School of Liberal Arts, SeoulTech, 232 Gongneung-ro, Nowongu, Seoul, 01811, South Korea. *Remarks on cubic partitions.*

Cubic partitions are a special kind of bi-partitions of which name is motivated from a connection between cubic continued fraction and an arithmetic property of this bi-partition. Since H.-C. Chan investigated its arithmetic properties, there have been number of studies on cubic partition and its variants. In this talk, we survey recent results on them. In particular, we will introduce the roles of  $\theta$ -operator and some questions. (Received September 16, 2016)

1125-11-1425 **Luca Candelori\***, lcandelori@lsu.edu, and **Cameron Franc.** *Graded modules of vector-valued modular forms and directed graphs.*

Vector-valued modular forms are holomorphic functions on the upper half-plane taking values in complex representations of subgroups of the modular group, together with a certain ‘weight’ and growth conditions at the cusps. In this talk we study the structure of the graded modules of vector-valued modular forms, and discuss the question of when these modules are projective over the ring of classical modular forms. We also describe the correspondence between modules of vector-valued modular forms and representations of certain quivers (i.e. directed graphs), via tilting theory. (Received September 16, 2016)

1125-11-1430 **Victor Hugo Moll\*** (vhm@tulane.edu), Department of Mathematics, Tulane University, New Orleans, LA 70118, **Aashita Kesharwani** (akesarwa@tulane.edu), Department of Mathematics, Tulane University, New Orleans, LA 70118, and **Xiao Guan** (xguan1@tulane.edu), Department of Mathematics, Tulane University, New Orleans, LA 70118. *Tree structures coming from valuations.* Preliminary report.

The question of  $p$ -adic valuations of sequence of integers often can be given in terms of a tree structure. Examples of this phenomena include the valuations of Stirling numbers as well as sequences generated by polynomials. A detailed analysis of quadratic polynomials will be given. (Received September 16, 2016)

1125-11-1457 **Robert Styer\*** (robert.styer@villanova.edu), Dept of Mathematics and Statistics, 800 Lancaster Ave, Villanova, PA 19085, and **Reese Scott.** *Number of solutions to  $a^x + b^y = c^z$ .*

We show the following: For relatively prime integers  $a$  and  $b$  both greater than one and odd integer  $c$ , there are at most two solutions in positive integers  $(x, y, z)$  to the equation  $a^x + b^y = c^z$ . There are an infinite number of  $(a, b, c)$  giving exactly two solutions. Lastly, we outline some extensions. (Received September 16, 2016)

1125-11-1461 **Karen Taylor\***, karen.taylor@bcc.cuny.edu. *Applications of Hyperbolic Fourier Coefficients.* Preliminary report.

In this talk we will give examples of identities obtained from specializing the exact formula for the  $n$ th hyperbolic fourier coefficient of a cusp form to the full modular group. (Received September 16, 2016)

1125-11-1465 **Laura Hall-Seelig\*** (hallseelig1@merrimack.edu). *A history of studies of a certain pair of Diophantine equations.*

The search for solutions to the pair of Diophantine equations  $xyz = 1$  and  $x + y + z = k$  for rational integers  $k$  has captured the interest of number theorists since the 1960s, when it was first proven that this pair of equations has no nontrivial rational solutions with  $k = 1$ . This talk will present a history of the search for simultaneous solutions to this pair of equations in integers of number fields of small degree, with a particular emphasis on several collaborations with H.G. Grundman. (Received September 16, 2016)

1125-11-1466 **Djordje Milićević\*** (dmilicevic@brynawr.edu), Bryn Mawr College, Department of Mathematics, 101 North Merion Avenue, Bryn Mawr, PA 19010, and **Valentin Blomer**, **Étienne Fouvry**, **Emmanuel Kowalski**, **Philippe Michel** and **William F. Sawin.** *Analysis and arithmetic in moments of twisted  $L$ -functions.*

Central values of  $L$ -functions encode essential arithmetic information in contexts ranging from distribution of primes to elliptic curves and arithmetic manifolds. In this talk, I will present asymptotic formulas for moments in families of twisted  $L$ -functions with all primitive characters modulo  $q$ , with a power saving in  $q$ , obtained by a combination of analytic and arithmetic techniques ranging from spectral theory to arithmetic geometry. In

addition to providing statistical and intrinsic information about the underlying family of automorphic forms, such asymptotics are an essential ingredient in analytic approaches to questions of arithmetic importance such as upper bounds, nonvanishing, or extreme values, and I will also survey several of our applications. (Received September 17, 2016)

1125-11-1467 **Austin Daughton\*** (adaughto@fandm.edu). *Coefficients of Logarithmic Vector-Valued Poincaré Series.*

In 2004, Knopp and Mason computed the coefficients of vector-valued Poincaré series associated to a normal representation. Their expression for these coefficients strongly parallels the classical case and involves Bessel functions and ‘generalized’ Kloosterman sums. For logarithmic representations, Knopp and Mason wrote down a matrix-valued Poincaré series whose columns are logarithmic vector-valued modular forms, but the coefficients of these forms are not derived. However, for logarithmic representations where  $\rho(T)$  is a single Jordan block, we can instead construct a Poincaré series that is a natural analogue of Poincaré series associated to a normal representation. In this talk, I’ll discuss this construction and give an exact expression for the Fourier coefficients of these Poincaré series. (Received September 17, 2016)

1125-11-1489 **Samuel Gross** and **Joshua Harrington\*** (joshua.harrington@cedarcrest.edu), 100 College Drive, Allentown, PA 18104, and **Laurel Minott**. *Sums of Polynomial Residues.*

In an article in the Monthly from 1904, Orlando Stetson studied the sums of distinct residues of triangular numbers modulo a prime. Rather curiously, this sum is always the same residue class independent of the prime chosen. We extend Stetson’s theorem to all polygonal numbers and find similar phenomenon. Extensions to sums of residues of general polynomials will also be discussed. (Received September 17, 2016)

1125-11-1514 **Junxian Li\*** (jli135@illinois.edu), **Kyle Pratt** and **George Shakan**. *Heuristic and improvement on the least prime in an arithmetic progression.*

Fix  $k$  a positive integer, and let  $\ell$  be coprime to  $k$ . Let  $p(k, \ell)$  denote the smallest prime equivalent to  $\ell \pmod{k}$ , and set  $P(k) = \max_{(\ell, k)=1} p(k, \ell)$ . Assuming some weak dependence between primes, we give a heuristic for some statistics of  $P(k)$ . Applying modified sieve weights of Maynard and Tao on large gaps between primes, we improve a lower bound for the least prime in an arithmetic progression by Pomerance. In particular, we show that for almost every  $k$  one has  $P(k) \gg \phi(k) \log k \log_2 k \log_4 k / \log_3 k$ , answering a question of Ford, Green, Konyangin, Maynard, and Tao. (Received September 20, 2016)

1125-11-1518 **Daniel Schultz\***, dps23@psu.edu, and **Timothy Huber** and **Donxi Ye**. *Ramanujan-Sato Series for  $\frac{1}{\pi}$  Arising from the Monster Group.*

The Monster group is the largest sporadic simple group. Due to the settling of the Moonshine Conjecture by Borcherds, we now know that the McKay–Thompson series  $T_g$  of an element of the Monster group is the Hauptmodul of some subgroup of  $\mathrm{SL}_2(\mathbb{R})$ . I will present the theory of series for  $\frac{1}{\pi}$  associated to these Hauptmoduln, which generalize Ramanujan’s and the Chudnovsky brothers’ famous series for  $\frac{1}{\pi}$ . This is joint work with Tim Huber of The University of Texas–Pan American and Dong Xi Ye of The University of Wisconsin–Madison. (Received September 17, 2016)

1125-11-1535 **Saikat Biswas\*** (saikat.biswas@asu.edu), School of Mathematical & Statistical Sciences, Arizona State University, Tempe, AZ 85287. *Capitulation, unit groups, and the cohomology of  $S$ -idèle classes.* Preliminary report.

Suppose that  $L/K$  is a finite, cyclic extension of number fields with Galois group  $G$ . Let  $S$  be a finite set of primes of  $K$  that contains all the infinite primes. The extension of ideals from  $K$  to  $L$  induces the  $S$ -capitulation map, whose kernel classifies the  $S$ -ideal classes in  $K$  that become principal in  $L$ . In this talk, we first interpret the kernel and cokernel of the  $S$ -capitulation map in terms of  $C_{L,S}$ , the  $S$ -idèle class group of  $L$ . We then relate the arithmetic of  $C_{L,S}$  to that of  $U_{L,S}$ , the group of  $S$ -units of  $L$ . We show that many known results in algebraic number theory, in particular Hilbert’s Theorem 94, follow as a direct consequence of our idèle-theoretic results. (Received September 17, 2016)

1125-11-1542 **Jonathan D. Rehmert\*** (rehmertjonathan@gmail.com), PO Box 355, Point Lookout, MO 65726, and **Brandt Kronholm** (brandt.kronholm@utrgv.edu), Department of Mathematics, University of Texas Rio Grande Valley, Edinburg, TX 78539. *Unrestricted Congruence Properties of the Restricted Partition Function.* Preliminary report.

The partition function  $p(n, m)$  enumerates the number of partitions of  $n$  into at most  $m$  parts. Brandt Kronholm of University of Texas Rio Grande Valley has obtained results on infinite families of congruences for  $p(n, m)$  for

$m$  a prime number of parts modulo powers of  $m$ . In this presentation we prove similar results, but with no restrictions on the number of parts nor modulus. (Received September 17, 2016)

1125-11-1576 **Frank Garvan\*** ([fgarvan@uf1.edu](mailto:fgarvan@uf1.edu)). *Weighted partition identities and divisor sums.*

We prove two one-parameter  $q$ -series identities which specialize to four weighted partition identities. Three of the weighted partition identities have coefficients which are divisor sums. The other is an identity due to Alladi whose right side is Gauss's triangular number series. Some of the proofs depend on  $q$ -series results related to the arithmetic of  $\mathbb{Q}[\sqrt{2}]$  due to Lovejoy and due to Corson, Favero, Liesinger and Zubairy. (Received September 18, 2016)

1125-11-1585 **Frank Garvan\*** ([fgarvan@uf1.edu](mailto:fgarvan@uf1.edu)). *The Andrews spt-function mod 4.* Preliminary report.

The Andrews spt-function,  $\text{spt}(n)$ , is the number of smallest parts in the partitions of  $n$ . This function has a number of surprising arithmetic properties, and has been studied by a number of authors. The parity of  $\text{spt}(n)$  is completely determined by the prime factorisation of  $24n - 1$ . The parity result was proved by Andrews, the author and Liang, and corrected an earlier result of Folsom and Ono. In this talk we present a number of conjectures for  $\text{spt}(n) \pmod{4}$ . (Received September 18, 2016)

1125-11-1650 **Oleksiy Klurman\*** ([1klurman@gmail.com](mailto:1klurman@gmail.com)), 2920 Chemin de la Tour, Montreal, QC H3T 1J4, Canada. *Correlations of multiplicative functions and applications.*

We develop the asymptotic formulas for correlations

$$\sum_{n \leq x} f_1(P_1(n)) f_2(P_2(n)) \cdots f_m(P_m(n))$$

where  $f_1, \dots, f_m$  are bounded "pretentious" multiplicative functions, under certain natural hypotheses. We then deduce several desirable consequences: first, we characterize all multiplicative functions  $f: \mathbb{N} \rightarrow \{-1, +1\}$  with bounded partial sums. This answers a question of Erdős from 1957 in the form conjectured by Tao. Second, we show that if the average of the first divided difference of multiplicative function is zero, then either  $f(n) = n^s$  for  $\text{Re}(s) < 1$  or  $|f(n)|$  is small on average. This settles an old conjecture of Kátai. Finally, we discuss multidimensional analog of the results above and its applications to the study of Gowers norms of multiplicative functions. (Received September 18, 2016)

1125-11-1662 **Chad Awtrey\*** ([cawtrey@elon.edu](mailto:cawtrey@elon.edu)), **Jim Beuerle** and **Jade Schrader**. *Constructing Galois 2-extensions of the 2-adic numbers.*

Let  $\mathcal{K}^G$  denote the set of isomorphism classes of Galois extensions  $K$  of the 2-adic numbers whose Galois group is some 2-group  $G$ . Using the work of Yamagishi in 1995, it is possible to determine the size of  $\mathcal{K}^G$ . In this talk, we show how to compute a defining polynomial associated to each class. The algorithm makes use of subfields of index two; including their Galois groups and their quadratic extensions. We end with an application of our algorithm to classifying the 251 Galois 2-adic fields of degree 16. (Received September 18, 2016)

1125-11-1665 **Christelle Vincent\*** ([christelle.vincent@uvm.edu](mailto:christelle.vincent@uvm.edu)). *Abel-Jacobi maps and Riemann points on hyperelliptic Riemann surfaces.* Preliminary report.

Let  $X$  be a compact Riemann surface of genus  $g$  equipped with a choice of an Abel-Jacobi map. This choice determines a point  $r \in \mathbb{C}^g$  called the Riemann point, or the vector of Riemann constants. If  $X$  is a hyperelliptic curve, a further choice of labeling of the branch points of  $X$  with the symbols  $1, 2, \dots, 2g+1, \infty$  associates to this Riemann point a nonempty subset  $U$  of labels. In his seminal work on the characterization of hyperelliptic period matrices, Mumford shows that it is always possible to choose an Abel-Jacobi map such that the cardinality of  $U$  is  $g+1$ . In his own investigation of this same question, Poor shows that the cardinality of  $U$  must be congruent to  $g+1$  modulo 4. In this talk, we define the Riemann point  $r$ , show how to attach to it the set  $U$  when  $X$  is hyperelliptic, and show the converse of Poor's result: for any nonempty subset of  $\{1, 2, \dots, 2g+1, \infty\}$  of cardinality congruent to  $g+1$  modulo 4, there is a choice of Abel-Jacobi map such that this subset is the set  $U$  corresponding to the Riemann point. (Received September 18, 2016)

1125-11-1686 **Larry Ericksen\*** ([larryericksen@gmail.com](mailto:larryericksen@gmail.com)), PO Box 172, Millville, NJ 08332, and **Karl Dilcher** ([dilcher@mathstat.dal.ca](mailto:dilcher@mathstat.dal.ca)), 6264 Yukon Street, Halifax, NS B3L 1G1. *Continued fractions from Stern numbers and generalized polynomials.*

We derive finite and infinite continued fractions from Stern polynomials in two variables, as determined by their recursion formulas and generating functions.

Furthermore we identify algorithms that select elements in Stern's diatomic sequence, based on binary representations of their subscripts. We obtain recursion formulas with determinant weights that specify individual paths through the Stern number trees.

We then construct continued fractions from these generalized number sequences. We use a similar approach to extend our results to continued fractions in Stern polynomials.

Numerous examples are provided to highlight methodology and illustrate special continued fractions. (Received September 18, 2016)

1125-11-1702 **Dawn C. Nelson\***, Department of Mathematics, 2641 John F Kennedy Blvd, Jersey City, NJ 07306. *Generalized Zeckendorf Decompositions and Monovariants*. Preliminary report.

Using only quarters, dimes, and pennies what is the fewest number of coins that will total to 42 cents? Hint: Do NOT use the quarter. The "greedy" technique of using the largest possible coin does not give the correct answer in this problem.

In this talk we will show that when decomposing positive integers as sums of Fibonacci numbers, the greedy algorithm (which results in the Zeckendorf decomposition) yields the fewest number of summands. The same is true of the new Fibonacci Quilt sequence (a two-dimensional analog to Zeckendorf decompositions). As time allows we discuss other generalized Zeckendorf decompositions such as positive linear recurrence sequences. The main tools in our proofs are monovariants. (Received September 18, 2016)

1125-11-1770 **Maria Monica Nastasescu\*** (mnastase@math.brown.edu), Providence, RI. *Average central values of the degree 4 L-function on  $GS(4)/\mathbb{Q}$* .

I present some results towards proving an average result for the degree 4 L-function on  $GS(4)/\mathbb{Q}$  at the center using the relative trace formula. More specifically, I consider a suitable relative trace formula such that the spectral side is an average of central L-values of genus 2 holomorphic Siegel eigenforms of weight  $k$  and level  $N$  twisted by some fixed character. I then work on computing the corresponding geometric side. (Received September 19, 2016)

1125-11-1772 **Antara Mukherjee\***, mukherjeea1@citadel.edu. *GCD properties of Generalized Fibonacci Polynomials*. Preliminary report.

A sequence that satisfies the recurrence relation  $F_0(x) = 0$ ,  $F_1(x) = 1$  and  $F_n(x) = xF_{n-1}(x) + F_{n-2}(x)$  for  $n \geq 2$  is called the Fibonacci polynomial. The Generalized Fibonacci Polynomial (GFP) is a natural generalization of the above mentioned sequence. It is known that the greatest common divisor of two Fibonacci numbers is a Fibonacci number. However, this gcd property does not always hold for every GFP sequence. In this presentation I will provide a definition of generalized Fibonacci polynomials and classify them in two types depending on their Binet formula I will then give a complete characterization of those polynomials that satisfy the Fibonacci gcd property. I will also show that the polynomials that satisfy the Fibonacci gcd property are Fibonacci polynomials, Pell polynomials, Fermat polynomials, Chebyshev polynomials of second kind, Jacobsthal polynomials and one type of Morgan-Voyce polynomials while the polynomials that do not satisfy the Fibonacci gcd property are: Lucas polynomials, Pell-Lucas polynomials, Fermat-Lucas polynomials, Chebyshev polynomials of first kind, Jacobsthal-Lucas polynomials and second type of Morgan-Voyce polynomials. These last set of polynomials partially satisfy the mentioned property. (Received September 19, 2016)

1125-11-1778 **J. Larry Lehman\*** (llehman@umw.edu), Department of Mathematics, University of Mary Washington, 1301 College Avenue, Fredericksburg, VA 22401. *A Continued Fraction Algorithm for Quadratic Numbers, Forms, and Ideals*. Preliminary report.

We demonstrate an algorithm for constructing the continued fraction expansion of an arbitrary irrational quadratic number  $v$ , that is, a root of a degree two polynomial in  $\mathbb{Z}[x]$  having positive discriminant  $\Delta$ , not a square. The method associates to  $v$  a pair of integers  $a$  and  $k$  for which  $a$  divides  $\phi(k)$ , where  $\phi(x)$  is a particular quadratic polynomial of discriminant  $\Delta$ . These same pairs  $a$  and  $k$  produce all quadratic forms of discriminant  $\Delta$ , and all ideals of a domain  $D_\Delta$  of quadratic integers. We show that this algorithm also allows us to determine all distinct classes of these quadratic forms and ideals. (Received September 19, 2016)

1125-11-1783 **Evan P. Dummit\*** (evan.dummit@gmail.com), 915 Hylan Building, University of Rochester, Rochester, NY 14627, and **David S. Dummit** and **Hershly Kisilevsky**. *Characterizations of Quadratic, Cubic, and Quartic Residue Matrices*.

A recent paper of D. Dummit, Granville, and Kisilevsky showed the existence of unusually large biases in a number of prime-counting problems. While investigating this phenomenon, the following question arose: given  $n$  odd primes  $p_1, \dots, p_n$  where  $p^*$  denotes  $(-1)^{(p-1)/2}p$ , how many possible configurations are there for the splitting

behavior of  $p_i$  in  $\mathbb{Q}(\sqrt{p_j^*})$  for the possible pairs  $(i, j)$ ? A natural way to organize this information is via the “quadratic residue matrix” of Legendre symbols  $\frac{p_i}{p_j}$ , which is a seemingly natural object that does not appear to have been previously studied. In my talk, I will give a simple characterization of these quadratic residue matrices along with natural generalizations to the cubic and quartic cases. (Received September 19, 2016)

1125-11-1796 **Krishnaswami Alladi\*** (alladik@uf1.edu), Department of Mathematics, University of Florida, Gainesville, FL 32611. *The local distribution of the number of small prime factors - variations of the classical theme.* Preliminary report.

Let  $f(n, y)$  (resp.  $F(n, y)$ ) denote the number of prime factors of  $n$  that are  $< y$ , where  $f(n, y)$  (resp.  $F(n, y)$ ) counts prime factors singly (resp. with multiplicity). We discuss the number of integers up to  $x$  for which  $f(n, y) = k$  (resp.  $F(n, y) = k$ ), where both  $y$  and  $k$  could vary with  $x$ . When  $y$  is small, the enumeration involves partitions/compositions. As  $y$  gets larger an interesting variation of the classical situation emerges, the variation being maintained until  $y$  is quite close to  $x$  in a certain sense. The finer aspect of the local distribution of  $f(n, y)$  requires analytic tools and these will be indicated along with the type of results that can be proven with such methods. Details of the analysis is being carried out in the PhD thesis of my student Todd Molnar. Tenenbaum has shown in correspondence how some of our estimates could be improved. (Received September 19, 2016)

1125-11-1815 **Chi Huynh** (nhuynh30@gatech.edu), **Carsten Peterson** (carsten.peterson@yale.edu) and **Yen Nhi Truong Vu\*** (ytruongvu17@amherst.edu). *On Summand Minimality of Generalized Zeckendorf Decompositions.*

Zeckendorf states that every number can be uniquely represented as a sum of non-consecutive Fibonacci numbers, paralleling a base  $d$  representation. Given a recurrence  $a_n = c_1 a_{n-1} + \dots + c_t a_{n-t}$ , the tuple  $(c_1, \dots, c_t)$  is called the signature. Miller and Wang, and independently Hamlin, proved that given a non-negative linear recurrence with  $c_1 \geq 1$ , each number has a unique representation with respect to the recurrence sequence, called the generalized Zeckendorf decomposition (GZD), composed of “digits” from an allowable finite list. We prove for all  $n$ , the GZD uses the fewest summands of all representations if and only if the signature is weakly decreasing. To prove sufficiency, we construct an algorithm to arrive at any number’s unique GZD and show the number of summands decreases over the algorithm’s course. To prove necessity we handle a few distinct cases. When  $c_1 > 1$ , we give an example of a non-GZD representation of a number that has fewer summands than the GZD. When  $c_1 = 1$ , we non-constructively prove the existence of a counterexample using the irreducibility of certain polynomials and growth rate arguments. Joint work with Katherine Cordwell, Max Hlavacek, Steven J. Miller and Eyvindur A. Palsson. (Received September 19, 2016)

1125-11-1940 **Timothy J. Huber\*** (timothy.huber@utrgv.edu). *Modular  $p$ -midiation principles.*

I will discuss generalizations of Jacobi’s principle of dimidiation and the Borweins’ principle of trimidiation. Level  $p$  generalizations arise from a special basis construction for  $\mathcal{M}_1(\Gamma(p))$ , the vector space of weight one modular forms for the principal congruence subgroup of level  $p$ . At higher levels, where the number of regular cusps for  $X(p)$  is bounded by  $2(\text{genus} - 1)$ , the dimension of  $\mathcal{M}_1(\Gamma(p))$  is unknown, and the construction generates spanning sets of theta quotients for the image of  $\mathcal{M}_1(\Gamma_1(p))$  in  $\mathcal{M}_1(\Gamma(p))$  under the map  $\tau \mapsto \tau/p$ . The spanning sets further decompose into a direct sum of  $p$ -dissection components given by quotients of theta functions. Dissections of combinatorial generating functions may be given in closed form in terms of these quotients. Congruences for coefficients may be extracted from the dissections. (Received September 19, 2016)

1125-11-1965 **Justin DeBenedetto** and **Jeremy Rouse\***, P.O. Box 7388, Winston-Salem, NC 27109. *Quadratic forms representing all integers coprime to 3.*

We prove that a positive-definite quadratic form with integer coefficients represents all positive-integers coprime to 3 if and only if it represents the integers in  $\{1, 2, 5, 7, 10, 11, 13, 14, 17, 19, 22, 23, 26, 29, 31, 34, 35, 37, 38, 46, 47, 55, 58, 62, 70, 94, 110, 119, 145, 203, 290\}$ . This result is similar to the 290-theorem of Bhargava and Hanke, and the proof uses results about ternary quadratic forms, and properties of modular forms and the Petersson inner product. (Received September 19, 2016)

1125-11-2005 **Jean-Francois Biasse\***, 4202 E Fowler Ave, Tampa, FL 33620, and **Claus Fieker** and **Michael John Jacobson**. *Using lattice reduction to improve isogeny evaluation.*

We present novel algorithms for finding small relations and ideal factorizations in the ideal class group of an order in an imaginary quadratic field, where both the norms of the prime ideals and the size of the coefficients involved are bounded. We show how our methods can be used to improve the computation of large-degree isogenies and endomorphism rings of ordinary elliptic curves defined over finite fields. We obtain improved heuristic complexity

results in almost all cases for these problems, and significantly improved performance in practice, especially in situations where the ideal class group can be computed in advance. (Received September 19, 2016)

1125-11-2013 **James Ricci\*** ([jricci@daemen.edu](mailto:jricci@daemen.edu)), 4380 Main Street, Department of Mathematics and Computer Sci., Amherst, NY 14226. *Finiteness results for regular binary quadratic polynomials*. Preliminary report.

An integral quadratic polynomial is called regular if it represents every integer that is represented by the polynomial itself over the reals and over the  $p$ -adic integers for every prime  $p$ . In a joint paper with W.K. Chan in 2015, we show that there are only finitely many equivalence classes of positive primitive ternary regular complete quadratic polynomials with conductor  $c$ . This generalizes analogous finiteness results for positive definite regular ternary quadratic forms by Watson in 1954 and for ternary triangular forms by Chan and Oh in 2013. In this talk we will look at similar results for binary regular quadratic polynomials. (Received September 19, 2016)

1125-11-2077 **Zarathustra Elessar Brady\*** ([notzeb@gmail.com](mailto:notzeb@gmail.com)). *New Sifting Iterations*. Preliminary report.

Buchstab iteration is a well known technique, based on a simple combinatorial observation, which produces new sieve-theoretic bounds from old sieve-theoretic bounds. By applying Buchstab iteration to the Selberg sieve, Diamond, Halberstam, and Richert have achieved the best known sifting bounds when the sifting dimension is slightly greater than 1. We will describe a few variants of Buchstab iteration which slightly improve these bounds in this range, and conjecture the existence of an infinite sequence of similar iteration rules which can be used to describe the optimal sieves of dimension between 1 and  $3/2$ . (Received September 19, 2016)

1125-11-2081 **Nathan S McAnally\*** ([nmcanall@citadel.edu](mailto:nmcanall@citadel.edu)). *A Generalization of Identities from Fibonacci Numbers to the Generalized Fibonacci Polynomial*.

A sequence that satisfies the recurrence relation  $F_0(x) = 0$ ,  $F_1(x) = 1$  and  $F_n(x) = xF_{n-1}(x) + F_{n-2}(x)$  for  $n \geq 2$  is called the Fibonacci polynomial. The Generalized Fibonacci Polynomial (GFP) is a natural generalization of the above-mentioned sequence. Familiar examples of the GFP include Fibonacci polynomials, Lucas polynomials, Pell polynomials, Pell-Lucas polynomials, Fermat polynomials, Fermat-Lucas polynomials, both types of Chebyshev polynomials, Jacobsthal polynomials, Jacobsthal-Lucas polynomials and both types of Morgan-Voyce polynomials. We classify the GFP into two types, namely Fibonacci type and Lucas type. A Fibonacci type polynomial is equivalent to a Lucas type polynomial if they both satisfy the same recurrence relations. In this talk, we will discuss our process to generalize identities from Fibonacci numbers and Lucas numbers to the GFP. (Received September 19, 2016)

1125-11-2105 **Terrence Richard Blackman\*** ([tblackman@mec.cuny.edu](mailto:tblackman@mec.cuny.edu)), Department of Mathematics, Medgar Evers College, 1650 Bedford Ave., Brooklyn, NY 11225. *Spectral correspondences for Maass waveforms on quaternion groups*.

We prove that in most cases the Jacquet-Langlands correspondence between newforms for Hecke congruence groups and newforms for quaternion orders is a bijection. Our proof covers almost all cases where the Hecke congruence group is of cocompact type, i.e. when a bijection is possible. The proof uses the Selberg trace formula. (Received September 19, 2016)

1125-11-2125 **Luke Giberson\*** ([lgibers@g.clemson.edu](mailto:lgibers@g.clemson.edu)). *Average Twin Prime Conjecture for Elliptic Curves over Abelian Number Fields*.

Let  $E/\mathbb{Q}$  be an elliptic curve. For a prime  $p$  of good reduction, let  $\#E(\mathbb{F}_p)$  denote the number of  $\mathbb{F}_p$ -rational solutions to  $E$ . In 1988, Koblitz conjectured an asymptotic

$$\pi_E^{\text{twin}}(X) = \#\{p < X : p \text{ prime and } \#E(\mathbb{F}_p) \text{ prime}\} \sim C_E \cdot \frac{X}{\log^2 X},$$

where  $C_E$  is an explicit constant depending on the curve  $E$ . A recent paper of Balog, Cojocaru, and David proved this conjecture on average. In this work, the author obtains a similar average result for curves over an arbitrary abelian number field. (Received September 19, 2016)

1125-11-2148 **Caroline Turnage-Butterbaugh\*** ([ctb@math.duke.edu](mailto:ctb@math.duke.edu)). *On  $r$ -gaps between zeros of the Riemann zeta-function*. Preliminary report.

Denote by  $0 < \gamma_1 \leq \gamma_2 \leq \dots$  the imaginary part of the zeros of the Riemann zeta-function on the critical line. Selberg showed that for all positive integers  $r$  there exists an absolute constant  $c$  such that  $\limsup_{n \rightarrow \infty} (\gamma_{n+r} - \gamma_n) \frac{\log \gamma_n}{2\pi r} > 1 + c/\sqrt{r}$  and  $\liminf_{n \rightarrow \infty} (\gamma_{n+r} - \gamma_n) \frac{\log \gamma_n}{2\pi r} < 1 - c/\sqrt{r}$ . We continue the investigation into qualitative descriptions of  $r$ -gaps between zeros of the Riemann zeta-function in this preliminary report, which is joint work with Brian Conrey. (Received September 19, 2016)

1125-11-2155 **Liuquan Wang** ([wangliuquan@u.nus.edu](mailto:wangliuquan@u.nus.edu)) and **Ae Ja Yee\*** ([ayy2@psu.edu](mailto:ayy2@psu.edu)), Department of Mathematics, The Pennsylvania State University, University Park, PA 16802. *Some Hecke-Rogers type identities.*

In q-series, Hecke-Rogers type sums are interesting objects. Recently, a double series of Hecke-Rogers type has arisen from the work of Andrews, Dixit, Schultz, and Yee on partition functions associated with Ramanujan's third order mock theta function  $\omega(q)$ . In this talk, I will discuss the series with two further identities. This is joint work with Liuquan Wang from National University of Singapore. (Received September 19, 2016)

1125-11-2172 **Darren B Glass\*** ([dglass@gettysburg.edu](mailto:dglass@gettysburg.edu)), 300 N Washington Street, Gettysburg, PA 17325. *Counting Arithmetical Structures on Graphs.*

For any finite graph, Lorenzini defined the notion of an arithmetical structure on the graph inspired by some notions in algebraic geometry. One formulation of this definition is a labelling of the vertices of a graph with positive integers so that the label of each vertex is a divisor of the sum of the labels of all adjacent vertices. These structures are of interest for a number of reasons, but in this talk, I will emphasize the number theoretic questions they lead to and discuss recent work with various co-authors counting the number of structures on graphs in various families. (Received September 19, 2016)

1125-11-2190 **Tomas E Guardia\*** ([guardia@gonzaga.edu](mailto:guardia@gonzaga.edu)), Mathematics Department, 502 E. Boone Ave., MSC 2615, Spokane, WA 99258-0072, and **Douglas A Jimenez** ([dougjim@gmail.com](mailto:dougjim@gmail.com)), Av. Corpahuayco, Parque Tecnológico, Barquisimeto, Venezuela. *Fiboquadratic Sequences Raised From Rithmomachia.*

In this talk we will show that under an infinite extension of the board, the numbers of Rithmomachia become the first terms of a family of Fiboquadratic Sequences. Fiboquadratic Sequences have been studied in Discrete Mathematics, Combinatorics and Number Theory. We will see a general form of Cassini's Identity obtained by the properties of Fiboquadratic Sequences. (Received September 19, 2016)

1125-11-2197 **Ryan William Matzke\*** ([matzk053@umn.edu](mailto:matzk053@umn.edu)), 1920 South 1st Street, Unit 506, Minneapolis, MN 55454. *Looking for Sum-Freedom: The Maximum Size of  $(k, l)$ -sum-free Sets.* Preliminary report.

Let  $G$  be an Abelian group and let  $A \subseteq G$ . For any  $h \in \mathbb{N}_0$ , we define the  $h$ -fold sumset of  $A$  as

$$hA = \left\{ \sum_{i=1}^h a_i : a_i \in A \right\}.$$

For  $k, l \in \mathbb{N}_0$ , with  $k > l$ , we say that  $A$  is  $(k, l)$ -sum-free if  $kA \cap lA = \emptyset$ . Sets that satisfy this for  $k = 2$  and  $l = 1$  are often simply called sum-free sets. In 2005, Green and Ruzsa were able to find the maximum size of a sum-free subset of any finite Abelian group. Using results from Bajnok, Plagne, and Hamidoune, we can begin finding the maximum size of  $(k, l)$ -sum-free subsets of finite abelian groups through the use of  $(k, l)$ -sum-free arithmetic progressions. (Received September 19, 2016)

1125-11-2202 **Terence Coelho, Jongwon Kim and Matthew C. Russell\*** ([russell12@math.rutgers.edu](mailto:russell12@math.rutgers.edu)). *Companions and generalizations of Göllnitz's "big" partition theorem.* Preliminary report.

In the spirit of Göllnitz's "big" partition theorem of 1967, we present a new modulus 6 partition identity. Alladi, Andrews, and Gordon provided a refinement of Göllnitz's big theorem in 1995 via a "key identity" and the method of weighted words. Using this technique, two similar identities of Göllnitz type were discovered by Alladi (in 1999) and by Alladi and Andrews (in 2015). We finish the picture by presenting and proving the fourth and final possible modulus 6 identity. Furthermore, we provide a generalization of these for all moduli  $n \geq 6$ . (Received September 19, 2016)

1125-11-2246 **Piper Harron\***, 2565 McCarthy Mall (Keller Hall 401A), Honolulu, HI 96822, and **Manjul Bhargava**. *Equidistribution of Shapes of Number Fields of degree 3, 4, and 5.*

In her talk, Piper Harron will introduce the ideas that there are number fields, that number fields have shapes, and that these shapes are everywhere you want them to be. This result is joint work with Manjul Bhargava and uses his counting methods which we currently only have for cubic, quartic, and quintic fields. She will wave her hands in the general direction of a sketch of the proof of this result and beseech all interested parties to check out her thesis. (Received September 20, 2016)

1125-11-2249 **Nathan Kaplan\*** (nckaplan@math.uci.edu), **Gautam Chinta** (gchinta@ccny.cuny.edu) and **Shaked Koplewitz** (shaked.koplewitz@yale.edu). *Counting lattices by cotype.*

The short integer solution (SIS) problem asks, given  $m$  uniformly random elements  $g_1, \dots, g_m$  from  $(\mathbb{Z}/q\mathbb{Z})^n$  to find an integer vector  $(x_1, \dots, x_m)$  of small norm such that  $x_1g_1 + \dots + x_mg_m = 0$ . This problem plays an important role in the theory of worst-case to average-case reductions for lattice problems developed by Ajtai. This naturally leads to finding short vectors in sublattices  $L$  of  $\mathbb{Z}^m$  with  $\mathbb{Z}^m/L = (\mathbb{Z}/q\mathbb{Z})^n$ . In the generalized version of this problem we replace  $(\mathbb{Z}/q\mathbb{Z})^n$  with a more general finite abelian group  $G$ .

The cotype of an  $n$ -dimensional lattice  $L$  is the finite abelian group  $\mathbb{Z}^n/L$ . What properties do we expect for the cotype of a randomly chosen sublattice of  $\mathbb{Z}^n$ ? How many sublattices have cotype  $G$ ? We discuss these and other problems and explain a connection to the Cohen-Lenstra heuristics from number theory. (Received September 20, 2016)

1125-11-2255 **Cezar Lupu\*** (lupucezar@gmail.com), Pittsburgh, PA 15260. *The Riemann zeta function for integer values and evaluation of some multiple zeta values.* Preliminary report.

In this talk, we derive some new series representations involving odd values of the Riemann zeta function and Euler numbers. Using a well-known series representation for the Clausen function, we also provide some new representations of Apéry's constant. In particular cases, we recover some well-known series representations of  $\pi$ . Moreover, we use analytic methods to evaluate multiple zeta values and multiple  $t$ -values of Zagier type. Our formulas are related to the ones proved by Hoffman and Zagier back in the early days of the MZV. (Part of this is joint work with Derek Orr) (Received September 20, 2016)

1125-11-2265 **Vanessa Arcelia Aguirre\*** (aguirre7@hawaii.edu) and **Seneca Cox** (senecac@hawaii.edu). *Factorizations, elasticity, and Frobenius numbers of numerical monoids generated by a double arithmetic sequence.*

Much is known about numerical monoids with two generators as well as those with generators forming an interval or an arithmetic sequence. When the generators are not of these forms, numerical monoids exhibit more complicated behavior. We introduce a monoid generated by a double arithmetic sequence or the numerical monoid  $\langle a, b, a+s, b+s \rangle$ , and we assume that  $s \mid a$  and  $a < b$ . We give a general expression for the Frobenius number of these numerical monoids and provide a proof for when  $b = a + 1$  and  $s \geq a/s$ . In this case, the Frobenius number for monoids generated by a double arithmetic sequence is equal to the Frobenius number of  $\langle b, s \rangle$ , the monoid generated by  $b$  and  $s$ . In addition, we compare the elasticity of elements in  $\langle a, b \rangle$  with those in a monoid generated by a double arithmetic sequence  $\langle a, b, a+s, b+s \rangle$ . While it is known that the overall elasticity of the monoid generated double arithmetic sequence is larger than that of the monoid  $\langle a, b \rangle$ , we explore their differences in elasticity element-wise. (Received September 20, 2016)

1125-11-2325 **Padmavathi Srinivasan\*** (psrinivasan41@math.gatech.edu), 686 Cherry St, Atlanta, GA 30332-0160. *Conductors and minimal discriminants of hyperelliptic curves.*

Conductors and minimal discriminants are two measures of degeneracy of the singular fiber in a family of hyperelliptic curves. In the case of elliptic curves, the Ogg-Saito formula shows that (the negative of) the Artin conductor equals the naive minimal discriminant. In the case of genus two curves, Qing Liu showed that equality no longer holds in general, but the two invariants are related by an inequality. We extend Liu's inequality to hyperelliptic curves of arbitrary genus assuming that the Weierstrass points are rational. We also present explicit examples that suggest that Liu's inequality extends without any assumptions on the rationality of the Weierstrass points. We explain the difficulties in adapting the proof of the inequality in the case of rational Weierstrass points to the general case. (Received September 20, 2016)

1125-11-2334 **Paul Pollack\*** (pollack@uga.edu), Department of Mathematics, Boyd Graduate Studies Research Center, University of Georgia, Athens, GA 30602. *Torsion subgroups of CM elliptic curves.*

For each positive integer  $d$ , let  $T(d)$  denote the supremum of all orders of groups  $E(F)[\text{tors}]$  appearing for an elliptic curve  $E$  defined over a degree  $d$  number field  $F$ . A celebrated theorem of Merel asserts that  $T(d) < \infty$  for all  $d$ . However, the known quantitative results in this direction are far from the conjectured truth. Let  $T_{\text{CM}}(d)$  be defined the same way as  $T(d)$ , but with the restriction to CM elliptic curves. I will discuss some recent statistical results concerning  $T_{\text{CM}}(d)$  and related functions. Perhaps surprisingly, the "anatomy of integers" (as pioneered by Paul Erdős) plays a key role in the proofs. Joint work with Abbey Bourdon and Pete L. Clark. (Received September 20, 2016)

1125-11-2395 **DoYong Kwon\*** (doyong@jnu.ac.kr), Department of Mathematics, Chonnam National University, Gwangju, 500-757, South Korea. *Sturmian words and Cantor sets arising from unique expansions over ternary alphabets.*

Over a finite alphabet  $A$  of real numbers, unique expansions in base  $\beta$  are considered. A real number  $G_A$  called the generalized golden ratio is a border of situation of unique expansions. If  $\beta < G_A$  then there are only trivial unique expansions in base  $\beta$ , while we have non-trivial unique expansions in base  $\beta$  whenever  $\beta > G_A$ . Komornik, Lai, and Pedicini (2011) investigated the case where  $A$  consists of three real numbers, and demonstrated that Sturmian words curiously emerge out of the generalized golden ratio. The present talk focuses on Sturmian words under this context. For a given alphabet  $A = \{a_1, a_2, a_3\}$  with  $a_1 < a_2 < a_3$ , we give a complete characterization of the corresponding Sturmian words effectively and algorithmically, which reveals interesting structures behind the generalized golden ratios. (Received September 20, 2016)

1125-11-2425 **Annie Chen, T. Alden Gassert\*** (thomas.gassert@wne.edu) and **Katherine E. Stange.** *Index divisibility in dynamical sequences and cyclic orbits modulo  $p$ .*

Let  $\phi(x) = x^d + c$  be an integral polynomial of degree at least 2, and consider the sequence  $(\phi^n(0))_{n=0}^\infty$ , which is the orbit of 0 under iteration by  $\phi$ . Let  $D_{d,c}$  denote the set of positive integers  $n$  for which  $n \mid \phi^n(0)$ . We give a characterization of  $D_{d,c}$  in terms of a directed graph and describe a number of its properties, including its cardinality and the primes contained therein. In particular, we study the question of which primes  $p$  have the property that the orbit of 0 is a single  $p$ -cycle modulo  $p$ . We show that the set of such primes is finite when  $d$  is even, and conjecture that it is infinite when  $d$  is odd. (Received September 20, 2016)

1125-11-2439 **Thomas Garrity\*** (tgarrity@williams.edu), Department of Mathematics and Statistics, Williams College, Williamstown, MA 01267, and **Peter McDonald.** *Minkowski Question Mark Type Functions for a Family of Multi-dimensional Continued Fractions.* Preliminary report.

Over a century ago, Herman Minkowski developed his question mark function, which is a strictly increasing continuous function from the unit interval to the unit interval that sends the set of quadratic irrationalism to the set of rationals. A few decades after his work, it was shown that his function, though strictly increasing, had to have derivative zero almost everywhere, and thus is a naturally occurring singular function. Underlying all the properties of the question mark function is the theory of continued fractions. We will discuss generalizations of the question mark function to a family of multidimensional continued fractions, called triangle partition maps. These maps include the triangle map and the Mönkmayer map. Combinations of these maps include almost all known multidimensional continued fraction algorithms. We will see for each of these maps that there is only one potential candidate for a question mark type function. We mainly discuss which of these algorithms are, in a natural sense, singular and which have other key properties analogous to the original question mark function. (Received September 21, 2016)

1125-11-2482 **Hiram Golze\*** (golze2@illinois.edu).  *$t$ -Core Words and Their Use in Proving  $t$ -Core Identities.*

We introduce the notion of  $t$ -core words. By describing these, we can find tools for proving identities bijectively. These are shown to be especially useful in the case of  $t = 3$ . (Received September 20, 2016)

1125-11-2603 **Abbey Bourdon\*** (abourdon@uga.edu) and **Pete L. Clark.** *Torsion Points on CM Elliptic Curves.*

Let  $F$  be a number field, and let  $E/F$  be an elliptic curve with complex multiplication (CM) by an order  $\mathcal{O}$  in an imaginary quadratic field  $K$ . We provide an explicit description of the Weber function field of  $E$ , classically known in the case where  $\mathcal{O}$  is the full ring of integers in  $K$ , and we use this to prove a uniform open image theorem for  $K$ -CM elliptic curves whose endomorphisms are rationally defined. We apply these results to give a complete determination of the degrees of  $K$ -CM points on the modular curves  $X_1(N)_K$  for any positive integer  $N$ . (Received September 20, 2016)

1125-11-2733 **Jennifer Berg\*** (jb93@rice.edu). *Brauer Manin Obstruction for a family of surfaces.*

A problem of great interest in number theory is determining whether a polynomial equation has a rational or integral solution. A necessary condition is local solubility, however this is generally not sufficient to guarantee a "global" (integer or rational) solution. In order for local solutions to come from a common global solution, it turns out that they must satisfy certain compatibility conditions that can arise from quadratic reciprocity and higher reciprocity laws. These conditions are known as the Brauer-Manin obstruction. In this talk, I will describe recent work on computing this obstruction for families of affine surfaces (fibered over the affine line). (Received September 20, 2016)

1125-11-2758 **Daniel P. Wisniewski\*** ([daniel.wisniewski@desales.edu](mailto:daniel.wisniewski@desales.edu)), Department of Mathematics/Computer Science, DeSales University, 2755 Station Avenue, Center Valley, PA 18034. *Progress and Challenges Regarding Tetranomial Thue Equations.*

In 2000, Emery Thomas published results for counting the number of solutions to trinomial Thue equations. In his paper, Thomas suggested that his methods, which include solving a Diophantine approximation problem combined with a gap principle argument, along with an application of the Thue-Siegel principle, could be adapted to tetranomials. However, in so doing for the four-term Thue equation, certain additional conditions were necessary in order to find explicit numerical upper bounds for the number of solutions  $(p, q) \in \mathbb{Z}$ , with  $|pq| \geq 2$ , to the tetranomial Thue equation  $|F(x, y)| = 1$ , where

$$F(x, y) = ax^n + r^m y^{n-m} - rx^k y^{n-k} + ty^n,$$

with  $n > m > k > 0$  and  $a, r, s, t \in \mathbb{Z} - \{0\}$ . While highlighting certain challenges faced in this process, an overview of the development of these results will be given. (Received September 20, 2016)

1125-11-2832 **Wilson Harvey\*** ([harveywa@math.sc.edu](mailto:harveywa@math.sc.edu)) and **Michael Filaseta**. *Covering Subsets of the Integers.*

A covering system of the integers is a finite system of congruences where each integer satisfies at least one of the congruences. Two questions in covering systems posed by Erdős have been of particular interest in the mathematical literature. First is the minimum modulus problem, whether the minimum modulus of a covering system of the integers with distinct moduli can be arbitrarily large, and the second is the odd covering problem, whether a covering system of the integers with distinct moduli can be constructed with all moduli odd. We consider these and similar questions for subsets of the integers, such as the set of prime numbers, the Fibonacci numbers, and numbers that are the sums of two squares. For example, we show that there does exist an odd covering of the integers that are the sums of two squares, and that the minimum modulus problem can be answered in the affirmative for the Fibonacci numbers. (Received September 20, 2016)

1125-11-2837 **Renate Scheidler\*** ([rscheid1@ucalgary.ca](mailto:rscheid1@ucalgary.ca)). *Fake Real Quadratic Orders.*

In an unpublished note from 2014, Henri Cohen coined the term *fake real quadratic order* for the localization of a maximal imaginary quadratic order at a split prime ideal. The name was motivated by the surprising fact that fake real quadratic orders behave very much like maximal orders of real quadratic fields; in particular, they have unit rank 1 and tend to have an enormously large fundamental unit and a very small class number. This invites the tantalizing question of whether certain well-known conjectures formulated for actual real quadratic orders also hold in fake real quadratic orders. Two such conjectures include the widely believed Cohen-Lenstra heuristic which asserts that approximately 75% of all real quadratic fields are expected to have class number one, and the more controversial Ankeny-Artin-Chowla conjecture which claims that if  $q$  is a prime congruent to 1 (mod 4) and  $\epsilon = (t + u\sqrt{q})/2$  is the fundamental unit of  $\mathbb{Q}(\sqrt{q})$ , then  $q \nmid u$ . In this talk, we present extensive numerical data that speak to these two conjectures in the setting of fake real quadratic orders. This is joint work with Mike Jacobson and our jointly supervised graduate student Hongyan Wang. (Received September 20, 2016)

1125-11-2852 **Alia Hamieh** ([alia.hamieh@uleth.ca](mailto:alia.hamieh@uleth.ca)), University of Lethbridge, Department of Mathematics and Computer Scienc, Lethbridge, AB T1K 3M4, Canada, and **Naomi Tanabe\*** ([naomi.tanabe@dartmouth.edu](mailto:naomi.tanabe@dartmouth.edu)), Kemeny 6188, Dartmouth College, Hanover, NH 03755. *Non-vanishing of central L-values for Rankin-Selberg convolutions.*

In this talk, we present some nonvanishing results on central critical values of Rankin-Selberg L-functions associated to Hilbert modular forms. We use a mollification method to establish the results. This is an ongoing joint project with Alia Hamieh. (Received September 20, 2016)

1125-11-2857 **Paul D Nelson\*** ([paul.nelson@math.ethz.ch](mailto:paul.nelson@math.ethz.ch)). *Microlocal analysis on representations and number-theoretic applications.*

I will talk about joint work with Akshay Venkatesh in which we study microlocal analysis on unitary representations of a Lie group. The motivating problem is to understand bounds for and moments of L-functions on higher-rank arithmetic groups. One interesting ingredient is an algebraic phenomenon whereby irreducible representations retain a form of irreducibility after microlocalizing. (Received September 20, 2016)

1125-11-2877 **Dennis Eichhorn\*** ([deichhor@math.uci.edu](mailto:deichhor@math.uci.edu)), Department of Mathematics, University of California, Irvine, Irvine, CA 92697. *The Combinatorics of the Divisibility of spt-overpartition Functions.*

A variety of authors have studied various spt-overpartition functions, which count the total number of smallest parts that appear among all (possibly restricted) overpartitions of  $n$ . Many of these spt-overpartition functions

have interesting congruences and divisibility properties, and all of these properties were originally proven using generating function techniques. In this talk, we discuss some of these divisibility properties combinatorially. In doing so, we hope to develop a better understanding of how these divisibility properties arise from the actual partitions involved, without having to rely solely upon their generating functions. Included in this talk will be at least two reasons why studying spt-overpartition functions is at least slightly less crazy than it seems. (Received September 20, 2016)

1125-11-2881 **Heidi E Goodson\*** ([hgoodson@haverford.edu](mailto:hgoodson@haverford.edu)). *Hypergeometric Functions and Relations to Arithmetic and Analytic Properties of Curves.*

In 1965, Manin proved that the rows of the Hasse-Witt matrix of an algebraic curve are solutions to the differential equations of the periods, thus making a connection between arithmetic and analytic properties of curves. In the case of elliptic curves, the Hasse-Witt matrix has a single entry: the trace of Frobenius. For an elliptic curve in the Legendre family, the trace of Frobenius can be expressed in terms of a finite field  ${}_2F_1$  hypergeometric function and the period of the curve can be expressed in terms of a matching classical  ${}_2F_1$  hypergeometric series. Furthermore, I have shown that these matching hypergeometric expressions are congruent modulo  $p$ . In this talk, I will give an overview of these results and extend them to a family of higher genus generalized Legendre curves. (Received September 20, 2016)

1125-11-2959 **Siegfred Baluyot\*** ([sbaluyot@ur.rochester.edu](mailto:sbaluyot@ur.rochester.edu)), Department of Mathematics, 915 Hylan Hall, Rochester, NY 14627. *On the zeros of Riemann's zeta-function on the critical line.*

In 1942, Selberg proved that a positive proportion of the zeros of the Riemann zeta-function  $\zeta(s)$  are on the critical line. Later, Levinson used a different approach to prove that the proportion is at least  $1/3$ . In this talk, we present a third approach to prove Selberg's theorem using an idea of Atkinson. A main ingredient of the proof is an estimate for the fourth moment of  $\zeta(s)$  times a mollifier. (Received September 20, 2016)

1125-11-3001 **Zhu Cao\*** ([zcaoc@kennesaw.edu](mailto:zcaoc@kennesaw.edu)) and **Yong Hu**. *Exact covering systems, Quadratic forms, and product identities for theta functions.*

In this talk, we discuss the connections among exact covering systems, quadratic forms, and product identities for theta functions. We show that most of the identities among Ramanujan's forty identities for the Rogers-Ramanujan functions can be proved using ECS and QF. We also apply these ideas to obtain new identities for products of three or more theta functions. (Received September 20, 2016)

1125-11-3012 **Malcolm Edward Rupert\*** ([mrupert@uidaho.edu](mailto:mrupert@uidaho.edu)). *An explicit theta lift from Hilbert modular forms to Siegel paramodular forms.*

Let  $E/\mathbb{Q}$  be a real quadratic extension for which 2 is unramified and let  $\pi_0$  be an irreducible, cuspidal automorphic representation of  $\mathrm{gl}(2, \mathbb{A}_E)$ . I provide a formula, locally at every prime, for an explicit theta lift which produces a Siegel paramodular newform from the data of  $\pi_0$ . After showing the connection to curves by modularity results, I will outline the steps necessary to produce this formula and sketch a proof that a given test vector satisfies the required invariance properties to produce a nonzero Siegel paramodular newform. (Received September 20, 2016)

1125-11-3022 **Enrique Treviño\*** ([trevino@mx.lakeforest.edu](mailto:trevino@mx.lakeforest.edu)), 555 N. Sheridan, Lake Forest College, Lake Forest, IL 60045. *Summer research projects for First Year students.*

During the past three years I've mentored 5 research projects for first year undergraduates. In this talk I will discuss these projects. I will give a brief description of how I selected the projects, what the projects were and how good these projects were for the students. The projects have the following titles: 1) What's so rational about the alphabet?, 2) Finding perfect polynomials mod 2, 3) On Tupper's self-referential formula, 4) First world-solutions to first-year problems, 5) Beatty sequences and the prime race. (Received September 20, 2016)

1125-11-3060 **Robert Harron\*** ([rharron@math.hawaii.edu](mailto:rharron@math.hawaii.edu)). *Equidistribution of shapes of cubic fields of fixed quadratic resolvent.*

Building upon work of Bhargava, P. Harron, and Shnidman, I will discuss results on the distribution of shapes of cubic fields  $K$  of fixed quadratic resolvent. The shapes depend on the trace zero form (that is the projection of the trace form to the trace zero space). For instance, I'll show that the shapes of complex cubic fields lie on the geodesic on the modular surface  $\mathrm{SL}(2, \mathbf{Z}) \backslash \mathfrak{H}$  determined by their trace zero form and that, in a fixed such geodesic, the shapes are equidistributed with respect to the natural hyperbolic measure. In the case of pure cubic fields (whose quadratic resolvent field is the third cyclotomic field), the corresponding geodesics have infinite length and the equidistribution must be considered in a regularized sense. That these geodesics are of infinite

length provides a reason behind the different asymptotic growth rates of pure cubic fields versus other fields of fixed quadratic resolvent seen in the work of Bhargava–Shnidman and Cohen–Morra. I’ll also discuss related results such as the fact that the shape is a complete invariant of complex cubic fields. (Received September 20, 2016)

1125-11-3112 **Farshid Hajir\*** ([hajir@math.umass.edu](mailto:hajir@math.umass.edu)), Mathematics & Statistics Lederle Tower, 710 N. Pleasant Street, UMass Amherst, Amherst, MA 01003, and **Christian Maire**. *Exponents of Class Groups in Towers of Number Fields*.

We discuss upper bounds on the exponents of class groups in certain unramified pro- $p$  towers of number fields. These come from a combination of group theory and the Tsfasman-Vladut generalization of the Brauer-Siegel theorem. (Received September 21, 2016)

1125-11-3153 **Keith R Matthews** and **John P Robertson\*** ([jpr2718@gmail.com](mailto:jpr2718@gmail.com)), 600 Gardenia Ter, Delray Beach, FL 33444. *On the Stolt Fundamental Solutions of  $x^2 - dy^2 = 4n$* . Preliminary report.

Stolt equivalence can give a more compact way of presenting solutions to many generalized Pell equations of the form  $x^2 - dy^2 = 4n$ , when compared with Nagell equivalence. We use modifications of the Lagrange-Matthews-Mollin (LMM) continued fraction algorithm to find the Stolt fundamental solutions. (Received September 21, 2016)

1125-11-3154 **Mojtaba Moniri\*** ([m-moniri@wiu.edu](mailto:m-moniri@wiu.edu)). *Diophantine questions on a familiar limit*.

This is a proposed UGR project on some plausible Diophantine-analytic identities as  $\frac{\sin x}{x}$  converges to 1 along certain values  $x \rightarrow 0$ . Using  $\csc^2 x = \frac{1}{x^2} + \frac{1}{3} + \frac{x^2}{15} + \mathcal{O}(x^4)$  about 0, as  $m \rightarrow \infty$  one has  $\frac{1}{4} \csc^2 \frac{\pi}{2m+2} = \frac{4^{m+1}}{\pi^2} + \frac{1}{12} + \frac{\pi^2}{15 \times 4^{m+3}} + \mathcal{O}(4^{-2m})$ . We wrote this with  $\frac{1}{4}$  and  $m+2$  on the left to go with  $\frac{1}{2 - \underbrace{\sqrt{2 + \sqrt{2 + \dots + \sqrt{2}}}}_{m \text{ times}}}$  =

$\frac{1}{4} \csc^2 \frac{\pi}{2m+2}$ . We ask whether

$$\lfloor \frac{1}{4} \csc^2 \frac{\pi}{2m+2} \rfloor =? \lfloor \frac{4^{m+1}}{\pi^2} + \frac{1}{12} \rfloor (?)$$

for all  $m$ . We similarly raise the questions  $\lfloor \frac{1}{4} \csc^2 \frac{3\pi}{2m+3} \rfloor =? \lfloor \frac{4^{m+2}}{9\pi^2} + \frac{1}{12} \rfloor (?)$ ,

$\lfloor \frac{1}{4} \csc^2 \frac{\pi}{3 \times 2^{m+2}} \rfloor =? \lfloor \frac{9 \times 4^{m+1}}{\pi^2} + \frac{1}{12} \rfloor (?)$ , and  $\lfloor \frac{1}{4} \csc^2 \frac{5\pi}{12 \times 2^m} \rfloor =? \lfloor \frac{9 \times 4^{m+1}}{25\pi^2} + \frac{1}{12} \rfloor (?)$ .

[Using Mathematica, we verified all four when  $m \leq 20,000$ .] (Received September 21, 2016)

## 12 ► Field theory and polynomials

1125-12-978 **Miguel A. Mendez\*** ([mmendezenator@gmail.com](mailto:mmendezenator@gmail.com)), Calle 39-18 #35, Apartamento 301, Bogotá, Cundinamar 111311, Colombia. *C-Monoids in Species and Posets*.

In the category of set species we review three monoidal operations, 1) Substitution 2) Product 3) Arithmetic product

They give rise to three monoidal categories. The first one is related to the substitution of formal power series, the second to the ordinary product, and the third to the product of Dirichlet series. Monoids with respect to the substitution are operads. Monoids with respect to the product are called ordinary monoids (closely related to associative algebras). The third one leads to a new kind of monoids that we call of Dirichlet type. In each of the three monoidal categories, monoids that satisfy a left cancellation law are called c-monoids. The c-monoids are important because interesting families of partially ordered sets can be constructed from them in a very natural way. Cohen-Macaulay property for posets associated to c-monoids in the first and second monoidal categories are respectively equivalent to the Koszul property of the corresponding operad or associative algebra. We explore Koszulness versus Cohen-Macaulay property in the Dirichlet case. (Received September 13, 2016)

1125-12-1660 **Chad Awtrey\*** ([cawtre@elon.edu](mailto:cawtre@elon.edu)). *Some open problems in computational Galois theory*. The work of 19th century mathematician Evariste Galois shows that roots of polynomials have inherent symmetries, which are encoded as permutations of the roots. In the language of abstract algebra, these symmetries form a group under function composition, called the polynomial’s Galois group. Naturally, a fundamental task in computational algebra is to determine the Galois group of a polynomial. In this talk, we will discuss the history and context of Galois’ work, recent computational results, as well as some potential open problems suitable for investigation by undergraduate students. (Received September 18, 2016)

1125-12-2181 **José William Velázquez\*** ([jose.velazquez16@upr.edu](mailto:jose.velazquez16@upr.edu)), hc 45 Box 9810 Road 730, Cayey, Puerto Rico, cayey, PR 00736. *2 error binary correcting codes through nonlinear algebra.*

The fundamental problem in communication is the fact that as we increase the amount of information we want to send through a channel the risk of the information being altered increases. The purpose of error correcting codes is to recover the information that was lost in the channel. Linear codes in general can be defined by a  $n \times n$  parity check matrix "H" where  $n$  is the block length of the codewords and  $k$  is the rank of the codes generator matrix. We study the application of nonlinear functions (focusing on the Gold and Kasami sequences) on the parity check matrix so that we can increase the minimum distance of a code and hence its error correcting capacity. In general, nonlinear functions defined over finite fields have very important applications in error correcting codes. The Gold and Kasami functions are related to 2-error correcting codes of length  $2^s - 1$  as studied by Van Lint, Wilson, Janwa and others. We use only the properties of the Gold and Kasami functions and their domains, as well as, multivariate polynomials related to these functions in order to determine parameters that help us identify 2 error correcting codes. (Received September 19, 2016)

### 13 ► Commutative rings and algebras

1125-13-277 **Nathan A. BeDell\*** ([nabedell@liberty.edu](mailto:nabedell@liberty.edu)). *Logarithms over a Real Associative Algebra.*

Given a real associative and finite dimensional unital algebra  $\mathcal{A}$ , a natural question to ask is how to generalize the standard theory of differential and integral calculus. In this talk, using such a theory to construct the exponential function over  $\mathcal{A}$ , I outline what can be said about the logarithmic functions in  $\mathcal{A}$ . In particular, I present the finding that for a large class of nilpotent algebras the exponential function is injective, and hence such algebras have a unique logarithm on the image of the exponential. In addition to this, I examine how Freese's work on generalized trigonometric functions may be used to find explicit formulas for logarithms analogous to the result that  $\text{Log}(z) = \log(|z|) + i\text{Arg}(z)$  from complex analysis. To conclude the talk, I discuss some relevant open problems to this line of inquiry, and discuss how the theory of logarithms might be applied to other areas of mathematics. (Received August 22, 2016)

1125-13-351 **Susan Loepp\*** ([sloep@williams.edu](mailto:sloep@williams.edu)), Bronfman Science Center, 18 Hoxsey St., Williamstown, MA 01267. *Understanding the Relationship Between Local Rings and Their Completions: Contributions by Undergraduates.* Preliminary report.

Given a Noetherian ring  $R$  with exactly one maximal ideal  $M$ , we first define a metric on  $R$  with respect to  $M$ . The completion of  $R$  with respect to this metric turns out to be a ring with surprisingly nice properties. One can often gain a better understanding of the ring  $R$  by studying its completion. However, the relationship between  $R$  and its completion is not well understood. After giving a general introduction to the topic of rings and their completions, we will highlight research by undergraduates over the last 20 years that has helped us to better understand this mysterious relationship. (Received August 29, 2016)

1125-13-565 **Lokendra Paudel and Simplice Tchamna\*** ([simplice.tchamna@gcsu.edu](mailto:simplice.tchamna@gcsu.edu)), Milledgeville, GA 31061. *Some properties of saturations of submodules.*

Let  $R$  be a subring of a ring  $S$ , and let  $A$  be an  $R$ -submodule of  $S$ . The saturation of  $A$  (in  $S$ ) by  $\tau$  is set  $A_{[\tau]} = \{x \in S : tx \in A \text{ for some } t \in \tau\}$ , where  $\tau$  is a multiplicative subset of  $R$ . We study properties of saturations of  $R$ -submodules of  $S$ . In particular, we investigate conditions under which the equality  $AR_{[p]} = A_{[p]}$  is satisfied. We construct an example of ring extension for which the relation  $AR_{[p]} = A_{[p]}$  is not always true. (Received September 06, 2016)

1125-13-600 **Scott T Chapman\*** ([scott.chapman@shsu.edu](mailto:scott.chapman@shsu.edu)), Sam Houston State University, Department of Mathematics and Statistics, Box 2206, Huntsville, TX 77341. *REU projects involving non-unique factorization in integral domains and monoids: Past, Present, and Future.* Preliminary report.

Since the late 1990's, REU projects involving the theory of non-unique factorizations in integral domains and monoids have helped introduce dozens of undergraduate students to the world of mathematical research. In this talk, I will outline the background material necessary for a student to begin such a project. Several results from past projects will be highlighted, some of which were recently completed. I will close with a list of potential problems appropriate for future REU students. (Received September 07, 2016)

- 1125-13-779 **Michael T. Annunziata, Courtney R. Gibbons and Cole Hawkins\***, colepshawkins@gmail.com, and **Alexander J. Sutherland**. *Rational Combinations of Betti Diagrams of Complete Intersection Modules*.

We investigate decompositions of Betti diagrams over a polynomial ring within the framework of Boij-Söderberg theory. That is, given a Betti diagram, we determine if it is possible to decompose it into the Betti diagrams of complete intersections. To do so, we determine the extremal rays of the cone generated by the diagrams of complete intersection modules and provide an exponential-time algorithm for decomposition. (Received September 11, 2016)

- 1125-13-974 **Sarah M. Fleming and Lena Ji\***, lji@math.princeton.edu, and **S. Loepp, Peter M. McDonald, Nina Pande and David Schwein**. *Strange Formal Fibers: A Counterexample*.

Let  $R$  be a local ring with maximal ideal  $\mathfrak{m}$ , and let  $\hat{R}$  be its completion with respect to  $\mathfrak{m}$ . Completion induces a morphism  $\text{Spec}\hat{R} \rightarrow \text{Spec}R$  given by  $\mathfrak{q} \mapsto \mathfrak{q} \cap R$ , and for each prime ideal  $\mathfrak{p} \in \text{Spec}R$ , the formal fiber of  $R$  at  $\mathfrak{p}$  is defined to be the preimage of  $\mathfrak{p}$  under this map. The dimension of the formal fiber of  $R$  at  $\mathfrak{p}$  – that is, the maximal length of a chain of prime ideals of  $\hat{R}$  lying over  $\mathfrak{p}$  – is denoted  $\alpha(R, \mathfrak{p})$ . In many cases, the dimensions of formal fibers are well-understood; for most rings,  $\alpha(R, \mathfrak{p}) = \dim R - \text{ht } \mathfrak{p} - 1$ .

Heinzer, Rotthaus, and Sally have asked, given an excellent local integral domain  $R$  such that  $\alpha(R, (0)) > 0$ , if the set of height one prime ideals  $\mathfrak{p}$  such that  $\alpha(R, \mathfrak{p}) = \alpha(R, (0))$  is finite. Given previous results, the expectation might be an affirmative answer. We construct a non-excellent counterexample where every height one prime ideal  $\mathfrak{p}$  of  $R$  has the property that  $\alpha(R, \mathfrak{p}) = \alpha(R, (0))$ . This talk is based on joint work completed at the Williams College REU with Sarah Fleming, S. Loepp, Peter McDonald, Nina Pande, and David Schwein. (Received September 13, 2016)

- 1125-13-1174 **Sarah M. Fleming, Lena Ji, S. Loepp, Peter M. McDonald, Nina Pande\*** (ngp3@williams.edu) and **David Schwein**. *Strange Formal Fibers: An Excellent Counterexample*.

The relationship between a local ring  $R$  and its completion  $\hat{R}$  is often studied through the natural map from the prime ideals of  $\hat{R}$  to the prime ideals of  $R$  given by  $\mathfrak{q} \mapsto \mathfrak{q} \cap R$ , where  $\mathfrak{q}$  is a prime ideal of  $\hat{R}$ . If  $\mathfrak{p}$  is a prime ideal of  $R$ , then the inverse image of  $\mathfrak{p}$  under this map is called the formal fiber of  $R$  at  $\mathfrak{p}$ . In general, this map is not well understood.

We discuss several properties of formal fibers and provide a counterexample to a question about formal fibers posed by Heinzer, Rotthaus, and Sally. Our counterexample is an excellent regular local ring whose formal fibers have an unexpected structure. We discuss the general technique for constructing a ring while controlling its formal fibers and some of the challenges associated with our problem. This research was carried out by five undergraduates and advised by Susan Loepp at the 2015 SMALL REU at Williams College. (Received September 15, 2016)

- 1125-13-1187 **Patrick M Phelps\*** (patrickphelps@mail.adelphi.edu) and **Branden Stone** (bstone@adelphi.edu). *Matroids on Commutative Rings*. Preliminary report.

Matroids are a well researched topic in graph theory; they let us abstract the ideas of independence found in linear algebra. It has been shown that a matroid can be formed over a Group, using the notion of additive inverses to form independent sets. In this talk we will extend these ideas to a commutative ring. With these new matroids in hand, we will analyze the generators of the toric ideals associated to them. In particular we are concerned with the degree in which the ideals are generated. (Received September 15, 2016)

- 1125-13-1198 **Rebecca R.G.\*** (rirebhuh@syr.edu). *Using closure operations to study singularities*.

In characteristic  $p > 0$ , many of the existing results on the singularities of commutative rings were proved using tight closure, a technique developed by Mel Hochster and Craig Huneke. There are also a number of results in equal characteristic 0 that have used reduction to characteristic  $p$  to take advantage of tight closure methods. In this talk, I will discuss a generalization of tight closure called a Dietz closure. The simplest Dietz closures come from tensor products with big Cohen-Macaulay modules and algebras. I will present results linking Dietz closures to singularities of commutative rings in various characteristics, and describe their relevance to the homological conjectures. (Received September 15, 2016)

- 1125-13-1222 **Branden Stone\*** (bstone@adelphi.edu). *Examples of Undergraduate Research in Commutative Algebra*. Preliminary report.

Introducing advanced research topics to undergraduate students can be a difficult task. In this talk, we will focus on two undergraduate projects that dealt with areas in commutative algebra; Boij-Söderberg theory and

toric ideals. We will discuss how these topics were introduced, how the students progressed, and what results were obtained. (Received September 15, 2016)

1125-13-1264 **Greg Oman** and **Adam Salminen\*** (as341@evanville.edu). *Polynomial and power series rings with finite quotients.*

We determine the rings  $R$  with the property that  $R[X]/I$  (respectively,  $R[[X]]/I$ ) is finite for every nonzero ideal  $I$  of  $R[X]$  (respectively,  $R[[X]]$ ). We also look at a one-sided analogue of these results. (Received September 15, 2016)

1125-13-1474 **Dana Fry**, **Zvi Rosen** and **Jessica Sidman\*** (jsidman@mholyoke.edu), Department of Mathematics and Statistics, Mount Holyoke College, South Hadley, MA 01075, and **Louis Theran** and **Cynthia Vinzant**. *Special positions of bar and joint frameworks.* Preliminary report.

Let  $G$  be a generically rigid bar-and-joint framework. The bars in the framework correspond algebraically to a system of quadratic equations in which we view the joint coordinates as variables and the (squared) distances as parameters. In this talk I will discuss how algebraic methods may be used to determine bar lengths for which such a framework  $G$  is in a special position, i.e., a position for which it has nontrivial internal motions. (Received September 17, 2016)

1125-13-1545 **Nic Ford**, **Jake Levinson\*** (jakelev@umich.edu) and **Steven Sam**. *Foundations of Boij-Söderberg Theory for Grassmannians.*

Boij-Söderberg theory characterizes the syzygies of graded modules (up to scalar) and relates them to sheaf cohomology on projective space. We extend the theory to the setting of  $GL_k$ -equivariant modules and sheaf cohomology on Grassmannians. Algebraically, we study modules over a polynomial ring in  $kn$  variables ( $k \leq n$ ), thought of as the entries of a  $k \times n$  matrix.

We give equivariant analogues of three important features of the ordinary theory: the Herzog-Kühl equations (linear constraints on Betti tables); the nonnegative bilinear pairing between Betti tables and cohomology tables; and the explicit polyhedral structure of the cone of Betti tables for square matrices. The latter cone is the target of the bilinear pairing; it serves as a base case for the theory.

Our statements specialize to those of Boij-Söderberg theory for graded modules when  $k = 1$ . Our proof of the equivariant pairing gives a new proof in the graded setting: it relies on finding perfect matchings on certain graphs associated to Betti tables. (Received September 17, 2016)

1125-13-1698 **Luigi Ferraro\*** (lferraro2@math.unl.edu). *A class of modules with infinite regularity.*

Let  $R$  be the graded ring  $k[x_1, \dots, x_n]/I$  where  $I$  is a homogeneous ideal of the polynomial ring with the standard grading. If the Castelnuovo-Mumford regularity of  $k$  is infinite we are going to construct a class of modules with infinite regularity. (Received September 18, 2016)

1125-13-1975 **Darrin Weber\*** (dweber3@vols.utk.edu). *The Classification of Zero-Divisor Graphs of Commutative Rings Without Identity.* Preliminary report.

The zero-divisor graph of a commutative ring  $R$ , denoted  $\Gamma(R)$ , is a graph whose vertices are the nonzero zero-divisors of the ring  $R$  with edges drawn between vertices  $x$  and  $y$  if and only if  $xy = 0$ . In a paper from 2006, Shane Redmond classified all finite rings with identity that had zero-divisor graphs on vertices  $\leq 14$ . We look at extending this work to commutative rings without an identity. (Received September 19, 2016)

1125-13-2513 **Jason M Lutz\*** (lutzj@gonzaga.edu). *Homological characterizations of quasi-complete intersections.*

Let  $R$  be a commutative Noetherian ring and  $I$  an ideal of  $R$ . The homology of a Koszul complex associated with  $I$  is an invariant of  $I$ , and if this homology vanishes in positive degree, then  $I$  is said to be a *complete intersection*. If the homology exhibits the structure of an exterior algebra, then  $I$  is said to be a *quasi-complete intersection*. Using Tate's "adjunction of variables", we obtain an extension of the Koszul complex; a result of Blanco, Majadas, and Rodicio yields that  $I$  is a quasi-complete intersection if and only if the homology of this infinite complex vanishes in positive degree. Our main results characterize quasi-complete intersections as those ideals for which the homology of the associated Tate construction vanishes in a finite band of sufficient size. (Received September 20, 2016)

1125-13-2570 **Roberto Barrera** ([rbarrera@math.tamu.edu](mailto:rbarrera@math.tamu.edu)), Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843, **Jeffrey Madsen** ([madsen4@purdue.edu](mailto:madsen4@purdue.edu)), Department of Mathematics, Purdue University, 150 N. University Street, West Lafayette, IN 47907, and **Ashley K. Wheeler\*** ([ashleykw@uark.edu](mailto:ashleykw@uark.edu)), Department of Mathematical Sciences, SCEN 309, University of Arkansas, Fayetteville, AR 72701. *Finiteness of Associated Primes of Local Cohomology Modules over Stanley-Reisner Rings.*

Local cohomology modules, even over a Noetherian ring  $R$ , are typically unwieldy. As such, it is of interest whether or not they have finitely many associated primes. We prove the affirmative in the case where  $R$  is a Stanley-Reisner ring, over a field, whose associated simplicial complex is a  $T$ -space. (Received September 20, 2016)

1125-13-2702 **Haydee Lindo\*** ([08hml@williams.edu](mailto:08hml@williams.edu)), 820 Main Street, Apt 2, Williamstown, MA 01267. *Trace ideals of modules and algebras over commutative rings.*

I will present some new results regarding trace ideals of modules and algebras over commutative rings. This continues the project begun in arXiv:1603.08576 relating the center of the endomorphism ring of a module  $M$ , over a commutative noetherian ring, to the endomorphism ring of the trace ideal of  $M$ . (Received September 20, 2016)

1125-13-3031 **Paul Baginski\*** ([pbaginski@fairfield.edu](mailto:pbaginski@fairfield.edu)), Fairfield University, Department of Mathematics, 1073 North Benson Rd., Fairfield, CT 06824, and **Gregory Knapp, Jad Salem** and **Gabrielle Scullard**. *Nonunique factorization in the ring of integer-valued polynomials.*

The ring of integer-valued polynomials  $\text{Int}(\mathbb{Z})$  is the set of polynomials with rational coefficients which produce integer values for integer inputs. Specifically,

$$\text{Int}(\mathbb{Z}) = \{f(x) \in \mathbb{Q}[x] \mid \forall n \in \mathbb{Z} f(n) \in \mathbb{Z}\}.$$

$\text{Int}(\mathbb{Z})$  constitutes an interesting example in algebra from many perspectives; for example, it is a natural example of a non-Noetherian ring. It is also a ring with nonunique factorization: given  $f(x) \in \text{Int}(\mathbb{Z})$ , there can be multiple ways to write  $f(x)$  as a product of irreducible integer-valued polynomials. Not only can we have  $f(x) = a_1(x) \cdots a_n(x) = b_1(x) \cdots b_m(x)$ , for different irreducibles  $a_i(x)$  and  $b_j(x)$ , we can have different numbers of irreducible factors, i.e.  $n \neq m$ . Thus, an element  $f(x)$  can have different factorization lengths  $n$ . Frisch recently demonstrated that in  $\text{Int}(\mathbb{Z})$ , you can find an element  $f(x)$  that has any factorization lengths you desire and you can even prescribe the number of factorizations of each length. The polynomials constructed in this way have high degree. We give a graded analysis, determining the possible elasticities and catenary degrees for a polynomial of bounded degree. (Received September 20, 2016)

1125-13-3134 **Basanti Sharma Poudyal\*** ([poudyal@uta.edu](mailto:poudyal@uta.edu)), 411 S. Nedderman Dr., Arlington, TX 76019. *Existence of totally reflexive modules in graded local rings with Hilbert series  $1 + et + (e - 1)t^2$ .*

Let  $(A, m)$  be a Noetherian local graded ring with Hilbert series  $1 + et + (e - 1)t^2$ . It is known that the existence of exact zero divisors implies the existence of non-free totally reflexive modules. We are interested in the existence of these modules in the absence of exact zero divisors. In a recent study, Vraciu and Atkins constructed an example of a ring of codimension 8 that does not have exact zero divisors, but has non-free totally reflexive modules. In this talk, we will give a class of rings of codimension 5 and higher admitting totally reflexive modules, but without having exact zero divisors. (Received September 21, 2016)

## 14 ► Algebraic geometry

1125-14-113 **Mee Seong Im\*** ([meeseongim@gmail.com](mailto:meeseongim@gmail.com)), 646 Swift Road, Department of Mathematical Sciences, Thayer Hall, Office 249, West Point, NY 10996. *On the geometry of filtered representations of quivers and connections to isospectral Hilbert schemes.*

We prove that varieties constructed via unipotent invariants of the Borel on filtered quiver representations are finite dimensional, which are important in the spec and the proj constructions in algebraic geometry. I will discuss their connections to quiver Grothendieck-Springer resolutions and isospectral Hilbert schemes. Other related topics such as quantum Hamiltonian reduction and their connections to Cherednik algebras, and the Borel generalization of the moment map of the Atiyah-Drinfeld-Hitchin-Manin (ADHM) construction will be discussed. This is joint with Lisa M. Jones at the University of Cambridge. (Received September 10, 2016)

- 1125-14-159 **Irene Bouw, Wei Ho, Beth Malmskog, Renate Scheidler, Padmavathi Srinivasan and Christelle Vincent\*** ([christelle.vincent@uvm.edu](mailto:christelle.vincent@uvm.edu)). *Curves with many automorphisms.* For  $p$  an odd prime, we study a certain class of Artin-Schreier curves. The automorphism group of these curves contains a large extra special group as a subgroup. Precise knowledge of this subgroup makes it possible to compute the zeta function of the curves after extending the base field to contain the field of definition of the automorphisms. We find that over fields of square cardinality, these curves are either maximal or minimal, and we classify which curves fall into which category. (Received August 05, 2016)
- 1125-14-230 **Douglas A Torrance\*** ([dtorrance@piedmont.edu](mailto:dtorrance@piedmont.edu)), P.O. Box 10, 1021 Central Ave, Demorest, GA 30535. *Generic forms of low Chow rank.* The least number of products of linear forms that may be added together to obtain a given form is the Chow rank of this form. The Chow rank of a generic form corresponds to the smallest  $s$  for which the  $s$ th secant variety of the Chow variety fills the ambient space. We show that, except for certain known exceptions, this secant variety has the expected dimension for low values of  $s$ . (Received August 15, 2016)
- 1125-14-289 **Emilio Bujalance, Antonio F. Costa and Milagros Izquierdo\*** ([milagros.izquierdo@liu.se](mailto:milagros.izquierdo@liu.se)), Department of Mathematics, Linköping University, 58183 Linköping, Ostergötla, Sweden. *Dessins d'enfants and a curve of Wiman.* A dessin d'enfant in a surface  $S$  is an embedded bipartite graph in the surface. The dessin determines the conformal structure of  $S$ . A surface admits a dessin d'enfant if and only if its is defined over a number field (as complex curve), and equivalently  $S$  is a covering on the projective line ramified on three points. Here we show that, with a few exceptions, a dessin d'enfant in a surface  $S_g$  of genus  $g$ , with rotational group of order  $4g$  determines Wiman's curve of type II:  $y^2 = x(x^{2g} - 1)$ . (Received August 24, 2016)
- 1125-14-392 **Emma Previato\*** ([ep@bu.edu](mailto:ep@bu.edu)), Department of Mathematics and Statistics, Boston University, Boston, MA 02215-2411. *Weierstrass Semigroups.* Weierstrass points play a major role in the theory of automorphisms of algebraic curves. We report on the existence of a  $\langle 6, 13, 14, 15, 16 \rangle$  Weierstrass semigroup (with J. Kameda and S. Matsutani, *Internat. J. Math.* **24**, 2013). For this class of curves, we give formulas that link algebraic and transcendental moduli, via Klein's higher-genus sigma function. For a trigonal curve, we give an explicit expression of the Riemann theta constant (with Kameda and Matsutani, *Arc. Math.* 2016), not previously known when the Weierstrass semigroup is not symmetric. We then review an application of the Kleinian sigma to hyperelliptic curves, which yields algebraic equations for points of finite order (in a suitable sense) on the Jacobian (with Y. Kodama and S. Matsutani, *Ann. Inst. Fourier* **63**, 2013). The finiteness condition reflects into Poncelet's classical porism, and we report on a generalization to Poncelet's theorem in space. Poncelet's traverses are motions in an algebraically completely integrable system. We propose an interpretation of the Jacobian fibration of integral manifolds analogous to the one devised by N.J. Hitchin in the elliptic case, which corresponds to the monodromy of a particular algebraic class of Painlevé VI equations. (Received August 31, 2016)
- 1125-14-493 **S. Allen Broughton\*** ([brought@rose-hulman.edu](mailto:brought@rose-hulman.edu)), **Antonio F. Costa** and **Milagros Izquierda**. *Subgroups of the Mapping Class Group Corresponding to 1-dimensional Strata in the Branch Locus of Moduli Space.* Preliminary report. The branch locus  $\mathcal{B}_\sigma \subset \mathcal{M}_\sigma$ , the moduli space of surfaces of genus  $\sigma$ , is the subspace of surfaces with a non-trivial automorphism group. The branch locus admits a stratification by finitely many, irreducible, complex algebraic varieties, corresponding to surfaces whose automorphism groups have topologically equivalent actions. In turn, each stratum determines a conjugacy classes of finite subgroups of the mapping class of genus  $\sigma$ . The correspondence between strata and conjugacy classes of finite subgroups is not 1-1, but is fairly close to 1-1. The strata of dimension 0 correspond to quasiplatonic surfaces, which are very well studied. In this talk we take the next step and study the subgroups corresponding to strata of dimension 1, where the quotient surface  $S/\text{Aut}(S)$  is either a sphere with four branch points or a torus with one branch point. We discuss the topology of these strata in terms of the structure of the group. (Received September 04, 2016)
- 1125-14-533 **Aaron Landesman\*** ([aaronlandesman@stanford.edu](mailto:aaronlandesman@stanford.edu)) and **Anand Patel**. *Association with Interpolation.* I will tell the story of my experiences with interpolation. Interpolation is the problem of finding curves or higher dimensional objects passing through a given collection of points. I will focus the history of interpolation in algebraic geometry and how I came to learn about it. The talk is aimed at a very general mathematical audience. (Received September 05, 2016)

1125-14-535 **Nate Strawn\*** ([nate.strawn@georgetown.edu](mailto:nate.strawn@georgetown.edu)), Department of Mathematics and Statistics, Georgetown University, Washington, DC 20007, **Jameson Cahill** ([jamesonc@nmsu.edu](mailto:jamesonc@nmsu.edu)), Department of Mathematical Sciences, New Mexico State University, Las Cruces, NM 88003, and **Dustin Mixon** ([dustin.mixon@afit.edu](mailto:dustin.mixon@afit.edu)), Department of Mathematics and Statistics, Air Force Institute of Technology, Wright-Patterson AFB, Dayton, OH 45433. *Connectivity and Irreducibility of Finite Unit-Norm Tight Frame Varieties.*

Finite Unit-Norm Tight Frames (FUNTFs) are redundant linear dictionaries with applications in compression and the robust transmission of signals. As algebraic varieties, FUNTF varieties are intersections of Stiefel manifolds and products of spheres, and Eigensteps (or Gelfand-Tsetlin patterns) provide useful local parameterizations. Exploiting these parameterizations, we demonstrate that FUNTF varieties are connected, verifying a conjecture by Dykema et al. from 2006. After introducing this initial connectivity result, we refine our technique to further demonstrate the connectivity of the sets of nonsingular points of the FUNTF varieties, which is in turn used to show that FUNTF varieties are irreducible. One corollary of this last result is that a generic FUNTF has full spark, which provides theoretical support for compressed sensing applications of random FUNTFs. (Received September 05, 2016)

1125-14-596 **Dustin Moody\*** ([dustin.moody@nist.gov](mailto:dustin.moody@nist.gov)), NIST, 100 Bureau Drive, Computer Security Division, Gaithersburg, MD 20877, and **Daniel Shumow**, Microsoft. *Isogenies of Edwards Curves.*

Isogenies are the structure preserving maps between elliptic curves. As such, isogenies play a key role many areas of elliptic curve cryptography. For example, they have been proposed as a mathematical primitive in the construction of hash functions, pseudo-random generators, as well as post-quantum public key cryptosystems. In this work, we present a new isogeny formula for elliptic curves known as Edwards curves. The new formula is twice as efficient than using the standard Velu formula for isogenies. We examine the potential applications of the Edwards isogeny formula in cryptography. (Received September 12, 2016)

1125-14-619 **Sara Shirinkam\***, [sara.shirinkam@utsa.edu](mailto:sara.shirinkam@utsa.edu), **Adel Alaeddini**, [adel.alaeddini@utsa.edu](mailto:adel.alaeddini@utsa.edu), and **Elizabeth Gross**, [elizabeth.gross@sjsu.edu](mailto:elizabeth.gross@sjsu.edu). *Numerical algebraic geometry for identifying the number of components in Gaussian Mixture Models.*

Gaussian Mixture Models (GMM) are among most statistically mature methods for clustering and density estimation with numerous successful applications in science and engineering. GMM parameters are typically estimated from training data using the iterative Expectation-Maximization (EM) algorithm, which requires the number of Gaussian components a priori. In this study we proposed a numerical algebraic geometry approach to identify the optimal number of Gaussian components in GMM. The proposed approach transforms GMM models with various number of components into equivalent polynomial regression splines and uses homotopy continuation methods to find the model or equivalently the number of components which is most compatible with training data. The proposed approach also identifies the location of all local maxima of the equivalent polynomial regression model which accurately estimates the location of Gaussian components centers. We compare the performance of the proposed approach against popular methods in the literature which are based on Akaike information criterion (AIC) and Bayesian Information Criterion (BIC) using extensive simulation. (Received September 08, 2016)

1125-14-646 **Sean F Ballentine**, **Rebecca Black\*** ([rblack1@math.umd.edu](mailto:rblack1@math.umd.edu)), **Ariella Kirsch**, **Adam Lizzi** and **Robert Maschal**. *Locally Recoverable Trace Codes.* Preliminary report.

The field of locally recoverable codes (LRC codes) comes from the application of coding theory to cloud and distributed storage systems. Constructions from algebraic geometry have contributed many interesting classes of such codes using curves over finite fields, often times not a prime field. This talk will explore the preservation of recoverability of these codes when the trace map is applied to the codes generated from some of the algebraic geometry constructions. (Received September 08, 2016)

1125-14-863 **Cristian Lenart\*** (c1enart@albany.edu), Department of Mathematics and Statistics, State University of New York at Albany, 1400 Washington Avenue, Albany, NY 12222, **Kirill Zainoulline**, Department of Mathematics and Statistics, University of Ottawa, 585 King Edward Street, Ottawa, Ontario K1N 6N5, Canada, and **Changlong Zhong**, Department of Mathematics and Statistics, State University of New York at Albany, 1400 Washington Avenue, Albany, NY 12222. *Parabolic Kazhdan-Lusztig basis, Schubert classes, and equivariant oriented cohomology.*

This talk is concerned with Schubert calculus for an arbitrary oriented cohomology theory in the sense of Levine-Morel. Beyond  $K$ -theory, the classes associated to Schubert varieties depend on the chosen Bott-Samelson desingularizations; therefore, a natural problem is to define canonical classes. We offer a solution in the case of the equivariant oriented cohomology theory corresponding to the 2-parameter Todd genus, which works for any partial flag variety. It is based on a new interpretation of Deodhar's construction of the parabolic Kazhdan-Lusztig basis. We make a conjecture about the relationship of the canonical classes with smoothness of Schubert varieties, and prove it in several special cases. (Received September 12, 2016)

1125-14-868 **Christopher Allen Manon\*** (cmanon@gmu.edu), 4400 University Drive, Fairfax, VA 22030. *Newton-Okounkov cones of free group character varieties.*

A Newton-Okounkov cone is a convex set which behaves as a combinatorial snapshot of the coordinate algebra of a variety. I will describe how these objects have been used to answer questions about the coordinate algebras of free group character varieties, and give an update on an ongoing experimental project to use Newton-Okounkov cones to find polynomial presentations of these algebras. (Received September 12, 2016)

1125-14-870 **Shmuel Friedland** (friedlan@uic.edu) and **Li Wang\*** (liwang8@uic.edu). *Geometric measure of entanglement of symmetric d-qubits is polynomial-time computable.*

We give a simple formula for finding the spectral norm of  $d$ -mode symmetric tensor in two variables over the complex or real numbers in terms of the complex or real roots of a corresponding polynomial in one complex variable. This result implies that the geometric measure of entanglement of symmetric  $d$ -qubits is polynomial-time computable. (Received September 12, 2016)

1125-14-993 **Martha Precup\*** (mprecup@math.northwestern.edu). *The Betti numbers of regular Hessenberg varieties and the dot action.*

Regular semisimple Hessenberg varieties are smooth subvarieties of the flag variety that carry an action of the Weyl group on their cohomology. Although the action is defined Lie theoretically, this representation has arisen in the work of Shareshian and Wachs on chromatic quasisymmetric functions. Brosnan and Chow have proven that this representation is determined by the Betti numbers of other regular Hessenberg varieties in Lie type  $A$ . In this talk, we discuss their results and give another formula which proves that the Betti numbers of these varieties are palindromic in all Lie types. (Received September 13, 2016)

1125-14-1015 **Sean D Lawton\*** (slawton3@gmu.edu), Department of Mathematical Sciences, 4400 University Dr, Exploratory Hall, room 4400, Fairfax, VA 22030, and **Christopher Manon** (cmanon@gmu.edu), Department of Mathematical Sciences, 4400 University Dr, Exploratory Hall, room 4400, Fairfax, VA 22030. *Character Varieties of Free Groups are Gorenstein but not always Factorial.*

Fix a rank  $g$  free group  $F$  and a connected reductive complex algebraic group  $G$ . Let  $X(F,G)$  be the  $G$ -character variety of  $F$ . When the derived subgroup  $DG$  in  $G$  is simply connected we show that  $X(F,G)$  is factorial (which implies it is Gorenstein), and provide examples to show that when  $DG$  is not simply connected  $X(F,G)$  need not even be locally factorial. Despite the general failure of factoriality of these moduli spaces, using different methods, we show that  $X(F,G)$  is always Gorenstein. (Received September 14, 2016)

1125-14-1016 **Sean D Lawton\*** (slawton3@gmu.edu), Department of Mathematical Sciences, 4400 University Dr, Exploratory Hall, 4400, Fairfax, VA 22030, and **Krishnendu Gongopadhyay** (krishnendu@iisermohali.ac.in), Knowledge City, Sector 81, SAS Nagar, Manauli PO 140306, Mohali, India. *Invariants of pairs in  $SL(4, C)$  and  $SU(3,1)$ .*

We describe a minimal global coordinate system of order 30 on the  $SL(4,C)$ -character variety of a rank 2 free group. Using symmetry within this system, we obtain a smaller collection of 22 coordinates subject to 5 further real relations that determine conjugation classes of generic pairs of matrices in  $SU(3,1)$ . (Received September 14, 2016)

1125-14-1056 **Natalie Hobson\*** ([nhobson@uga.edu](mailto:nhobson@uga.edu)), UGA Dept. of Mathematics, Boyd Research Center, Athens, GA 30602. *Identities between first Chern classes of vector bundles of Conformal blocks.*

Given a simple Lie algebra  $\mathfrak{g}$ , a positive integer  $\ell$ , and an  $n$ -tuple  $\vec{\lambda}$  of dominant integral weights for  $\mathfrak{g}$  at level  $\ell$ , one can define a vector bundle on  $\overline{M}_{g,n}$  known as a *vector bundle of conformal blocks*. These bundles are nef in genus  $g = 0$  and so this family provides potentially an infinite number of elements in the nef cone of  $\overline{M}_{0,n}$  to analyze. Result relating these divisors with different data is thus significant in understanding these objects. In this talk, we use correspondences of these bundles with products in quantum cohomology in order to classify when a bundle with  $\mathfrak{sl}_2$  or  $\mathfrak{sp}_{2\ell}$  is rank one. We show this is also a necessary and sufficient condition for when these divisors are equivalent. (Received September 14, 2016)

1125-14-1127 **Kristin E. Lauter\*** ([klauter@microsoft.com](mailto:klauter@microsoft.com)). *The Future of Curves in Cryptography (?)*. Over the last decade, the use of Elliptic Curve Cryptography (ECC) has become standard across the industry, used for example for digital signatures to assure identity and authenticity, and for key exchange to set up secure browser sessions. ECC was proposed 3 decades ago, but widespread adoption was prompted by government-issued Suite B requirements in 2006. In the meantime, practical aspects of using higher genus curves for cryptography have also been explored, and it has been demonstrated that Jacobians of genus 2 curves can even match the performance of ECC at the same security levels. But recently, government-issued recommendations (CNSA Suite) have shifted to require larger key sizes for RSA and ECC-based systems based on the threat of quantum attacks, while awaiting the outcome of an international competition to select new “Post-Quantum” Cryptographic (PQC) systems. This talk will discuss ongoing work on assessing security of ECC and higher genus curves against both quantum and classical attacks. (Received September 15, 2016)

1125-14-1263 **Allen Knutson\*** ([allenk@math.cornell.edu](mailto:allenk@math.cornell.edu)). *The cyclic Bruhat decomposition of flag manifolds.*

The positroid stratification on the Grassmannian arises in many ways (from positive real, or characteristic  $p$ , or Poisson geometry) that generalize to give the Richardson stratification on the flag manifold. The way that doesn’t generalize similarly is to see the positroid stratification as the common refinement of the  $n$  cyclic shifts of the Bruhat decomposition. I’ll describe the corresponding decomposition on the full flag manifold  $GL_n/B$  – which, unfortunately, is *not* a stratification – and give some evidence that the strata are smooth and irreducible of predictable dimension. (Received September 15, 2016)

1125-14-1267 **Max B. Kutler\*** ([kutler@uoregon.edu](mailto:kutler@uoregon.edu)). *Faithful tropicalization of hypertoric varieties.* An arrangement  $\mathcal{A}$  of affine hyperplanes defines a *hypertoric variety*  $\mathfrak{M}_{\mathcal{A}}$ , the geometry of which is controlled by the combinatorics of  $\mathcal{A}$ . The variety  $\mathfrak{M}_{\mathcal{A}}$  admits a natural tropicalization, induced by its embedding in a Lawrence toric variety. We explicitly describe the polyhedral structure of this tropicalization. Using a recent result of Gubler, Rabinoff, and Werner, we prove that there is a continuous section of the tropicalization map. (Received September 15, 2016)

1125-14-1271 **Andrew Obus\*** ([andrewobus@gmail.com](mailto:andrewobus@gmail.com)), University of Virginia, Department of Mathematics, Kerchof Hall, 141 Cabell Drive, Charlottesville, VA 22904, and **Holly Krieger, John Doyle, Lloyd West, Rachel Pries** and **Simon Rubinstein-Salzedo**. *Good and bad reduction of dynamomic modular curves.*

The dynamomic modular curves are curves defined over  $\mathbb{Z}$  parameterizing dynamical systems on  $\mathbb{P}^1$  along with periodic points (or periodic orbits). For *quadratic* dynamical systems  $x^2 + c$ , the corresponding modular curves are smooth in characteristic zero. We give several results about when these curves have good/bad reduction, as well as when the reduction is irreducible. These results are motivated by various uniform boundedness conjectures in arithmetic dynamics. (Received September 15, 2016)

1125-14-1327 **Anthony Weaver\*** ([anthony.weaver@bcc.cuny.edu](mailto:anthony.weaver@bcc.cuny.edu)). *A combinatorial approach to the conjugacy class count for elementary abelian subgroups of the mapping class group.*

It is notoriously difficult to count conjugacy classes of subgroups of mapping classes in a given genus  $g$ , even when one restricts to the simplest kinds of groups such as elementary abelian  $p$  groups. One reason to be interested in such counts is the bijection between conjugacy classes of finite subgroups of the mapping class group and topological equivalence classes of actions of the corresponding group on compact surfaces of genus  $g$ . Here we study fully ramified actions of  $Z_p^k$ , the elementary abelian  $p$  group of rank  $k \geq 2$ . We adopt a constructive/combinatorial approach, and obtain explicit counts. We introduce a new topological invariant, the *partition type*, which encodes, in the form of an additive partition, the number of points fixed by each non-trivial

cyclic subgroup in a given  $Z_p^k$  action. It turns out to be easier to count topological types when the partition type is fixed in advance. We focus on the case  $k = 2$ .

This talk is based on joint work with Mariela Carvacho, Universidad Tecnica Federico Santa Monica, Chile. (Received September 16, 2016)

1125-14-1334 **Dino J Lorenzini\*** ([lorenzini@uga.edu](mailto:lorenzini@uga.edu)), Department of Mathematics, Boyd Graduate Studies Research Center, Athens, GA 30602. *Regular models of curves*. Preliminary report. Let  $K$  be the field of fractions of a complete discrete valuation ring  $R$ . Let  $X/K$  be a smooth proper geometrically connected curve. Let  $L/K$  be an extension such that  $X_L/L$  has semi-stable reduction. We will discuss conjectures, and some partial results towards these conjectures, pertaining to the following two questions. Given a regular model of  $X/K$ , what is it possible to infer on the extension  $L/K$  and the minimal regular model of  $X_L/L$ ? Given the extension  $L/K$  and a regular model of  $X_L/L$ , what information can be obtained on the minimal regular model of  $X/K$ . Such questions are most difficult in the case where the extension  $L/K$  is wild. (Received September 16, 2016)

1125-14-1374 **Brett Frankel\***, Mathematics Department, 2033 Sheridan Road, Evanston, IL 60208.  $\mathbb{F}_q$ -Local Systems on Abelian Varieties of Low  $p$ -rank.

For an abelian variety  $A$  with small  $p$ -torsion, we count the number of representations of the étale fundamental group of  $A$  to  $GL_n(q)$ , where  $q$  is a power of  $p$ . This count (for fixed  $n$ ) turns out to be a polynomial in  $q$ . The space of such representations is not a scheme, but does have the structure of a constructible set. We give an explicit formula for this polynomial, then state a few theorems which elucidate its features. In particular, we state a new result which generalizes to cosets a theorem of Frobenius about the number of solutions to  $x^n = 1$  in a finite group. (Received September 16, 2016)

1125-14-1383 **Frank Sottile\*** ([sottile@math.tamu.edu](mailto:sottile@math.tamu.edu)), Department of Mathematics, Mailstop 3368, Texas A&M University, College Station, TX 77843-3368, **Anton Leykin** ([leykin@math.gatech.edu](mailto:leykin@math.gatech.edu)), School of Mathematics, Georgia Institute of Technology, 686 Cherry Street, Atlanta, GA 30332-0160, and **Jose Israel Rodriguez** ([joisro@uchicago.edu](mailto:joisro@uchicago.edu)), Department of Statistics, University of Chicago, Chicago, IL 60637. *Trace Test*.

We give a brief derivation of the trace test to verify completeness of a witness set of an irreducible variety in affine or projective space. We then consider the trace test for subvarieties of products of projective spaces working with multihomogeneous witness sets. We show how a dimension reduction based on Bertini's Theorem leads to a practical trace test in this case involving a curve in a small-dimensional affine space. (Received September 16, 2016)

1125-14-1553 **Han-Bom Moon\*** ([hmoon8@fordham.edu](mailto:hmoon8@fordham.edu)) and **Sang-Bum Yoo**. *Classical invariant theory and birational geometry of moduli spaces of parabolic bundles*.

Invariant theory is the study of the invariant subring of a given ring equipped with a group action. Describing the invariant subring was one of the central mathematical problems in the 19th century and many great algebraists such as Cayley, Clebsch, Hilbert, and Weyl had contributed to it. Still invariant theory is an active part of modern algebraic geometry, and one reason is that there are many interesting connections between invariant theory and modern birational geometry of moduli spaces. In this talk I will explain a concrete example of this connection, in the case of the moduli space of parabolic vector bundles on the projective line. (Received September 17, 2016)

1125-14-1584 **Bruce Reznick\*** ([reznick@illinois.edu](mailto:reznick@illinois.edu)). *Sums of powers of binary quadratic forms*. Preliminary report.

We will discuss the representation of binary forms as a sum of powers of binary forms. Particular attention will be given to characterizing those binary sextic forms which are a sum of two cubes of quadratic forms. We give an algorithm for representing any binary sextic as a sum of three cubes of quadratic forms. (Received September 18, 2016)

1125-14-1590 **Maria Angelica Cueto, Hannah Markwig and Ralph Morrison\*** ([10rem@williams.edu](mailto:10rem@williams.edu)), 181 The Knolls, Apt 3, Williamstown, MA 01267. *Tropical curves of genus 2*.

Classically, a curve of genus 2 is a hyperelliptic curve, and can be defined by an equation  $y^2 = f(x)$  where  $f$  has degree 5 or 6. However, tropicalizing a curve in this form does not in general give rise to a faithful tropicalization.

Using tropical modification, I will show you how to re-embed such curves to yield faithful tropicalizations. This is joint work with Maria Angelica Cueto and Hannah Markwig. (Received September 18, 2016)

1125-14-1607 **Sandra Di Rocco, Kelly Jabbusch and Gregory G. Smith\***  
(ggsmith@mast.queensu.ca). *Polytopes and toric vector bundles.*

To each torus-equivariant vector bundle over a smooth complete toric variety, we associate a collection of convex polytopes, called a parliament of polytopes. We will explore the correspondence between features of the toric vector bundle and properties of the parliament of polytopes. (Received September 18, 2016)

1125-14-1623 **Emilie Dufresne, Parker Edwards and Heather A Harrington\***,  
harrington@maths.ox.ac.uk, and **Jonathan D Hauenstein**. *Topological data analysis of real algebraic varieties.*

We study real algebraic varieties using topological data analysis. Topological data analysis (TDA) provides a growing body of tools for computing geometric and topological information about spaces from a finite sampling of points. We present a new adaptive algorithm for finding provably dense samples of points on real algebraic varieties given a set of defining polynomials. The algorithm utilizes methods from numerical algebraic geometry to give formal guarantees about the density of the sampling and it also employs geometric heuristics to minimize the sampling. Since TDA methods consume significant computational resources that scale poorly in the number of sample points, our sample minimization makes applying TDA methods more feasible. We demonstrate our algorithm with examples and present our findings. (Received September 18, 2016)

1125-14-1666 **Daniel Brake\*** (dbrake@nd.edu), **Jonathan Hauenstein, Lloyd N Trefethen and Charles Wampler**. *Regularizing closed cells of numerical real decompositions.*

Real algebraic components are computable by a variety of methods, including Cylindrical Algebraic Decomposition, ray tracing, and the numerical cellular decomposition. Bertini\_real implements the numerical method, which uses a combination of homotopy continuation and symbolics such as isosingular deflation to decompose real curves and surfaces in any reasonable ambient dimension. Using linear projection values as parameters for the implicit function theorem, Bertini\_real's output is a set of cells consisting of lower-dimensional boundary and generic cells, and a homotopy connecting them. This talk will discuss regularization of the cell homotopy for closed 1- and 2-cells of numerical real decompositions. (Received September 18, 2016)

1125-14-1705 **Tony Shaska\***, Department of Mathematics and Statistics, Oakland University, Rochester, MI 48309, and **Andreas Malmendier**, Department of Mathematics and Statistics, Utah State University, Logan, UT 84322. *A universal pair of genus-two curves.*

Let  $\mathfrak{p}$  be any point in the moduli space of genus-two curves  $\mathcal{M}_2$  and  $K$  its field of moduli. We provide a universal pair of genus-two curves over its minimal field of definition  $K[d]$  where  $d^2$  is given in terms of  $\mathfrak{p}$ . Furthermore,  $K$  is a field of definition if and only if  $d^2$  is a complete square in  $K$ . (Received September 18, 2016)

1125-14-1755 **Anton Leykin\*** (leykin@math.gatech.edu). *Solving polynomial systems via monodromy and trace test.*

We revisit two ideas that together with numerical homotopy continuation techniques lead to a methodology for finding subsets of complex solutions of systems of polynomial equations.

**Monodromy** action can be used to extend a solution subset (e.g., a subset of the so-called witness set of an irreducible positive-dimensional solution component).

**Trace test** verifies the completeness of a partial solution set (e.g., the completeness of a witness set).

We highlight the recent theoretical developments that lead to new practical algorithms.

[Based on works with Duff, Hill, Jensen, Lee, Rodriguez, Sommars, and Sottile.] (Received September 19, 2016)

1125-14-1854 **Luca Weihs\*** (luca@uw.edu) and **Mathias Drton** (md5@uw.edu). *Combinatorial Conditions for Generic Identification in Structural Equation Models.* Preliminary report.

Linear structural equation models relate the components of a random vector using linear interdependencies and Gaussian noise. Such models can be naturally associated with a mixed graph, a graph containing both directed and bidirected edges, where the graph's vertices correspond to the components of the random vector, the directed edges represent the linear relationships between components, and the bidirected edges encode unobserved confounding. Our question of interest is that of generic identifiability, whether a generic choice of linear and confounding effects can be uniquely recovered from the joint covariance matrix of the random variables. An existing combinatorial criteria for establishing generic identifiability is the Half-Trek Criterion; this criterion uses the existence of trek systems in the mixed graph to discover generically invertible linear equations. By

iteratively exploiting such systems, a sufficient condition for generic identifiability can be checked in polynomial time. By considering additional combinatorial properties of mixed graphs, we show how we may discover new invertible equation systems and, thereby, extend the applicability of the Half-Trek Criterion. We then consider both polynomial and exponential time algorithms leveraging our observations. (Received September 19, 2016)

1125-14-1903 **Elizabeth Milićević\*** (emilicevic@haverford.edu), Dept. of Mathematics, 370 Lancaster Avenue, Haverford, PA 19041, and **Arun Ram**. *The Peterson Isomorphism: Moduli of Curves and Alcove Walks*.

This talk will describe a labeling of the points of the moduli space of genus 0 curves in the complete complex flag variety using the combinatorial machinery of alcove walks. Following Peterson, this geometric labeling explains the “quantum equals affine” phenomenon which relates the quantum cohomology of this flag variety to the homology of the affine Grassmannian. Time permitting, we will discuss how this labeling yields a rational parameterization of certain Gromov-Witten varieties. (Received September 19, 2016)

1125-14-1962 **Nathan Pflueger\*** (npflueger@math.brown.edu). *Special divisors on general  $k$ -gonal curves*.

We discuss a version of the Brill-Noether theorem for algebraic curves with a degree  $k$  map to the projective line. Our result gives an upper bound on the dimensions of the spaces of special divisors on a general such curve, which holds with equality in a wide range of cases. The proof is based on tropical geometry; specifically, we completely describe the special divisors on metric graphs consisting of a chain of loops, extending work of Cools, Draisma, Payne, and Robeva. (Received September 19, 2016)

1125-14-2021 **Martin Helmer\*** (martin.helmer@berkeley.edu), 966 Evans Hall, Berkeley, CA 94720-3840, and **Jose Israel Rodriguez** and **Serkan Hosten**. *Topological Invariants and the Maximum Likelihood Degree of a Toric Variety*. Preliminary report.

Let  $X_A$  be the projective toric variety defined by an integer  $d \times n$  matrix  $A$  of rank  $d$  and let  $c$  be a general element of the associated dimension  $n$  complex algebraic torus. We show that the maximum likelihood (ML) degree of the variety  $c \cdot X_A$  obtained by the torus action of  $c$  on  $X_A$  can be seen as a coefficient of a particular component of the Chern-Mather class of  $X_A$ . This realization allows us to determine a, so called, ML discriminant for  $c \cdot X_A$ . In particular we show that if the principal A-determinant,  $E_A$ , of  $X_A$  does not vanish at  $c$  we have that  $\text{MLdegree}(c \cdot X_A)$  is given by the normalized volume of the polytope obtained by taking the convex hull of the matrix  $A$ . Using this we also confirm a known relation between the ML degree of  $c \cdot X_A$  and the Euler characteristic of an associated very affine variety. (Received September 19, 2016)

1125-14-2035 **Erik A Insko\*** (einsko@fgcu.edu) and **Martha Precup**. *Singularities of Hessenberg Varieties*. Preliminary report.

Semisimple Hessenberg varieties are a family of subvarieties of the flag variety with important connections to representation theory, algebraic geometry, and combinatorics. In this talk, we will investigate the singularities of semisimple Hessenberg varieties by analyzing the combinatorial commutative algebra of patch ideals. We will describe a combinatorial criterion that identifies the singular loci of semisimple Hessenberg varieties for a subfamily of these varieties, and time permitting, share some interesting cases in which the algorithm breaks down for other cases. (Received September 19, 2016)

1125-14-2062 **Andreas Malmendier\*** (andreas.malmendier@usu.edu), 3900 Old Main Hill, Logan, UT 84322, and **Tony Shaska**, Department of Mathematics and Statistics, Rochester, MI 48309. *The Satake sextic in elliptic fibrations on  $K3$* .

We describe explicit formulas relevant to the F-theory/heterotic string duality that reconstruct from a specific Jacobian elliptic fibration on the Shioda-Inose surface covering a generic Kummer surface the corresponding genus-two curve using the level-two Satake coordinate functions. We derive explicitly the rational map on the moduli space of genus-two curves realizing the algebraic correspondence between a sextic curve and its Satake sextic. We will prove that it is not the original sextic defining the genus-two curve, but its corresponding Satake sextic which is manifest in the F-theory model, dual to the  $\mathfrak{so}(32)$  heterotic string with an unbroken  $\mathfrak{so}(28) \oplus \mathfrak{su}(2)$  gauge algebra. (Received September 19, 2016)

1125-14-2114 **Grigoriy Blekherman**, **Daniel Plaumann**, **Rainer Sinn** and **Cynthia Vinzant\*** (clvinzan@ncsu.edu). *Polynomials as sums of few squares*.

The classes of polynomials for which every nonnegative polynomial is a sum of squares is an important subject dating back to Hilbert. I will discuss the number of squares needed in these representations and give a tight bound in terms of the number of variables. This generalizes the celebrated result by Hilbert that every real

nonnegative ternary quartic is a sum of three squares. Applying this theory to polynomials called biforms gives low-rank factorizations of positive semidefinite bivariate matrix polynomials. For polynomials in two variables, we can also count the number of representations as a sum of few squares. (Received September 19, 2016)

1125-14-2210 **Luca Schaffler\***, Department of Mathematics, The University of Georgia, 1023 D. W. Brooks Drive, Athens, GA 30602. *The KSBA compactification of the moduli space of  $D_{1,6}$ -polarized Enriques surfaces.*

We describe the moduli compactification by stable pairs of a 4-dimensional family of Enriques surfaces, which arise as the  $\mathbb{Z}_2^2$ -cover of the blow up of  $\mathbb{P}^2$  at three general points branched along a configuration of six lines. The divisor is chosen to be an appropriate multiple of the ramification locus. We study the degenerations parametrized by the boundary, its stratification, and we construct a morphism from this compactification to the Baily-Borel compactification of the same 4-dimensional family of Enriques surfaces. The boundary of the compactification obtained using stable pairs turns out to be respectively toroidal, Baily-Borel and a mixture of these two in a neighborhood of the preimage of the three 0-cusps of the Baily-Borel compactification. The toroidal part of the boundary has a combinatorial interpretation in terms of certain polyhedral subdivisions of the unit cube. (Received September 19, 2016)

1125-14-2238 **Serkan Hosten\*** ([serkan@sfsu.edu](mailto:serkan@sfsu.edu)), 1600 Holloway Avenue, San Francisco, CA 94132, and **Carlos Amendola, Nathan Bliss, Isaac Burke, Courtney Gibbons, Martin Helmer, Evan Nash, Jose Rodriguez and Daniel Smolkin.** *Maximum Likelihood Degree of Toric Varieties and Discriminants.*

We consider the maximum likelihood estimation problem on a toric variety. The family of all varieties obtained by a torus action on a toric variety can be stratified with respect to the maximum likelihood degree of the members of the family. For a generic member the maximum likelihood degree is equal to the degree of the variety. We determine that those members with a deficient maximum likelihood degree correspond to the points of the principal A-determinant. We will present examples of toric varieties for which we can compute the stratification with respect to the maximum likelihood degree. (Received September 20, 2016)

1125-14-2273 **Mahir Bilen Can\*** ([mcan@tulane.edu](mailto:mcan@tulane.edu)), 4909 1/2 Carondelet St., New Orleans, LA 70115. *Bruhat orders on wonderful embeddings of symmetric varieties of type A.* Preliminary report.

We will present a (new) combinatorial rule for comparing two Borel orbits in the variety of complete quadrics. If time permits we will present applications and explain how to generalize our results to the other two cases. (Received September 20, 2016)

1125-14-2292 **Justin Sawon\*** ([sawon@email.unc.edu](mailto:sawon@email.unc.edu)), Department of Mathematics, University of North Carolina, Chapel Hill, NC 27599-3250. *Holomorphic Lagrangian fibrations.*

A hyperkahler manifold is a Riemannian manifold  $M$  with a triple of complex structures  $I, J, K$  that make it into a Kahler manifold in many different ways. If we fix one complex structure, say  $I$ , on a compact hyperkahler manifold  $M$ , and consider holomorphic fibrations with respect to  $I$ , then a surprising result of Matsushita states that these fibrations are Lagrangian with respect to the Kahler forms of  $J$  and  $K$ . In this talk I will describe some examples of these ‘holomorphic Lagrangian fibrations’. The construction and study of dual fibrations, which arise in Mirror Symmetry, is a major area of investigation. (Received September 20, 2016)

1125-14-2339 **Lek-Heng Lim** ([lekheng@galton.uchicago.edu](mailto:lekheng@galton.uchicago.edu)) and **Jose Israel Rodriguez\*** ([joisro@uchicago.edu](mailto:joisro@uchicago.edu)). *Numerical algebraic geometry in econometrics and the GMM degree.*

In econometrics, the generalized method of moments (GMM) combines data with population moment conditions to estimate the unknown population parameters. For Pearson’s classic method of moments, one determines the common root(s) of  $n$  nonlinear polynomials, called cost functions, where  $n$  is the number of population parameters. In GMM, one has more cost functions than population parameters. To account for this, the common root condition is replaced by minimizing a positive definite quadratic form of cost functions. The quadratic form may have multiple local minima. However, the number of local extrema is bounded above by the GMM degree, a generalization of maximum likelihood and method of moments degree. (Received September 20, 2016)

1125-14-2417 **Timothy Duff, Cvetelina Hill, Anders Jensen, Kisun Lee, Anton Leykin\*** (leykin@math.gatech.edu) and **Jeffrey Sommars**. *Solving polynomial systems via homotopy continuation and monodromy*.

We develop an algorithm to find all solutions of a generic system in a family of polynomial systems with parametric coefficients using numerical homotopy continuation and the action of the monodromy group. We argue that the expected number of homotopy paths that this algorithm needs to follow is roughly linear in the number of solutions. We demonstrate that our software implementation is competitive with the existing state-of-the-art methods implemented in other software packages. (Received September 20, 2016)

1125-14-2423 **James S Wolper\*** (wolpjame@isu.edu), Dept. of Mathematics and Statistics, Idaho State University, 921 S. 8th Ave., MS8085, Pocatello, ID 83209. *Are hard questions in algebraic geometry approximately easy?* Preliminary report.

Is a Prym the jacobian of a curve? Can an abelian variety contain a hyperelliptic curve? Is the intermediate jacobian of a cubic threefold the jacobian of a curve?

A new way to approach these and similar questions is through studying the statistics of the periods, which can be done numerically using current day software. Can statistical and numerical techniques makes these hard questions approximately easy? (Received September 20, 2016)

1125-14-2543 **Sudhir R Ghorpade** (srg@math.iitb.ac.in), Department of Mathematics, Indian Institute of Technology Bombay, Powai, Mumbai, 400076, India, **Fernando Piñero** (pinerofernando@gmail.com), Department of Mathematics, University of Puerto Rico - Ponce, 2151 Ave. Santiago de los Caballeros, Ponce, PR 00716-9996, and **Prasant Singh\*** (psinghprasant@gmail.com), Department of Mathematics, Indian Institute of Technology Bombay, Powai, Mumbai, 400076, India. *Maximal linear sections of 2-step flag varieties and quadratic Veronese of Grassmannians over finite fields*. Preliminary report.

Let  $V$  be a vector space of dimension  $m$ . For  $1 \leq \ell_1 \leq \ell_2 < m$ , consider

$$\mathcal{F}(\ell; m) = \{(V_1, V_2) \in G_{\ell_1}(V) \times G_{\ell_2}(V) : V_1 \subseteq V_2\}$$

with its Plücker-Segre embedding in  $\mathbb{P}(\wedge^{\ell_1} V \otimes \wedge^{\ell_2} V)$ . We consider the following questions:

- Determine the least positive integer  $k \leq \binom{m}{\ell_1} \binom{m}{\ell_2}$  such that  $\mathcal{F}(\ell; m)$  embeds in  $\mathbb{P}^{k-1}$ .
- Determine the maximal number of  $\mathbb{F}_q$ -rational points in  $\mathcal{F}(\ell; m) \cap H$ , where  $H$  varies over hyperplanes in  $\mathbb{P}^{k-1}$ .

Questions such as these have been considered by several mathematicians in the case of Veroneseans and Grassmannians. For the line-hyperplane incidence variety  $\mathcal{F}(\ell; m)$  where  $\ell = (1, m-1)$ , both these questions were answered by F. Rodier (2003). Subsequently, there has been some partial progress by G. Hana (2005), but the general case appears to be open. We answer the first question using some combinatorial representation theory. We also give an explicit lower bound, and in some cases the exact value, for the maximum sought by the second question. (Received September 20, 2016)

1125-14-2714 **Linda Chen\***, lchen@swarthmore.edu. *Equivariant cohomology and counting tableaux*.

I will discuss applications of equivariant cohomology to combinatorial formulas for numbers of tableaux. (Received September 20, 2016)

1125-14-2780 **Neal Livesay\*** (nlives1@lsu.edu), 2150 Duncan Drive, Baton Rouge, LA 70802. *Moduli spaces of flat GSp-connections*. Preliminary report.

A fundamental problem in the study of differential equations is the classification of first-order singular differential operators up to gauge equivalence. A modern version of this problem, rephrased in the language of algebraic geometry, involves the construction of moduli spaces of meromorphic  $G$ -connections (or, equivalently, flat  $G$ -bundles) on  $\mathbb{C}\mathbb{P}^1$ , for  $G$  a reductive group. P. Boalch (2001) has constructed moduli spaces for  $GL_n$ -connections in the case that the connection matrix at each singularity - an element of the formal loop algebra  $\mathfrak{gl}_n(\mathbb{C}((z)))$  - is diagonalizable. More recently, C. Bremer and D. Sage (2012) have used representation-theoretic methods to develop a new approach to studying  $G$ -connections. Furthermore, they have used this approach to construct moduli spaces of  $GL_n$ -connections with certain classes of non-diagonalizable connection matrices (such as the generalized Airy connection matrices). In this talk, I will describe my recent work to further refine this theory for the study of  $GSp_{2n}$ -connections, and demonstrate the theory for some small rank examples. (Received September 20, 2016)

1125-14-2803 **Neriman Tokcan\*** (tokcan2@illinois.edu). *Relative Ranks of Binary Forms.*

Suppose  $f(x, y)$  is a binary form of degree  $d$  with coefficients in a field  $K \subseteq \mathbb{C}$ . The  $K$ -rank of  $f$ ,  $L_K(f)$ , is the smallest number of  $d$ -th powers of linear forms over  $K$  of which  $f$  is a  $K$ -linear combination. We prove that for  $d \geq 5$ , there always exists a form of degree  $d$  with at least three different ranks over various fields. We also find lower bounds for  $\mathbb{C}$ -rank and  $\mathbb{R}$ -rank of binary forms depending on their factorizations. (Received September 20, 2016)

1125-14-2886 **Caleb McKinley Shor\*** (cshor@wne.edu), WNE Math Department, 1215 Wilbraham Rd., Springfield, MA 01119. *Higher-order Weierstrass points on certain hyperelliptic curves, superelliptic curves, and associated towers.*

Higher-order Weierstrass points (or  $q$ -Weierstrass points) are special points on algebraic curves of genus  $g > 1$  which have associated weights. They are closely related to curve automorphisms.

In this talk, we consider  $q$ -Weierstrass points on curves in a few settings. We will look at certain hyperelliptic curves of genus 2 and 3. Considering various possible automorphism groups, we will see the distributions of weights for  $q = 2, 3$ . We will also look at superelliptic curves, which are curves of the form  $y^n = f(x)$  for  $f(x)$  a separable polynomial of degree  $d > n \geq 2$ . On such curves, all branch points are  $q$ -Weierstrass points for all  $q \in \mathbb{N}$ , and we will see how to compute their weights.

Finally, we'll consider towers of curves iteratively defined by equations of superelliptic curves. We will see how to compute the  $q$ -weights of certain points in these towers. Interestingly, the  $q$ -weight computations lead to some new results in the field of numerical semigroups. In particular, given a numerical semigroup generated by a geometric sequence of integers, for any  $m \geq 0$ , we will compute the sum of the  $m$ th powers of the natural numbers which are not in the semigroup.

This talk involves work done in collaboration with Tony Shaska and T. Alden Gassert. (Received September 20, 2016)

1125-14-2910 **Madeline Brandt, David J. Bruce, Taylor Brysiewicz, Robert Krone\*** (rk71@queensu.ca) and **Elina Robeva**. *The degree of the special orthogonal group.*

The set of rotation matrices,  $SO(n)$ , forms an irreducible algebraic variety. A formula for the degree of this variety can be derived from the work of Kazarnovskii. However, the computations involved in deriving the formula are complicated and prone to error. Moreover symbolic algorithms could only determine the degree for  $n$  at most 5, which is too small to adequately verify the formula in the even case. With a numerical computation using monodromy we show the the degree of  $SO(6)$  is 4768, confirming the formula value. (Received September 20, 2016)

1125-14-2998 **Fiona Knoll\*** (fknoll@clermson.edu), **Shuhong Gao** and **Michael Burr**. *Precise Dimensions That Guarantee a Transformation Preserving the Euclidean Distance.*

Johnson and Lindenstrauss (1984) proved that any finite set of data in a high dimensional space can be projected into a low dimensional space with the Euclidean metric information of the set being preserved within any desired accuracy, provided the projected dimension lies above a certain threshold. Kane and Nelson (2011) proved such a projection does not exist if the projected dimension lies below another threshold.

In this presentation, we will discuss the bounds of the projected dimension of those transformations preserving the Euclidean distance and the existence of a gap between these bounds. This is joint work with Shuhong Gao and Michael Burr. (Received September 20, 2016)

1125-14-3122 **David Yang\***, Department of Mathematics, 77 Massachusetts Avenue, Massachusetts Institute of Technology, Cambridge, MA 02139-4307. *Rational Curves on Hypersurfaces.*

A hypersurface can be as the zero locus of a single multivariate polynomial. They are some of the simplest geometric spaces appearing in algebraic geometry. We survey what is known about the geometry of hypersurfaces, in particular about spaces of lines and their higher degree analogues, rational curves, in the hypersurface. (Received September 21, 2016)

1125-14-3137 **Steven Rayan\*** (rayan@math.usask.ca), Department of Mathematics and Statistics, University of Saskatchewan, McLean Hall, Saskatoon, SK S7N 5E6, Canada. *The quiver at the bottom of the twisted nilpotent cone on  $\mathbb{P}^1$ .*

For the moduli space of Higgs bundles on a Riemann surface of positive genus, critical points of the natural Morse-Bott function lie along the nilpotent cone of the Hitchin fibration and are representations of A-type quivers in a twisted category of holomorphic bundles. The fixed points that globally minimize the function are representations of  $A_1$ . For twisted Higgs bundles on the projective line, the quiver describing the bottom of the cone is more complicated. We determine it. We show that the moduli space is topologically connected whenever

the rank and degree are coprime, thereby verifying conjectural lowest Betti numbers coming from high-energy physics. (Received September 21, 2016)

## 15 ► *Linear and multilinear algebra; matrix theory*

1125-15-49 **Paul R. Bouthellier\*** (pbouthe@pitt.edu), 504 East Main Street, Titusville, PA 16354.  
*Visualizing Gimbal Lock and Quaternions with the HTML5 Canvas.*

Quaternions are always discussed in the context of gimbal lock. However, as there exist a number of different definitions of gimbal lock, there is often a great deal of confusion in discussions of the topic. In this talk we shall look at several different definitions of gimbal lock, including the general aerospace and computer graphics versions. We shall look at the mathematics behind the concepts, what gimbal lock means in each field, and illustrate these definitions with interactive web pages using the HTML5 canvas. We then shall examine why quaternions avoid gimbal lock when used properly, and can still result in gimbal lock when used improperly. The concepts will again be illustrated with interactive web pages using the HTML5 canvas. (Received June 22, 2016)

1125-15-252 **Stephan Ramon Garcia\*** (stephan.garcia@pomona.edu), Department of Mathematics, Pomona College, 610 N College Ave, Claremont, CA 91711. *A tale of two matrices.*

For square  $A$  and  $B$ , the matrices  $AB$  and  $BA$  are closely related, yet they are not necessarily similar. We discuss this intriguing relationship and highlight several sufficient conditions that ensure the similarity of  $AB$  and  $BA$ . (Received August 18, 2016)

1125-15-334 **Leslie Hogben\*** (hogben@iastate.edu). *Generalizations of the Strong Arnold Property and the inverse eigenvalue problem of a graph.*

For a given graph  $G$  and an associated class of real symmetric matrices whose off-diagonal entries are nonzero exactly where  $G$  has edges, the inverse eigenvalue problem of  $G$  is to determine the collection of all possible spectra for such matrices. Inverse eigenvalue problems and the background of this problem will be described, together with techniques such as the fundamental work of Colin de Verdière and the Strong Arnold Property. Two extensions of the Strong Arnold Property that target a better understanding of all possible spectra and their associated multiplicities will be presented, referred to as the Strong Spectral Property and the Strong Multiplicity Property. Applications of these properties to the inverse eigenvalue problem of a graph will be discussed.

This talk is based on joint work with W. Barrett, S. Butler, S. Fallat, H. T. Hall, J. C.-H. Lin, B.L. Shader, and M. Young. (Received August 27, 2016)

1125-15-428 **Steven J Miller\*** (sjm1@williams.edu). *Random Matrix Ensembles with Split Limiting Behavior.*

Random matrix theory has successfully modeled a variety of systems, from energy levels of heavy nuclei to zeros of the Riemann zeta function. One of the central results is Wigner's semi-circle law: the distribution of normalized eigenvalues for ensembles of real symmetric matrices converge to the semi-circle density (in some sense) as the matrix size tends to infinity. We introduce a new family of  $N \times N$  random real symmetric matrix ensembles, the  $k$ -checkerboard matrices, whose limiting spectral measure has two components. All but  $k$  eigenvalues are in the bulk, and their behavior, appropriately normalized, converges to the semi-circle as  $N \rightarrow \infty$ ; the remaining  $k$  are tightly constrained near  $N/k$  and their distribution converges to the  $k \times k$  hollow GOE ensemble (this is the density arising by modifying the GOE ensemble by forcing all entries on the main diagonal to be zero). Similar results hold for complex and quaternionic analogues. We are able to isolate each regime separately through appropriate choices of weight functions for the eigenvalues and then an analysis of the resulting combinatorics. This is joint work with Paula Burkhardt, Peter Cohen, Jonathan Dewitt, Max Hlavacek, Carsten Sprunger, Yen Nhi Truong Vu, Roger Van Peski, and Kevin Yang. (Received September 01, 2016)

1125-15-442 **Alan C. Krinik\*** (ackrinik@cpp.edu), **Uyen Nguyen, Ali Oudich, Pedram Ostadhassanpanjehali** and **Ryan Kmet.** *Exploring a Class of Finite, Tridiagonal, Stochastic Matrices.*

We begin by applying some nice results of Kouachi (2008) that characterize the general form of eigenvalues for two different classes of finite, tridiagonal, stochastic matrices having  $q$  on the lower diagonal and  $p$  on the upper diagonal with  $p, q > 0$  and  $p + q < 1$ .

•We first consider an absorbing birth-death Markov chain having state space  $\{0,1,2,\dots,H\}$  where states 0 and  $H$  are absorbing states and states  $1,2,3,\dots,H-1$  each have constant one-step (nonzero) transition probabilities:  $p$  for going up one step,  $q$  for going down one step and  $r$  the chance of returning to the same state in one step. Formulas for the  $n$ -step transition probabilities and the finite-time gambler's ruin probability are presented and discussed.

•Next, we consider a recurrent birth-death Markov chain having state space  $\{0,1,2,\dots,H\}$  where states  $1,2,3,\dots,H$  have nonzero probability  $q$  of going down by one step and states  $0,1,2,3,\dots,H-1$  have nonzero probability  $p$  of going up by one step. We again assume  $p + q < 1$  Formulas for the  $n$ -step transition probabilities are presented and discussed when  $H$  is odd.

Further generalizations and applications are discussed as time allows. (Received September 02, 2016)

1125-15-469 **Alexander Belton, Dominique Guillot, Apoorva Khare\*** (khare@stanford.edu) and **Mihai Putinar**. *Schur polynomials and linear matrix inequalities for Hadamard powers.*

We characterize the polynomials of degree  $N > 0$ , which when applied entrywise to the cone of positive  $N \times N$  matrices, preserve positivity. This result provides a quantitative version in fixed dimension of Schoenberg's celebrated theorem. By recasting our result as a tight linear matrix inequality for matrix pencils, we derive asymptotically sharp bounds for the matrix cube problem for Hadamard powers. The proof of our result makes use of a novel determinantal identity involving Schur polynomials. (Received September 02, 2016)

1125-15-730 **Emily J Evans\*** (ejevans@mathematics.byu.edu), **Jeffery Humpherys** and **Nathaniel Merrill**. *A Novel Technique for Calculating the Effective Resistance of an Undirected Graph*. Preliminary report.

In this talk we introduce a way of determining the effective resistance of an undirected graph by considering an absorbing Markov chain and using the Drazin inverse. The idea behind our result is to scale the adjacency matrix of the graph to become the transition probability matrix of a Markov chain, and then modify that matrix to become an absorbing Markov chain. We then use the Drazin inverse to obtain the effective resistance of the graph. Not only does our technique correctly recover the effective resistance of an undirected graph, it provides promising new measure to be used in the link predication problem for directed graphs. (Received September 10, 2016)

1125-15-887 **Chris Godsil\***, Combinatorics and Optimization, University of Waterloo, Waterloo, Ontario N2L 3G1, Canada. *Average mixing of quantum walks.*

Let  $X$  be a graph with adjacency matrix  $A$ . The family of matrices  $U(t) = \exp(itA)$  (for  $t \geq 0$ ) determines what physicists call a *quantum walk*. For matrices  $M$  and  $N$ , let  $M \circ N$  denote the Schur product. Then

$$\widehat{M} := \lim_{T \rightarrow \infty} \int_0^T U(t) \circ U(-t) dt$$

is the *average mixing matrix* of the walk. If  $E_1, \dots, E_m$  are the idempotents in the spectral decomposition of  $A$ , then

$$\widehat{M} = \sum_r E_r \circ E_r.$$

and so  $\widehat{M}$  also has an algebraic definition. We can view it as a graphical invariant. In my talk I will discuss some of the properties of this matrix, and some of the related open questions. (Received September 12, 2016)

1125-15-897 **Charles R Johnson** and **Ilya M Spitkovsky\*** (ims20nyu.edu). *On matrices subordinate to a tree.*

Matrices subordinate to trees are considered. An efficient normality characterization for any such matrix is given, and several consequences (not valid for general normal matrices) of it are established. In addition, the existence (and enumeration) of flat portions on the boundary of the field of values of matrices subordinate to a tree is characterized.

The talk is based on the results obtained jointly with our students M. Stevanovic (NYUAD) and M. Turnansky (William & Mary). (Received September 13, 2016)

1125-15-947 **Richard A Brualdi\*** (brualdi@math.wisc.edu), Department of Mathematics, UW-Madison, 480 Lincoln Drive, Madison, WI 53706. *Combinatorial Matrix Theory modulo  $k$* . Preliminary report.

Combinatorial Matrix Theory (CMT) is now an important and thriving area of mathematics concerned with combinatorial properties of matrices and the use of matrix theory in combinatorial problems. In this expository talk we shall introduce the idea of combinatorial matrix theory modulo  $k$  (CMT-mod  $k$ ) and discuss some

analogues of well-known theorems of CMT. This talk is based on joint work with Seth Meyer. (Received September 13, 2016)

1125-15-1189 **Jun Ji\***, Department of Mathematics, Kennesaw State University, 1100 S. Marietta Pkwy, Marietta, GA 30060, and **Yimin Wei**. *Weighted Moore-Penrose Inverses and the Fundamental Theorem of Even-Order Tensors with Einstein Product*. Preliminary report.

We treat even-order tensors with Einstein product as linear operators from tensor space to tensor space, define the null spaces and the ranges of tensors, and study their relationship. We consider the weighted Moore-Penrose inverse of an even-order tensor and extend the fundamental theorem of linear algebra for matrix spaces to tensor spaces. Using the new relationship, we characterize the least-squares ( $\mathcal{M}$ ) solutions to a multilinear system and establish the relationship between the minimum-norm ( $\mathcal{N}$ ) least-squares ( $\mathcal{M}$ ) solution of a multilinear system and the weighted Moore-Penrose inverse of its coefficient tensor. (Received September 15, 2016)

1125-15-1278 **Xavier Martinez-Rivera\*** (xaviermr@iastate.edu). *The epr-sequence over a field of characteristic 2*.

The enhanced principal rank characteristic sequence (epr-sequence) of an  $n \times n$  symmetric matrix over a field  $\mathbb{F}$  was recently defined as  $\ell_1 \ell_2 \cdots \ell_n$ , where  $\ell_k$  is either A, S, or N based on whether all, some (but not all), or none of the order- $k$  principal minors of the matrix are nonzero; that is,  $\ell_k = \mathbf{A}$  if all the principal minors of order  $k$  of the matrix are nonzero,  $\ell_k = \mathbf{N}$  if none are nonzero (i.e., all are zero), and  $\ell_k = \mathbf{S}$  if some (but not all) are nonzero. There is no known characterization of the epr-sequences that are attainable by symmetric matrices over the real or any other field of characteristic *not* 2. However, for a field of characteristic 2 such a characterization is now known, and it is presented in this talk. (Received September 15, 2016)

1125-15-1341 **Grigoriy Blekherman\*** (greg@math.gatech.edu) and **Rainer Sinn**. *Maximum Likelihood Threshold of Gaussian Graphical Models*. Preliminary report.

Solution to positive definite matrix completion problem determines the existence of maximal likelihood estimate for a Gaussian graphical model. I will discuss two geometric invariants: the maximum likelihood threshold and the Gaussian rank of a graph. We will give several interpretations of these invariants and the first example of a graph where these two invariants are different. (Received September 16, 2016)

1125-15-1482 **S. Cioaba, R. Elzinga, M. Markiewitz and K. Vander Meulen\*** (kvanderm@redeemer.ca), Department of Mathematics, Redeemer University College, Ancaster, ON L9K1J4, Canada, and **T. Vanderwoerd**. *On the addressing problem for triangular graphs*.

Graham and Pollak introduced the problem of addressing the vertices of a graph using  $t$ -tuples with entries in a symbol set  $\{0, a, b\}$  so that the distance between any two vertices is the number of positions in which the one address has symbol  $a$  and the other has symbol  $b$ . For a given graph  $G$ , the goal is to find the minimum  $t$  such that  $G$  has an addressing with  $t$ -tuples. A well-known lower bound is obtained from the the inertia of the distance matrix of  $G$ . We describe a matrix technique to improve the lower bound, applying it to triangular graphs. (Received September 17, 2016)

1125-15-1544 **Maria A. A. Freitas\*** (maguieiras@im.ufrj.br), **Celso M. Silva Jr** and **Renata R. Del-Vecchio**. *Distance Laplacian eigenvalues and forbidden subgraphs*.

The study of the distance Laplacian matrix is recent. Some of the results in literature relate eigenvalues and structural properties of graphs. We will present relations between multiplicities of distance Laplacian eigenvalues and forbidden subgraphs. (Received September 17, 2016)

1125-15-1620 **Louis Deaett** and **Colin M Garnett\*** (colin.garnett@bhsu.edu). *Combinatorial conditions that preclude SAPpiness*.

It is well known that a complex zero-nonzero pattern cannot be spectrally arbitrary if its digraph doesn't have at least two loops and at least one two cycle, or at least three loops. This talk focuses on several other combinatorial conditions on the digraph that preclude it from being spectrally arbitrary. In particular we are sometimes able to reduce the number of unknown entries to be below the threshold of  $2n - 1$ . Other techniques will be discussed, including finding that one coefficient is a multiple of another. (Received September 18, 2016)

1125-15-1645 **Louis Deaett\*** (louis.deaett@quinnipiac.edu) and **Colin Garnett**. *Algebraic conditions induced by matrix patterns*. Preliminary report.

Replacing each nonzero entry of a matrix with the symbol  $*$  gives its *zero-nonzero pattern*. We may ask whether this combinatorial object captures any information about the eigenvalues of the matrix. Sometimes there exists a multiset of eigenvalues that can be ruled out for every matrix with that pattern; we say that the pattern fails

to be *spectrally arbitrary*. It has been conjectured that an  $n \times n$  spectrally arbitrary pattern must have at least  $2n$  nonzero entries.

The combinatorics of a particular zero-nonzero pattern (perhaps described by a digraph) may imply algebraic conditions on the coefficients of each characteristic polynomial belonging to a matrix with that pattern. Certain such algebraic conditions imply that the pattern cannot be spectrally arbitrary.

Is there a succinct set of algebraic conditions of this sort that apply to every pattern with fewer than  $2n$  nonzero entries? We show that the answer is ‘yes’ for  $n \leq 6$ , and show that the question for  $n = 7$  can be narrowed down to a very small number of patterns. (Received September 18, 2016)

1125-15-1757     **Lei Cao\*** (leicaomath@gmail.com), **Selcuk Koyuncu** and **Timothy Ryan Parmer**.  
*A Minimal Completion of Doubly Substochastic Matrix.*

Let  $B$  be an  $n \times n$  doubly substochastic matrix and let  $s$  be the sum of all entries of  $B$ . In this paper we show that  $B$  has a sub-defect of  $k$ , which can be computed by taking the ceiling of  $n - s$ , if and only if there exists an  $(n + k) \times (n + k)$  doubly stochastic extension containing  $B$  as a submatrix and  $k$  minimal. We also propose a procedure constructing a minimal completion of  $B$ , and then express it as a convex combination of partial permutation matrices. (Received September 20, 2016)

1125-15-1833     **Prakash Ghimire\***, Louisiana State University of Alexandria, Mulder 338, Alexandria, LA 71302. *Derivations of the Lie algebra of strictly block upper triangular matrices.*

Let  $F$  be a field,  $M_n$  the Lie algebra of all  $n \times n$  matrices over a field  $F$ ,  $N$  the Lie subalgebra of  $M_n$  consisting of all strictly block upper triangular matrices in  $M_n$  with a given partition. In this talk, we give an explicit description of any derivations of the Lie algebra  $N$  over  $F$ . (Received September 19, 2016)

1125-15-1973     **Huajun Huang** and **Luke Oeding\*** (oeding@auburn.edu). *Symmetrization of Principal Minors and Cycle Sums.*

We solve the Symmetrized Principal Minor Assignment Problem, that is we show how to determine if for a given vector  $v \in \mathbb{C}^n$  there is an  $n \times n$  matrix that has all  $i \times i$  principal minors equal to  $v_i$ . We use a special isomorphism (a non-linear change of coordinates to cycle-sums) that simplifies computation and reveals hidden structure. We use the symmetries that preserve symmetrized principal minors and cycle-sums to treat 3 cases: symmetric, skew-symmetric and general square matrices. We describe the matrices that have such symmetrized principal minors as well as the ideal of relations among symmetrized principal minors / cycle-sums. We also connect the resulting algebraic varieties of symmetrized principal minors to tangential and secant varieties, and Eulerian polynomials. (Received September 19, 2016)

1125-15-2042     **Xiaofeng Chen, Wei Fang, Wei Gao, Yubin Gao, Guangming Jing, Zhongshan Li\*** (zli@gsu.edu), **Yanling Shao** and **Lihua Zhang**. *Essential sign change numbers of full sign pattern matrices.*

A sign pattern (matrix) is a matrix whose entries are from the set  $\{+, -, 0\}$  and a sign vector is a vector whose entries are from the set  $\{+, -, 0\}$ . A sign pattern or sign vector is full if it does not contain any zero entries. The minimum rank of a sign pattern matrix  $\mathcal{A}$  is the minimum of the ranks of the real matrices whose entries have signs equal to the corresponding entries of  $\mathcal{A}$ . The notions of essential row sign change number and essential column sign change number are introduced for full sign patterns and condensed sign patterns. By inspecting the sign vectors realized by a list of real polynomials in one variable, a lower bound on the essential row and column sign change numbers is obtained. Using point-line configurations on the plane, it is shown that even for full sign patterns with minimum rank 3, the essential row and column sign change numbers can differ greatly and can be much bigger than the minimum rank. Some open problems concerning square full sign patterns with large minimum ranks are discussed. (Received September 19, 2016)

1125-15-2133     **Zhongshan Li\*** (zli@gsu.edu), **Fuzhen Zhang** and **Xiaodong Zhang**. *On the number of vertices of the stochastic tensor polytope.*

This talk is devoted to the study of lower and upper bounds for the number of vertices of the polytope of  $n \times n \times n$  stochastic tensors (all of whose entries are nonnegative real numbers and the sum of entries in every line is 1). By using known results on polytopes (i.e., the Upper and Lower Bound Theorems), we present some new lower and upper bounds. We show that the new upper bound is tighter than the one recently obtained by Chang, Paksoy and Zhang, and also sharper than the one by Linial and Luria. We demonstrate that the analog of the lower bound obtained in such a way, however, is no better than the existing ones. (Received September 19, 2016)

1125-15-2169 **Marina Arav\***, Department of Mathematics and Statistics, Georgia State University, 30 Pryor Street, Atlanta, GA 30303, and **Frank Hall, Hein van der Holst** and **Zongshan Li**. *Signed graphs with maximum nullity at most two.*

A signed graph is a pair  $(G, \phi)$ , where  $G = (V, E)$  is a graph (in which parallel edges are permitted, but loops are not) and  $\phi : E \rightarrow \{-, +\}$ . By  $S(G, \phi)$ , we denote the set of all symmetric  $n \times n$  matrices  $A = [a_{i,j}]$  such that if  $a_{i,j} < 0$ , then  $i$  and  $j$  are connected by at least one  $-$  edge, if  $a_{i,j} > 0$ , then  $i$  and  $j$  are connected by at least one  $+$  edge, and if  $a_{i,j} = 0$ , then  $i$  and  $j$  are nonadjacent or  $i$  and  $j$  are connected by a  $+$  and by a  $-$  edge. The parameters  $M(G, \phi)$  and  $\xi(G, \phi)$  of a signed graph  $(G, \phi)$  are the largest nullity of any matrix  $A \in S(G, \phi)$  and the largest nullity of any matrix  $A \in S(G, \phi)$  that has the Strong Arnold Property, respectively. In this talk, we discuss the characterizations of the classes of signed graphs  $(G, \phi)$  with  $M(G, \phi) \leq 1$ , of the class of signed graphs  $(G, \phi)$  with  $\xi(G, \phi) \leq 1$ , of the class of 2-connected signed graphs  $(G, \phi)$  with  $M(G, \phi) \leq 2$ , and of the class of 2-connected signed graphs  $(G, \phi)$  with  $\xi(G, \phi) \leq 2$ . (Received September 19, 2016)

1125-15-2402 **Selcuk Koyuncu\*** (skoyuncu@ung.edu) and **Lei Cao** (leicaomath@gmail.com). *Sub-defect of Product of Doubly Substochastic Matrices.*

The sub-defect of an  $n \times n$  doubly substochastic matrix  $S$ , denoted by  $sd(S)$ , is defined to be the smallest integer  $k$  such that there exists an  $(n+k) \times (n+k)$  doubly stochastic matrix containing  $S$  as a submatrix. Let  $A$  and  $B$  be arbitrary doubly substochastic matrices. We show that  $AB$  is also a doubly substochastic matrix and  $\max\{sd(A), sd(B)\} \leq sd(AB) \leq \min\{n, sd(A) + sd(B)\}$ . (Received September 20, 2016)

1125-15-2411 **Mehmet Gumus\*** (mgumus@siu.edu) and **Jianhong Xu**. *A New Characterization of Simultaneous Lyapunov Diagonal Stability via Hadamard Products.*

A well-known characterization by Kraaijevanger (LAA 151: 245–254) for Lyapunov diagonal stability states that a real, square matrix  $A$  is Lyapunov diagonally stable if and only if  $A \circ S$  is a  $P$ -matrix for any positive semidefinite  $S$  with nonzero diagonal entries. This result is extended here to a new characterization involving similar Hadamard multiplications for simultaneous Lyapunov diagonal stability on a set of matrices. Among the main ingredients for this extension are a new concept called  $\mathcal{P}$ -sets and a recent result regarding simultaneous Lyapunov diagonal stability by Berman, Goldberg, and Shorten (Contem. Math. 619: 19–29). (Received September 20, 2016)

1125-15-2428 **Tulay Ayyildiz Akoglu\*** (tayyild@ncsu.edu), 2311 Stinson Drive, Raleigh, NC 27695. *Constructing exact Hermite matrices using approximate roots.* Preliminary report.

Let  $\mathbf{f} = (f_1, \dots, f_N)$  be a system of multivariate polynomials in over rationals with common roots  $\xi_1, \xi_2, \dots, \xi_k$  and assume that  $I = \langle f_1, \dots, f_N \rangle$  is a zero dimensional ideal. Hermite matrix of  $\mathbf{f}$  with respect to an auxiliary polynomial  $g$  is defined by

$$H_g(\mathbf{f}) := V^T D_g V$$

where  $V$  is a Vandermonde matrix and  $D_g$  is a diagonal matrix such that  $[D_g]_{ii} = g(\xi_i)$ . The signature of Hermite matrices gives important information on the signs of the polynomial  $g$  on the common real roots of  $\mathbf{f}$ . Now assume that we are only given approximate roots of  $\mathbf{f}$  and want to find the exact Hermite matrix with respect to  $g$ . First, we compute an approximate Hermit matrix  $H_g(\mathbf{f})$  using given approximate roots of  $\mathbf{f}$  and a Vandermonde matrix. Second, we rationalize each entry of the approximate Hermite matrix using rational number reconstruction (via continued fractions) with a preset bound on the denominators. Then we describe a certification procedure to decide whether the rationalized Hermite matrix is the exact one. (Received September 20, 2016)

1125-15-2442 **Robert Erra\***, 14-16 rue Voltaire, 94270 Le Kremlin Bicetre, France, and **Marwan Burelle, Alexandre Letois** and **Mark Angoustures**. *A new spectral graph entropy.* Preliminary report.

We propose here a new graph entropy. Let  $G$  be an undirected graph of size  $N$ ; the now classical Von Neumann entropy of a graph is defined as:  $S(G) = -\sum_{i=1}^N \lambda_i \log(\lambda_i)$ , with  $Sp(G) = \{\lambda_1, \dots, \lambda_N\}$  the spectrum of the adjacency matrix of  $G$ ; we can also use the spectrum of the normalized combinatorial Laplacian of  $G$ .

Let  $k$  be the number of distinct eigenvalues, let  $M = \{m_1, \dots, m_k\}$  be the vector of multiplicities of all eigenvalues and let  $\mathcal{M} = \{\omega_1 = m_1/N, \dots, \omega_k = m_k/N\}$  be the normalized vector of multiplicities. So we have  $\sum_{i=1}^k \omega_i = 1$ , we can then compute  $H_S(G) = -\sum_{i=1}^k \omega_i \log(\omega_i)$ , and we will call this the *spectral graph entropy*. This is the Shannon entropy of the normalized vector of eigenvalue multiplicities.

This entropy has a nice property: it can be computed for directed or undirected graphs, for weighted or unweighted graphs. We can also compute the graph entropy with the normalized combinatorial Laplacian of  $G$  instead of using the adjacency matrix.

We will present a simple algorithm to compute exactly this spectral entropy. We will give some of its properties and we will present some open problems. (Received September 20, 2016)

1125-15-2490 **Xin Li\***, Department of Mathematics, University of Central Florida, Orlando, FL 32816, and **Aritra Dutta**. *On the asymptotic behavior of the solutions to the general weighted low rank approximation as one block of the weights approach to infinity*. Preliminary report.

We will study the asymptotic behavior of the solutions to the following weighted low rank approximations of matrices:

$$\min_{X_1, X_2: \text{rank}(X_1, X_2) \leq r} \{ \|(A_1 - X_1) \odot W_1\|_F^2 + \|(A_2 - X_2) \odot W_2\|_F^2 \}$$

as  $W_1 \rightarrow \infty$  and  $W_2 \rightarrow 1$ , where  $\odot$  denotes the Hadamard (element-wise) multiplication. We show that, under proper assumptions, the limit exists and in the special case when  $W_2 = 1$ , the rate of convergence is also established in terms of  $\lambda = \max_{i,j} \{(W_1)_{i,j}\}$ . (Received September 20, 2016)

1125-15-2495 **Jephian C.-H. Lin\*** ([chlin@iastate.edu](mailto:chlin@iastate.edu)). *The minimum rank problem on loop graphs*.

A loop graph is obtained from a simple graph by designating each vertex as having or not having a loop. The minimum rank problem for a graph  $\mathcal{G}$  is to determine the smallest possible rank among symmetric matrices whose  $i, j$ -entry ( $i = j$  is possible) is nonzero whenever  $i$  is adjacent to  $j$ , and zero otherwise. The zero forcing number, which is defined through a color-change game on graphs, provides a lower bound for the minimum rank. This talk will discuss different variations of the zero forcing numbers and introduce the odd cycle zero forcing number. (Received September 20, 2016)

1125-15-2890 **Boyko Gyurov** ([bgyurov@ggc.edu](mailto:bgyurov@ggc.edu)) and **Daniel Pinzon\*** ([dpinzon@ggc.edu](mailto:dpinzon@ggc.edu)), 1000 University Center Ln, Lawrenceville, 30043. *The Determinant of Graphs Joined by Edges*.

Given the disjoint union of any two (di)graphs,  $G$  and  $H$ , we denote by  $G \overset{j}{\succ} H$  the graph given by adding  $j$  edges that connect distinct vertices of  $G$  to distinct vertices of  $H$ . We will show a procedure to decompose the determinant of  $G \overset{j}{\succ} H$  into the sum of the determinants of modifications of the original graphs. These modifications are vertex deletions and directed graph handles. We will show that the determinant calculations of certain joined graphs are greatly simplified using this method and mention some algebraic properties that result from the decomposition. We will also discuss many avenues for undergraduate research. (Received September 20, 2016)

1125-15-2934 **Chung Y Wong\*** ([wong24@tcnj.edu](mailto:wong24@tcnj.edu)), 2000 Pennington Road, Department of Mathematics and Statistics, Science Complex SCP 231, Ewing, NJ 08628, and **Hugo J Woerdeman** ([hugo@math.drexel.edu](mailto:hugo@math.drexel.edu)), Drexel University, Department of Mathematics, 15 S. 33rd Street, Philadelphia, PA 19104. *Related problems to the Bernstein-Szegő moment problem in two variables*.

The Bernstein-Szegő measure moment problem asks when a given finite list of complex numbers form the Fourier coefficients of the spectral density function of a stable polynomial in the one-variable case. Szegő proved in 1919 that it is possible if and only if the Toeplitz matrix form by these numbers is positive definite. Bernstein later proved in 1930 a real line analog of the problem.

The question remained open in two variables until Geronimo and Woerdeman stated and proved the necessary and sufficient condition in 2004. Unlike the solution in one variable, it does not suffice to write down a single matrix and check whether it is positive definite. A positive definite completion condition is also required.

We further pursue the moment problem in two variables and beyond. We first enhance the two-variable results by identifying the eigenstructure of matrices that arise from the theory. We then create a method that allows us to compute the Fourier coefficients in a given infinite region by using a finite portion of the coefficients. In addition, we compute the asymptotics for the Fourier coefficients and later generalize the result to higher dimensions. (Received September 20, 2016)

1125-15-3000 **Troy Banks\*** ([tvbanks@salisbury.edu](mailto:tvbanks@salisbury.edu)), Salisbury University, 1101 Camden Ave, Salisbury, MD 21801. *On the structure of certain class of Hankel-like Kernels*.

We investigate a certain class of Hankel-like positive definite kernels using their associated orthogonal polynomials and provide calculation of the coefficients of the Hankel matrices corresponding to the parameters of the Gegenbauer polynomials. (Received September 20, 2016)

1125-15-3061 **Charles Johnson\*** ([crjohn@wm.edu](mailto:crjohn@wm.edu)). *The Nonnegative Single Inverse Eigenvalue Problem for Row Stochastic and Doubly Stochastic Matrices*.

The Nonnegative Inverse Eigenvalue Problem (NIEP) asks which spectra occur for  $n$ -by- $n$  nonnegative (equivalently, row stochastic matrices). In celebrated work, Karpelevich described all possible complex numbers that

can be a single eigenvalue of a row stochastic matrix. However, the description is not entirely explicit. The corresponding, more restrictive, problem for doubly stochastic matrices is still open. Here, we give a highly explicit description of the Karpelevich region and some new ideas stemming from recent work about the doubly stochastic problem. (Received September 20, 2016)

## 16 ► Associative rings and algebras

1125-16-106 **Jacob Laubacher\*** (jlaubac@bgsu.edu), **Mihai D. Staic** and **Alin Stancu**. *The Bar Simplicial Module*.

The Bar resolution is an important tool in the study of Hochschild cohomology and other related concepts. In this talk we present a bar-like complex  $\mathcal{B}(A, B, \varepsilon)$ , which appears naturally in the study of the secondary Hochschild cohomology. We prove that  $\mathcal{B}(A, B, \varepsilon)$  is a simplicial module over a certain simplicial algebra. We also show how it can be used to define the secondary Hochschild homology as well as the secondary cyclic (co)homologies. (Received July 28, 2016)

1125-16-274 **Daniel O Yee\*** (doyee@uwm.edu), 2727 N. Maryland Avenue, Apt. 308, Milwaukee, WI 53211. *Global Dimension of Connected Hopf Algebras*. Preliminary report.

Wang, Zhang, and Zhuang have studied Gelfand-Kirillov (GK) dimension of connected Hopf algebras (2015) and classified such Hopf algebras of GK dimension 4 over an algebraically closed field of characteristic zero. We study global dimension on connected Hopf algebras whether a similar classification can be obtained. We find that there is a positive answer with some restriction in particular when the space of primitive elements is a finite dimensional completely solvable Lie algebra over a field of characteristic zero. (Received August 25, 2016)

1125-16-276 **James S Cook\*** (jcook4@liberty.edu). *An Invitation to  $\mathcal{A}$ -Calculus for Undergraduates*. Let  $\mathcal{A}$  denote a real associative unital algebra of finite dimension. The calculus for  $\mathcal{A} = \mathbb{R}$  or  $\mathcal{A} = \mathbb{C}$  are widely known, however, for other choices of  $\mathcal{A}$  there are open questions which are accessible to undergraduates. In this talk I plan to discuss the rich history of the subject, summarize what we currently know, and outline possible directions for future research projects. I also describe some past projects by undergraduates: W.S. Leslie's of Laplace's equation for a commutative semisimple algebra, Bailu Zhang's work in Maple to verify Leslie's conjecture in noncommutative case, Daniel Freese's  $k$ -thagorean Theorem for the  $n$ -complex or  $n$ -hyperbolic numbers, and Nathan BeDell's novel solution techniques for  $\mathcal{A}$ -ODEs. (Received August 22, 2016)

1125-16-290 **James S Cook\*** (jcook4@liberty.edu).  *$\mathcal{A}$ -Calculus on a Real Associative Algebra*. Let  $\mathcal{A}$  denote a real associative unital algebra of dimension  $n$ . If  $f : \mathcal{A} \rightarrow \mathcal{A}$  is Frechet differentiable at  $p$  and if differential  $d_p f$  is right- $\mathcal{A}$ -linear then we say  $f$  is  $\mathcal{A}$ -differentiable at  $p$ . If  $\mathcal{A}$  is the quaternions then  $\mathcal{A}$ -differentiability gives us a method to differentiate with respect to a quaternionic variable. We contrast our definition to that which defines the derivative by a limit modulo the zero-divisors in  $\mathcal{A}$ . We also show how to construct  $n - 1$  conjugate derivatives for which the  $\mathcal{A}$ -differentiable functions have trivial conjugate derivatives. Generalized Laplace equations derived by Wagner and later by Waterhouse are reproduced here by a simple computation. Integration over  $\mathcal{A}$  is also similar to complex analysis: there is a natural integral over  $\mathcal{A}$  which allows the FTC, Goursat's Theorem and even Cauchy's Integral Theorem. However, Cauchy's Integral Formula is not generally available. We describe a joint work with Nathan BeDell where the application of abstract algebraic techniques and integral  $\mathcal{A}$ -calculus provide solutions of  $\mathcal{A}$ -ODEs. Zero-divisors produce strange results. (Received August 24, 2016)

1125-16-476 **Mee Seong Im\*** (meeseong.im@usma.edu), 646 Swift Road, Department of Mathematical Sciences, United States Military Academy, West Point, NY 10996. *Higher Schur-Weyl duality for Lie supersubalgebras*. Preliminary report.

We consider a big Kac module, which is an induced module from a simple module, tensored with  $d$  copies of a vector representation of a Lie supersubalgebra  $\mathfrak{p}(n)$ . We study the  $\mathfrak{p}(n)$ -centralizer subalgebra of the endomorphism algebra of this module and relate it to a modified construction of the Brauer algebra.

This is joint with Inna Entova Aizenbud, Martina Balagovic, Zajj Daugherty, Maria Gorelik, Iva Halacheva, Johanna Hennig, Gail Letzter, Emily Norton, Vera Serganova, and Catharina Stroppel. (Received September 03, 2016)

1125-16-526 **Agustín García Iglesias\*** (aigarcia@famaf.unc.edu.ar). *Hopf algebras of diagonal type*. After the classification of the finite-dimensional Nichols algebras of diagonal type by Heckenberger, the determination of its defining relations and the verification of the generation in degree one conjecture by Angiono, there

is still one step missing in the classification of complex finite-dimensional pointed Hopf algebras with abelian group of group-like elements, without restrictions on the order this group: the computation of all deformations or liftings. A technique towards solving this question, built on cocycle deformations, was developed previously by the author in collaboration with Andruskiewitsch, Angiono, Masuoka and Vay. In this talk, we shall discuss an article in collaboration with Andruskiewitsch and Angiono in which we elaborate further this technique and present an explicit algorithm to compute the liftings. In the same work, we applied this algorithm to classify all liftings of finite-dimensional Nichols algebras of Cartan type  $A$ , over a cosemisimple Hopf algebra. We shall also discuss an analogous result for Cartan type  $G_2$ , in collaboration with Jury-Giralaldi. (Received September 05, 2016)

1125-16-722 **James Zhang\*** ([zhang@math.washington.edu](mailto:zhang@math.washington.edu)), Department of Mathematics, Box 354350, University of Washington, Seattle, WA 98195. *Noncommutative Auslander Theorem and McKay correspondence.*

We survey on recent developments concerning Auslander theorem and McKay correspondence in the noncommutative setting. (Received September 09, 2016)

1125-16-892 **Daniel J Freese\*** ([dfreese3@liberty.edu](mailto:dfreese3@liberty.edu)), 5706 Wainwright Avenue, Rockville, MD 20851. *Calculus on a Real Associative Algebra.*

We generalize complex analysis by studying calculus on a general real associative algebra. We develop the notions learned in first and second semester calculus, extended to algebras, including limits, differentiation, and integration. Power series on an algebra allows us to study special functions on it, such as the exponential, sine, cosine, and hyperbolic sine and cosine. Using power series, we also define algebra-specific functions from the construction of a specific algebra, which generalize the standard trigonometric and hyperbolic functions. These new functions, for a special family of algebras, have many remarkable properties, including satisfying a general Pythagorean identity. (Received September 13, 2016)

1125-16-906 **Kenneth Chan, Ellen Kirkman and Chelsea Walton\*** ([notlaw@temple.edu](mailto:notlaw@temple.edu)), 1805 N. Broad Street, Philadelphia, PA 19122, and **James Zhang**. *McKay Correspondence for Hopf actions.*

We establish a more general version of the McKay correspondence for actions of semisimple Hopf algebras on noncommutative regular algebras of global dimension two. (Received September 13, 2016)

1125-16-998 **Julia Plavnik\*** ([julia@math.tamu.edu](mailto:julia@math.tamu.edu)) and **Sarah Witherspoon**. *Projectivity and tensor products for some Hopf algebras.* Preliminary report.

In this talk, we will pose some questions of projectivity and tensor products of modules for finite dimensional Hopf algebras. To give some answer to the questions, we will construct many classes of examples in which tensor powers of non-projective modules are projective and tensor products of modules in one order are projective but not in the other order. Some of these examples coming from groups were found in work of Benson and Witherspoon, and more recently were put into a general context with other examples in a joint work with Witherspoon. One of the fundamental tools that we use to prove some properties of the modules is the theory of support varieties for these Hopf algebras, and we will mention some of the main ideas if the time allows. (Received September 13, 2016)

1125-16-1319 **Jason Gaddis\*** ([gaddisjd@wfu.edu](mailto:gaddisjd@wfu.edu)), **Ellen Kirkman** ([kirkman@wfu.edu](mailto:kirkman@wfu.edu)), **W. Frank Moore** ([moorewf@wfu.edu](mailto:moorewf@wfu.edu)) and **Robert Won** ([wonrj@wfu.edu](mailto:wonrj@wfu.edu)). *Auslander's Theorem for permutation actions on  $(-1)$ -skew polynomial rings.* Preliminary report.

Let  $\mathbb{k}$  be an algebraically closed field of characteristic zero. If  $G$  is a finite subgroup of  $GL_n(\mathbb{k})$  containing no reflections and acting naturally on  $A = \mathbb{k}[x_1, \dots, x_n]$ , then then the skew group ring  $A\#G$  is isomorphic to  $\text{End}_{A^G}(A)$  as algebras precisely when  $G$  contains no reflections. Bao, He, and Zhang have developed the notion of pertinency and were successful in extending Auslander's Theorem to certain noncommutative algebras. In this talk, I will report on joint progress with Kirkman, Moore, and Won in extending Auslander's Theorem to the permutation action of  $S_n$  on  $\mathbb{k}_{-1}[x_1, \dots, x_n]$ . (Received September 16, 2016)

1125-16-1349 **Van C. Nguyen\*** ([v.nguyen@northeastern.edu](mailto:v.nguyen@northeastern.edu)), Department of Mathematics, Northeastern University, Boston, MA 02115, and **Xingting Wang**. *On the structures of Hopf algebras in prime characteristic.*

The goal of this talk is to present some recent progress in the classification of finite-dimensional Hopf algebras, over any algebraically closed field of prime characteristic  $p > 0$ . In particular, we discuss the classification of pointed  $p^3$ -dimensional Hopf algebras  $H$ , focusing on the cases when  $H$  is pointed but is not connected nor a

group algebra. Structures of Nichols algebras over  $p$ -groups in characteristic  $p$  as well as non-primitively generated braided Hopf algebras over  $C_p$  occur in this work. Our results provide many new examples of (parametrized) non-commutative and non-cocommutative finite-dimensional Hopf algebras in positive characteristic. Necessary background will be given. (Received September 16, 2016)

1125-16-1355 **Emil Horozov** (horozov@fmi.uni-sofia.bg), Department of Mathematics and Informatics, Sofia University, 5 J. Bourchier Blvd., 1126 Sofia, Bulgaria, and **Milen Yakimov\*** (yakimov@math.lsu.edu), Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803. *Adelic grassmannians for finite dimensional algebras.*

The scalar bispectral problem of Duistermaat and Grunbaum asks for classifying ordinary differential operators whose family of eigenfunctions are also eigenfunctions in the spectral parameter. It originated from applications to tomography and time-band limiting, but was later related to various problems in representation theory, integrable systems and algebraic geometry. We will present a classification of the bispectral operators with values in an arbitrary finite dimensional algebra for operators (which in a certain technical sense) are of rank 1. This associates a Wilson type adelic Grassmannian (an algebraic ind variety) to every finite dimensional algebra  $R$ , which is the moduli space of  $R$ -valued bispectral operators. (Received September 16, 2016)

1125-16-1368 **Ben Webster\*** (webster@virginia.edu), Department of Mathematics, University of Virginia, Charlottesville, VA 22903. *The discreet charm of the Coulomb branch.*

Braverman, Finkelberg and Nakajima have recently given a mathematical definition of the Coulomb branch of an  $N=4$  3-dimensional gauge theory. Even if you don't recognize any of the words in the previous sentence, I'd like to show you that Coulomb branches are a very friendly collection of symplectic singularities with a beautiful associated representation theory. They give a fresh perspective on Cherednik algebras and the categorification of representations of simple Lie algebras, and duality of symplectic singularities. (Received September 16, 2016)

1125-16-1386 **Alexandru Chirvasitu, Chelsea Walton and Xingting Wang\*** (xingting@temple.edu), Department of Mathematics, Temple University, 1805 N. Broad Street, Philadelphia, PA 19122. *Quantum groups associated to a pair of preregular forms.* Preliminary report.

In this talk, we study universal quantum groups that simultaneously coact on a pair of  $N$ -Koszul Artin-Schelter regular algebras. This work leads to a recovery of many well-known examples of quantum groups defined by various authors in the literature. Moreover, we show that these quantum groups have surprisingly nice presentations in terms of the twisted superpotentials associated to the underlining graded algebras, respectively. In particular, we will discuss the universal quantum group associated to a pair of three-dimensional Sklyanin algebras, whose ring-theoretic and homological behaviors need further investigation. This is a joint work with Alexandru Chirvasitu and Chelsea Walton. (Received September 16, 2016)

1125-16-1485 **Xin Tang\*** (xtang@uncfsu.edu), 1200 Murchison Road, Fayetteville, NC 28301. *The Cancellation Problem for Some Quantum Algebras.* Preliminary report.

The study of noncommutative Zariski cancellation problems has been initiated by Bell and Zhang. A  $k$ -algebra  $A$  is said to be cancellative if  $A[t] \cong B[t]$  for any  $k$ -algebra  $B$  implies  $A \cong B$ . Bell and Zhang have established several useful criteria for an algebra  $A$  to be either universally cancellative, or strongly cancellative, or cancellative. In particular, they have proved that many PI algebras with effective discriminants are strongly cancellative; and any  $k$ -algebra  $A$  with a trivial center is universally cancellative. In this talk, we first explore the connection between the group of unipotent automorphisms and the cancellation property for any connected graded  $k$ -algebra  $A$ . Assume that  $k$  is a field of characteristic zero and  $A$  is a  $k$ -algebra of finite Gelfand-Kirillov dimension. We prove that if  $\text{Aut}_{\text{uni}}(A) = \{\text{id}\}$ , then  $A$  is cancellative. Second, we study the cancellation problem for many classes of CGL extensions. Let  $A$  be a symmetric saturated CGL extension. Under a mild condition on  $A$ , we show that  $A$  is strongly cancellative and thus cancellative. Finally, we present some results on the cancellation problem for some polynomial-based quantum Weyl algebras. (Received September 17, 2016)

1125-16-1499 **Daniel P. Bossaller\*** (db684513@ohio.edu) and **Sergio R. López-Permouth** (lopez@ohio.edu). *Associativity and Infinite Matrices.* Preliminary report.

Given an infinite dimensional vector space  $V$  with basis  $\mathcal{B} = \{e_i : i < \omega\}$  over a field  $k$ , it is well known that the ring of left endomorphisms of  $V$  is isomorphic to the ring of column finite matrices  $\text{CFM}(k)$ , an associative ring. It is also well known that associativity does not hold for three arbitrary infinite matrices. In this talk I will give necessary and sufficient conditions for the associativity of the product of three arbitrary infinite matrices, and explore an applications of this theory to infinite-dimensional linear algebra. This is joint work with Sergio R. López-Permouth. (Received September 17, 2016)

1125-16-1501 **Karin Baur** and **Eleonore Faber\***, Department of Mathematics, 530 Church Street, Ann Arbor, MI 48109, and **Sira Gratz**, **Khrystyna Serhiyenko** and **Gordana Todorov**. *Mutation of  $A_n$  friezes*. Preliminary report.

A frieze is a grid of integers with a finite number of infinite rows satisfying a certain rule. Introduced in the 1970's by Conway and Coxeter, the interest in them gained fresh momentum in the last decade, when strong relations to cluster theory were discovered: in particular, there exists a bijection between friezes and cluster tilted algebras of type  $A$ .

In cluster theory, the key concept is that of mutation. In this talk we consider mutations of Conway-Coxeter friezes which are compatible with mutations of cluster-tilting objects in the associated cluster category of Dynkin type  $A$ . More precisely, we provide a formula, relying solely on the shape of the frieze, describing how each individual entry in the frieze changes under cluster mutation. We provide a combinatorial formula for the number of submodules of a string module, and with that a simple way to compute the frieze associated to a fixed cluster-tilting object in a cluster category of Dynkin type  $A$  in the sense of Caldero and Chapoton. (Received September 17, 2016)

1125-16-1564 **Yorck Sommerhäuser\*** ([sommerh@mun.ca](mailto:sommerh@mun.ca)), Memorial University of Newfoundland, Department of Mathematics and Statistics, St. John's, NL A1C 5S7, Canada. *Yetter-Drinfel'd Hopf Algebras and Their Associated Algebras*.

With each Yetter-Drinfel'd Hopf algebra, one can associate a variety of algebras, most prominently the Radford biproduct, but sometimes also a Hopf algebra with triangular decomposition, which appears in the construction of deformed enveloping algebras, its dual, and similar algebras. In the talk, we discuss these algebras and their interrelation. (Received September 18, 2016)

1125-16-1621 **Lauren Grimley\*** ([lgrimley@shc.edu](mailto:lgrimley@shc.edu)) and **Christine Uhl**. *Deformations of the skew group algebras of truncated quantum polynomial rings*. Preliminary report.

Deformations of an associative algebra are dictated by the Hochschild cohomology of that algebra. In this talk, we investigate the Hochschild cohomology of skew group algebras arising from a finite group action on truncated quantum polynomial rings, with a view towards deformations. We compare the cohomological conditions on deformations to the algebraic conditions of Poincare-Birkhoff-Witt deformations. By combining perspectives, we produce examples of truncated quantum Drinfeld Hecke algebras. (Received September 18, 2016)

1125-16-1749 **Iva Halacheva\*** ([i.halacheva@lancaster.ac.uk](mailto:i.halacheva@lancaster.ac.uk)), **Joel Kamnitzer**, **Leonid Rybnikov** and **Alex Weekes**. *A monodromy action on crystals and the cactus group*. Preliminary report.

The shift of argument algebras associated to a finite-dimensional reductive Lie algebra  $\mathfrak{g}$  are certain maximal commutative subalgebras of  $U(\mathfrak{g})$ . They are parametrized by a moduli space  $M_{\mathfrak{g}}$ , which in type  $A$  coincides with the Deligne-Knudson-Mumford moduli space of stable real curves of genus 0 with  $n + 1$  marked points. Furthermore, they have simple spectrum when acting on an irreducible highest-weight representation, and so produce a covering of  $M_{\mathfrak{g}}$ . We show that the resulting monodromy action coincides with a combinatorial action of the cactus group on  $\mathfrak{g}$ -crystals, realized via Schützenberger involutions. (Received September 19, 2016)

1125-16-1769 **Xingting Wang\*** ([xingting@temple.edu](mailto:xingting@temple.edu)), Temple University Department of Mathematics, 1805 N. Broad Street, Philadelphia, PA 19122, and **Xiaolan Yu** and **Yinhua Zhang**. *Non-noetherian Hopf algebras versus noetherian Hopf algebras*. Preliminary report.

In this talk, we will talk about how to use the monoidal Takeuchi-Morita equivalence as a bridge between noetherian and non-noetherian Hopf algebras to study various properties of the latter. In particular, we will discuss homological behaviors such as twisted Calabi-Yau property. Many examples will be provided during the talk including the study of quantum automorphism groups associated to Artin-Schelter regular algebras. This is a joint work with Xiaolan Yu and Yinhua Zhang. (Received September 19, 2016)

1125-16-1794 **Cris Negron\*** ([negronc@mit.edu](mailto:negronc@mit.edu)). *Hopf algebras, tensor categories, and gauge invariants*. I will define a Hopf algebra and explain how any Hopf algebra gives rise to a so called tensor category. I will explain how, from a number of perspectives, the tensor category is a more fundamental object than the Hopf algebra. With this in mind, I will discuss a number of invariants which help us to study tensor categories directly. (Some of this is joint work with Richard Ng.) (Received September 19, 2016)

1125-16-1808 **Cris Negron\*** ([negronc@mit.edu](mailto:negronc@mit.edu)) and **Richard Ng** ([rng@math.lsu.edu](mailto:rng@math.lsu.edu)). *Gauge invariants from the antipode of a finite dimensional Hopf algebra.*

Due to a result of Radford, we know that the antipode of a finite dimensional Hopf algebra is always a finite order operator. We show that, for finite dimensional Hopf algebras over a field of characteristic 0 with the Chevalley property, the order of the antipode is a gauge invariant. That is to say, if two such Hopf algebras admit an equivalence between their associated tensor categories of representations, then their antipodes have the same order. In fact, a more refined result holds: the trace of any given power of the antipode is a gauge invariant. I will discuss how this result relates to other invariants, such as the indicators and quasi-exponent. (Received September 19, 2016)

1125-16-1828 **Jieru Zhu\*** ([jieru.zhu-1@ou.edu](mailto:jieru.zhu-1@ou.edu)), 3001 Oak Tree Ave Apt L-12, Norman, OK 73072, and **Jonathan Kujawa**. *Presenting cyclotomic Schur algebras.*

A classical result states that the action of  $\mathfrak{gl}(V)$  and the symmetric group on  $d$  letters mutually centralize each other on the  $d$ -fold tensor of  $V$ . If  $V$  admits an action by  $\mathbb{Z}/r\mathbb{Z}$ , it induces an action of the wreath product of  $\mathbb{Z}/r\mathbb{Z}$  and the symmetric group on  $d$  letters. A Levi Lie subalgebra of  $\mathfrak{gl}(V)$  gives the full centralizer of this action, and we showed a presentation for the centralizing algebra (the cyclotomic Schur algebra.) When  $r = 2$ , this becomes a presentation for the Type B hyperoctahedral Schur algebra defined by Richard Green. (Received September 19, 2016)

1125-16-1850 **Juan Cuadra\*** ([jcdiaz@ual.es](mailto:jcdiaz@ual.es)), University of Almeria, Dpt. Mathematics, 04120 Almeria, Almeria, Spain. *On Hopf orders and Kaplansky's sixth conjecture.*

A theorem of Frobenius states that the degree of any complex irreducible representation of a finite group  $G$  divides the order of  $G$ . The proof uses that the group algebra  $\mathbb{C}G$  is defined over  $\mathbb{Z}$ .

Kaplansky's sixth conjecture predicts that this theorem holds for complex semisimple Hopf algebras. There are several partial results in the affirmative. Compared to the case of groups, the main difficulty to prove this conjecture (if true) is that it is not guaranteed that a complex semisimple Hopf algebra  $H$  is defined over  $\mathbb{Z}$  or, more generally, over a number ring. If it would be so, Larson proved that  $H$  satisfies Kaplansky's sixth conjecture. The question whether every complex semisimple Hopf algebra can be defined over a number ring has always been behind this conjecture.

In this talk we will answer this question in the negative. The family of examples that we will handle, constructed by Galindo and Natale, are Drinfeld twists of certain group algebras. The key fact is that the twist contains a scalar fraction, which makes impossible to define such Hopf algebras over a number ring.

This result is part of a joint work with Ehud Meir (University of Hamburg) published in *Trans. Amer. Math. Soc.* and available at [arXiv.org](https://arxiv.org). (Received September 19, 2016)

1125-16-1871 **María D. Vega\*** ([maria.vega@usma.edu](mailto:maria.vega@usma.edu)), Department of Mathematical Sciences, United States Military Academy, West Point, NY 10996. *Hopf algebras, Indicators and Twisted Extensions.*

In this talk, I will describe twisted exponents as well as twisted extensions constructed from a group acting on a Hopf algebra  $A$  as Hopf automorphisms. I will show connections between the twisted and classical Frobenius-Schur indicators and exponents for the smash coproduct produced by twisted extensions to those of the original Hopf algebra  $A$ . I will also describe a formula for twisted indicators and use it to prove periodicity and rationality statements. (Received September 19, 2016)

1125-16-1934 **Siân Fryer\*** ([sianfryer@math.ucsb.edu](mailto:sianfryer@math.ucsb.edu)). *From prime ideals to physics, via total nonnegativity.*

We start with a very classical question in representation theory: obtain a description of the prime and primitive ideals in a given algebra. For certain types of nice noncommutative algebra (e.g. the families known as quantum algebras and CGL algebras) this can be achieved by first studying finitely many " $H$ -primes". If we restrict our attention further to just the algebras coming from the standard quantization of the coordinate rings of matrices, there are remarkable connections between the  $H$ -primes in these algebras and cells of totally nonnegative real matrices (matrices in which every minor is nonnegative).

Recently the combinatorial behaviour of totally nonnegative cells has also been linked to several questions in physics, in particular the computation of scattering amplitudes in the quantum field theory  $N = 4$  SYM. I'll talk about ways we can use techniques developed to study  $H$ -primes in order to tackle some of these questions about scattering amplitudes. (No prior physics knowledge required!) (Received September 19, 2016)

1125-16-2002 **Benjamin C Tharp\*** (btharp@math.ou.edu). *On the representation theory of the marked Brauer algebra.*

The marked Brauer algebra is a diagram algebra which generalizes the ordinary Brauer algebra and is in Schur-Weyl duality with the type  $\mathfrak{p}$  Lie superalgebra. Like the ordinary Brauer algebra, this marked version has a very rich representation theory. We will discuss some of the interesting combinatorics which arise from studying certain representations of the marked Brauer algebra. (Received September 19, 2016)

1125-16-2026 **Hazar M. Abu-Khuzam\*** (hazar@aub.edu.lb), Department of Mathematics, American University of Beirut, Beirut, Lebanon. *Structure of certain von Neumann regular rings.*

We study the structure of certain classes of von Neumann regular rings with certain constraints, such as, multiplicatively generated by certain subsets, or having a semiprime center. Such rings with other constraints on elements are also studied. (Received September 19, 2016)

1125-16-2130 **Ellen E. Kirkman\*** (kirkman@wfu.edu), **Jianmin Chen** (chenjianmin@xmu.edu.cn) and **James J. Zhang** (zhang@math.washington.edu). *Invariant subrings of noetherian graded down-up algebras under group coactions.*

Let  $\mathbb{k}$  be a field of characteristic zero,  $\mathbb{D} = \mathbb{D}(\alpha, \beta)$  be a noetherian down-up algebra that is graded by a finite group  $G$ , and  $H = \text{Hom}_{\mathbb{k}}(\mathbb{k}G, \mathbb{k})$  be the  $\mathbb{k}$ -linear dual of the group algebra  $\mathbb{k}G$ . The fixed subring  $\mathbb{D}^H$  under the Hopf algebra  $H$  can be identified with the identity component  $\mathbb{D}_e$  under the  $G$ -grading. We prove that  $\mathbb{D}$  is rigid in the sense that  $\mathbb{D}^H$  is never AS regular (so  $\mathbb{D}^H$  is not isomorphic to  $\mathbb{D}$ ), and hence each  $\mathbb{D}$  has no dual reflection group. We provide further results for coactions on the down-up algebra  $\mathbb{D} := \mathbb{D}(\alpha = 0, \beta = 1)$ . As one example, when the homological determinant of the  $H$ -action on  $\mathbb{D}$  is trivial, we have Auslander's Theorem: the smash product  $\mathbb{D} \# H$  is isomorphic to  $\text{End}_{\mathbb{D}H}(\mathbb{D})$ , as  $\mathbb{k}$ -algebras. (Received September 19, 2016)

1125-16-2158 **Taiki Shibata\*** (shibata@ualberta.ca), Dept. of Math. and Stat. Sci., University of Alberta, Edmonton, Alberta T6G 2G1, Canada. *A Hopf-algebraic approach to the study of algebraic supergroups.*

An *algebraic supergroup* is a group-valued functor on the category of commutative superalgebras represented by a finitely-generated commutative Hopf superalgebra. It has been known that representations of algebraic supergroups can be applied in non-super (modular) representation theory.

Serganova (2011) introduced the notion of *quasireductive supergroups* as a super version of the notion of split reductive groups. This is an interesting and important class of algebraic supergroups including queer supergroups  $Q(n)$  whose Lie superalgebra is a queer superalgebra  $\mathfrak{q}(n)$ . She constructed irreducible representations of quasireductive supergroups over an algebraically closed field of characteristic zero in terms of their Lie superalgebras. In this talk, I introduce a Hopf-algebraic approach to the study of quasireductive supergroups and, as an application, give a generalization of Serganova's result to the case where the base field is arbitrary. The main tool of this approach is the *super-hyperalgebra*  $\text{hy}(G)$  of an algebraic supergroup  $G$  which is a refinement of the notion of the Lie superalgebra  $\text{Lie}(G)$  of  $G$ . (Received September 19, 2016)

1125-16-2235 **Briana Foster-Greenwood\*** (brianaf@cpp.edu) and **Cathy Kriloff** (krilcath@isu.edu). *Drinfeld orbifold algebras for symmetric groups.*

Since their introduction in the 1980's, graded Hecke algebras have appeared in various guises and settings ranging from symplectic reflection algebras in orbifold theory to rational Cherednik algebras used to prove results in algebraic combinatorics. Graded Hecke algebras, and their recent generalization to Drinfeld orbifold algebras, arise from modifying commutator relations in a skew group algebra and supply a piece to the puzzle of determining all ways a skew group algebra may be deformed. In this talk, we describe an infinite family of Drinfeld orbifold algebras arising from the permutation representation of a symmetric group. (Received September 20, 2016)

1125-16-2253 **Bradley S. McQuaig\*** (bsm0012@auburn.edu), Auburn University, Department of Mathematics and Statistics, 221 Parker Hall, Auburn, AL 36849. *Strongly Non-Singular Rings, Morita-Equivalence, and the Maximal Ring of Quotients.*

In 2005, Albrecht, Dauns, and Fuchs classified rings for which the classes of torsion-free and non-singular right  $R$ -modules coincide. Here, a right  $R$ -module  $M$  is *torsion-free* if  $\text{Tor}_1^R(M, R/Rr) = 0$  for every  $r \in R$ , and a right  $R$ -module  $M$  is *non-singular* if  $xI$  is nonzero for every nonzero  $x \in M$  and every essential right ideal  $I$  of  $R$ . We extend this to classify the rings  $R$  for which the classes of torsion-free and non-singular right  $S$ -modules coincide for every ring  $S$  Morita-equivalent to  $R$ . We then look to characterize rings whose  $n \times n$  matrix rings are Baer-rings. A ring is *Baer* if every right (or left) annihilator ideal is generated by an idempotent. Central to

these discussions is the maximal ring of quotients, and we consider relevant results and examples. (Received September 20, 2016)

1125-16-2375 **Yevgenia Kashina\***, Department of Mathematical Sciences, DePaul University, Chicago, IL 60614. *Semisimple Hopf algebras of dimension 32 with large abelian groups of grouplikes*. Preliminary report.

In this talk we will finish the classification of semisimple Hopf algebras of dimension 32 with abelian group of grouplike elements of order 16. (Received September 20, 2016)

1125-16-2769 **Hannah Elizabeth Downs\*** ([hedowns42@students.tntech.edu](mailto:hedowns42@students.tntech.edu)). *Clifford Algebras as Hopf Algebras and the Connection Between Cocycles and Walsh Functions*.

Group algebras and twisted group algebras can be shown as Hopf algebras, and we can connect Clifford algebras to these Hopf algebras through twisted group algebras. We show that Walsh functions of Clifford algebras correlate to the cocycles described for these twisted group algebras. (Received September 20, 2016)

1125-16-2797 **Gus Lonergan\***, [gusl@mit.edu](mailto:gusl@mit.edu). *Frobenius and embedded Grassmannians*. Preliminary report.

Let  $G$  be a reductive algebraic group over  $\mathbb{C}$ . Geometric Satake gives an equivalence between spherical perverse sheaves mod  $p$  on the affine Grassmannian  $Gr$  and representations of the dual group  $G^\vee$  defined over  $\mathbb{F}_p$ . It is natural to ask: does the Frobenius endofunctor  $F$  of  $Rep(G^\vee)$  correspond to something geometric under this equivalence?

In fact,  $F$  can be described categorically, and thus geometrically in terms of nearby cycles and monodromy, using a version of the Beilinson–Drinfeld Grassmannian.

Now we make a parallel observation.  $Gr$  contains many embedded copies of  $Gr$  (and other partial affine flag varieties), for instance appearing as the components of the fixed-point set for the action of loop rotation by the  $p^{\text{th}}$  roots of unity. In fact, this ‘fractal’ nature of  $Gr$  seems analogous to the well-known ‘fractal’ nature of  $Rep(G^\vee)$ , and it is natural to ask if they are in any way compatible via geometric Satake.

We find evidence for such a compatibility by comparing the (equivariant) homologies of the relevant spaces. Finally, we speculate on the exact form such a compatibility might take, by globalizing these subspaces to the Beilinson–Drinfeld Grassmannian considered earlier, and fitting  $F$  into the picture. (Received September 20, 2016)

1125-16-2957 **Kevin Gerstle, Miodrag C Iovanov\*** ([miodrag-iovanov@uiowa.edu](mailto:miodrag-iovanov@uiowa.edu)) and **Gerard Koffi**. *Monoidal categories arising from representations of finite dimensional algebras, and their representation rings*.

For a monoidal abelian category  $\mathcal{C}$ , its Grothendieck or representation rings represent interesting invariants; in the other direction, given a ring  $R$ , finding a monoidal category whose Grothendieck or representation ring is isomorphic to  $R$  is a question of active interest and the main problem of Categorification. This becomes especially relevant when the category  $\mathcal{C}$  is small, and in particular, when  $\mathcal{C}$  has finitely many objects. We survey existing results and present new ones on Hopf algebras having (locally) finitely many indecomposable representations; these include interesting classes of quantum groups, such as Hopf algebras whose category of comodules is equivalent to those of chain complexes and whose representation ring categorifies generalized Fibonacci polynomials. At the same time, we show that many important classes of finite dimensional algebras which have some finiteness in representation type, that is, that have only finitely many representations of a certain type (including algebras of finite representation type), admit weak bialgebra structures and give rise to monoidal categories which have representation rings that categorify various types of semigroup algebras. This is based partly on joint separate works G. Koffi and K. Gerstle. (Received September 20, 2016)

1125-16-2965 **Jesse S. F. Levitt\*** ([jslevitt@usc.edu](mailto:jslevitt@usc.edu)) and **Milen Yakimov**. *Discriminants of Polynomial Identity Quantized Weyl Algebras*.

We present explicit formulas for the the discriminants of Polynomial Identity (PI) quantized Weyl algebras over a general class of central subalgebras along with several applications. Following a program proposed by Chan, Young and Zhang for investigating certain filtered PI algebras. We first classify the centers of PI quantized Weyl algebras, and examine the case where these algebras are then free over their centers. Two distinct approaches arise for calculating their discriminants, with one coming from deformation theory and Poisson geometry, while the other is based in the methods of quantum cluster algebras. Both formulations allow all such algebras to be classified and the discriminant is found to be both locally dominating and effective with applications to the automorphism and isomorphism problems for tensor products of these algebras. (Received September 20, 2016)

## 17 ► *Nonassociative rings and algebras*

1125-17-218      **Corinne A. Manogue\*** ([corinne@physics.oregonstate.edu](mailto:corinne@physics.oregonstate.edu)), Dept of Physics, Oregon State University, Corvallis, OR 97331, and **Tevian Dray** ([teviaan@math.oregonstate.edu](mailto:teviaan@math.oregonstate.edu)), Dept of Mathematics, Oregon State University, Corvallis, OR 97331. *Division algebra descriptions of rotation groups, with applications to physics.*

Quaternions are often used to describe rotations in 3 (Euclidean) dimensions. Alternatively, a description of the Lorentz group in 4 (spacetime) dimensions in terms of complex numbers can be extended to a quaternionic description in 6 dimensions—and an octonionic description in 10 dimensions. These identifications unify the description of vector and spinor representations of these groups, leading to a natural language for describing the massless Dirac equation in 3, 4, 6, and 10 dimensions—precisely the dimensions in which classical supersymmetry is possible. (Received August 12, 2016)

1125-17-219      **Tevian Dray\*** ([teviaan@math.oregonstate.edu](mailto:teviaan@math.oregonstate.edu)), Dept of Mathematics, Oregon State University, Corvallis, OR 97331, and **Corinne A. Manogue** ([corinne@physics.oregonstate.edu](mailto:corinne@physics.oregonstate.edu)), Dept of Physics, Oregon State University, Corvallis, OR 97331. *The eigenvalue problem for quaternionic and octonionic matrices.*

The eigenvalues of complex Hermitian matrices are real, but what if the matrix is quaternionic? We consider several eigenvalue problems for Hermitian matrices over both the quaternions and octonions, showing to what extent it is possible to recover the properties expected by analogy with the complex case. There are nevertheless some interesting surprises along the way. Foremost among them is the fact that the octonionic projective space  $\mathbb{O}P^2$  in fact consists of quaternionic elements. (Received August 12, 2016)

1125-17-312      **Angelo Bianchi** and **Evan Wilson\***, 415 Harrisburg Ave, Lancaster, PA. *Gröbner basis for local Weyl modules for the hyper and truncated current  $\mathfrak{sl}_2$ -algebras.*

We use the theory of Gröbner bases for ideals to construct linear bases for graded local Weyl modules for the (hyper) current and the truncated current algebras associated to the finite-dimensional complex simple Lie algebra  $\mathfrak{sl}_2$ .

The main result is a characteristic-free construction of a linear basis for this important family of modules for the hyper current  $\mathfrak{sl}_2$ -algebra. In the positive characteristic setting this work represents the first construction giving an expected basis. In the characteristic zero setting, it results in a different construction of the basis obtained by Chari and Pressley and also in the construction of a new basis for local Weyl modules for the current  $\mathfrak{sl}_2$ -algebra which allows us to obtain a basis for the local Weyl modules for truncated current  $\mathfrak{sl}_2$ -algebras. (Received August 25, 2016)

1125-17-335      **Naihuan Jing** and **Chad R. Mangum\*** ([cmangum@niagara.edu](mailto:cmangum@niagara.edu)), Dunleavy 333, 5795 Lewiston Rd, Niagara University, NY 14109, and **Kailash C. Misra**. *New Realization of Twisted Toroidal Lie Algebras.*

The representation theory of Lie algebras is a vibrant field of research and has been significant in various areas of mathematics and physics for several decades. In this talk, we will discuss a recent advance in part of this theory, namely twisted (2-)toroidal (Lie) algebras, which we view as universal central extensions of twisted multi-loop algebras. The usual loop algebra realization generalizes the familiar realization of affine Kac-Moody algebras. We will discuss a new realization of these algebras given by generators and relations, based on a similar realization by Moody, Rao, and Yokonuma in the untwisted case. This has the advantage of being more amenable than the loop algebra realization to studying the representation theory. This is joint work with Dr. Kailash Misra and Dr. Naihuan Jing. (Received August 27, 2016)

1125-17-373      **Emilie B Wiesner\*** ([ewiesner@ithaca.edu](mailto:ewiesner@ithaca.edu)) and **Matthew Ondrus**. *Automorphisms, derivations, and subalgebras of the insertion-elimination algebra.*

The insertion-elimination algebra is a Lie algebra that can be realized in terms of inserting and eliminating operations on the set of rooted trees. It arises in mathematical physics and shares interesting features with the Virasoro algebra. In this talk, I'll present, joint with Matthew Ondrus, our work on some fundamental results on the insertion-elimination algebra and how they compare to the Virasoro algebra. These results will include descriptions of the automorphism group, derivations, and finite-dimensional subalgebras. (Received August 30, 2016)

- 1125-17-630 **Christopher M Drupieski**, Dept. of Mathematical Sciences, DePaul University, 2320 N Kenmore Ave, Chicago, IL 60614-3210, and **Jonathan R Kujawa\***, Dept. of Mathematics, University of Oklahoma, Norman, OK 73019-3103. *Support varieties for Lie superalgebras and supergroups in positive characteristic.*
- Cohomological support varieties are a fundamental tool in the study of representations of finite groups, restricted Lie algebras, finite group schemes, and related structures. In the super (i.e.  $\mathbf{Z}/2\mathbf{Z}$ -graded) setting these varieties were introduced and investigated by Boe-Kujawa-Nakano in characteristic zero. In this talk I will discuss ongoing work with Chris Drupieski to investigate cohomological varieties for graded objects over a field of positive characteristic. Our results reveal interesting generalizations of the classical theory. (Received September 08, 2016)
- 1125-17-635 **Pamela E Harris\*** (peh2@williams.edu), Bronfman #204, 18 Hoskey Street, Williamstown, MA 01267, and **Erik Insko** and **Mohamed Omar**. *The  $q$ -analog of Kostant's partition function and the highest root of the simple Lie algebras.*
- Kostant's partition function counts the number of ways to represent a particular vector (weight) as a nonnegative integral sum of positive roots of a Lie algebra. For a given weight the  $q$ -analog of Kostant's partition function is a polynomial where the coefficient of  $q^k$  is the number of ways the weight can be written as a nonnegative integral sum of exactly  $k$  positive roots. In this talk, we present generating functions for the  $q$ -analog of Kostant's partition function when the weight in question is the highest root of the classical Lie algebras of types  $B$ ,  $C$ , and  $D$ , and the exceptional Lie algebras of type  $G_2$ ,  $F_4$ ,  $E_6$ ,  $E_7$ , and  $E_8$ . (Received September 08, 2016)
- 1125-17-725 **Gail Letzter\*** (gletzter@verizon.net), National Security Agency, 9800 Savage Road, Fort Meade, MD 20755-6844. *Cartan Subalgebras for Quantum Symmetric Pair Coideals.* Preliminary report.
- Recent discoveries show that the coideal subalgebras used to form quantum symmetric pairs play a fundamental role in the representation theory of quantized enveloping algebras. However, there is still no general theory of finite-dimensional modules for these coideal subalgebras. In this talk, we establish an important step in this direction: we show that every quantum symmetric pair coideal subalgebra admits a quantum Cartan subalgebra which is an extension of a commutative ring and specializes to its classical counterpart. The construction builds on Kostant and Sugiura's classification of Cartan subalgebras for real semisimple Lie algebras via strongly orthogonal root systems. We show that the quantum Cartan subalgebras act semisimply on a large family of finite-dimensional modules and describe the corresponding weight spaces in a number of examples. (Received September 16, 2016)
- 1125-17-1194 **Stefanie Wang\*** (sgwang@iastate.edu), 396 Carver Hall, Mathematics Department, Iowa State University, Ames, IA 50011. *Super-Catalan numbers and rooted binary trees.* Preliminary report.
- A quasigroup is a structure equipped with nonassociative binary operations of multiplication, left division, and right division. Rooted binary trees with  $n$  leaves represent the number of ways to bracket a word with  $n$  arguments involving a single nonassociative binary operation. While Catalan numbers count rooted binary trees with  $n$  leaves, we are interested in the super-Catalan numbers that count the quasigroup words with  $n$  arguments. We will also discuss the relationship between super-Catalan numbers and the quasigroup of integer-valued functions on the vertex set of an infinite tree of valency four. (Received September 15, 2016)
- 1125-17-1255 **Nathaniel Schwartz\*** (nschwartz2@washcoll.edu) and **John Hutchens**. *On the  $k$ -Involutions of  $G_2$  over fields of Characteristic 2.* Preliminary report.
- Let  $C$  be an octonion algebra defined over a field of characteristic 2. In this talk, we discuss a classification of isomorphism classes of automorphisms of order 2 of algebraic groups of the form  $\text{Aut}(C)$ . We do this by describing automorphisms of  $C$  fixing a 4-dimensional subalgebra, and then describe those which induce an inner automorphism of order 2. For fields of characteristic not 2, a similar classification has been carried out by Hutchens. (Received September 15, 2016)
- 1125-17-1676 **Michael K. Lau\*** (michael.lau@mat.ulaval.ca), Département de mathématiques, 1045 av. de la Médecine, Pavillon Vachon, local 1056, Québec, Québec G1V0A6, Canada. *Weight modules for current algebras.*
- Current algebras are Lie algebras of regular maps from an affine variety to a finite-dimensional simple Lie algebra. We will discuss the classification of simple weight modules (with finite dimensional weight spaces) for current algebras. In particular, any such module is parabolically induced from a simple admissible evaluation

representation for a Levi subalgebra. Conversely, all modules obtained in this way have finite weight multiplicities. (Received September 18, 2016)

1125-17-1800 **John M. Duse1\*** (jdusel@whittier.edu), Department of Mathematics, Whittier College, 13406 Philadelphia St., Whittier, CA 90608. *Crystal folding.*

We introduce a procedure to fold a crystal  $B$  of simply-laced Cartan type  $\mathcal{C}$  by the action of an automorphism  $\sigma$ , which produces a crystal  $B_\sigma$  for the folded Langlands dual datum  $\mathcal{C}^{\sigma^\vee}$ . This construction preserves normality and the Weyl group action, and is compatible with Kashiwara's tensor product rule.

Combinatorial properties of the folding of  $B(\infty)$  reflect the structure of subalgebra of the quantum group  $U_q^-(\mathcal{C})$ . In particular, this subalgebra admits a  $U_q^-(\mathcal{C}^{\sigma^\vee})$ -module structure via Berenstein and Greenstein's machinery of quantum folding, which is encoded by the  $\mathcal{C}^{\sigma^\vee}$ -crystal structure of  $B(\infty)_\sigma$ . We find that  $B(\infty)_\sigma$  is generated by a set of highest-weight elements over the monoid of lowering operators. The highest-weight set of  $B(\infty)_\sigma$  identifies with a monoid admitting a unique finite  $\subset$ -minimal generating set, in finite type, and a subset of the Weyl group called the balanced parabolic quotient is in one-to-one correspondence with this generating set in type  $D$ . (Received September 19, 2016)

1125-17-1993 **Rebecca L. Jayne\*** (rjayne@hsc.edu), Box 187, Hampden-Sydney, VA 23943. *Multiplicities of maximal dominant weights of integrable modules.*

For  $n, k \geq 2$ , we study the multiplicities of certain maximal dominant weights of the irreducible highest weight  $\widehat{sl}(n)$ -module  $V(k\Lambda_0)$ . We give the multiplicity of the weight  $k\Lambda_0 - \sum_{i=0}^{\ell} (\ell - i)(\alpha_i + \alpha_{n-i})$  by the number of certain admissible sequences of  $k - 1$  lattice paths in a colored  $\ell \times \ell$  square. In turn, we find that the number of such admissible sequences of lattice paths is given by the sum of squares of the number of standard Young tableaux of shape  $\lambda \vdash \ell$  with  $l(\lambda) \leq k$ , a value that can be calculated using the well known Frame-Robinson-Thrall hook length formula. This is a joint work with Kailash C. Misra. (Received September 19, 2016)

1125-17-2191 **Asilata Bapat\*** (asilata@uga.edu) and **Robin Walters** (r.walters@neu.edu). *Bernstein–Sato polynomials and monodromy conjectures for Weyl arrangements.*

To a singularity of an algebraic hypersurface, one can associate a subtle invariant called the Bernstein–Sato polynomial or the  $b$ -function. The roots of the  $b$ -function recover several other known singularity invariants, so it is useful to understand the  $b$ -functions of interesting hypersurfaces.

We will consider the case of a Weyl hyperplane arrangement, which is the arrangement of roots in the root system of a semi-simple Lie algebra. For these hypersurfaces, we establish a link (conjectured by Denef–Loeser in the monodromy conjectures) between the  $b$ -function and another known invariant, namely the topological zeta function. (Received September 19, 2016)

1125-17-2343 **Jonathan I Hall\*** (jhall@math.msu.edu), Department of Mathematics, Michigan State University, 619 Red Cedar Road, East Lansing, MI 48824. *Geometric presentations of algebras.*

While most of the sporadic finite simple groups were constructed via permutation representation, this was not always possible. Griess constructed the Monster via its representation on an algebra, and other groups have effective constructions via algebra representation. We discuss some recent results in this area, in part motivated by connection with vertex operator algebras. (Received September 20, 2016)

1125-17-2403 **J Hennig\*** (jhennig1@ualberta.ca) and **S Sierra** (s.sierra@ed.ac.uk). *Path algebras of quivers and representations of locally finite Lie algebras.*

We explore the (noncommutative) geometry of representations of locally finite Lie algebras. Let  $L$  be one of these Lie algebras, and let  $I \subseteq U(L)$  be the annihilator of a locally simple  $L$ -module. We show that for each such  $I$ , there is a quiver  $Q$  so that locally simple  $L$ -modules with annihilator  $I$  are parameterized by “points” in the “noncommutative space” corresponding to the path algebra of  $Q$ . We classify the quivers that occur and along the way discover a beautiful connection to characters of the symmetric groups  $S_n$ . (Received September 20, 2016)

1125-17-2472 **Michael Penn\*** ([michael.penn@coloradocollege.edu](mailto:michael.penn@coloradocollege.edu)), Colorado College, Colorado Springs, CO 80903, **Christopher Sadowski**, Ursinus College, and **Gautam Webb**, University of Oregon. *Principal Subspaces of Twisted Modules of Lattice Vertex Operator Algebras.*

Given an even lattice,  $L$ , with a certain positivity condition, and an automorphism that fixes the principal subalgebra  $WL$  of the lattice vertex algebra  $V_L$ , we explore the principal subspace of the associated twisted  $V_L$ -module. We describe this twisted module in terms of the quotient of a polynomial algebra by an ideal generated by certain quadratic relations. In addition, the graded dimensions are found. (Received September 20, 2016)

1125-17-2478 **Eric N Sommers\*** ([esommers@math.umass.edu](mailto:esommers@math.umass.edu)). *Functions on nilpotent orbit covers.*

We discuss a method to calculate the G-module of regular functions on covers of even nilpotent orbits. The proof involves new results on the cohomology of line bundles on the cotangent bundle of a partial flag variety. (Received September 20, 2016)

1125-17-2527 **Joseph B Timmer\*** ([joseph.timmer@colorado.edu](mailto:joseph.timmer@colorado.edu)), University of Colorado, Department of Mathematics, Campus Box 395, Boulder, CO 80309-0395. *Automorphisms of Drinfeld Doubles and Bismash Products.* Preliminary report.

Recent work of Marc Keilberg and Peter Schauenburg can be used to describe the structure of the automorphism group of the Drinfeld double  $\mathcal{D}(G)$  for a finite group  $G = C \times H$  with  $C$  abelian and  $H$  non-abelian with no non-trivial abelian direct factor.

In this talk, we will outline the methods and recent advances towards a more general theorem. We also present recent results concerning bismash products and questions that still need to be resolved. (Received September 20, 2016)

1125-17-2572 **Olivia Chandrasekhar\*** ([olivia.chandrasekhar@coloradocollege.edu](mailto:olivia.chandrasekhar@coloradocollege.edu)), **Hanbo Shao** and **Michael Penn**. *Invariants of the Free-Fermion Vertex Algebra under the Action of  $\mathbb{Z}/2$ .*

Many authors, most famously H. Weyl in the early 20<sup>th</sup> century, have studied rings of polynomial invariants. More recently, A. Linshaw and co-authors adapted classical invariant theory to study the invariance of vertex algebras. Drawing on Linshaw's methods, our work describes a linear isomorphism from classically invariant polynomial rings to quantum operator algebras that allows us to apply the first fundamental theorem of invariant theory. Specifically, we study the invariance of the rank  $n$  free-fermion vertex algebra under the action of the  $\mathbb{Z}/2$  group and obtain its minimal generating set. (Received September 20, 2016)

1125-17-2575 **Ben Lewis Cox\*** ([coxbl@cofc.edu](mailto:coxbl@cofc.edu)), Charleston, SC, and **Kaiming Zhao**, , Canada. *Certain families of Polynomials arising in the study of hyperelliptic Lie algebras.*

The associative ring  $R(P(t)) = \mathbb{C}[t^{\pm 1}, u \mid u^2 = P(t)]$ , where  $P(t) = \sum_{i=0}^n a_i t^i = \prod_{k=1}^n (t - \alpha_k)$  with  $\alpha_i \in \mathbb{C}$  pairwise distinct, is the coordinate ring of a hyperelliptic curve. The Lie algebra  $\text{Der}(R(P(t)))$  of derivations is called the hyperelliptic Lie algebra associated to  $P(t)$  and they are a particular type of Krichever-Novikov algebra. In this talk we describe the universal central extension of  $\text{Der}(R(P(t)))$  in terms of certain families of polynomials which in a particular case are associated Legendre polynomials. In our description of these families of polynomials, we employ the use of Fáa di Bruno's formula (or Arbogast's formula) and Bell polynomials. (Received September 20, 2016)

1125-17-2762 **J Hennig\*** ([jhennig1@ualberta.ca](mailto:jhennig1@ualberta.ca)). *Real forms of finitary Lie algebras.* Preliminary report.

We classify the real forms of the complex, simple finitary Lie algebras  $\mathfrak{sl}(\infty)$ ,  $\mathfrak{sp}(\infty)$ , and  $\mathfrak{so}(\infty)$  using methods from nonabelian Galois cohomology. (Received September 20, 2016)

1125-17-2782 **William Graham**, **Martha Precup** and **Amber Russell\*** ([acrusse3@butler.edu](mailto:acrusse3@butler.edu)). *Lusztig's Generalized Springer Correspondence and Graham's Variety in Type A.* Preliminary report.

The Springer correspondence relates irreducible representations of the Weyl group for a reductive Lie algebra to a subset of simple perverse sheaves on the nilpotent cone for that Lie algebra. Essential to this result is the Springer resolution and its fibers.

In his generalization of the Springer correspondence, Lusztig relates each simple perverse sheaf on the nilpotent cone with an irreducible representation of a relative Weyl group. In this talk, I will discuss a map defined by William Graham in Type A which plays a role in Lusztig's generalized setting that is similar to that of the

Springer resolution in the classical version. I will focus primarily on the combinatorics used to study the fibers of Graham's map. (Received September 20, 2016)

## 18 ► *Category theory; homological algebra*

1125-18-83 **Noson S. Yanofsky\*** ([noson@sci.brooklyn.cuny.edu](mailto:noson@sci.brooklyn.cuny.edu)). *A categorical approach to the Cayley-Dickson construction*. Preliminary report.

The Cayley-Dickson construction takes one from the reals to the complex numbers, from the complex to the quaternions, from the quaternions to the octonions, from the octonions to . . . etc. Each time a certain property of the algebraic structure is lost. A sketch is a categorical way of describing algebraic structures. We define a functor from the category of sketches to the category of sketches that takes a description of an algebraic structure and outputs the description of the Cayley-Dickson construction on that structure. With this we will have a clearer way of seeing what properties are lost with the Cayley-Dickson construction. (Received July 14, 2016)

1125-18-553 **Zhenghan Wang\*** ([zhenghwa@microsoft.com](mailto:zhenghwa@microsoft.com)), 2237 CNSI Bldg. Microsoft Station Q, Santa Barbara, CA 93106. *On Classification of Super-modular Categories by Rank*. Preliminary report.

We will discuss the problem of classifying low-rank super-modular categories as inspired by fermionic topological phases of matter. This is a preliminary report of a on-going joint project. (Received September 06, 2016)

1125-18-626 **Michael Robinson\*** ([michaelr@american.edu](mailto:michaelr@american.edu)), Department of Mathematics and Statistics, American University, 4400 Massachusetts Ave NW, Washington, DC 20016. *Local topological analysis of complex systems*.

Complex predictive models are notoriously hard to construct and to study. Sheaf theory provides a toolbox for constructing predictive models described by systems of equations. Without referring to the models directly – only that a model consists of spaces and maps between them – the most readily apparent feature of a multi-model system is its topology. This topology should be modeled first, and then the spaces and maps of the individual models be specified in accordance with the topology. The power of this approach is that complex models can be assembled from smaller, easier-to-construct models. This talk will explain how a disciplined, diagrammatic process (co)sheafifies continuous dynamical systems, partial differential equations, probabilistic graphical models, and discrete approximations of these models. Shadows of the sheaf theoretic perspective are apparent in a variety of disciplines, for instance in the construction of volume meshers (which construct pullbacks and pushforwards of sheaves of functions), finite element solvers (which construct the space of global sections of a sheaf), and loopy belief propagation (which iteratively determines individual global sections). (Received September 08, 2016)

1125-18-950 **Scott Morrison and David Penneys\*** ([penneys.2@osu.edu](mailto:penneys.2@osu.edu)), 100 Math Tower, 231 West 18th Avenue, Columbus, OH 43210-1174. *Tensor categories enriched in braided tensor categories*.

Fusion categories generalize the representation categories of quantum groups, and thus we think of fusion categories as objects which encode quantum symmetry. Recently, there has been a lot of interest in super fusion categories, which are enriched in super vector spaces. These objects are examples of tensor categories enriched in symmetric tensor categories. In this talk, I'll discuss an ongoing project with Morrison in which we study tensor categories enriched in a braided fusion category  $V$ , which is not assumed to be symmetric. We classify  $V$ -fusion categories in terms of oplax braided tensor functors from  $V$  to the centers of ordinary fusion categories. Under this correspondence, strong braided tensor functors correspond to  $V$ -complete  $V$ -fusion categories. (Received September 13, 2016)

1125-18-1506 **Qing Zhang\*** ([zhangqing2513@tamu.edu](mailto:zhangqing2513@tamu.edu)). *Super Modular Categories from Quantum Groups*.

A super-modular category is a unitary ribbon fusion category with Müger center equivalent to the symmetric ribbon category of super-vector spaces. For modular categories, Ng and Schauenburg showed that the images of the representations of the mapping class group of a torus are always finite. In the super-modular setting, one has a representation of the spin mapping class group of the torus. We conjecture that the images are also finite. In this talk, we will provide evidence by looking at the super modular categories arising from quantum groups at roots of unity. (Received September 17, 2016)

1125-18-1525 **Radmila Sazdanovic\*** ([rsazdan@ncsu.edu](mailto:rsazdan@ncsu.edu)), Department of Mathematics NCSU, Box 8205, Raleigh, NC 27695, and **Mikhail Khovanov**. *Diagrammatic categorification of the polynomial ring  $Z[x]$* . Preliminary report.

We develop a diagrammatic categorification of the polynomial ring  $Z[x]$  that leads to categorification of some families of orthogonal polynomials. Our categorifications satisfy a version of the Bernstein-Gelfand-Gelfand reciprocity property with the indecomposable modules corresponding to the monomials and standard modules to the orthogonal polynomials in the Grothendieck ring. (Received September 17, 2016)

1125-18-1534 **Radmila Sazdanovic\*** ([rsazdan@ncsu.edu](mailto:rsazdan@ncsu.edu)), Department of Mathematics NCSU, Box 8205, Raleigh, NC 27695, and **Mikhail Khovanov**. *Diagrammatic categorification of the Chebyshev polynomials*. Preliminary report.

Representations of  $sl(2)$  provide a categorification of the Chebyshev polynomials of the second kind with the irreducible  $(n+1)$ -dimensional representation corresponding to the  $n$ -th Chebyshev polynomial. We describe a novel categorification of the Chebyshev polynomials based on a diagrammatic non-commutative idempotented algebra. (Received September 17, 2016)

1125-18-1582 **Karthik Yegnesh\*** ([karthik.yegnesh@gmail.com](mailto:karthik.yegnesh@gmail.com)), 2293 Weigner Road, Lansdale, PA 19446. *Categorified Geometry through Stacks of 2-Rings*. Preliminary report.

There have been several approaches to developing the foundations of a 2-categorical analog of algebraic geometry, notably by Balmer, Rosenberg, and Chirvasitu/Johnson-Freyd. These approaches differ mainly in their notion of a "commutative 2-ring." For example, Balmer uses tensor triangulated categories, while Chirvasitu and Johnson-Freyd roughly define a commutative 2-ring as a suitable symmetric monoidal locally presentable category. In this talk, we outline an approach to "2-algebraic geometry" that utilizes the notion of a "stack of 2-rings," i.e a stack over a Grothendieck 2-site that takes values in the 2-category of 2-rings in the sense of Chirvasitu/Johnson-Freyd. This can be seen as a vertically categorified sheaf of rings and is a natural extension of the notion of a 2-ring. We show that some basic notions in classical sheaf theory can be generalized to the 2-categorical setting in a straightforward manner using this framework. The notion of a stack of 2-rings therefore provides a particularly flexible framework for 2-algebro-geometric constructions, which can be largely attributed to the clarity of Chirvasitu/Johnson-Freyd's definition. We also discuss future avenues for generalizing other algebro-geometric notions using our theory of stacks of 2-rings. (Received September 18, 2016)

1125-18-1736 **Kenichi Shimizu\*** ([kshimizu@shibaura-it.ac.jp](mailto:kshimizu@shibaura-it.ac.jp)), 307 Fukasaku, Minuma-ku, Saitama-shi, Saitama, 337-8570, Japan. *Integral theory in finite tensor categories*.

Integrals for Hopf algebras play an important role in the study of Hopf algebras. In this talk, I will introduce the notion of an integral and a cointegral of a finite tensor category, and then show that several results on finite-dimensional Hopf algebras, such as Maschke's theorem, can be formulated and extended to finite tensor categories by using integrals and cointegrals. I will also give a short review of the character theory internal to finite tensor categories, and explain the roles of integrals and cointegrals in the theory. (Received September 19, 2016)

1125-18-1812 **Daniel Creamer\*** ([dan1010c@math.tamu.edu](mailto:dan1010c@math.tamu.edu)). *A Computational Approach to Classifying Rank 6 Modular Tensor Categories*. Preliminary report.

I attempt to classify all rank 6 Modular Tensor Categories using the admissibility criteria given in papers of Bruillard, Ng, Rowell, and Wang [BNRW]. Specifically, I use Groebner bases to help classify the possible modular data. From there I use the methods developed by [BNRW] to classify explicit categories. (Received September 19, 2016)

1125-18-2004 **Carlos M Ortiz Marrero\*** ([carlos.ortiz@pnnl.gov](mailto:carlos.ortiz@pnnl.gov)) and **Paul Bruillard** ([paul.bruillard@pnnl.gov](mailto:paul.bruillard@pnnl.gov)). *Rank 5 Premodular Categories*. Preliminary report.

In recent years, researchers have found it useful to classify fusion categories based on a numeric parameter known as their rank. Premodular categories are a class of fusion categories that are amenable to classification. However, they are degenerate to some degree and thus more difficult to understand than modular categories. Premodular categories have previously been classified through rank 4, while modular categories are completely classified through rank 5 and partially characterized through rank 11. In this talk we will survey a number of classification tools developed in recent years that exploit the degeneracy of a premodular category. We will discuss how these tools can be employed to classify pseudounitary rank 5 premodular categories up to Grothendieck equivalence. (Received September 19, 2016)

1125-18-2063 **Andrew Schopieray\*** ([schopier@uoregon.edu](mailto:schopier@uoregon.edu)). *Finiteness of quantum subgroups of  $SU(n)$* . Preliminary report.

Connected étale algebras in a modular tensor category can be used to identify the relations generated by its class in the Witt group of nondegenerate braided fusion categories or to construct its module categories and modular invariants. All such algebras have been classified for the categories  $\mathcal{C}(\mathfrak{g}, k)$  constructed from quantum groups at roots of unity where  $k \in \mathbb{Z}_{\geq 0}$  and  $\mathfrak{g}$  is either  $\mathfrak{sl}_2$  or  $\mathfrak{sl}_3$ . In these cases there is an ADE classification scheme with Type A and Type D being a predictable family of algebras and Type E being the exceptional algebras which do not fall into the previous classes. Here we present proof that there is a finite bound on the level  $k$  for which exceptional connected étale algebras of  $\mathcal{C}(\mathfrak{sl}_4, k)$  can exist and discuss a generalization of this method to general  $\mathfrak{sl}_n$ . (Received September 19, 2016)

1125-18-2086 **Tom Braden\*** ([braden@math.umass.edu](mailto:braden@math.umass.edu)). *A sufficient condition for a category of perverse sheaves to be highest weight*. Preliminary report.

A number of important categories in representation theory (category  $\mathcal{O}$ , rational representations of algebraic groups, etc.) have geometric realizations as categories of perverse sheaves on associated geometric spaces. Many of these categories are also highest weight, however up to now there has been no geometric proof of this. Vilonen gave a criterion for a category of perverse sheaves to be highest weight, but in practice it seems to be difficult to apply. We show that for singularities with symplectic resolutions, a torus action can be used to give a more tractable sufficient condition. We show that this condition holds at least for hypertoric varieties, giving another proof of a result of the speaker and Carl Mautner. (Received September 19, 2016)

1125-18-2833 **Daniel Barter, Corey Jones and Henry Tucker\*** ([htucker@usc.edu](mailto:htucker@usc.edu)), San Diego, CA. *Eigenvalues of rotations in ribbon fusion categories*.

Formulae for the eigenvalues of rotation operators in ribbon fusion categories are given in terms of the categorical Frobenius-Schur indicators. These yield some applications in various classification problems for fusion categories, most importantly in the braided setting. (Received September 20, 2016)

## 19 ► *K*-theory

1125-19-348 **Roger Plymen\*** ([r.j.plymen@soton.ac.uk](mailto:r.j.plymen@soton.ac.uk)), Mathematical Sciences, Southampton, SO17 1BJ, United Kingdom. *Extended affine Weyl groups, the Baum-Connes correspondence and Langlands duality*.

Let  $G$  be a compact connected semisimple Lie group. Let  $W'_a(G)$  be the associated extended affine Weyl group. We investigate the  $K$ -theory of the reduced  $C^*$ -algebra of  $W'_a(G)$ . We show that rationally this  $K$ -theory does not change if we replace  $G$  by its Langlands dual. Of special interest is the  $A_{n-1}$  tower of groups  $SU_n(\mathbb{C})/C_k$  where  $k$  divides  $n$ . We investigate with many numerical examples how the  $K$ -theory groups change as we work our way through such a tower. Joint work with Graham Niblo and Nick Wright. (Received September 16, 2016)

1125-19-505 **John Roe\*** ([john.roe@psu.edu](mailto:john.roe@psu.edu)). *Index Equals Coarsening Equals Heat Flow Equals Loss of Control*.

I will talk about Baum's perspective on index theory and how it relates to the geometric idea of "loss of control". (Received September 04, 2016)

1125-19-711 **Paul Baum, Erik Guentner and Rufus Willett\*** ([rufus@math.hawaii.edu](mailto:rufus@math.hawaii.edu)), 2565 McCarthy Mall, Keller 401A, Honolulu, HI 96822. *Exactness and the Baum-Connes conjecture*.

The Baum-Connes conjecture (due to Paul Baum and Alain Connes) posits a surprising connection between topological and analytic aspects of group theory; it has important applications to manifold topology and representation theory, among other things. Although true in many cases, there are counterexamples to strong forms of the conjecture. I'll survey joint work with Paul Baum and Erik Guentner about the role of 'exactness' in these counterexamples, and possible reformulations on the analytic side that remove the counterexamples. (Received September 09, 2016)

1125-19-1146 **Jonathan M. Rosenberg\*** ([jmr@math.umd.edu](mailto:jmr@math.umd.edu)), Department of Mathematics, University of Maryland, William E. Kirwan Hall, College Park, MD 20742-4015. *The Baum-Connes Conjecture and matching of D-brane charges*.

Quantum field theories in physics (especially string theories, but there are other cases as well) are often related by what physicists call T-dualities. (The "T" stands for "target space.") These relate a theory on one spacetime

$X$  to a theory on another spacetime  $X'$ , often with a change of certain auxiliary fields. Such a duality is accompanied by a matching of charges of “D-branes” in one theory to those in the other. Mathematically, this is represented by an isomorphism of  $K$ -theories (possibly with twisting) on  $X$  and  $X'$ , usually with a degree shift. In many cases, this  $K$ -theory isomorphism arises from a special case of the Baum-Connes Conjecture. We will discuss this phenomenon, with examples. (Received September 15, 2016)

1125-19-1426 **Peter Haskell\*** (phaskell@math.vt.edu). *Toeplitz operators on singular spaces.* Preliminary report.

This paper uses  $KK$ -theory to define Toeplitz operators on unions of codimension-one subspaces of complex vector spaces. The Fredholm indices of these operators are discussed as examples of index theory on singular spaces. This is joint work with B. Ordonez-Delgado. (Received September 16, 2016)

## 20 ► *Group theory and generalizations*

1125-20-100 **Ronald Solomon\*** (solomon.1@osu.edu), Department of Mathematics, The Ohio State University, 231 W. 18th Avenue, Columbus, OH 43210. *Update on the GLS project.*

The GLS project refers to a project initiated by Danny Gorenstein and Richard Lyons in 1982 to produce a unified treatment of the proof of the Classification of the Finite Simple Groups. To date, six volumes have been published in the AMS Mathematical Surveys and Monographs series. I shall report on the status of Monograph 7, which is nearing completion, and also on the anticipated Monographs 8, 9, and 10. (Received July 27, 2016)

1125-20-203 **Luise-Charlotte Kappe\*** (menger@math.binghamton.edu), Binghamton University, Department of Mathematical Sciences, Binghamton, NY 139026000, and **H. Heineken** and **R. F. Morse**. *A GAP-conjecture and its solution: isomorphism classes of capable special  $p$ -groups of rank 2.*

A group is said to be capable if it is a central quotient group and a  $p$ -group is special of rank 2 if its center is elementary abelian of rank 2 and equal to its commutator subgroup. In 1990, Heineken showed that if  $G$  is a capable special  $p$ -group of rank 2, then  $p^5 \leq |G| \leq p^7$ . Over a decade ago we asked GAP to determine the number of isomorphism classes of capable special  $p$ -groups of rank 2 for small primes  $p$ . GAP told us that in these cases, the number of isomorphism classes of special  $p$ -groups of rank 2 grows with  $p$ . However, for the capable among them the number of isomorphism classes is independent of the prime  $p$ . Finally, we were able to show that what GAP conjectured is true for all primes  $p$ . (Received August 11, 2016)

1125-20-446 **Jane Gilman\*** (gilman@rutgers.edu). *Adjoining Roots and Rational Powers of Generators in  $PSL(2, \mathbb{R})$  and Discreteness.*

We give necessary and sufficient discreteness conditions when a root or a rational power of an algorithmic stopping generator is adjoined to a discrete free rank two subgroup of  $PSL(2, \mathbb{R})$ . We also present results for non-free groups. (Received September 02, 2016)

1125-20-550 **M. A. Escobar-Ruiz, E. G. Kalnins** and **W. Miller, Jr.\*** (miller@ima.umn.edu). *2D 2nd order Laplace superintegrable systems, Heun equations, QES and Böcher contractions.*

Second-order conformal quantum superintegrable systems in 2 dimensions are Laplace equations on a manifold with an added scalar potential and 3 independent 2nd order conformal symmetry operators. They encode all the information about 2D Helmholtz or time-independent Schrödinger superintegrable systems in an efficient manner: Each system admits a quadratic symmetry algebra and is multiseparable. The separation equations comprise all the types of hypergeometric and Heun equations in full generality, and they yield all of the 1D Schrödinger exactly solvable and QES systems related to the Heun operator. The different systems are related by Stäckel transforms, by the symmetry algebras and by Böcher contractions of the conformal algebra  $so(4, C)$  to itself, which enables all systems to be derived from a single generic one. Distinct separable bases for a Laplace system are related by interbasis expansion coefficients, themselves special functions, such as the Wilson polynomials. Applying Böcher contractions to expansion coefficients for ES systems one can derive the Askey scheme for hypergeometric orthogonal polynomials. This approach facilitates a unified view of special function theory, incorporating hypergeometric and Heun functions in full generality. (Received September 06, 2016)

1125-20-554 **Valerie Peterson** and **Jacob Russell\***, (jrussell1@ccny.cuny.edu), and **Aaron Wootton**. *Maximal Group Actions on Compact Oriented Surfaces.*

We consider the problem of when a cyclic group of orientation preserving automorphisms  $C_p$  of prime order  $p$  on a compact oriented surface  $S$  of genus  $\sigma \geq 2$  is finitely maximal, meaning there is no nontrivial finite

supergroup  $G > C_p$  of orientation preserving automorphisms of  $S$ . This is equivalent to determining when a given conjugacy class of the mapping class group,  $MCG(S)$ , isomorphic to  $C_p$  is finitely maximal. We show that such a supergroup always exists unless the number of fixed points of the action is maximal (or equivalently, the quotient genus  $S/C_p$  is minimal). Moreover, we exhibit an infinite sequence of genera within which  $C_p$  is never finitely maximal. (Received September 06, 2016)

1125-20-564 **Ian Hogan\*** (ihogan@kent.edu), 631 Silver Meadows Blvd, Kent, OH 44240, and **Michael Decker**. *The Brauer Complex and Decomposition Numbers of Symplectic Groups*. Preliminary report.

We follow the development of L. Chastkofsky and J. E. Humphreys to construct the Brauer Complex of  $Sp_4(q)$  and  $Sp_6(q)$ . The complex is an open simplicial complex derived from the alcove geometry of the associated affine Weyl group together with Deligne-Lusztig character labels. The Deligne-Lusztig characters of  $Sp_4(q)$ , odd  $q$ , were computed by B. Srinivasan and of  $Sp_6(q)$  even  $q$  by F. Lübeck. As a consequence we gain information about the decomposition matrices for these groups in defining characteristic. (Received September 06, 2016)

1125-20-593 **Lisa Rose Hendrixson\*** (lhendrix@kent.edu) and **Mark L. Lewis** (lewis@math.kent.edu). *Connections Between the Number of Constituents and the Derived Length of a Group*.

I will focus on bounding the derived length of a finite solvable group  $G$  in two different situations. I will consider an irreducible character  $\chi$  of  $G$  and study the case when  $\chi\bar{\chi}$  has two nonprincipal irreducible constituents and the case when it has three nonprincipal irreducible constituents. In the first case, I will provide the best possible bound. In the second case, I will provide a bound for the derived length of  $G$ . I will also provide a theorem that will yield a starting point when looking for other examples. (Received September 07, 2016)

1125-20-604 **Joshua Maglione\*** (maglione@math.colostate.edu), Department of Mathematics, 1874 Campus Delivery, Fort Collins, CO 80523. *New characteristic structure from filters*.

We introduce a general method to make a series of isomorphism invariant subgroups. Even in the extreme case where the only classically known characteristic subgroup is the commutator, this method can produce maximal characteristic series. We tested this on a half billion 2-groups of class 2, the examples for which isomorphism invariants are least known. We succeeded in identifying new subgroups in about 97% of the groups we surveyed, and our approach can be adapted for further improvement. Our strategy begins with a structure theorem on the cohomology of nilpotent groups and techniques in nonassociative algebra. We report on individual and joint work with J.B. Wilson. (Received September 07, 2016)

1125-20-659 **Martha Kilpack**, Dept. of Mathematics, Brigham Young University, Provo, UT 84602, and **Arturo Magidin\*** (magidin@member.ams.org), Department of Mathematics, University of Louisiana at Lafayette, P.O. Box 43568, Lafayette, LA 70504-3568. *The lattice of algebraic closure operators on infinite subgroup lattices*.

Let  $G$  be a group, and let  $L = \text{subgrps}(G)$  be the lattice of subgroups of  $G$ . A closure operator  $\phi$  on  $L$  is algebraic if for every  $H \leq G$ ,  $\phi(H)$  is the subgroup generated by the closures  $\phi(K)$ , where  $K$  ranges over all finitely generated subgroups of  $H$ . The lattice of algebraic closure operators on  $L$ ,  $\text{aco}(L)$ , is an algebraic lattice.

In prior work we determined that if  $G$  is finite, then  $\text{a.c.o.}(L)$  is isomorphic to a subgroup lattice if and only if  $G$  is cyclic of prime power order; and we extended this to arbitrary finite lattices  $L$  (whether or not they are themselves subgroup lattices). We now investigate the case where  $G$  is infinite. We settle the case in which  $G$  has torsion, and the torsionfree case in which  $G$  has a nontrivial abelian normal subgroup (including the case when  $G$  is itself abelian). We conjecture that for infinite groups  $G$ ,  $\text{a.c.o.}(\text{subgrps}(G))$  is isomorphic to a subgroup lattice if and only if  $G$  is isomorphic to the Prüfer  $p$ -group for some prime  $p$ . (Received September 08, 2016)

1125-20-695 **Mark W Bissler\*** (mbissle2@kent.edu), Stow, OH 44224, and **Mark L Lewis**. *A family of graphs that cannot occur as character degree graphs of solvable groups*.

We investigate character degree graphs of solvable group. In particular, we provide general results that can be used to eliminate which degree graphs can occur a solvable groups. Finally, we show a specific family of graphs cannot occur as a character degree for any solvable group. (Received September 09, 2016)

1125-20-710 **Alexandru Chirvasitu\*** ([chirva@uw.edu](mailto:chirva@uw.edu)), Department of Mathematics, University of Washington, Seattle, WA 98195, and **Souleiman Omar Hoche** and **Pawel Kasprzak**. *Lattices of subgroups for locally compact quantum groups.*

The lattice of subgroups of a discrete group is the subject of numerous results revolving around the central theme of decomposing the group into "chunks" (subquotients) that can then be compared to one another in various ways.

Examples of results in this class would be the Noether isomorphism theorems, Zassenhaus' butterfly lemma, the Schreier refinement theorem for subnormal series of subgroups, the Dedekind modularity law, and last but not least the Jordan-Holder theorem.

We discuss analogues of the above-mentioned results in the context of quantum groups of two flavors: compact (cast mathematically as objects dual to certain well-behaved cosemisimple Hopf algebras) and discrete, which are Pontryagin duals of the former. Time permitting, the talk will also touch on the difficulties of extending such results to the more general setting of locally compact quantum groups (which simultaneously generalizes the two cases).

(joint with Souleiman Omar Hoche and Pawel Kasprzak) (Received September 09, 2016)

1125-20-720 **Violeta Vasilevska\*** ([violeta.vasilevska@uvu.edu](mailto:violeta.vasilevska@uvu.edu)), 800 W University Parkway, Orem, UT 84058. *Comparing the large scale structure of Cayley graphs of the group of integers.*

In this presentation the following problem (posed by Richard E. Schwartz) will be discussed: Are the Cayley graphs  $C_2 = \text{Cay}(\{Z\}, \{\pm 2^k\})$  and  $C_3 = \text{Cay}(\{Z\}, \{\pm 3^k\})$ , for  $k = 0, 1, 2, \dots$ , quasi-isometric? Namely, the problem asks if these two spaces are alike ("the same") when their large-scale shape structure is compared. We present what has been known about these two spaces in regards to this still-open problem. Then a particular class of maps will be considered and we will prove that these maps are not quasi-isometries between  $C_2$  and  $C_3$ . Possibilities for further work will also be discussed.

This work was supported by a CURM grant and was conducted as an undergraduate research project with Daniel Adams and Daniel Gulbrandsen. (Received September 09, 2016)

1125-20-735 **Sang-hyun Kim, Thomas Koberda\*** ([thomas.koberda@gmail.com](mailto:thomas.koberda@gmail.com)) and **Yash Lodha**.  *$N$ -chain groups of homeomorphisms of one-manifolds.*

We study the algebraic structure of finitely generated groups of homeomorphisms of one-manifolds. We concentrate on groups generated by homeomorphisms, each supported on a single interval lying in a chain of intervals, subject to some mild dynamical conditions. These groups have many remarkable properties, including either having a simple commutator subgroup or having a wandering interval. (Received September 19, 2016)

1125-20-746 **Cristina M Ballantine\*** ([cballant@holycross.edu](mailto:cballant@holycross.edu)), **Brooke Feigon, Radhika Ganapathy, Janne Kool, Kathrin Maurischat** and **Amy Wooding**. *Explicit constructions of Ramanujan Bigraphs.*

We construct explicitly an infinite family of Ramanujan graphs which are bipartite and biregular. Our construction starts with the Bruhat-Tits building of an inner form of  $SU(3, \mathbb{Q}_p)$ . To make the graphs finite, we take successive quotients by infinitely many discrete co-compact subgroups of decreasing size. (Received September 10, 2016)

1125-20-875 **Skip Garibaldi\*** ([skip@member.ams.org](mailto:skip@member.ams.org)) and **Robert M. Guralnick**. *Generically free representations and essential dimension.*

You might think of the essential dimension of an algebraic group  $G$  as the number of parameters needed to specify a  $G$ -torsor. (For example, when  $G$  is the orthogonal group in  $n$  variables, a  $G$ -torsor is an  $n$ -dimensional quadratic form, which can be diagonalized, so the essential dimension is  $n$ .) The essential dimension of  $G$  is at most the dimension of a generically free representation. This talk is about joint work with Bob Guralnick, where we prove new upper bounds on essential dimension by proving that certain representations are generically free over all fields. (Received September 12, 2016)

1125-20-905 **Martha Lee H Kilpack\*** ([mlhkilpack@mathematics.byu.edu](mailto:mlhkilpack@mathematics.byu.edu)) and **Arturo Magidin**. *For what finite lattices does the lattice of closure operators form a subgroup lattice?*

A lattice  $L$  is called algebraic if it is generated by the compact elements of  $L$ . For each algebraic lattice  $L$  there exists an algebraic structure  $A$  such that the lattice of subalgebras of  $A$  is algebraic to  $L$ . We will call the algebraic lattice  $L$  a subgroup lattice if  $L$  is isomorphic to a subgroup lattice  $\text{Sub}(G)$  for some group  $G$ .

For a finite lattice  $L$  the closure operators which act on  $L$  form an algebraic lattice  $\text{c.o.}(L)$ . Extending the results for when  $L$  is a finite subgroup lattices, we will show for any finite lattice  $L$  that  $\text{c.o.}(L)$  is a subgroup lattice if and only if  $L$  is a chain (totally ordered set). (Received September 13, 2016)

1125-20-919 **Amanda A Schaeffer Fry\***, [aschae6@msudenver.edu](mailto:aschae6@msudenver.edu), Denver, CO 80217, and **Jay Taylor**. *On a Conjecture Of Navarro and Self-Normalizing Sylow 2-Subgroups in Type A*. Navarro has conjectured a necessary and sufficient condition for a finite group  $G$  to have a self-normalising Sylow 2-subgroup, which is given in terms of the ordinary irreducible characters of  $G$ . The first-named author has reduced the proof of this conjecture to showing that certain related statements hold when  $G$  is quasisimple. We show that these conditions are satisfied when  $G/Z(G)$  is  $\mathrm{PSL}_n(q)$ ,  $\mathrm{PSU}_n(q)$ , or a simple group of Lie type defined over a finite field of characteristic 2. (Received September 13, 2016)

1125-20-1006 **Veny Liu\*** ([venyliu@hawaii.edu](mailto:venyliu@hawaii.edu)) and **Jon M Corson**. *Inverse Semigroupoids and Their Free Objects*. Inverse semigroupoids are generalizations of inverse semigroups and groupoids. To investigate inverse semigroupoids, we define a special type congruence and homomorphism. Furthermore, free objects in the category of inverse semigroupoids are interesting themselves and they form the foundation for studying inverse semigroupoids combinatorially. A free inverse semigroupoid has a (symmetric) basis, and it turns out to be unique. (Received September 14, 2016)

1125-20-1036 **William Hardesty**, Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803, **Daniel K. Nakano\***, Department of Mathematics, University of Georgia, Athens, GA 30602, and **Paul Sobaje**, Department of Mathematics, University of Georgia, Athens, GA 30602. *On the existence of mock injective modules for algebraic groups*.

Let  $G$  be an affine algebraic group scheme over an algebraically closed field  $k$  of characteristic  $p > 0$ , and let  $G_r$  denote the  $r$ -th Frobenius kernel of  $G$ . Motivated by recent work of Friedlander, we investigate the class of mock injective  $G$ -modules, which are defined to be those rational  $G$ -modules that are injective on restriction to  $G_r$  for all  $r \geq 1$ . In this talk we provide necessary and sufficient conditions for the existence of non-injective mock injective  $G$ -modules, thereby answering a question raised by Friedlander. Furthermore, we investigate the existence of non-injective mock injectives with simple socles. Interesting cases are discovered that show that this can occur for reductive groups, but will not occur for their Borel subgroups. (Received September 14, 2016)

1125-20-1099 **Jay Taylor\*** ([jaytaylor@math.arizona.edu](mailto:jaytaylor@math.arizona.edu)), Department of Mathematics, University of Arizona, 617 N. Santa Rita Ave., Tucson, AZ 85721. *Action of Automorphisms on Irreducible Characters of Symplectic Groups*.

In recent years a new approach has been developed to several long standing conjectures in the representation theory of finite groups; such as the McKay conjecture. These conjectures are stated for all finite groups but the recent approach has reduced these conjectures to checking certain conditions on quasisimple finite groups (often referred to as inductive conditions). This, in theory, makes the conjecture more manageable thanks to the classification of finite simple groups. However, the downside to this is that one requires information about how automorphisms act on the irreducible characters of quasisimple finite groups. In this talk we present new results in this direction concerning the finite symplectic groups  $\mathrm{Sp}_{2n}(q)$  where  $q$  is an odd prime power. Specifically we completely describe the action of the automorphism group on the ordinary irreducible characters of these groups. Our methods quintessentially involve Kawanaka's theory of Generalised Gelfand-Graev Representations (GGGRs) and the Springer correspondence. (Received September 14, 2016)

1125-20-1128 **David J. Benson** and **Jon F. Carlson\*** ([jfc@math.uga.edu](mailto:jfc@math.uga.edu)), Department of Mathematics, University of Georgia, Athens, GA 30602. *Nilpotence and generation in the stable module category*.

Nilpotence has been studied in stable homotopy theory and algebraic geometry. We study the corresponding notion in modular representation theory of finite groups, and apply the discussion to the study of ghosts, and generation of the stable module category. In particular, we show that for a finitely generated  $kG$ -module  $M$ , the tensor  $M$ -generation number and the tensor  $M$ -ghost number are both equal to the degree of tensor nilpotence of a certain map associated with  $M$ . (Received September 15, 2016)

1125-20-1160 **Mark C Hunnell\*** ([hunnellm@wssu.edu](mailto:hunnellm@wssu.edu)). *Generalized Symmetric  $k$ -Varieties Corresponding to Finite Order Automorphisms*. Preliminary report.

For a field  $k$ , symmetric  $k$ -varieties are a generalization of the real reductive symmetric spaces with applications in many areas of mathematics including representation theory, number theory, and geometry. Symmetric varieties are the homogeneous spaces  $G/H$ , where  $G$  is a reductive algebraic group and  $H$  is an open subgroup of the fixed points of an involutorial automorphism, the symmetric  $k$ -varieties generalize these spaces to arbitrary fields. We

provide some results when  $H$  is replaced by the fixed points of an arbitrary finite order automorphism. (Received September 15, 2016)

1125-20-1182 **Alexandre Turull\*** ([turull@ufl.edu](mailto:turull@ufl.edu)), Department of Mathematics, University of Florida, Gainesville, FL 32611. *Character triple isomorphisms and elements of the Brauer-Clifford group.*

We adapt the definition of *character triple isomorphisms* to take into account rationality properties of the characters, such as fields of definition and Schur indices, as well as, blocks, Brauer characters, and decompositions number, when appropriate. We discuss how equality of certain elements of the Brauer-Clifford group implies the existence of endoisomorphisms from one finite group to the other, and how each such endoisomorphisms yields a unique character triple isomorphism in the new sense. The character triple isomorphisms can be manipulated through composition, direct sums, restrictions, field extensions, and these operations reflect parallel operations at the level of endoisomorphisms.

The existence of these character triple isomorphisms can be used to obtain results on character theory of finite groups where the fields of definition, Schur indices, or blocks are important. For example, the author has obtained a refinement of the Dade Projective Conjecture for all finite  $p$ -solvable groups using these results. (Received September 15, 2016)

1125-20-1236 **Randall D. Helmstutler\*** ([rhelmstu@umw.edu](mailto:rhelmstu@umw.edu)), Department of Mathematics, 1301 College Avenue, Fredericksburg, VA 22401. *Generalized dihedral groups in non-commutative cryptographic protocols.* Preliminary report.

Given an abelian group  $A$ , we may construct the associated dihedral group  $D(A)$  by allowing the group of order two to act on  $A$  by inversion. When not all elements of  $A$  have order two, the group  $D(A)$  is non-abelian. This gives a convenient taxonomy to many unnamed non-abelian groups, these groups being amenable to analysis under several key exchange protocols in non-commutative cryptology. We will provide an overview of the most commonly studied non-commutative protocols, showing how these protocols behave under generalized dihedral groups. In particular, we highlight an example of a recently completed undergraduate research project, wherein Ko-Lee key exchange was shown to be susceptible to a quadratic-time attack. By studying these protocols using different classes of platform groups, similar open problems arise that are within reach of the undergraduate student with a solid course in group theory. (Received September 15, 2016)

1125-20-1330 **Tara Brendle** and **Dan Margalit\*** ([margalit@math.gatech.edu](mailto:margalit@math.gatech.edu)). *Models for Mapping Class Groups.*

A celebrated theorem of Nikolai Ivanov states that the automorphism group of the mapping class group is again the mapping class group. The key ingredient is his theorem that the automorphism group of the complex of curves is the mapping class group. After many similar results were proved, Ivanov made a metaconjecture that any “sufficiently rich object” associated to a surface should have automorphism group the mapping class group. In joint work with Tara Brendle, we show that the typical normal subgroup of the mapping class group has automorphism group the mapping class group. To do this, we show that a large family of complexes associated to a surface has automorphism group the mapping class group. (Received September 16, 2016)

1125-20-1350 **Thomas M. Fiore\***, [tmfiore@umich.edu](mailto:tmfiore@umich.edu), and **Thomas Noll**, [thomas.mamuth@gmail.com](mailto:thomas.mamuth@gmail.com). *Voicing Transformations and a Linear Representation of Uniform Triadic Transformations.*

We first revisit Joe Straus’ interpretation of Webern, Concerto for Nine Instruments, Op. 24, Second Movement and re-interpret its retrograde inversion enchaining operation as a composite of a voicing reflection and a permutation. This motivates our investigation of the matrix group  $\mathcal{J}$  generated by voicing reflections, its extension by permutations, and its utility in more economic descriptions of flip-flip cycles. We sketch a Structure Theorem for the group  $\mathcal{J}$  and apply it to find a matrix with orbit the diatonic falling fifth sequence, as well as four matrices that each realize the alternating  $PLP$  and  $L$  flip-flop in Clampitt’s interpretation of the famous Grail sequence. We also discuss some implications of our Structure Theorem for a linear representation of Hook’s uniform triadic transformations group. This is joint work with Thomas Noll. (Received September 16, 2016)

1125-20-1411 **Rachel Davis\*** ([rachel.davis@wisc.edu](mailto:rachel.davis@wisc.edu)). *Fox calculus in Galois theory.*

Given  $C$ , a smooth genus  $g$  curve with  $n$  punctures defined over  $\mathbb{Q}$ , and a prime  $\ell$ , there exists an exterior Galois representation (terminology of Nakamura) from  $G_{\mathbb{Q}}$  to the outer automorphism group of the pro- $\ell$  part of the fundamental group of  $C \otimes \overline{\mathbb{Q}}$ . The image is known to lie in the pro- $\ell$  mapping class group  $\Gamma_{g,n}$ . The goal of this talk is to describe how Fox calculus and the Magnus representation can be used to further analyze the image. (Received September 16, 2016)

1125-20-1494 **Paul Sobaje\*** ([sobaje@uga.edu](mailto:sobaje@uga.edu)), 166 Oak Meadow Dr., Athens, GA 30605. *On Springer Isomorphisms For Algebraic Groups*. Preliminary report.

Let  $G$  be a simple and simply connected algebraic group over an algebraically closed field  $k$ . A 'Springer isomorphism' is a  $G$ -equivariant isomorphism from the nilpotent variety of the Lie algebra to the unipotent variety of the algebraic group. In this talk we will consider the following issues: definition of these isomorphisms over subfields, uniqueness results, and finding appropriate scheme structures for the nilpotent and unipotent varieties. (Received September 17, 2016)

1125-20-1599 **Spencer J Gerhardt\*** ([sgerhard@usc.edu](mailto:sgerhard@usc.edu)). *Generation problems for classical algebraic groups*.

Let  $G$  be a classical algebraic group over an uncountable algebraically closed field of characteristic  $p \geq 0$ . Let  $C_1, \dots, C_e$  be conjugacy classes of  $G$ . In this talk we consider the following question: when do there exist elements  $x_1, \dots, x_e$  in  $C_1, \dots, C_e$  such that  $\langle x_1, \dots, x_e \rangle$  is Zariski dense in  $G$ ? Results for different types of classes  $C_1, \dots, C_e$  of  $G$  are given, depending on the group  $G$ . Applications to random generation of finite groups of Lie type, and the representation theory of simple algebraic groups are then discussed. (Received September 18, 2016)

1125-20-1640 **Rachel Skipper\*** ([skipper@math.binghamton.edu](mailto:skipper@math.binghamton.edu)), Department of Mathematical Sciences, Binghamton University, PO Box 6000, Binghamton, NY 13902-6000. *A generalization of the Hanoi towers group*.

The Hanoi Towers group is a self-similar, regular branch group which acts as automorphisms of the infinite rooted ternary tree and models the game "the Towers of Hanoi".

We generalize the algebraic construction of this group to produce a new infinite family of self-similar, regular branch groups and explore their algebraic and geometric properties. (Received September 18, 2016)

1125-20-1771 **Lindsey Heiberger\*** ([lheiberger13@stac.edu](mailto:lheiberger13@stac.edu)), Sparkill, NY, and **Heather Palmer, Daniel Viaud and Meghan De Witt**. *Symmetries of the Hypercube*.

We look at the construction of  $n$ -dimensional hypercubes from  $(n-1)$ -dimensional hypercubes and look at various ways of visualizing the first five dimensions of hypercubes using 2D and 3D modeling. We explore the relationship between the external dimension of a hypercube and its internal elements when increasing either the external dimension or internal dimension by one. Let  $X_{n,m}$  be the number of  $m$ -dimensional faces inside of an  $n$ -dimensional hypercube. We prove, by induction on  $k=n+m$ , the following recursive formula:  $X_{n,m} = 2X_{n-1,m} + X_{n-1,m-1}$ . We explore the symmetric group of the  $n$ -dimensional hypercube, and discuss the relationship between the internal makeup of an  $n$ -dimensional hypercube and the size of the hypercube's symmetric group. We then describe the alteration in the size of the multiplication table if different dimensions are allowed during the hypercube's rotations. Last, we will examine the relationship between the multiplication table of a square and higher dimensional hypercubes. (Received September 19, 2016)

1125-20-1843 **Christopher P Bendel, Daniel K Nakano and Cornelius Pillen\*** ([pillen@southalabama.edu](mailto:pillen@southalabama.edu)), Department of Mathematics and Statistics, University of South Alabama, Mobile, AL 36608. *Tensor Products and Filtrations*. Preliminary report.

Let  $G$  be a simple algebraic group over an algebraically closed field of prime characteristic  $p$  which is split over the prime field. If  $p$  is roughly twice the size of the Coxeter number or larger it is well-known that the tensor product of a Steinberg module and any simple  $G$ -module with restricted highest weight is a tilting module. Not much is known about these tensor products for smaller primes. This question is of interest because it is closely related to two long-standing conjectures by Stephen Donkin. The first concerns the lifting of projective modules of the Frobenius kernels to  $G$ . The second deals with various types of filtrations and their connection. This talk is a survey of known results together with explicit examples. (Received September 19, 2016)

1125-20-1857 **Dan Rossi\***, 617 N Santa Rita Ave., PO Box 210089, Tucson, AZ 85721. *Rational Characters and Rational Conjugacy Classes in Finite Groups*.

Let  $G$  be a finite group. A character  $\chi \in \text{Irr}(G)$  is called *rational* if  $\chi(g) \in \mathbb{Q}$  for every  $g \in G$ , and an element  $g \in G$  is called *rational* if  $\chi(g) \in \mathbb{Q}$  for every  $\chi \in \text{Irr}(G)$ . If  $g \in G$  is rational then we say the conjugacy class  $cl_G(g)$  is rational. Write  $\text{Irr}_{\mathbb{Q}}(G)$  and  $Cl_{\mathbb{Q}}(G)$ , respectively, for the sets of rational irreducible characters and rational conjugacy classes of  $G$ . Extending work of Navarro-Tiep (2008) we show that when  $G$  is non-solvable either  $|\text{Irr}_{\mathbb{Q}}(G)| = 3$  if and only if  $|Cl_{\mathbb{Q}}(G)| = 3$  or else the composition factors of  $G$  are under very tight control. (Received September 19, 2016)

- 1125-20-2038 **Catherine A. Buell\*** (cbuell1@fitchburgstate.edu), **Aloysius Helminck, Vicky Kilma, Jennifer Schaefer, Carmen Wright and Ellen Ziliak.** *Orbit decomposition of the generalized symmetric spaces of  $SL_2(F_q)$ .* Preliminary report.  
The talk will discuss the decomposition of the double-coset space  $H_k \backslash G_k/H_k$  for  $G = SL_2(k)$  and  $H$  the fixed-point group of an involution  $\theta$  of  $G$  with  $k$  any finite field. This decomposition, which plays a role in representation theory, is central in the study of symmetric spaces and their generalizations. Computing, characterizing, and eventually generalizing the  $H_k$ -orbits of the individual elements and the maximal tori of  $G_k/H_k$  will ultimately classify the double cosets of various subgroup actions (like parabolic or Borel) on the generalized symmetric space. This decomposition which has been highly studied over the real numbers and algebraically closed fields but with little known for finite fields. We will establish which conventions hold or fail over finite fields. (Received September 19, 2016)
- 1125-20-2050 **Muhammad Inam\*** (minam@westga.edu), 97 B Tillman Drive, Carrolltan, GA 30117. *The word problem for some classes of Adian semigroups, Adian inverse semigroups and Adian groups.*  
We show that the word problem is decidable for some classes Adian semigroups, Adian inverse semigroups and Adian groups. (Received September 19, 2016)
- 1125-20-2163 **Pramod N. Achar\*** (pramod@math.lsu.edu), **Shotaro Makisumi, Simon Riche and Geordie Williamson.** *Modular Koszul duality for Kac–Moody groups, part II.*  
This talk is a sequel to Shotaro Makisumi’s talk (but I will try to make it self-contained). I will introduce the monoidal category of “free-monodromic tilting sheaves” on a Kac–Moody flag variety. The main result is that this category is equivalent to the monoidal category of parity sheaves on the Langlands dual Kac–Moody flag variety. This result implies the Riche–Williamson conjecture on characters of tilting modules of reductive groups. This is joint work with Shotaro Makisumi, Simon Riche, and Geordie Williamson. (Received September 19, 2016)
- 1125-20-2203 **Pramod N. Achar, Shotaro Makisumi\*** (makisumi@stanford.edu), **Simon Riche and Geordie Williamson.** *Modular Koszul duality for Kac–Moody groups, part I.*  
This talk is a prequel to Pramod Achar’s talk. I will discuss a new approach to the modular Koszul duality of Pramod Achar and Simon Riche, a derived equivalence relating Langlands dual flag varieties, analogous to the Koszul duality of Beilinson–Ginzburg–Soergel. The key ingredient is a new construction of a left monodromy action in the modular setting. This is partly based joint work with Pramod Achar, Simon Riche, and Geordie Williamson. (Received September 19, 2016)
- 1125-20-2302 **David J. Hemmer\*** (dhemmer@math.buffalo.edu), 211 Mathematics Building, University at Buffalo, Buffalo, NY 14260, and **Harald Ellers** (hellers@allegheny.edu), Allegheny College, Department of Mathematics, 20 N. Main St., Meadville, PA 16335. *Gelfand–Zetlin lattices in Specht modules for symmetric groups.* Preliminary report.  
The irreducible representations of the symmetric group  $\Sigma_n$  in characteristic 0 are labelled by partitions  $\lambda$  of  $n$ . They have a very nice basis, called the Gelfand–Zetlin (G–Z) basis. This basis is indexed by standard tableaux of shape  $\lambda$  and is perfectly adapted to restriction to smaller symmetric groups, in the sense that the direct sum decomposition of the restricted module results from partitioning the basis vectors. It also has the property that the Jucys–Murphy elements  $L_t = (1, t) + (2, t) + \cdots + (t - 1, t)$  act diagonally on it with integer eigenvalues.  
The complex irreducible module labelled by  $\lambda$  contains many different integer lattices, which often produce non-isomorphic modules on reduction to characteristic  $p$ . The most popular choice of lattice dates back to Specht, and the corresponding module is called the Specht module. The study of these modules was pioneered by Gordon James. This particular lattice has the property that it is generated by a Gelfand–Zetlin basis vector of “highest weight”. We investigate lattices generated by other GZ-basis vectors, drawing parallels with the theory of maximal and minimal admissible lattices and Weyl modules for the general linear group. (Received September 20, 2016)
- 1125-20-2328 **Siu-Hung Ng\*** (rng@math.lsu.edu), Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803. *Gauge invariants from the powers of antipodes.*  
Let  $S$  be the antipode of a finite-dimensional Hopf algebra  $H$ , which is not necessarily semisimple. It has been known that  $Tr(S)$  and  $Tr(S^2)$  are invariant of the finite tensor category  $Rep(H)$  of the representations of  $H$ . However, it remains unclear whether  $Tr(S^n)$  is an invariant for each integer  $n$ . An affirmative answer of this question immediately implies the invariance of the orders of  $S$  and  $S^2$ . In this talk, we will show that they are

invariants of  $\text{Rep}(H)$  when the Jacobson radical of  $H$  is a Hopf ideal. This is a joint work with Cris Negron. (Received September 20, 2016)

1125-20-2359 **Casey W Wynn\*** (cwynn2@kent.edu) and **Mark L Lewis** (lewis@math.kent.edu). *Supercharacter Theories of Semirectspecial  $p$ -Groups and Frobenius Groups.*

In this talk, we describe all supercharacter theories of semirectspecial  $p$ -groups and Frobenius groups. We first review the constructions which will classify these supercharacter theories. We then show that the supercharacter theories of these groups, as well as the more general Camina pairs, must have nontrivial  $C$ -normal subgroups. Next, we examine the superclass structure to show that the supercharacter theory can be constructed from supercharacter theories of subgroups of the group and its quotient groups. We can apply this to count the number of supercharacter theories for the group. (Received September 20, 2016)

1125-20-2538 **Pramod N. Achar, Maitreyee Kulkarni and Jacob P. Matherne\*** (matherne@math.umass.edu). *A combinatorial Fourier transform for quiver representation varieties in type A.* Preliminary report.

For a given dimension vector, we consider the space of representations of the linearly-oriented type A quiver  $\bullet \rightarrow \bullet \rightarrow \cdots \rightarrow \bullet$ . This affine space has a stratification by orbits for a product of general linear groups, so we can study the equivariant constructible derived category of sheaves on it. The Fourier–Sato transform gives an equivalence between this derived category and the derived category for the reversed quiver. We give a combinatorial algorithm for computing the Fourier–Sato transform in this setting by introducing certain triangular arrays of nonnegative integers. This is joint (in progress) work with Pramod N. Achar and Maitreyee Kulkarni. (Received September 20, 2016)

1125-20-2552 **Drew Tomlin\*** (drewtillis@my.unt.edu) and **J. Matthew Douglass** (matthew.douglass@unt.edu). *A decomposition of the group algebra of a hyperoctahedral group.*

In the 1980's, Lehrer and Solomon showed that the representation of the  $n^{\text{th}}$  symmetric group on the singular cohomology of the space of  $n$ -tuples of distinct complex numbers is a direct sum of representations induced from linear characters of the centralizers of a complete set of conjugacy class representatives. They conjectured that the same result holds for a general finite Coxeter group. Douglass, Roehrl, and Pfeiffer have shown that the Lehrer–Solomon decomposition in cohomology is connected with a decomposition of the regular representation of the symmetric group arising from Solomon's descent algebra.

The Mantaci–Reutenauer algebra of a hyperoctahedral group is a subalgebra of the group algebra with properties similar to those of descent algebras of symmetric groups. In this talk, I will describe how the Mantaci–Reutenauer algebra of a hyperoctahedral group leads to a decomposition of the regular representation as a direct sum of representations induced from linear characters of the centralizers of a complete set of conjugacy class representatives. This answers the question raised by Bonnafé of whether such a decomposition exists.

This is joint work with J. Matthew Douglass. (Received September 20, 2016)

1125-20-2592 **Jack Wagner\*** (jw5788@stu.armstrong.edu) and **Sabrina Hessinger.** *Monomial Galois Groups of Homogeneous Linear Differential Equations of Arbitrary Order.*

Relative to many fields in mathematics, Differential Galois Theory is a new area which beautifully integrates many mathematical disciplines, including not only abstract algebra and differential equations, but also topology, linear algebra, and representation theory. In this talk, we will briefly present the essential elements of differential Galois theory, couching these elements in the familiar context of undergraduate ordinary differential equations, and pointing out similarities and differences with the more familiar Galois theory of polynomials. Our current work is situated in the case when a differential equation has monomial Galois group. In the spirit of earlier work of J. Kovacic, M. Singer and F. Ulmer, we begin by understanding the form of such groups and their representations. We then make use of a correspondence between reducibility properties of group representations and factorizations of associated linear differential equations to investigate implications on the solutions of the differential equation. (Received September 20, 2016)

1125-20-2655 **Joshua Roberts\*** (jroberts7@ggc.edu), 1000 University Center Ln, Lawrenceville, GA 30043. *A Spectral Sequence Reduction for Low Dimensional Group Homology.*

A motivational problem for group homology is a conjecture of Quillen, a version of which can be reformulated to state that the second homology of the general linear group over  $R = \mathbb{Z}[1/p, \zeta_p]$ , for  $p$  an odd prime, is isomorphic to the second homology of the group of units of  $R$ , where the homology calculations are over the field of order  $p$ . Moreover, this group has a finite presentation. Calculation of the homology of this group is difficult. We show by explicit calculation that the problem may be simplified by an application of the Hochschild–Serre

Spectral Sequence of an appropriate group extension. More precisely, the problem of calculating the second degree homology, with the coefficients above, reduces to a calculation of a certain transgression map in the spectral sequence. (Received September 20, 2016)

1125-20-2819 **Samuel J Ivy\*** ([samuel.ivy@usma.edu](mailto:samuel.ivy@usma.edu)), United States Military Academy, West Point, NY 10996. *Generalizing Parabolic Subsets from Involutorial Automorphisms.*

This exposition focuses on the algebraic and combinatorial structures of symmetric spaces of Lie groups emphasizing the action of involutions on the underline root systems. The characterization of the orbits of parabolic subgroups acting on these symmetric spaces involves the action of both the symmetric space involution on the maximal  $k$ -split tori (and their root system) with an opposing involutorial automorphism. This work characterizes the resulting parabolic subsets within the corresponding root system. (Received September 20, 2016)

## 22 ► *Topological groups, Lie groups*

1125-22-89 **Shiang Tang\*** ([tang@math.utah.edu](mailto:tang@math.utah.edu)), 1400 east, 155 south, RM 233, Salt Lake City, UT 84112. *Action of intertwining operators on pseudo-spherical  $K$ -types and automorphic forms on metaplectic groups.*

In this paper, we give a concrete description of the two-fold cover of a simply connected, split real reductive group and its maximal compact subgroup as Chevalley groups. We study a small genuine representation of the maximal compact subgroup called pseudospherical representation, which appear with multiplicity one in the principal series representation. We introduce a family of canonically defined intertwining operators and compute the action of them on pseudospherical  $K$ -types, obtaining explicit formulas of the Harish-Chandra  $c$ -function. It has potential applications in the study of automorphic forms on metaplectic groups. (Received July 18, 2016)

1125-22-278 **Vignon S Oussa\*** ([voussa@bridgew.edu](mailto:voussa@bridgew.edu)), 131 Summer street, Bridgewater, MA 02325. *Discrete frames arising from irreducible solvable actions.* Preliminary report.

In this presentation, we will provide a unified method which is exploited to construct reproducing systems arising from unitary irreducible representations of solvable Lie groups. More precisely, we will show how a careful study of differential geometric properties of coadjoint orbits leads to a systematic and explicit construction to discrete frames and smooth frames of compact supports. In contrast to well-known techniques such as the coorbit theory and other discretization schemes, we make no assumption on the integrability of the representations of interest. Additionally, we will present various examples which illustrate that our method handles a variety of groups relevant to wavelet and time-frequency analysis experts. For example, the  $ax+b$  group, the Heisenberg groups, the generalized Heisenberg groups, the shearlet groups, solvable extensions of vector groups and various solvable extensions of non-commutative nilpotent Lie groups are just a few examples of groups that can be handled (in a unified fashion) by our method. (Received August 22, 2016)

1125-22-378 **George Lusztig** ([gyuri@math.mit.edu](mailto:gyuri@math.mit.edu)), 77 Massachusetts Ave, Cambridge, MA 02139, and **Zhiwei Yun\*** ([zhiweiyun@gmail.com](mailto:zhiweiyun@gmail.com)), 10 Hillhouse Ave, New Haven, CT 06511. *Perverse sheaves arising from cyclically graded Lie algebras and DAHA.*

Let  $G$  be a simple and simply-connected algebraic group whose Lie algebra  $\mathfrak{g}$  carries a  $\mathbf{Z}/m\mathbf{Z}$ -grading. The grading gives a subgroup  $G_0$  which acts on each graded piece  $\mathfrak{g}_i$ . Consider the derived category of  $G_0$ -equivariant sheaves on  $\mathfrak{g}_i$  that are supported on the nilpotent cone. In special cases, this category contains Fourier transforms of character sheaves and canonical bases arising from quivers.

We give a block decomposition of this category in terms of cuspidal data in the same spirit as the generalized Springer correspondence. To each block, we also attach a graded DAHA with unequal parameters and construct modules of it from objects in the block. (Received August 30, 2016)

1125-22-408 **William M. McGovern\*** ([mcgovern@math.washington.edu](mailto:mcgovern@math.washington.edu)), Department of Mathematics, Box 354350, University of Washington, Seattle, WA 98195. *Annihilators and associated varieties of Harish-Chandra modules for  $Sp(p, q)$ .*

We present combinatorial recipes for computing annihilators and associated varieties of simple Harish-Chandra modules of trivial infinitesimal character for the group  $Sp(p, q)$ , using domino and signed tableaux, respectively (Received August 31, 2016)

1125-22-585 **Toshiyuki Kobayashi\*** ([toshi@ms.u-tokyo.ac.jp](mailto:toshi@ms.u-tokyo.ac.jp)), The University of Tokyo, Komaba, Tokyo, Japan, Tokyo, 153-8914, Japan. *Conformally Covariant Symmetry Breaking Operators on Differential Forms and Some Applications.*

We discuss restrictions of irreducible representations to reductive subgroups with focus on geometric question:

1. Classification of conformally covariant symmetry breaking operators on differential forms.
- If time permits, we shall also discuss the following related questions for orthogonal groups of rank one:
2. Period of irreducible unitary representations.
  3. Occurrence of complementary series representations in branching laws.

Refereces

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1125-22-708 **Chal Benson\*** ([bensonf@ecu.edu](mailto:bensonf@ecu.edu)), Department of Mathematics, East Carolina University, Greenville, NC 27858. *Antiholomorphic involutions and multiplicity free representations.*

Let  $K$  be a compact Lie group acting unitarily on a finite dimensional hermitian vector space  $V$ . We form the associated representation of  $K$  in the polynomial ring  $\mathbb{C}[V]$  and call  $K : V$  a (linear) *multiplicity free action* when this associated representation is multiplicity free. A result due to Faraut and Thomas shows that this will be the case whenever there exists an antiholomorphic involution on  $V$  preserving  $K$ -orbits. Independent results of Akhiezer and Sasaki show, conversely, that such an involution necessarily exists whenever  $K : V$  is multiplicity free. The talk will survey these results and outline elementary proofs under restricted hypotheses. (Received September 09, 2016)

1125-22-724 **Pierre Clare\*** ([cclare@math.dartmouth.edu](mailto:cclare@math.dartmouth.edu)), HB6188, Kemeny Hall, 27 N Main Street, Hanover, NH 03755. *Geometric pictures of intertwining operators.*

Under certain circumstances, intertwining operators between principal series of real reductive groups can be realized through geometrically meaningful transformations. In this talk, we will review applications of that fact in various contexts. (Received September 09, 2016)

1125-22-1034 **Bent Ørsted\***, [orsted@imf.au.dk](mailto:orsted@imf.au.dk). *Homomorphisms between Verma modules and small representations.* Preliminary report.

For a semisimple Lie group and a minimal parabolic subgroup we study certain systems of partial differential equations on the corresponding nilradical. When the system corresponds to a homomorphism between Verma modules, the solution space may be related to representations of the semisimple group. We shall give examples where one obtains small unitary representations in such a way. This is based on joint work with T. Kubo. (Received September 14, 2016)

1125-22-1046 **Mark Colarusso\*** ([colaruss@uwm.edu](mailto:colaruss@uwm.edu)). *The Gelfand-Zeitlin integrable system on complex orthogonal Lie algebras.* Preliminary report.

Kostant and Wallach introduced the Gelfand-Zeitlin (GZ) integrable system on  $\mathfrak{gl}(n, \mathbb{C})$  and studied the Lagrangian flows and generic fibres of the moment map of the system. In this talk, we discuss the analogous integrable system on  $\mathfrak{g} = \mathfrak{so}(n, \mathbb{C})$ . We study the geometry of this integrable system by studying the adjoint action of the symmetric subgroup  $K = SO(n-1, \mathbb{C})$  on  $\mathfrak{g}$ . We use the theory of  $K$ -orbits on the flag variety of  $\mathfrak{g}$  to describe the nilfibre of the geometric invariant theory quotient  $\mathfrak{g} \rightarrow \mathfrak{g}/K$ . Using our description of the nilfibre and the Luna slice theorem, we develop an analogue of the classical Jordan decomposition for the  $K$ -action on  $\mathfrak{g}$  and use it to describe the points in the moment map fibres of the GZ integrable system where the flows are Lagrangian. If time permits, we will briefly discuss our approach to understanding the geometry of the moment fibres at singular points of the integrable system using the theory of flat deformations of schemes. This is joint work with Sam Evens. (Received September 14, 2016)

1125-22-1581 **Andrew Zimmer\*** ([aazimmer@uchicago.edu](mailto:aazimmer@uchicago.edu)). *(unmarked) Length spectrum rigidity of representations.*

The (unmarked) length spectrum of a hyperbolic surface  $S$  is the sequence of the lengths of all non-oriented primitive curves on  $S$  arranged in ascending order. A construction of Sunada implies that the length spectrum does not determine the surface: there exists non-isometric closed Riemann surfaces with the same length spectrum. However, a theorem of Wolpert says that a generic closed Riemann surface is determined up to isometry

by its length spectrum. In this talk I will describe how Wolpert's result generalizes to convex real projective structures on closed surfaces and more generally to representation varieties of finitely generated groups. This is joint work with Thomas Barthelmé and Ludovic Marquis. (Received September 18, 2016)

1125-22-1605 **Thomas C. Hales** and **Julia Gordon\*** ([gor@math.ubc.ca](mailto:gor@math.ubc.ca)), 121-1984 Mathematics Rd., Vancouver, BC V6T 1Z2, Canada, and **Sharon Frechette** and **Lance Robson**. *Transfer of Transfer*.

In the WIN project, we proved that Shalika germs for the special linear and for symplectic Lie algebras are "motivic" (as explained in Sharon Frechette's talk). In this talk, I will explain the broader context for this project, in particular, how Shalika germs appear in the proof of the Fundamental Lemma, and discuss subsequent work with T. C. Hales on transferring the Fundamental Lemma for smooth functions to positive characteristic. Our WIN project left some open questions in the general case; they can be circumvented for the transfer of the Fundamental lemma, but an answer would greatly simplify the proof; I will survey these questions. (Received September 18, 2016)

1125-22-1681 **Benjamin Harris\*** ([benjamin.harris@simons-rock.edu](mailto:benjamin.harris@simons-rock.edu)) and **Yoshiki Oshima**. *Vogan-Zuckerman Characters and Semisimple Coadjoint Orbits*.

Vogan and Zuckerman constructed an irreducible, unitary representation  $\pi(\mathcal{O}, \Lambda)$  of a real, reductive algebraic group  $G_{\mathbb{R}}$  for every semisimple orbital parameter  $(\mathcal{O}, \Lambda)$  in the good range. In this talk, we present an integral formula for the character of  $\pi(\mathcal{O}, \Lambda)$  in terms of the geometry of the orbital parameter  $(\mathcal{O}, \Lambda)$ . Special cases of this formula were previously obtained by Harish-Chandra and Kirillov when  $G_{\mathbb{R}}$  is compact and by Duflo and Rossmann when  $\mathcal{O}$  is of maximal dimension. (Received September 18, 2016)

1125-22-1748 **Henrik Schlichtkrull\*** ([schlicht@math.ku.dk](mailto:schlicht@math.ku.dk)). *Real spherical spaces and their classification*.

A homogeneous space  $G/H$  of a real reductive Lie group  $G$  is called spherical if a minimal parabolic subgroup of  $G$  admits an open orbit on it. All symmetric spaces are spherical, but the property is shared also by other spaces. Classifications are known of complex spherical spaces and of real symmetric spaces, but up to now not of real spherical spaces. A classification for the case with  $G$  simple and  $H$  reductive, recently obtained in joint work with F. Knop, B. Krötz and T. Pecher, will be presented. (Received September 19, 2016)

1125-22-1768 **Alan McLeay\***, [a.mcleay.1@research.gla.ac.uk](mailto:a.mcleay.1@research.gla.ac.uk). *Ivanov's Metaconjecture: Automorphism Groups of Sufficiently Rich Complexes of Regions for Surfaces with Punctures*.

It is a well-known and fundamental result of Ivanov that the curve complex of an orientable surface with punctures has automorphism group isomorphic to the extended mapping class group of the surface. It was subsequently shown that the equivalent statement is true for a number of other complexes, among them the pants complex (Margalit) and the separating curve complex (Brendle-Margalit, Kida). Such results led Ivanov to make a metaconjecture: all sufficiently rich complexes related to the surface will have automorphism group isomorphic to the extended mapping class group. A result by Brendle-Margalit shows this to be true for a broad class of complexes for closed surfaces. In this talk I will give the more general result for complexes relating to surfaces with punctures. (Received September 19, 2016)

1125-22-1792 **Firas Hindelah\*** ([hindelef@gvsu.edu](mailto:hindelef@gvsu.edu)), 1 Campus Dr, Allendale, MI 49401, and **Anthony Pecoraro**. *Classifying Seven Dimensional Lie Algebras With Nilradical Isomorphic to  $A_{5,2} \oplus \mathbb{R}$* . Preliminary report.

Low dimensional solvable Lie Algebra were completely classified up to dimension six. A general theorem asserts that if  $g$  is a solvable Lie Algebra of dimension  $n$ , then the dimension of its nilradical is at least  $n/2$ . For the seven dimensional algebras, the nilradical's dimension could be 4, 5, 6 or 7. The four and seven dimensional nilradical cases were classified. We examine the six dimensional nilradical case. We first looked for the six dimensional nilpotent algebras and found 32 algebras. In this talk we focus on the class where the nilradical is isomorphic to a direct sum of the five-dimensional algebra  $A_{5,2}$  and the one dimensional algebra denoted by  $A_{5,1} \oplus \mathbb{R}$ . (Received September 19, 2016)

1125-22-1795 **Markus Hunziker** ([markus\\_hunziker@baylor.edu](mailto:markus_hunziker@baylor.edu)), **John A. Miller\*** ([john\\_miller5@baylor.edu](mailto:john_miller5@baylor.edu)) and **Mark R. Sepanski** ([mark\\_sepanski@baylor.edu](mailto:mark_sepanski@baylor.edu)). *Szygies and covariant differential operators*.

We provide explicit descriptions of minimal free resolutions that appear in classical invariant theory. More precisely, using Howe duality and recent work by Enright, Hunziker, and Pruet, we interpret these resolutions as

Bernstein–Gelfand–Gelfand resolutions of unitary highest weight representations and then describe the differentials explicitly in terms of covariant differential operators on Hermitian symmetric spaces that were introduced by Jakobsen in the 1980's. (Received September 19, 2016)

1125-22-1875 **Jose A. Franco** ([jose.franco@unf.edu](mailto:jose.franco@unf.edu)) and **Markus Hunziker\*** ([markus\\_hunziker@baylor.edu](mailto:markus_hunziker@baylor.edu)). *The Tricomi equation and complementary series representations of  $SL(2, \mathbb{R})$* . Preliminary report.

The Lie algebra of infinitesimal symmetries of the Tricomi equation  $\eta u_{\xi\xi} + u_{\eta\eta} = 0$  is isomorphic to  $\mathfrak{sl}(2, \mathbb{R})$  plus an infinite-dimensional piece reflecting the fact that the equation is linear. A priori, the action of  $\mathfrak{sl}(2, \mathbb{R})$  on solutions does not globalize to an action of the group  $SL(2, \mathbb{R})$ . However, by restricting the space of solutions to a certain distinguished subspace, we show that we do indeed obtain a globalization and that the resulting representation is a complementary series representation of  $SL(2, \mathbb{R})$ . (Received September 19, 2016)

1125-22-2312 **William Graham\*** ([wag@uga.edu](mailto:wag@uga.edu)) and **Victor Kreiman** ([kreiman@uwp.edu](mailto:kreiman@uwp.edu)). *Cominuscule points in Schubert varieties*.

Cominuscule flag varieties (for example, Grassmannians) have the property that multiplicities and Hilbert series of torus-invariant subvarieties (such as Schubert varieties) at torus fixed points can be calculated using equivariant cohomology and  $K$ -theory. We introduce the notion of cominuscule points in varieties with torus actions, and show how this notion applies to calculate multiplicities and Hilbert series at certain points of Schubert varieties in more general flag varieties. We also describe some of the associated combinatorics. (Received September 20, 2016)

1125-22-2381 **Laura Rider\*** ([laurajoy@uga.edu](mailto:laurajoy@uga.edu)). *Tilting Modules and the centralizer of a regular nilpotent*. Preliminary report.

In this talk, I'll discuss the representation theory of a reductive group in positive characteristic with an emphasis on the role played by tilting modules. We will consider their restriction to the centralizer of a regular nilpotent and how this relates to some equivalences in geometric representation theory. (Received September 20, 2016)

1125-22-2418 **Marcus J Slupinski** and **Robert J Stanton\*** ([stanton.2@osu.edu](mailto:stanton.2@osu.edu)). *Duality of geometries on flag manifolds associated to maximal parabolic subgroups in split  $G_2$* . Preliminary report.

The real split linear Lie group  $G_2$  has two flag manifolds associated to non-conjugate maximal parabolic subgroups. Each of them has an interesting geometric structure which will be recalled. By means of a natural incidence relation and a thorough analysis of cross products in seven dimensions we shall identify an explicit geometric correspondence between these geometric structures. (Received September 20, 2016)

1125-22-2556 **Anna Wienhard\*** ([wienhard@uni-heidelberg.de](mailto:wienhard@uni-heidelberg.de)). *A tale of rigidity and flexibility - discrete subgroups of higher rank Lie groups*.

Discrete subgroups of Lie groups play an important role in various areas of mathematics. Lattices, discrete subgroups of finite covolume, are fairly well understood, revealing a dichotomy of flexibility and rigidity. Lattices in  $SL(2, \mathbb{R})$  are flexible, each such lattice has a deformation space of positive dimension, which is closely related to the Teichmüller space of a surface. Lattices in  $SL(n, \mathbb{R})$  with  $n \geq 2$  are super-rigid, due to a celebrated theorem of Margulis. It is rather difficult to get a handle on discrete subgroups which are not lattices. I will discuss new developments in geometry, low-dimensional topology, number theory, analysis and representation theory that led to the discovery of several interesting families of discrete subgroups which are not lattices, but - quite surprisingly - admit an interesting structure theory, which arises from a combination of flexibility and rigidity. A particular exciting aspect is the discovery of higher Teichmüller spaces and their relation to various areas in mathematics, such as analysis, algebraic geometry, geometry, dynamics, representation theory. (Received September 20, 2016)

## 26 ► Real functions

1125-26-765 **George A Anastassiou\*** ([ganastss2@gmail.com](mailto:ganastss2@gmail.com)), Department of Mathematical Sciences, University of Memphis, Memphis, TN 38152. *Most General Fractional Self Adjoint Operator Representation formulae and Operator Poincaré and Sobolev type and other basic Inequalities*. Preliminary report.

We give here many very general fractional self adjoint operator Poincaré and Sobolev type and other basic inner product inequalities to various di-rections. Initially we give several very general fractional representation

formulae in the self adjoint operator sense. Inequalities are based in the self adjoint operator order over a Hilbert space. (Received September 11, 2016)

1125-26-1894 **Amalia Culiuc\*** (amalia@math.gatech.edu), **Francesco Di Plinio** and **Yumeng Ou**.  
*Uniform sparse domination of singular integrals via dyadic shifts.*

We give a simple proof that  $L^2$ -bounded dyadic shifts are dominated by positive sparse forms with linear growth in the complexity of the shift. Using Hytönen's dyadic representation theorem, our estimate leads to the positive sparse domination of a wide class of singular integrals both in the scalar and in the vector-valued function setting. Furthermore, we obtain an operator norm bound on the matrix weighted space  $L^2(W; \mathbb{R}^n)$  depending on the power  $3/2$  of the the matrix  $A_2$  characteristic, which is the best known dependence to date. (Received September 19, 2016)

1125-26-2897 **Udita N. Katugampola\*** (udita@udel.edu), Department of Mathematical sciences, University of Delaware, Newark, DE 19716. *Can we generalize the limit-definition of the derivative? II.* Preliminary report.

We consider the operator defined by

$$\mathcal{D}_\omega f(x) = \lim_{\epsilon \rightarrow 0} \frac{f(x + \epsilon \omega(x)) - f(x)}{\epsilon},$$

relative to a weight function  $\omega$ , which acts as a **catalyst** and controls the convergence rate. Such a derivative obeys familiar properties such as the product rule, quotient rule, power rule, chain rule, Rolle's Theorem and Mean Value Theorems. It is interesting to note that the case of  $\omega(x) = x^{1-\alpha}$ ,  $\alpha \in \mathbb{R}$ , is now known as the **conformable fractional derivative** though it lacks some key properties of a standard fractional derivative. Newly defined derivative is a close resemblance of several existing derivatives, namely, the directional derivative, the Fréchet and Gâteaux derivatives. We also define the corresponding weighted  $\omega$ -integral in a weighted-space  $X_\omega$  and discuss some properties related to differential equations governed by  $\omega$ -derivatives. It is also interesting to observe that such an integral appears in several unrelated topics such as *Stochastic integrals*, the *Brunn-Minkowski theory*, *Wulff shapes*, *Bihari's inequality* and *Variational Calculus*. (Received September 20, 2016)

1125-26-2920 **Fernando Lopez Garcia\*** (fernando.lopezgaracia@ucr.edu). *Weighted Korn inequality and solutions of the divergence on John domains.*

We show a weighted version of Korn inequality on bounded John domains, where the weights are nonnegative powers of the distance to the boundary. We also provide an estimate of the constant involved in the inequality which depends on the power that appears in the weight and a geometric condition on the domain. The proof uses a local-to-global argument based on a certain decomposition of functions.

In addition, we prove the solvability in weighted Sobolev spaces of  $\operatorname{div} u = f$  on the same class of domains. This result is fundamental for the variational analysis of the Stokes equations. In this case, the weights are nonpositive powers of the distance to the boundary. The constant in this problem is also estimated. (Received September 20, 2016)

## 28 ► Measure and integration

1125-28-1721 **Jonathan Breuer\*** (jbreuer@math.huji.ac.il), Einstein Institute of Mathematics, Edmond J. Safra Campus, The Hebrew University of Jerusalem, 9190401 Jerusalem, Israel.  
*A Transfer Matrix Approach to Scaled Limits of Christoffel-Darboux Kernels.*

We present an approach to computing the scaling limits of Christoffel-Darboux kernels using transfer matrix evolution. (Received September 19, 2016)

1125-28-1732 **Dong Hyun Cho\*** (j94385@kyonggi.ac.kr), Department of Mathematics, Kyonggi University, Youngtong-Gu, Suwon, Kyonggido 16227. *Scale transforms of unbounded functions on an analogue of Wiener space.*

Let  $C[0, T]$  denote the space of real-valued continuous functions on  $[0, T]$ . Let  $a$  be in  $C[0, T]$  and let  $h$  be of bounded variation with  $h \neq 0$  a.e. on  $[0, T]$ . Define  $Z : C[0, T] \times [0, T] \rightarrow \mathbb{R}$  by  $Z(x, t) = (h\chi_{[0, t]}, x) + x(0) + a(t)$ . For a partition  $0 = t_0 < t_1 < \dots < t_n < t_{n+1} = T$  of  $[0, T]$ , define random vectors  $Z_n : C[0, T] \rightarrow \mathbb{R}^{n+1}$  and  $Z_{n+1} : C[0, T] \rightarrow \mathbb{R}^{n+2}$  by  $Z_n(x) = (Z(x, t_0), Z(x, t_1), \dots, Z(x, t_n))$  and  $Z_{n+1}(x) = (Z(x, t_0), Z(x, t_1), \dots, Z(x, t_n), Z(x, t_{n+1}))$ . With the conditioning functions  $Z_n$  and  $Z_{n+1}$ , we evaluate the conditional Fourier-Feynman transforms and convolution products of the functions given by

$$f((v_1, Z(x, \cdot)), \dots, (v_r, Z(x, \cdot))) \int_{L_2[0, T]} \exp\{i(v, Z(x, \cdot))\} d\sigma(v),$$

where  $\sigma \in \mathcal{M}(L_2[0, T])$  and  $f \in L_p(\mathbb{R}^r)$ . We show that the conditional Fourier-Feynman transform of the conditional convolution product for the functions can be expressed by the product of transforms of each function with a change of scale. Finally the effects of drift will be investigated on the polygonal function of  $a$ . (Received September 19, 2016)

1125-28-2504 **James T. Gill\*** (jgi115@slu.edu), 220 N. Grand Blvd., Saint Louis, MO 63103, and **Panu Lahti** and **Nageswari Shanmugalingam**. *Asymptotic Properties of Finite Perimeter Sets in Metric Spaces*. Preliminary report.

We discuss research in progress by the authors. In 1999 Cheeger succeeded in finding a differential structure in metric measure spaces which are doubling and support a Poincaré inequality. This is a  $p > 1$  theory. We explore the  $p = 1$  theory which corresponds to so-called BV functions and finite perimeter sets. (Received September 20, 2016)

## 30 ► Functions of a complex variable

1125-30-185 **Rosihan M Ali\*** (rosihan@usm.my), School of Mathematical Sciences, Universiti Sains Malaysia, 11800 Penang, Malaysia, and **Zhen Chuan Ng**, School of Mathematical Sciences, Universiti Sains Malaysia. *Bohr inequality in hyperbolic geometry*.

The classical Bohr inequality is generalized to the class of analytic functions mapping between two disks centered at the origin of arbitrary sizes. Bohr inequalities are next obtained for analytic functions in the hyperbolic unit disk in both the Poincaré disk and Poincaré half-plane models. The Bohr radius for both models are shown to be  $\tanh(1/2)/3$ . (Received August 10, 2016)

1125-30-288 **Emilio Bujalance**, **Antonio F. Costa** and **Milagros Izquierdo\*** (milagros.izquierdo@liu.se), Department of Mathematics, Linköping University, 58183 Linköping, Ostergötla, Sweden. *Uniparametric families of compact Riemann surfaces with large symmetry*.

By the work of Hurwitz we know that a compact Riemann surface of genus  $g$  has at most  $84(g-1)$  automorphisms. Finding surfaces with large symmetry is not easy. In 1968 Accola and Maclachlan found isolated surfaces, in all genera, having  $8g + 8$  automorphisms, they describe the symmetry of the surface; i.e. the unique conjugacy class of a finite subgroup of the mapping class group of order  $8g + 8$ . Using Riemann-Hurwitz one gets that the maximal number of automorphisms of a uniparametric family of compact Riemann surfaces, in all genera, is  $4g + 4$ . Here we determine and describe the conjugacy classes of subgroups with order  $4g$  of the mapping class group. As in Accola-Maclachlan case the symmetry of surfaces with  $4g$  automorphisms is rigid. (Received August 24, 2016)

1125-30-386 **Christian Hokaj** and **Kenneth Plante\*** (kenplante4@gmail.com), 165 South Elm Street, Windsor Locks, CT 06096. *Topological Properties of Classical Multiplier Sequences, n-sequences, and Extensions*.

We consider topological properties of the space of classical  $n$ -sequences and the space of classical multiplier sequences, and show that both spaces are complete and contractible. We classify the boundary points of these sets (as subsets of all real sequences) in terms of the zeros of an associated polynomial, and give both necessary, and sufficient conditions for the extendability of an  $n$ -sequence. (Received August 30, 2016)

1125-30-549 **Matthew Chasse**, **Tamas Forgacs\*** (tforagacs@csufresno.edu) and **Andrzej Piotrowski**. *Towards the classification of Legendre multiplier sequences*. Preliminary report.

Let  $\{\gamma_k\}_{k=0}^{\infty}$  be a sequence of real numbers, and let  $T : \mathbb{R}[x] \rightarrow \mathbb{R}[x]$  be defined by  $T[P_n(x)] = \gamma_n P_n(x)$  ( $n = 0, 1, 2, \dots$ ), where  $P_n(x)$  is the  $n$ th Legendre polynomial. If  $T$  is a hyperbolicity preserver, we call the associated sequence a Legendre multiplier sequence. In this talk we present a proof of a recent conjecture, which states that if  $p \in \mathbb{R}[x]$  is a polynomial and  $\{p(k)\}_{k=0}^{\infty}$  is a Legendre multiplier sequence, then  $\deg p = 2m$  for some  $m \in \mathbb{N}$ . In addition, we show that  $p$  must be a polynomial in  $x^2 + x$ , and discuss further properties  $p$  must possess. (Received September 06, 2016)

1125-30-590 **Michael Dorff\*** (mdorff@math.byu.edu), **Samaneh Hamidi**, **Jay Jahangiri** and **Elif Yasar**. *Convolutions of univalent harmonic strip mappings*.

Complex-valued harmonic mappings can be regarded as generalizations of analytic functions and are related to minimal surfaces that are beautiful geometric shapes with intriguing properties. In this talk we discuss

some results concerning the preservation of the property of univalence (or one-to-oneness) when combining two harmonic mappings through convolutions or Hadamard products. In particular, let  $f_k$  (where  $k = 1, 2$ ) be univalent harmonic functions that are shears of the analytic map  $h_k - g_k = \frac{1}{2} \ln \left( \frac{1+z}{1-z} \right)$  with dilatation  $\omega_k = e^{i\theta_k} z^k$ . If the convolution  $f_1 * f_2$  is locally one-to-one and sense-preserving, then  $f_1 * f_2 \in S_H^O$  is convex in the direction of the real axis and hence univalent. (Received September 07, 2016)

1125-30-599 **Zair Ibragimov\***, 800 N. State College Blvd, 154 McCarthy Hall, Fullerton, CA 92831. *A Möbius invariant Cassinian metric.*

We discuss a new Möbius invariant,  $\delta$ -hyperbolic metric  $\tau_D$ , called Möbius invariant Cassinian metric, for domains  $D$  in  $\mathbb{R}^n$ , which can be considered as a Möbius invariant analogue of the scale-invariant Cassinian metric  $\tilde{\tau}_D$  recently introduced by the author. We discuss basic properties of  $\tau_D$  including its connections  $\tilde{\tau}_D$ , Seittenranta's metric and the hyperbolic metric. We show that  $\tau_D$  is monotonic with respect to domains, its density is the same as the density of Ferrand's metric and that the  $\tau_D$ -isometries of twice-punctured spaces are Möbius maps. (Received September 07, 2016)

1125-30-806 **Tao Cheng, Huiqiang Shi and Shanshuang Yang\*** (syang@mathcs.emory.edu), Department of Math and CS, Emory University, Atlanta, GA 30322. *Sewing homeomorphisms and conformal invariants.* Preliminary report.

For a Jordan domain  $\Omega$  in the extended complex plane  $\bar{\mathbb{C}}$ , let  $f_1$  and  $f_2$  map  $\Omega$  and  $\Omega^* = \bar{\mathbb{C}} \setminus \bar{\Omega}$  conformally onto the unit disk  $\mathbb{D}$  and  $\mathbb{D}^* = \bar{\mathbb{C}} \setminus \bar{\mathbb{D}}$ , respectively. Extending  $f_1$  and  $f_2$  homeomorphically to the boundary, one can define a homeomorphism of the unit circle as  $h_\Omega = f_2 \circ f_1^{-1}|_{\partial\mathbb{D}}$ , which is called a *sewing homeomorphism* induced by the Jordan domain  $\Omega$ . In this talk, we explore some connections between the analytic properties of the sewing homeomorphism  $h_\Omega$  and the geometric properties of a Jordan domain  $\Omega$ . In particular, using conformal invariants such as harmonic measure, extremal distance, and reduced extremal distance, we give several necessary and sufficient conditions for the sewing homeomorphism to be bi-Lipschitz or bi-Hölder. (Received September 12, 2016)

1125-30-860 **Alexei Poltoratski\*** (alexep@math.tamu.edu). *Toeplitz Order.*

We discuss a partial ordering of the set of inner functions induced by Toeplitz operators. A study of the Toeplitz ordering connects several well-known problems of analysis, including the two-weight Hilbert problem, the Beurling-Malliavin problem on completeness of exponential functions, spectral problems for differential operators, etc. In my talk I will discuss classical results, translated in terms of Toeplitz ordering, as well as new results and further questions. (Received September 12, 2016)

1125-30-878 **Maksym Derevyagin\*** (mderevya@olemiss.edu), MS. *On the Wall transformation and linear pencils of Jacobi matrices.*

I'm going to talk about a transform that establishes a one-to-one correspondence between Verblunsky coefficients and two sequences  $\{g_n\}$  and  $\{r_n\}$  of real numbers such that

$$0 < g_n < 1, \quad -\infty < r_n < \infty.$$

As a matter of fact, the transform was introduced by Wall and it can be thought of as a transform between Schur functions, Carathéodory functions, and Nevanlinna functions. So, to be more precise the goal of the talk is to demonstrate that the sequences  $\{g_n\}$  and  $\{r_n\}$  also give rise to a very special class of linear pencils of Jacobi matrices. Consequently, the Wall transform shows that the theory of polynomials orthogonal on the unit circle is just a tip of the iceberg of linear pencils of Jacobi matrices. In particular, since orthogonal polynomials on the unit circle are very well studied, the transform in question helps to build up a certain intuition towards the general theory of linear pencils of Jacobi matrices (e.g. behavior of the entries from the properties of the corresponding measure and vice versa).

Finally, using the developed framework I will show how one can handle Cauchy-type distributions on the real line, which cannot be dealt with by means of orthogonal polynomials on the real line due to the non-existence of moments. (Received September 12, 2016)

1125-30-1133 **C Bénéteau, D Khavinson, C Liaw, D Seco\*** (dseco@mat.uab.cat) and **B Simanek.** *Zeros of optimal polynomial approximants and Jacobi matrices.*

I present a recent work with Beneteau, Khavinson, Liaw and Simanek where we study the structure of the zeros of polynomials appearing in the study of cyclicity in Hilbert spaces of analytic functions. We find the minimum possible modulus of occurring zeros by solving a nonlinear extremal problem associated with norms of Jacobi matrices. (Received September 15, 2016)

1125-30-1148 **C. David Minda\*** (minda@ucmail.uc.edu), Department of Mathematical Sciences, Cincinnati, OH 45221-0025. *Quotients of hyperbolic metrics*. Preliminary report.

Suppose that  $\Omega_j$  is a hyperbolic region in  $\mathbb{C} \cup \{\infty\}$  with hyperbolic metric  $\lambda_j$ ,  $j = 1, 2$ , and  $\Omega_1 \subsetneq \Omega_2$ . Sharp elementary upper and lower bounds for  $\lambda_1(z)/\lambda_2(z)$  in terms of the hyperbolic distance relative to  $\Omega_2$  from  $z$  to  $\partial\Omega_1 \cap \Omega_2$  are discussed. These bounds were originally established and employed in complex dynamics. We show that these bounds have important consequences for the theory of the hyperbolic metric. For instance, it is plausible that  $\lambda_1$  and  $\lambda_2$  have similar behavior near the common boundary  $\partial\Omega_1 \cap \partial\Omega_2 \neq \emptyset$ . The bounds imply that  $\lambda_1(z)/\lambda_2(z) \rightarrow 1$  when  $z$  tends to an appropriate part of  $\partial\Omega_1 \cap \partial\Omega_2$ . (Received September 15, 2016)

1125-30-1183 **Michael Dorff** (mdorff@math.byu.edu), Department of Mathematics, Brigham Young University, Provo, UT 84602, **Samaneh G. Hamidi\*** (s.hamidi@mathematics.byu.edu), Department of Mathematics, Brigham Young University, Provo, UT 84602, **Jay M. Jahangiri** (jjahangi@kent.edu), Department of Mathematical Sciences, Kent State University, OH 44021, and **Elif Yasar** (elifyasar@uludag.edu.tr), Department of Mathematics, Faculty of Arts and Sciences, Uludag University, 16059 Bursa, Turkey. *Convolutions Of Univalent Harmonic Mappings*. Preliminary report.

Harmonic univalent mappings are a collection of univalent (i.e., 1-1) complex-valued functions,  $f = u + iv$ , where  $u$  and  $v$  satisfy Laplace's equation but not necessarily the Cauchy-Riemann equations. There has been much research done lately on the convolution of harmonic mappings. We prove some results concerning the convolution of univalent harmonic convex mappings provided that it is locally univalent and sense-preserving. In particular, let  $f_k$  (where  $k = 1, 2$ ) be univalent harmonic functions that are shears of the analytic map  $h_k - g_k = \frac{1}{2} \ln \left( \frac{1+z}{1-z} \right)$  with dilatation  $\omega_k = e^{i\theta_k} z^k$ . If the convolution  $f_1 * f_2$  is locally one-to-one and sense-preserving, then  $f_1 * f_2 \in S_H^O$  is convex in the direction of the real axis. In this talk we will discuss the proof of this result. (Received September 15, 2016)

1125-30-1248 **Catherine Beneteau\*** (cbenetea@usf.edu). *Optimal polynomial approximants in Dirichlet spaces*.

In this talk, I will discuss the problem of finding, for a given function  $f$  in a Dirichlet-type space, the polynomial  $p$  that is optimal in the sense that  $pf$  is the best approximation of 1 in norm, among all polynomials of degree at most  $n$ . I will examine how these optimal polynomials are related to classical objects of complex analysis such as weighted reproducing kernels and orthogonal polynomials and discuss related current open problems. This talk is based on a series of papers that have appeared over the last few years, with various co-authors, including Alberto Condoni, Dmitry Khavinson, Conni Liaw, Daniel Seco, Brian Simanek, and Alan Sola. (Received September 15, 2016)

1125-30-1312 **Catherine Beneteau\*** (cbenetea@usf.edu), **Dmitry Khavinson**, **Constance Liaw**, **Daniel Seco** and **Brian Simanek**. *Jentzsch-type theorems for optimal polynomial approximants*.

Recall the theorem of Jentzsch that states that if a Taylor series for a function  $f$  has radius of convergence 1, then every point on the unit circle is a limit point of the zeros of the corresponding Taylor polynomials. In this talk, I will examine analogous results examining accumulation points of the zeros of optimal polynomial approximants to reciprocals of functions in certain Hilbert spaces of analytic functions in the unit disk. (Received September 16, 2016)

1125-30-1380 **Tim Ferguson\*** (tjferguson1@ua.edu) and **William T Ross**. *The Range and Valence of Real Smirnov Functions*.

We give a characterization of the ranges of real Smirnov functions, which are functions in the Smirnov class of the unit disc with real boundary values almost everywhere. In addition, we discuss the valence of such functions and construct real Smirnov functions with certain valences. (Received September 16, 2016)

1125-30-1392 **Pritha Chakraborty\*** (pritha.chakraborty@tamucc.edu) and **Alexander Solynin**. *An Extremal Problem Approach to Korenblum's Maximum Principle in Bergman Spaces*.

B. Korenblum conjectured and W. Hayman proved that for  $f, g \in \mathcal{A}^2(\mathbb{D})$ , there is a constant  $c$ ,  $0 < c < 1$ , such that if  $|f(z)| \leq |g(z)|$  for all  $z$  in  $c \leq |z| < 1$ , then  $\|f\|_2 \leq \|g\|_2$ , where  $\mathcal{A}^2(\mathbb{D})$  is the set of square integrable analytic functions in the unit disc  $\mathbb{D}$ . The largest possible value of such  $c$  is called the Korenblum's constant. The exact value of this constant, which is denoted by  $\kappa$ , remains unknown. We define the Korenblum's problem in extremal setting to discuss a new approach towards finding  $\kappa$ . The focus of this talk will lie on some non-linear extremal problems in Bergman spaces and properties of extremal functions. (Received September 16, 2016)

1125-30-1695 **Jeremy T. Tyson\*** ([tyson@illinois.edu](mailto:tyson@illinois.edu)), 1409 West Green St., Urbana, IL 61801.  
*Distortion of dimension by Sobolev and quasiconformal mappings.*

I will discuss recent results on the distortion of Hausdorff dimension of generic elements in parameterized families of subsets under Sobolev or quasiconformal mappings. The talk is based on joint work with Zoltan Balogh, Roberto Monti, Pertti Mattila and Kevin Wildrick. (Received September 18, 2016)

1125-30-1701 **Khang D Tran\*** ([khangt@csufresno.edu](mailto:khangt@csufresno.edu)), Khang Tran, 5245 North Backer Avenue M/S PB108 Fresno, California State University, Fresno, Fresno, CA 93740, and **Tamas Forgacs**.  
*Zeros of polynomials generated by rational functions with a hyperbolic polynomial type denominator.*

This talk investigates the location of the zeros of a sequence of polynomials generated by a rational function with a denominator of the form  $G(z, t) = P(t) + zt^r$ , where the zeros of  $P$  are positive and real. We show that every member of a family of such generating functions - parametrized by the degree of  $P$  and  $r$  - gives rise to a sequence of polynomials  $\{H_m(z)\}_{m=0}^\infty$  that is eventually hyperbolic. Moreover, when  $P(0) > 0$  the real zeros of the polynomials  $H_m(z)$  form a dense subset of an interval  $I \subset \mathbb{R}^+$ , whose length depends on the particular values of the parameters in the generating function (Received September 18, 2016)

1125-30-1728 **See Keong Lee\*** ([sklee@usm.my](mailto:sklee@usm.my)), School of Mathematical Sciences, Universiti Sains Malaysia, 11800 USM, Penang, Malaysia. *Integral characterization and growth properties of analytic functions defined by a differential inequality.* Preliminary report.

Let  $\mathbb{D} = \{z : |z| < 1\}$  be the unit disc on the complex plane  $\mathbb{C}$ . The class  $\mathcal{U}(\lambda, \mu)$ , which consists of analytic functions  $f$  satisfying

$$\left| \left( \frac{z}{f(z)} \right)^2 f'(z) - \mu \right| < \lambda,$$

for  $z \in \mathbb{D}$ ,  $0 < \lambda \leq 1$  and  $\mu \in \mathbb{C}$ , will be studied. Among the properties to be investigated are the analytic characterization of the function  $f$  in  $\mathcal{U}(\lambda, \mu)$ , the size of the parameters  $\mu$  and  $\lambda$  that ensure univalence, the growth and distortion theorems, the coefficient bounds and the links between  $\mathcal{U}(\lambda, \mu)$  with several important geometric subclasses of analytic functions. (Received September 19, 2016)

1125-30-2106 **Joan Lind\*** ([jlind@utk.edu](mailto:jlind@utk.edu)) and **Jessica Robins**. *Loewner deformations driven by the Weierstrass function.*

The Loewner differential equation provides a way of encoding growing families of sets into continuous real-valued functions. Most famously, Schramm-Loewner Evolution (SLE) are the growing random families of sets that are encoded via the Loewner equation by a multiple of Brownian motion. We consider the families of sets encoded by a multiple of the Weierstrass function, which is a deterministic analog of Brownian motion, and prove that there is a phase transition in this setting, just as there is in the SLE setting. (Received September 19, 2016)

1125-30-2326 **Lisa Lorentzen\*** ([lisa@math.ntnu.no](mailto:lisa@math.ntnu.no)), Department of Mathematical Sciences, NTNU, T, Norway. *Classical Convergence of Random Continued Fractions.*

Convergence theorems for continued fractions  $\mathbf{K}(a_n/b_n)$  with real or complex elements  $(a_n, b_n)$  where all  $a_n \neq 0$ , offer sufficient conditions on  $\{(a_n, b_n)\}_{n=1}^\infty$  for  $\mathbf{K}(a_n/b_n)$  to converge; that is, for  $\lim_{n \rightarrow \infty} f_n$  to exist in  $\widehat{\mathbb{C}} := \mathbb{C} \cup \{\infty\}$ , where  $f_n$  is the truncated continued fraction of length  $n$ . However, these conditions are by no means necessary. In the present talk we prove that if  $(a_n, b_n)$  is picked randomly and independently from  $\mathbb{C}^2 = \mathbb{C} \times \mathbb{C}$  (or from  $\mathbb{R}^2$ ) according to some probability measure on  $\mathbb{C}^2$ , then the resulting continued fraction  $\mathbf{K}(a_n/b_n)$  converges with probability 1 as long as the measure  $\mu$  has sufficient support and the expectation of

$$\log \left( |a| + \frac{1 + |b|^2}{|a|} \right)$$

is finite. This extends earlier results in this direction. (Received September 20, 2016)

1125-30-2460 **Stephanie P Neas\*** ([neass0285@my.uwstout.edu](mailto:neass0285@my.uwstout.edu)). *Anallagmatic Curves and Inversion About the Unit Hyperbola.*

We introduce inversion about the unit circle from a complex perspective. Using linear fractional transformations we construct curves in the plane that are self-inverse (anallagmatic). We provide a classification of complex rational functions whose real part has anallagmatic level sets. This framework is then transferred to the split-complex numbers where we develop the theory of anallagmatic curves about the unit hyperbola. (Received September 20, 2016)

1125-30-3049 **Eric D. Schippers\*** ([eric\\_schippers@umanitoba.ca](mailto:eric_schippers@umanitoba.ca)), Department of Mathematics, Machray Hall, 186 Dysart Rd, Winnipeg, Manitoba R3T 2N2, Canada, and **Wolfgang Staubach** ([wulf@math.uu.se](mailto:wulf@math.uu.se)), Department of Mathematics, Box 480, 751 06, Uppsala, Sweden. *Jump decomposition on quasicircles.*

The Plemelj-Sokhotski jump formula says that a reasonable complex function on a reasonable Jordan curve is the difference between the boundary values of holomorphic functions on the two connected components of the complement. The holomorphic functions are obtained by a Cauchy integral. Various versions exist in the literature for different meanings of "reasonable". For Jordan curves, we choose "reasonable" to be the set of functions which are the boundary values of complex harmonic functions of finite Dirichlet energy on one of the components. We show that for quasicircles, this set of functions is the same for both complements. We furthermore show that a Plemelj-Sokhotski jump decomposition holds on quasicircles for this class of functions if and only if the domain is a quasicircle. (Received September 20, 2016)

## 31 ► Potential theory

1125-31-734 **Dima Khavinson\*** ([dkhavins@usf.edu](mailto:dkhavins@usf.edu)), **Erik Lundberg** and **Hermann Render**. *Dirichlet's problem with entire data on ellipsoidal cylinder.*

We show that the Poisson integral solution of the Dirichlet problem with entire data of minimal type posed on an ellipsoidal cylinder is an entire harmonic function. (Received September 10, 2016)

1125-31-1747 **Habib Ammari**, **Francisco Romero** and **Matias Ruiz\*** ([matias.ruiz@ens.fr](mailto:matias.ruiz@ens.fr)). *Optical detection and heat generation for plasmonic nanoparticles.* Preliminary report.

Plasmon resonant nanoparticles have unique capabilities of enhancing the brightness and directivity of light, confining strong electromagnetic fields, and out-coupling of light into advantageous directions, when illuminated at the right frequency. In the past years, the field of plasmonics has witnessed a great deal of attention because of their increasing number of applications; among them, their use as labels in molecular biology and in therapy as nanometric heat-generators that can be activated remotely by external electromagnetic fields. It is, therefore, important to have efficient mechanisms for imaging plasmonic nanoparticles and understand the role of geometry and materials properties in the production of heat. This talk aims to give answers to the former. Thus, the objective is twofold: (i) To analytically investigate the far field behavior when a plasmonic nanoparticle, immersed in a random media, is illuminated with an incident wave at the plasmonic resonance, in order to construct a robust imaging functional; (ii) To derive an asymptotic formula for the temperature in the border of plasmonic nanoparticles of arbitrary shape. (Received September 19, 2016)

1125-31-2028 **Douglas P. Hardin**, **Edward B. Saff** and **Oleksandr V. Vlasniuk\*** ([oleksandr.vlasniuk@vanderbilt.edu](mailto:oleksandr.vlasniuk@vanderbilt.edu)), Vanderbilt University, Department of Mathematics, 1326 Stevenson Center, Nashville, TN 37240. *Generating point configurations via hypersingular Riesz energy with an external field.*

For a compact  $d$ -dimensional rectifiable subset of  $\mathbb{R}^p$  we study asymptotic properties as  $N \rightarrow \infty$  of  $N$ -point configurations minimizing the energy arising from a Riesz  $s$ -potential  $1/r^s$  and an external field in the hypersingular case  $s \geq d$ . Results on separation and covering properties of such discrete minimizers are given. Formulas for the weak\* limit of normalized counting measures and the first-order asymptotic values of minimal energy are obtained. As an application, we derive a method for generating configurations whose normalized counting measures converge to a given absolutely continuous measure supported on a rectifiable subset of  $\mathbb{R}^p$ . Our theorems are illustrated with several numerical examples. (Received September 19, 2016)

1125-31-2849 **Lucio M-G Prado\***, Department of Mathematics, BMCC, The City University of New York, 199 Chambers Street, New York, NY. *The problem of  $p$ -Laplacian and its Surjectivity on Infinite Graphs.* Preliminary report.

Given a connected locally finite graph  $G$  with vertex set  $V$ , a discrete version of the  $p$ -Laplacian can be adapted to from Riemannian manifold. In this talk, I will give overview of some basic concepts and techniques with principal focus on the investigation of the surjectivity of the  $p$ -Laplacian in terms of  $p$  on infinite graphs. It is important to point out that this problem has direct relation to the existence of solution of Poisson equations. (Received September 20, 2016)

## 32 ► *Several complex variables and analytic spaces*

1125-32-306 **Leah Buck** and **Kelly Emmrich\*** ([emmrch.kelly@uwlax.edu](mailto:emmrch.kelly@uwlax.edu)), Department of Mathematics and Statistics, University of Wisconsin, La Crosse, 1725 State St., La Crosse, WI 54601, and **Tamas Forgacs**. *Sufficient Conditions for a Linear Operator on  $\mathbb{R}[x]$  to be Monotone*. Preliminary report.

We demonstrate that being a hyperbolicity preserver does not imply monotonicity for infinite order differential operators on  $\mathbb{R}[x]$ , thereby settling a recent conjecture in the negative. We also give several conditions, some necessary, some sufficient, for such operators to be monotone. (Received August 25, 2016)

1125-32-799 **Kelly Bickel**, Department of Mathematics, Bucknell University, Lewisburg, PA 17837, and **Alan A Sola\*** ([sola@math.su.se](mailto:sola@math.su.se)), Department of Mathematics, Stockholm University, 106 91 Stockholm, Sweden. *Weighted Dirichlet integrals in the bidisk*. Preliminary report.

We examine a number of equivalent expressions for Dirichlet-type norms on the bidisk, and discuss applications to membership problems for certain subclasses of functions. (Received September 12, 2016)

1125-32-869 **Clifford J. Earle** and **Sudeb Mitra\*** ([sudeb.mitra@qc.cuny.edu](mailto:sudeb.mitra@qc.cuny.edu)), Department of Mathematics, Queens College, CUNY, 65-30 Kissena Boulevard, Flushing, NY 11367-1597. *Holomorphic families of isomorphisms of Möbius groups*. Preliminary report.

This is an ongoing joint work with Clifford J. Earle. Let  $V$  be a simply connected complex Banach manifold with a basepoint. Let  $G$  be a subgroup of  $\mathrm{PSL}(2, \mathbb{C})$ ,  $E$  be a subset of the Riemann sphere (containing at least 3 points), and suppose that  $E$  is invariant under  $G$ . In this talk, we will discuss a new result on holomorphic families of isomorphisms of  $G$ . Our result crucially depends on the conformal naturality and real-analyticity of “Douady-Earle” sections (also called “barycentric sections”) of some generalized Teichmüller spaces. (Received September 12, 2016)

1125-32-1284 **Greg Knese\*** ([geknese@wustl.edu](mailto:geknese@wustl.edu)), One Brookings Drive, Department of Mathematics, Campus Box 1146, St. Louis, MO 63130. *Extreme points and saturated polynomials*. Preliminary report.

What are the extreme points in the convex set of analytic functions on the polydisk with positive real part (normalized to map 0 to 1)? In one variable this is well known and powerful (simply knowing the extreme points allows one to prove the Herglotz integral formula). In several variables no characterization is known and it is unlikely that a simple characterization exists. We shall focus on characterizing the extreme points in a special subclass of rational functions in two variables built out of polynomials with no zeros in the bidisc. Saturated polynomials, those with many zeros on the boundary of the bidisc, will play a key role. (Received September 15, 2016)

1125-32-2431 **Snehalatha Ballamoole\*** ([sb1244@msstate.edu](mailto:sb1244@msstate.edu)), **Thomas Len Miller**, **Vivien Glass Miller** and **Mathew Scott McBride**. *Cesàro-type operators on Hardy and Bergman space of analytic functions on the unit ball*.

Let  $\mathcal{H}(\mathcal{B}_n)$  denote the space of all holomorphic functions on the unit ball of  $C^n$ . We consider the integral operators

$$T_g f(z) = \int_0^1 f(tz) (\mathcal{R}g)(tz) \frac{dt}{t}$$

where  $g, f \in \mathcal{H}(\mathcal{B}_n)$ ,  $z \in \mathcal{B}_n$  and  $\mathcal{R}$  represents the radial derivative. We obtain boundedness and spectrum of these operators on the Hardy and weighted Bergman spaces in the unit ball.

(Received September 20, 2016)

1125-32-3062 **Sofia Ortega-Castillo\*** ([sofia.ortega@cimat.mx](mailto:sofia.ortega@cimat.mx)), Jalisco S/N, Col. Valenciana, 36000 Guanajuato, Guanajuato, Mexico. *Strong pseudoconvexity in infinite dimension*.

After an introduction to some relevant complex geometric analysis concepts, I will give a motivation to study strong pseudoconvexity in infinite dimension. Then I will present a general definition, as well as equivalent ones under special conditions, inspired by concepts from Banach space theory, such as strict and uniform convexity. Also, I will provide some examples and counterexamples to strict pseudoconvexity in the infinite-dimensional setting, that are actually balls of some classical Banach spaces. Finally, I will discuss some relationships between strong pseudoconvexity and the cluster value problem in Banach spaces. (Received September 20, 2016)

## 33 ► *Special functions*

1125-33-138 **Siddhartha Sahi\*** ([sahi@math.rutgers.edu](mailto:sahi@math.rutgers.edu)), Department of Mathematics, Rutgers University, New Brunswick, NJ 08903. *Macdonald hypergeometric functions.*

Hypergeometric functions for symmetric matrices were introduced in the 1950's by Herz, who gave an inductive definition using the Laplace transform. Subsequently Constantine obtained an explicit series expansion in terms of the zonal polynomials. These have found considerable applications in multivariate statistics, especially in the theory of non-central distributions.

In the 1980's I.G. Macdonald proposed a one-parameter generalization of this theory, replacing zonal polynomials by Jack polynomials. However many of the results in this more general theory remained conjectural. We will present solutions to several of these conjectures, obtained in joint work with Gestur Olafsson. As a further application of the theory, we derive a generalization of Ramanujan's "master theorem" to the setting of Jack polynomials. (Received August 03, 2016)

1125-33-571 **Marc Chamberland\*** ([chamberl@grinnell.edu](mailto:chamberl@grinnell.edu)), 1116 8th Ave., Department of Mathematics, Grinnell, IA 50112. *Combinatorial Identities via Matrix Factorization.*

The LU factorization is an effective tool in numerical linear algebra. By performing matrix factorizations symbolically, however, beautiful identities can be constructed. This talk will showcase this approach and the formulas produced connected to the Fibonacci numbers, binomial coefficients, q-series, Legendre polynomials, and more. (Received September 06, 2016)

1125-33-634 **Maria Angeles Garcia-Ferrero, David Gomez-Ullate and Robert Milson\*** ([robert.milson@dal.ca](mailto:robert.milson@dal.ca)), Dept. Mathematics & Statistics, Dalhousie University, 6316 Coburg Road, Halifax, NS B3H4R2, Canada. *Towards the classification of Exceptional Orthogonal Polynomials.*

Exceptional Orthogonal Polynomials are orthogonal polynomial families that arise as solutions for second-order eigenvalue problems. They generalize the classical families of Hermite, Laguerre, and Jacobi because they allow for polynomial sequences with a finite number of missing degrees. The fundamental technique for constructing such objects is the Darboux transformation, which can relate one of the above classical families with a family of orthogonal polynomials with a finite number of exceptional degrees. We will present a foundational theorem in this subject that asserts that *all* exceptional orthogonal polynomials arise in precisely this fashion. This result is an essential component of the ongoing classification programme for EOP. (Received September 08, 2016)

1125-33-683 **Mourad E. H. Ismail\*** ([mourad.eh.ismail@gmail.com](mailto:mourad.eh.ismail@gmail.com)), Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Some questions related to  $R_{II}$  continued fractions.*

We will discuss some old and new questions about the theory of  $R_{II}$  fractions, contiguous relations, monotonicity of zeros of polynomials, and certain functions. Connections to doubly infinite Jacobi matrices will be mentioned. (Received September 09, 2016)

1125-33-788 **Barry Simon\*** ([bsimon@caltech.edu](mailto:bsimon@caltech.edu)), Mathematics 253-37, Caltech, Pasadena, CA 91125. *Chebyshev Polynomials on the Real Line.*

This describes joint work with Jacob Christiansen and Maxim Zinchenko on Chebyshev polynomials,  $T_n$ , on a compact subset,  $\epsilon \subset \mathbb{R}$ . For any Parreau–Widom set,  $\epsilon$ , we prove an upper bound of the form  $\|T_n\|_\epsilon \leq QC(\epsilon)^n$  generalizing a bound of Totik and Widom for finite gaps sets. We obtain Szegő–Widom asymptotics for the polynomials of the set  $\epsilon$  thereby establishing a 45 year old conjecture of Widom. (Received September 11, 2016)

1125-33-979 **Willard Miller Jr** ([miller@ima.umn.edu](mailto:miller@ima.umn.edu)), 513 Vincent Hall, 206 Church St. SE, Minneapolis, MN 55455, and **Adrian Mauricio Escobar Ruiz\*** ([escob049@umn.edu](mailto:escob049@umn.edu)), 302 Vincent Hall, 206 Church St. SE, Minneapolis, MN 55455. *Conformal Laplace superintegrable systems in 2D: polynomial invariant subspaces.*

2nd-order conformal superintegrable systems in  $n$  dimensions are Laplace equations on a manifold with an added scalar potential and  $2n-1$  independent 2nd order conformal symmetry operators. They encode all the information about Helmholtz (eigenvalue) superintegrable systems in an efficient manner: there is a 1-1 correspondence between Laplace superintegrable systems and Stackel equivalence classes of Helmholtz superintegrable systems. In this talk we deal with superintegrable systems in two dimensions,  $n = 2$ , where there are 44 Helmholtz systems, corresponding to 12 Laplace systems. For each Laplace equation we determine the possible 2-variate polynomial subspaces that are invariant under the action of the Laplace operator, thus leading to families of

polynomial eigenfunctions. We also show the behavior of the polynomial invariant subspaces under a Stackel transform. The hidden  $gl_3$ -algebraic structure is exhibited for the  $gl_3$ -algebraic structure is exhibited for the exact and quasi-exact systems. Finally, for all Helmholtz superintegrable solvable systems we give a unified construction of 1D and 2D quasi-exactly solvable potentials possessing polynomial solutions, and a construction of new 2D PT-symmetric potentials is presented. (Received September 14, 2016)

1125-33-1290 **Sarah Post\*** ([spost@hawaii.edu](mailto:spost@hawaii.edu)), 2565 McCarthy Mall, Honolulu, HI 96822.

*Generalizations of the Askey-Wilson Algebra.*

In this talk, I will present extensions of Zhedanov's algebras QAW(3) and QR(3) from three different viewpoints. These algebras, originally defined in terms of integrable systems, have long been understood as the algebras generated by the three term recurrence and eigenfunctions relations for hypergeometric orthogonal polynomials (OPs). They also arise as the symmetry algebras for superintegrable systems with such OPs as overlap coefficients for different bases. From a purely algebraic perspective, the algebras can also be understood in terms of decompositions of triple tensor products of Lie algebras and their q-deformations.

The extensions of the algebras arise naturally from each of these viewpoints: in terms of bivariate OPs, extensions of the Hamiltonian systems in higher dimensions and quadruple tensor products. We will discuss these new algebras in detail and give indications of how to generalize to arbitrary dimensions. (Received September 20, 2016)

1125-33-1387 **Bernard Deconinck\*** ([deconinck@uw.edu](mailto:deconinck@uw.edu)) and **Benjamin L Segal.** *Analyzing the*

*stability spectrum for elliptic solutions of the focusing NLS equation.*

The one-dimensional focusing cubic nonlinear Schroedinger (NLS) equation is one of the most important integrable equations, arising in a multitude of applications. The stability of the stationary periodic solutions of NLS is well studied, leading to, for instance, the iconic figure-eight spectrum for its cnoidal wave solutions. We present an explicit expression for the linear stability spectrum of both the trivial- and nontrivial-phase solutions. We use this expression to generate many explicit results about the spectrum. (Received September 16, 2016)

1125-33-1473 **Walter Van Assche\*** ([walter@wis.kuleuven.be](mailto:walter@wis.kuleuven.be)), Department of Mathematics, KU Leuven, Celestijnenlaan 200 B box 2400, 3001 Leuven, Belgium. *Majorization results for zeros of orthogonal polynomials.*

Majorization is an important concept in statistics and linear algebra, used for instance to describe the location of eigenvalues of matrices in comparison to eigenvalues of the matrix after deleting some rows and columns. I will give some results for the majorization of zeros of orthogonal polynomials and associated polynomials on the real line when compared to zeros of higher degree polynomials. These results extend the well known interlacing property. (Received September 17, 2016)

1125-33-1714 **Jorge Arvesú\*** ([jarvesu@math.uc3m.es](mailto:jarvesu@math.uc3m.es)), Department of Mathematics, Avda. de la Universidad 30, 28911 Leganes, Madrid, Spain. *Vector equilibrium problem with constraint and nth root asymptotics for some multiple orthogonal polynomials.*

The nth root asymptotic behavior of some discrete multiple orthogonal polynomials is presented. Two main ingredients of the proposed approach for the study of the aforementioned asymptotic behavior are discussed; namely, an algebraic function formulation for the solution of the equilibrium problem with constraint to describe their zero distribution and the limiting behavior of the coefficients of the recurrence relations for multiple orthogonal polynomials. (Received September 19, 2016)

1125-33-2195 **Jongsil Lee** and **James Mc Laughlin\***, [jmclaughlin@wcupa.edu](mailto:jmclaughlin@wcupa.edu), and **Jaebum Sohn.**

*Applications of the Heine and Bauer-Muir transformations to Rogers-Ramanujan type continued fractions.*

It is shown that various continued fractions for the quotient of general Ramanujan functions  $G(aq, b, \lambda q)/G(a, b, \lambda)$  may be derived from each other via Bauer-Muir transformations. The separate convergence of numerators and denominators play a key part in showing that the continued fractions and their Bauer-Muir transformations converge to the same limit.

We also show that these continued fractions may be derived from Heine's continued fraction for a ratio of  ${}_2\phi_1$  functions and other continued fractions of a similar type, and by this method derive a new continued fraction for  $G(aq, b, \lambda q)/G(a, b, \lambda)$ .

New versions of some beautiful continued fraction expansions of Ramanujan are derived, for example:

If  $|q| < 1$  and  $|bq| < |a| < 1/|b|$ , then

$$\frac{(a^2q, b^2q; q^4)_\infty}{(a^2q^3, b^2q^3; q^4)_\infty} \\ = 1 + ab - \frac{(a+bq)(b+aq)}{(1+q^2)} + \frac{(a-bq)(b-aq)q^2}{(1+q^4)} - \frac{(a+bq^3)(b+aq^3)q^2}{(1+q^6)} + \frac{(a-bq^3)(b-aq^3)q^4}{(1+q^8)} - \dots$$

(Received September 19, 2016)

1125-33-2777 **Ahmad El-Guindy\*** (ahmad.el-guindy@qatar.tamu.edu), Texas A&M University at Qatar, Science Program, Doha, Qatar, and **Mourad E.H. Ismail**, University of Central Florida, Orlando, FL. *Unilateral and bilateral identities of Rogers-Ramanujan type.*

In this talk, we derive some sum-to-product identities in the theory of  $q$ -series. The formulas are related to a generalized version of the  $q$ -Airy function which was recently introduced by Ismail and Zhang. Along the way, we obtain some interesting results on the vanishing and non-vanishing of certain finite  $q$ -sums when twisted by arbitrary roots of unity with implication to both unilateral and bilateral infinite  $q$ -series. (Received September 20, 2016)

1125-33-2873 **Charles T Fulton\*** (cfulton@fit.edu), Dept of Mathematical Sciences, Florida Institute of Technology, 150 W University Blvd., Melbourne, FL 32901, and **Heinz Langer**. *Special Functions arising from Sturm-Liouville Equations.* Preliminary report.

Typically special functions are studied with complex arguments. Here we take a different approach which formulates definitions of special functions associated with the Sturm-Liouville equation

$$-(p(x)y'(x))' + q(x)y(x) = \lambda r(x)y(x)$$

as functions of two variables,  $y(x, \lambda)$ , where  $x$  is real and  $\lambda$  is complex. For singular SL equations having endpoints of  $LC/N$  and  $LP/N$  type ( $N$ =nonoscillatory for all real  $\lambda$ ), which are also RSPs, we formulate the definitions of Frobenius solutions,  $i = 1, 2$ , so that the following properties hold: (i)  $y_i(x, \bar{\lambda}) = \overline{y_i(x, \lambda)}$  and (ii)  $y_i(x, \lambda)$  entire in  $\lambda$  for fixed  $x$ . We then employ such special functions in the spectral analysis of Bessel, H-Atom and Associated Legendre equations. [See Fulton and Langer, Sturm-Liouville Operators with Singularities and Generalized Nevanlinna functions, Complex Anal and Oper Theory 4 (2010), 179-243; Fulton, Langer, Luger, The Associated Legendre Equation and Generalized Nevanlinna Functions; and Fulton, The Connection Problem for Solutions of Sturm-Liouville Problems with Two Singular Endpoints, and its Relation to  $m$ -Functions, Oberwolfach Report 1/2015 (DOI: 10.4171/OWR/2015/1).] (Received September 20, 2016)

1125-33-2933 **Joshua P. Bowman\*** (joshua.bowman@pepperdine.edu), Natural Science Division, Pepperdine University, 24255 Pacific Coast Highway, Malibu, CA 90263-4321. *Chebyshev-like maps in several and infinite variables.* Preliminary report.

Analogues of Chebyshev polynomials have been defined as maps  $\mathbb{C}^n \rightarrow \mathbb{C}^n$  by several authors. Like their single-variable cousins, these maps have interesting algebraic, analytic, and dynamical properties. In this talk we will study recursive formulas for these maps, as well as infinite-dimensional versions. (Received September 20, 2016)

1125-33-3002 **Atilla Sit\*** (atilla.sit@eku.edu) and **Daisuke Kihara**. *Three dimensional Krawtchouk descriptors for local comparison of protein surfaces.*

Direct comparison of images or 3D structures is computationally expensive and inefficient due to problems such as scaling and rotation. Also, in many applications, information about specific local structures rather than the entire image is of more interest. Thus, finding invariant descriptors that can retrieve local image patches or subimages becomes necessary. We have recently developed a set of 2D moment invariants based on discrete orthogonal Krawtchouk polynomials for comparison of local image patches. In this work, we extend them to 3D and construct three dimensional Krawtchouk descriptors (3DKD) that are invariant under translation, rotation, and scaling. We present the new formulation of 3DKDs and apply them to local comparison of protein surface patches for detecting ligand binding sites. (Received September 20, 2016)

1125-33-3076 **Constanze Liaw, Jessica S. Kelly** and **John M. Osborn\*** (john\_osborn@baylor.edu). *Moment Representations of Exceptional  $X_1$  Orthogonal Polynomials.*

We generalize the representations of  $X_1$  exceptional orthogonal polynomials through determinants of matrices that have certain adjusted moments as entries. We start out directly from the Darboux transformation, allowing for a universal perspective, rather independent of the particular system (Jacobi or Type of Laguerre polynomials). We include three ways to compute these adjusted moments — via a recursion formula, a matrix system, and as a hypergeometric function???.

Throughout we relate to the various examples of  $X_1$  exceptional orthogonal polynomials. We especially focus on the Jacobi and the Type III Laguerre case, as they seem to be less prevalent in literature.

We lastly include a preliminary discussion explaining that the higher co-dimension setting becomes more involved. (Received September 20, 2016)

## 34 ► Ordinary differential equations

1125-34-28 **Tiernan R. Fogarty\*** ([tiernan.fogarty@oit.edu](mailto:tiernan.fogarty@oit.edu)). *Successful modeling motivation for beam equations.*

Present the power of student collected data in motivating solution strategies for differential equations for beam analysis, referring to the published Modeling Scenario in SIMIODE and to other modeling experiences in differential equations coursework. (Received May 26, 2016)

1125-34-150 **Christopher S. Goodrich\*** ([cgood@prep.creighton.edu](mailto:cgood@prep.creighton.edu)), 7400 Western Ave., Omaha, NE 68114. *Coercive Nonlocal Elements in Fractional Differential Equations.*

I will discuss the use of coercive linear functionals in deducing existence results for fractional differential equations with nonlocal boundary conditions. As a specific example of this methodology, I will consider the fractional boundary value problem

$$\begin{aligned} -[D_{0+}^{\nu}y](t) &= \lambda f(t, y(t)), \quad 0 < t < 1 \\ y^{(i)}(0) &= 0, \quad 0 \leq i \leq n-2 \\ [D_{0+}^{\alpha}y](1) &= H(\varphi(y)), \end{aligned}$$

where  $n \in \mathbb{N}_4$ ,  $n-1 < \nu \leq n$ ,  $\alpha \in [1, n-2]$ , and  $\lambda > 0$  is a parameter. I will demonstrate that by utilizing a new order cone, one can generate existence results with minimal assumptions on the functions  $f$  and  $H$ . (Received August 04, 2016)

1125-34-208 **Ian W. Knowles\*** ([iknowles@uab.edu](mailto:iknowles@uab.edu)), Department of Mathematics, University of Alabama at Birmingham, Birmingham, AL 35294. *An inverse problem for the zeta function and the Cramer prime gap conjecture.* Preliminary report.

It is known that the location of the zeros of the Riemann zeta function directly encodes distribution properties of the prime numbers. We show how to inversely create a Sturm-Liouville equation with the property that the location of the zeros of the zeta function directly relates to asymptotic properties of solutions of the differential equation. This connection is then used to investigate an old conjecture of Cramer that, if  $p_n$  denotes the  $n$ -th prime, then  $p_{n+1} - p_n = O((\log p_n)^2)$ . (Received September 12, 2016)

1125-34-228 **Rachel Louise Bayless\*** ([rbayless@agnesscott.edu](mailto:rbayless@agnesscott.edu)), 141 E. College Ave, Decatur, GA 30030. *Modeling Feral Cat Control.*

Many communities in the United States have experienced a rapid growth in feral cat populations, and there is much debate among experts about the best methods for controlling these populations. The two most commonly used methods are trap-euthanize and trap-neuter-return. In this talk, we present a modeling project comparing the efficacy of these two control methods, and show that under certain assumptions the minimum number of traps needed to produce a decline in the population is the same for both models. All models presented are modifications of a standard exponential growth model, and thus this project is appropriate for the beginning of a differential equations course or an advanced assignment in calculus II. (Received August 15, 2016)

1125-34-293 **Keith A Landry\*** ([klandry@georgiasouthern.edu](mailto:klandry@georgiasouthern.edu)), P.O. Box 7995, Statesboro, GA 30460, and **Brian Winkel**. *Modeling in an Engineering Mathematics Class – Tuned Mass Dampers.*

The short presentation will focus on the intuitive mathematical modeling used with civil and mechanical engineering students to present two applications of systems of ordinary differential equations concerning excitation of a structure through resonance. Analysis includes investigation of the peak-frequency responses and stabilization by means of a tuned mass damper (TMD). The physics, engineering, terminology and mathematics associated with these phenomena are presented and discussed along with illustrations of how to use this material with students. (Received August 24, 2016)

1125-34-314 **Brian J Winkel** ([brianwinkel@simiode.org](mailto:brianwinkel@simiode.org)), 26 Broadway, Cornwall, NY 12518, and **Chris McCarthy\*** ([cmccarthy@bmcc.cuny.edu](mailto:cmccarthy@bmcc.cuny.edu)), Borough of Manhattan Community College CUNY, Department of Mathematics, 199 Chambers Street, New York, NY 10007. *SIMIODE – a supportive community for teaching and learning differential equations using the “modeling first” approach: from the classroom to preparing students for research.*

We present an overview of SIMIODE - Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations ([www.simiode.org](http://www.simiode.org)). We will discuss the educational resources and the opportunities for collaboration and publication (double-blind peer-reviewed, all open source) provided by SIMIODE.

We will discuss the modeling first approach to teaching differential equations in both the classroom and research setting.

The authors are part of an interdisciplinary research group studying bioremediation. One of our projects, that we will discuss, involves repurposing a common organic waste product, spent tea leaves, as a filter material that is able to remove heavy metal pollutants from water.

Our research group includes students, some of whom have never studied differential equations or statistics. A most efficient way to prepare these students, so that they can understand the material (differential equations and non-linear regression) on the level necessary to contribute, is to use the “modeling first” approach.

Funding acknowledgements: CUNY Community College Collaborative Incentive Research Grant. US Department of Education MSEIP Grant. (Received September 20, 2016)

1125-34-324 **John T. Sieben\*** ([jsieben@tlu.edu](mailto:jsieben@tlu.edu)), Texas Lutheran University, 1000 West Court St., Seguin, TX 78155. *Creating and using videos in support of modeling in differential equations course work.*

Videos can be a great aid to teaching and motivating students in differential equations. Our focus is on modeling and often a video can do a better job of clarifying the model than can text and static images. Another use of video in the modeling first approach is to allow students to gather data without having to physically carry out experiments. For example, a video might show a tank leaking water into a graduated cylinder next to a stopwatch showing the elapsed time. The students collect the data by starting and stopping a video and recording time and water remaining. They get their data but don't have to find a way to set up the physical experiment. Several examples will be given in the presentation. Finally, there are several tools for modelers that are video based and are now available. One example is an application called Video Physics. Applications of Video Physics will be presented. Not so many years ago videos were expensive and difficult to create, but not so today. In this talk the author's recommendations for best tools and practices will be explored. (Received August 26, 2016)

1125-34-330 **Sougata Dhar\*** ([sdhar@niu.edu](mailto:sdhar@niu.edu)), 1305 N Annie Glidden Road, Apt 201, DeKalb, IL 60115, and **Qingkai Kong** and **Michael McCabe**. *Fractional boundary value problems and Lyapunov-type inequalities with fractional integral boundary conditions.*

We discuss boundary value problems for Riemann-Liouville fractional differential equations with certain fractional integral boundary conditions. Such boundary conditions are different from the widely considered pointwise conditions in the sense that they allow solutions to have singularities, and different from other conditions given by integrals with a singular kernel since they arise from well-defined initial value problems. We derive Lyapunov-type inequalities for linear fractional differential equations and apply them to establish nonexistence, uniqueness, and existence-uniqueness of solutions for certain linear fractional boundary value problems. Parallel results are also obtained for sequential fractional differential equations. An example is given to show how computer programs and numerical algorithms can be used to verify the conditions and to apply the results. (Received August 27, 2016)

1125-34-371 **Patrice G Tiffany\*** ([patrice.tiffany@manhattan.edu](mailto:patrice.tiffany@manhattan.edu)), Manhattan College, Mathematics Department, Manhattan College Parkway, Bronx, NY 10471, and **Rosemary C Farley** ([rosemary.farley@manhattan.edu](mailto:rosemary.farley@manhattan.edu)), Manhattan College, Mathematics Department, Manhattan College Parkway, Bronx, NY 10471. *Bringing the Differential Course into the Twenty-First Century With a Computer Algebra System and a Modeling First Approach.*

This presentation will explain how we used a computer algebra system together with a modeling first approach to transform our sections of differential equations. There were two factors that played important roles in this decision.

First, the 2015 report of the Differential Equations Working Group of the CUPM of the MAA stated: "The ODE course is easily the course in the introductory undergraduate mathematics curriculum in which the use of technology is most essential." It also stated that with technology, modeling problems that were previously

inaccessible become accessible. Second, we attended a SIMIODE workshop where we learned about a community dedicated to modeling first in differential equations.

We decided to devote one-third of our class time to a modeling first approach. We held this class in a computer lab and used Maple. We discovered that the modeling first approach works very well when students use a CAS to solve previously inaccessible problems and get answers that make sense. We will provide several examples of modeling problems that worked well and report on some of the expected, and rather unexpected, benefits of this approach. (Received August 30, 2016)

1125-34-441 **Dina M. Yagodich\*** ([dyagodich@frederick.edu](mailto:dyagodich@frederick.edu)). *Baby steps for introducing Modeling First with teaching Differential Equations.*

Changing a teaching style is often a daunting task, especially for a course such as Differential Equations. With so much subject matter to cover, trying to move to a modeling-first method all at once can dissuade instructors from the attempt.

Dina Yagodich will discuss how she added the Modeling First method, starting from a single first-day-of-class M&M modeling activity to integrating modeling more consistently through the course over four semesters, from traditional face-to-face classes to online courses. In addition to modeling, Dina also switched to OER textbooks, which allowed for more flexibility in content. (Received September 01, 2016)

1125-34-450 **B Malcolm Brown\*** ([malcolm@cs.cf.ac.uk](mailto:malcolm@cs.cf.ac.uk)), **Marco Marletta** and **Juan Reves**. *Uniqueness for an Inverse problem in electromagnetism with partial data.*

A uniqueness result for the recovery of the electric and magnetic coefficients in the time-harmonic Maxwell equations from local boundary measurements is shown. No special geometrical conditions are imposed on the inaccessible part of the boundary of the domain, apart from that the boundary of the domain is  $C^{1,1}$ . The coefficients are assumed to coincide on a neighbourhood of the boundary: a natural property in many applications. (Received September 02, 2016)

1125-34-502 **Bilyana Tzolova\*** ([btzolov1@jhu.edu](mailto:btzolov1@jhu.edu)), **Abigail Hickok**, **Justyna Tafoya**, **Robert Loek Van Heyningen**, **Omer Tekin**, **Frank Graziani** and **Chris Scullard**. *Adaptive Polynomial Expansion Method for the Numerical Solution of the Landau Equation.* Preliminary report.

The mathematical description of plasma is a very important area of research with applications in stellar astrophysics and fusion research. The distributions of electrons and protons within plasma can be modeled using the quantum Lenard-Balescu equation. We study a system in which the two species of particles begin at different initial temperatures and evolve to the same equilibrium temperature. In order to solve the differential equations that govern this model, we build upon existing work which use Laguerre polynomials. In particular, we fix a numerical instability which occurs when the initial temperature of the particles are too far from the equilibrium temperature of the system by using an adaptive method that periodically updates the underlying polynomials. (Received September 04, 2016)

1125-34-530 **J. Diego Ramirez\*** ([ramirez@d.savannahstate.edu](mailto:ramirez@d.savannahstate.edu)), 3219 College St., Department of Mathematics, Savannah, GA 31404, and **Zachary Denton**, Greensboro, NC 27411. *Monotone iterative techniques for Riemann Liouville fractional integro-differential equations with initial condition.*

In this work we considered integro-differential initial value problems with Riemann Liouville derivative of order  $q$ ,  $0 < q \leq 1$ . We will define several types of lower and upper solutions and develop two monotone iterative techniques by constructing two sequences that converge uniformly and monotonically to minimal and maximal solutions. In the first result we will construct two natural sequences and in the second result we will construct two intertwined sequences. We will finish this presentation by illustrating our results with an example. (Received September 05, 2016)

1125-34-832 **John R Graef**, **Lingju Kong**, **Qingkai Kong** and **Min Wang\***, Department of Mathematics, Rowan University, Glassboro, NJ 08028. *The existence of positive solutions of a fractional boundary value problem.*

In this paper, the authors study a nonlinear fractional boundary value problem. A method to construct the associated Green's function as a series of functions is presented. Criteria for the existence and uniqueness of positive solutions are then established based on it. (Received September 12, 2016)

1125-34-836 **Ahmed Ghatasheh\*** ([ghatash@uab.edu](mailto:ghatash@uab.edu)), UAB Department of Mathematics, Campbell Hall, 1300 University Boulevard, Birmingham, AL 35294, and **Rudi Weikard**, UAB Department of Mathematics, Campbell Hall, 1300 University Boulevard, Birmingham, AL 35294. *Generalized Sturm-Picone Comparison Theorems.*

The original Sturm-Picone comparison theorem considers two equations of the form  $-(pu')' + qu = 0$  where  $p, q$  are continuous on  $[a, b]$  and  $p > 0$  on  $(a, b)$ . We modify Picone identity to develop Sturm-Picone type comparison theorems for two equations of the form  $-(p(u'+su))' + r p(u'+su) + qu = 0$  where  $1/p, q, r, s$  are integrable on  $(a, b)$  and  $p > 0$  almost everywhere on  $(a, b)$ . We show that there are multiple tests that can be used to do Sturm-Picone type comparison for two such equations. We show that the original Sturm-Picone comparison theorem is a simple case of our new results. (Received September 12, 2016)

1125-34-859 **Jeffrey W. Lyons\*** ([jlyons@nova.edu](mailto:jlyons@nova.edu)), Nova Southeastern University, FAR-MCT, 3301 College Ave, Ft. Lauderdale, FL 33025, and **Samantha A. Major**. *Continuous Dependence and Differentiating Solutions of an  $n$ th Order Boundary Value Problem with Integral Conditions.*

Using a few conditions, continuous dependence, and a result regarding smoothness of initial conditions, we show that derivatives of solutions with respect to the various boundary data of an  $n$ th order boundary value problem with integral boundary conditions solve the associated variational equation with interesting boundary conditions. A corollary is provided. (Received September 12, 2016)

1125-34-1103 **Bhuvaneswari Sambandham\*** ([buna@suu.edu](mailto:buna@suu.edu)) and **Aghalaya S. Vatsala**. *Numerical results for Sequential Caputo fractional boundary value problem.* Preliminary report.

Recently, we developed the generalized monotone method for sequential Caputo fractional boundary value problem with mixed boundary conditions which are in terms of Caputo fractional derivative. We have obtained a representation form for the corresponding linear Caputo sequential boundary value problem in terms of the Green's function. In addition, we have obtained a linear comparison result for sequential Caputo fractional differential inequalities with the mixed boundary conditions. The comparison result is useful in proving the monotonicity of iterates as well as the uniqueness of the solution of the nonlinear sequential boundary value problem. Our method yields, the integer results as a special case. In this work, we will develop the numerical methods to solve a linear sequential boundary value problem which will be used as a tool to develop the numerical methods to solve a nonlinear problem by a generalized monotone iterative process in a later work. (Received September 14, 2016)

1125-34-1169 **Liancheng Wang\*** ([lwang5@kennesaw.edu](mailto:lwang5@kennesaw.edu)), Department of mathematics, Kennesaw State University, 1100 South Marietta Pkwy, Marietta, GA 30060. *Hopf Bifurcation Analysis for a Predator-prey Model with Two Delays.*

In this research, we consider a Predator-prey mathematical model with two delays. The stability of the unique positive equilibrium point is investigated. The Hopf bifurcation analysis is performed and the conditions for Hopf bifurcation are established. The bifurcation diagram is described in terms of the delays and numerical simulations are provided to illustrate the theoretical results. (Received September 15, 2016)

1125-34-1280 **Seshadev Padhi** and **Jaffar Ali\***, Dept. of Mathematics, Fort Myers, FL 33965. *Positive Solutions of first order boundary value problems with nonlinear nonlocal boundary conditions.* Preliminary report.

We consider the existence of positive solutions of the nonlinear first order problem with a nonlinear nonlocal boundary condition given by

$$x'(t) = r(t)x(t) + \sum_{i=1}^n f_i(t, x(t)), t \in [0, 1]$$

$$\lambda x(0) = x(1) + \sum_{j=1}^n \Lambda_j(\tau_j, x(\tau_j)), \tau_j \in [0, 1]$$

where  $r : [0, 1] \rightarrow [0, \infty)$  is continuous and nonlinear functions  $f_i$  and  $\tau_j$  are continuous mappings from  $[0, 1] \times [0, \infty) \rightarrow [0, \infty)$  for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ . Here  $\lambda > 0$  is a parameter and nonlocal points satisfy  $0 \leq \tau_1 < \tau_2 < \dots < \tau_n \leq 1$ . We use Leray-Schauder theorem and Leggett-Williams fixed point theorem to prove our results. (Received September 15, 2016)

1125-34-1308 **Jon Paynter\***, Department of Mathematical Sciences, United States Military Academy, West Point, NY. *Remodel Your Differential Equations Course.*

Traditional differential equations courses emphasize solution techniques, and along the way introduce application problems. This talk outlines a way to transform a differential equations course by using application problems as the primary mode of introducing and reinforcing ODE concepts. Methods include the use of dedicated lessons for scenario introduction and modeling, in-class application assessments, modeling based course projects, and adjusted homework assignments. Specifically, this talk highlights the use of homework assignments that emphasize model construction in lieu of practicing isolated solution techniques. Throughout the course, we collected data on assessment performance and student perception to compare sections with and without these homework assignments. The talk includes details about two of the assignments and the results of the study. Results indicate that instructor emphasis on modeling approaches increases students' ability and confidence in modeling, with no loss in students' performance of traditional solution techniques. (Received September 16, 2016)

1125-34-1354 **Pavel Winternitz\*** ([wintern@crm.umontreal.ca](mailto:wintern@crm.umontreal.ca)), Centre de recherches mathématiques, Université de Montréal, Montreal, QC H3C 3J7, Canada. *Lie group classification of delay ordinary differential equations.*

A Lie group classification is given of first-order delay ordinary differential systems of the form

$$\dot{y} = f(x, y, y_-), \quad x_- = g(x, y, y_-),$$

where  $f$  and  $g$  are arbitrary smooth functions and  $y_- = y(x_-)$  is the value of  $y$  at the "delay point"  $x_-$ . If  $f$  is linear in  $y$  and  $y_-$  and  $g$  depends on  $x$  alone, the Lie symmetry algebra is infinite-dimensional, otherwise it can have dimensions  $0 \leq n \leq 3$ . The aim of the program is to extend the applicability of Lie group methods from differential and difference equations to delay differential ones. This is joint work with V. Dorodnitsyn and R. Kozlov. (Received September 16, 2016)

1125-34-1385 **Aghalaya S. Vatsala\*** ([vatsala@louisiana.edu](mailto:vatsala@louisiana.edu)), Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70504. *Riemann Liouville Versus Caputo Fractional Dynamic equations Via Laplace Transform Methods.* Preliminary report.

Qualitative study such as existence and uniqueness of solutions of nonlinear sequential fractional differential equations with initial conditions are vital due to its application in science and engineering fields. Amongst them, fractional dynamic equations of Riemann-Liouville type and Caputo type are studied vastly because of its close relation to integer dynamic equation. In this work, we apply the Laplace transform method to linear sequential Riemann Liouville and Caputo dynamic equations. We demonstrate that all our results yield the integer dynamic equation results as a special case. This is not true if the corresponding fractional dynamic equation are not sequential. (Received September 16, 2016)

1125-34-1406 **Travis M. Metzger\*** ([tmm011@morningside.edu](mailto:tmm011@morningside.edu)), 25258 400 82nd Ave, Garreston, SD 57030. *Modeling the Zika Virus and Mitigating Microcephaly Outcomes.*

Recently the CDC has issued travel advisories and warnings to women who are, or may become pregnant, in Latin American and Caribbean countries due to large scale outbreaks of the Zika virus. The Zika virus has been linked to microcephaly and Guillain-Barré syndrome. We present a deterministic model for the Zika virus using two coupled systems of different equations for the host and vector populations. Model parameters are obtained from literature. The model is used to evaluate two intervention programs, a birth control and a vector control program. We evaluate which of these programs would result in the largest reduction in microcephaly cases for Brazil and Haiti. (Received September 16, 2016)

1125-34-1626 **Marius Beceanu\*** ([mbeceanu@albany.edu](mailto:mbeceanu@albany.edu)), Department of Mathematics and Statistics, Earth Science 110, University at Albany SUNY, Albany, NY 12222, and **Avy Soffer**. *New methods for the study of supercritical semilinear wave equations.*

I'll present two methods for the study of supercritical wave equations: the decomposition of solutions into incoming and outgoing parts and a (positivity-based) comparison principle. Neither of these methods is completely new, but their application to the study of supercritical equations and the results we obtained are new. (Received September 18, 2016)

1125-34-1656 **Susmita Sadhu\*** ([susmita.sadhu@gcsu.edu](mailto:susmita.sadhu@gcsu.edu)). *Noise induced mixed-mode oscillations and distribution of outbreaks in a bi-trophic ecosystem.*

We study the effect of stochasticity, in the form of Gaussian white noise, in a predator-prey model with two distinct time-scales. The random perturbations, measuring environmental variations, are introduced in the birth

rate of the prey and in the death rate of the predator, leading to a system of stochastic slow-fast equations. We explore the effect of noise near the onset of the singular Hopf bifurcation. The stochastic model admits several kinds of noise driven mixed-mode oscillations that capture the intermediate dynamics between two cycles of population outbreaks of the prey. We study the distribution of the random variable  $N$ , representing the random number of small oscillations between two large oscillations, which can be related to the return time between the outbreaks. Finally, we estimate the probability of repeated outbreaks as a function of the noise intensity and distance to the Hopf bifurcation by transforming the model into a suitable form. (Received September 18, 2016)

1125-34-1893 **Minu Pilvankar\*** ([minu.pilvankar@okstate.edu](mailto:minu.pilvankar@okstate.edu)), 127 N Duck St, Apt F13, Stillwater, OK 74075, and **Ashlee N Ford-Versypt** and **Michele A. Higgins**. *Mathematical Modeling of Glucose Dependent Renin-Angiotensin System in Podocytes in Diabetic Kidney Disease*. Preliminary report.

Diabetic kidney disease (DKD) is the primary cause of end-stage renal failure. Hyperglycemia is known to initiate and worsen the pathophysiology of DKD. Podocytes are terminally differentiated renal cells, vital for maintenance of the renal filtration barrier. Podocytes express a local renin-angiotensin system (RAS) that is altered in hyperglycemia. Studies have shown that a RAS peptide, angiotensin II (ANG II), is modulated in hyperglycemic conditions, triggering podocyte loss. DKD progression can be slowed by controlling the ANG II levels to prevent irreversible podocyte loss. However, experimental evidence for glucose-dose-dependency of ANG II is scarce in the literature. Hence, we use mathematical modeling for a better insight into the underlying mechanism. The model describes local RAS network by a system of ordinary differential equations that triggers the synthesis of ANG II. Different parameterization approaches were analyzed to add glucose-dependency through the parameters. Sensitivity analysis was conducted to identify key ANG II-modulating biomarkers. The model was used to study the change in ANG II concentrations with varying glucose levels. The model can be used to study the effect of different ANG II modulating therapies which could be useful for drug development. (Received September 19, 2016)

1125-34-1911 **Paul Eloe**, 300 College Park, Dayton, OH 454692316, and **Tyler Masthay\*** ([tmasthay1@udayton.edu](mailto:tmasthay1@udayton.edu)), 300 College Park, Dayton, OH 454692316. *Uniqueness implies existence of solutions for three-point boundary value problems for fractional differential equations*.

Let  $a < b$  and assume  $2 < \alpha \leq 3$ . For each

$$a < x_1 < x_2 < x_3 < b, \quad y_1, y_2, y_3 \in \mathbb{R},$$

consider boundary value problems for the fractional differential equation

$$D_{*x_1}^\alpha y(x) = f(x, y(x), y'(x), y''(x)), \quad x_1 < x < b, \quad (1)$$

with boundary conditions

$$y(x_1) = y_1, \quad y(x_2) = y_2, \quad y'(x_2) = y_3, \quad (2)$$

or

$$y(x_1) = y_1, \quad y(x_2) = y_2, \quad y(x_3) = y_3, \quad (3)$$

where  $D_{*x_1}^\alpha y(x)$  denotes the Caputo fractional derivative of order  $\alpha$ . We obtain sufficient conditions such that if solutions of (1), (3) are unique when they exist, then for all  $a < x_1 < x_2 < x_3 < b$ ,  $y_1, y_2, y_3 \in \mathbb{R}$ , solutions of (1), (2) and solutions of (1), (3) exist.

As part of the development, a compactness criterion for families of solutions of (1) is obtained. (Received September 19, 2016)

1125-34-1924 **Paul Eloe\***, 300 College Park, Dayton, OH 454692316, and **Tyler Masthay**, 300 College Park, Dayton, OH 454692316. *Uniqueness implies existence of solutions for two-point boundary value problems for fractional differential equations*.

Let  $a < b$  and assume  $1 < \alpha \leq 2$ . For each  $a < x_1 < x_2 < b$ ,  $y_1, y_2 \in \mathbb{R}$ , we consider the two-point conjugate type boundary value problem for the fractional differential equation

$$D_{*x_1}^\alpha y(x) = f(x, y(x), y'(x)), \quad x_1 < x < b, \quad (1)$$

$$y(x_1) = y_1, \quad y(x_2) = y_2, \quad (2)$$

where  $D_{*x_1}^\alpha y(x)$  denotes the Caputo fractional derivative of order  $\alpha$ . We obtain the following analogue of a well-known result for ordinary differential equations: if

**(A):**  $f : (a, b) \times \mathbb{R}^2 \rightarrow \mathbb{R}$  is continuous,

**(B):** if for each  $a < x_1 < b$ ,  $y_1, m \in \mathbb{R}$ , solutions of (1) satisfying initial conditions

$$y(x_1) = y_1, \quad y'(x_1) = m,$$

are unique and extend to all of  $(x_1, b)$ , and

**(C):** if for all  $a < x_1 < x_2 < b$ ,  $y_1, y_2 \in \mathbb{R}$ , solutions of (1), (2) are unique, when they exist,

then for all  $a < x_1 < x_2 < b$ ,  $y_1, y_2 \in \mathbb{R}$ , solutions of (1), (2) exist. (Received September 19, 2016)

1125-34-2075 **Jeffrey Thomas Neugebauer\*** ([jeffrey.neugebauer@eku.edu](mailto:jeffrey.neugebauer@eku.edu)), 521 Lancaster Ave, 313 Wallace Building, Richmond, KY 40475. *Extremal points for a fractional boundary value problem with a fractional boundary condition.*

The theory of  $u_0$ -positive operators with respect to a cone in a Banach space is applied to study the boundary value problem for Riemann-Liouville fractional linear differential equation  $D_{0+}^\alpha u + p(t)u = 0$ ,  $0 < t < b$ , satisfying boundary conditions  $u^{(i)}(0) = 0$ ,  $i = 0, 1, \dots, n-2$ ,  $D_{0+}^\beta u(1) = 0$ ,  $b > 0$ ,  $n-1 < \alpha \leq n$ ,  $0 \leq \beta \leq n-1$ . The first extremal point, or conjugate point, of the conjugate boundary value problem is defined and criteria are established to characterize the conjugate point. (Received September 19, 2016)

1125-34-2126 **Lindsay K Bradley\*** ([bradley14@winthrop.edu](mailto:bradley14@winthrop.edu)), 2061 Dunlap Roddey Road, Rock Hill, SC 29730, and **Kristen Abernathy** and **Zachary Abernathy**. *A Mathematical Model of Chronic Myeloid Leukemia with Treatment.* Preliminary report.

Chronic Myeloid Leukemia (CML) is a prevalent type of cancer where the presence of cancer stem cells is well studied. In this talk, we modify existing Gompertzian growth models to study the dynamics of CML and the effects of treatment on CML. In the absence of treatment, we demonstrate that the cure state is always unstable. We then present conditions on treatment parameters to guarantee a locally stable cure state. We conclude with numerical simulations and remaining open questions. (Received September 19, 2016)

1125-34-2157 **Roger J Thelwell\*** ([thelwerj@jmu.edu](mailto:thelwerj@jmu.edu)), Department of Math and Statistics, MSC 1911, Harrisonburg, VA 22807. *Taylor Methods in Nonlinear Differential Equations.* Preliminary report.

Taylor series from the foundation for a wide variety of numerical methods in undergraduate numerical courses. Often, however, their apparent complexity when applied to nonlinear ordinary differential equations motivates the introduction of Runge-Kutta and other numerical integration schemes. In this talk, we re-examine the applicability of Taylor methods, thanks to modern Computer Algebra Systems such as Maple and Mathematica. Taylor methods can now be easily applied to problems ranging from initial value problems and boundary value problems, and make the analysis of stability and control a straightforward exercise. A number of open problems will be explored to demonstrate the approach. (Received September 19, 2016)

1125-34-2342 **Aladeen Al Basheer\*** ([albasha@clarkson.edu](mailto:albasha@clarkson.edu)), Clarkson University, 8 Clarkson Ave 5815, POTSDAM, NY 13699, **Rana D Parshad** ([rparshad@clarkson.edu](mailto:rparshad@clarkson.edu)), Clarkson University, 8 Clarkson Ave 5815, potsdam, NY 13699, **Emmanuel Quansah** ([quansae@clarkson.edu](mailto:quansae@clarkson.edu)), Clarkson University, 8 Clarkson Ave 5815, potsdam, NY, and **Suman Bhowmick** ([bhowmis@clarkson.edu](mailto:bhowmis@clarkson.edu)), Clarkson University, 8 Clarkson Ave 5815, potsdam, 13699. *Prey cannibalism alters the dynamics of Holling-Tanner-type predator-prey models.*

Cannibalism, which is the act of killing and consumption of conspecifics, has been considered primarily in the predator, despite strong ecological evidence that it exists among prey. In the current manuscript, we investigate both the ODE and spatially explicit forms of a Holling-Tanner model, with ratio dependent functional response, and show that cannibalism in the predator provides a stabilizing influence as expected. However, when cannibalism in the prey is considered, we show that it cannot stabilize the unstable interior equilibrium in the ODE case, in certain parameter regime, but can destabilize the stable interior equilibrium, leading to a stable limit cycle or “life boat” mechanism, for prey. We also show that prey cannibalism can lead to pattern forming Turing dynamics, which is an impossibility without it. (Received September 20, 2016)

1125-34-2458 **Darryl K Nester\*** ([nesterd@bluffton.edu](mailto:nesterd@bluffton.edu)), Bluffton University 52, 1 University Drive, Bluffton, OH 45817. *Slope and Direction Fields — Online, Without Java.*

Software to produce slope and direction fields is practically indispensable for teaching a differential equations course, but finding a suitable program can be tricky. The proprietary software included with some textbooks might not be compatible with all computers (and might not be updated as operating systems change). Online options like the jOde applet have become difficult to use because of increasing security restrictions on Java

applets; even if you can run it on your computer, will it work on the classroom computer? Or on your students' computers?

Faced with this problem while teaching DEs two years ago, I created [www.bluffton.edu/~nesterd/java/slopefields.html](http://www.bluffton.edu/~nesterd/java/slopefields.html). With helpful feedback from colleagues at other schools (who found it while searching for alternative to jOde), I've refined it and added a number of helpful features, and it is now listed as a resource by CODEE.org and SIMIODE.org. (Received September 20, 2016)

1125-34-2498 **Bo Yang\*** ([byang@kennesaw.edu](mailto:byang@kennesaw.edu)). *Upper and Lower Estimates for Positive Solutions of a Sixth Order Boundary Value Problem*. Preliminary report.

In this paper, we consider a sixth order boundary value problem. Upper and lower estimates for positive solutions of the problem are obtained. Sufficient conditions for the existence and nonexistence of positive solutions are established. (Received September 20, 2016)

1125-34-2620 **Vijay Jung Kunwar\*** ([vijay.kunwar@asurams.edu](mailto:vijay.kunwar@asurams.edu)) and **Mark van Hoeij** ([hoeij@math.fsu.edu](mailto:hoeij@math.fsu.edu)). *Solving Second Order Linear Differential Equations with Five Regular Singularities*.

Hypergeometric functions play a significant role on solving linear differential equations with regular singularities. Gauss hypergeometric function, also known as  ${}_2F_1$  function, is a solution of the second order linear differential equation:

$x(1-x)y'' + (c - (a+b+1)x)y' - aby = 0$ , also called Gauss hypergeometric differential equation which has 3 regular singularities at  $x = 0, 1, \text{ and } \infty$ . Based on this property, we develop algorithms to find hypergeometric solutions of differential equations which have (i)  $n$  regular singularities, or (ii) solutions with specific degree  $d$ . In this presentation we will talk about our algorithm designed to find hypergeometric solutions of second order linear differential equations with 5 regular singularities, at least one logarithmic. (Received September 20, 2016)

1125-34-2790 **Vardayani Ratti\*** ([vardayani.ratti@dartmouth.edu](mailto:vardayani.ratti@dartmouth.edu)), **Peter G Kevan** and **Hermann J Eberl**. *Modeling the interplay between Varroa destructor-Acute Bee Paralysis Virus infestation and division of labour in a honeybee colony*.

The western honeybees are vanishing. Recent years have seen honeybees in distress, with up to 35% of colonies breaking down annually. There is no single cause that is believed to be responsible for the colony losses. In this talk, a mathematical model for the honeybees-varroa mites-virus complex is presented in which, based on division of labour, the bee population is divided into two categories: hive bees and forager bees. The model consists of ordinary differential equations for the dependent variables: uninfected hive bees, uninfected foragers, infected hive bees, number of mites overall, and of mites carrying the virus. The main objective of the model is to study the interplay between disease propagation and division of labour in a honeybee colony. The model focuses on Acute Bee Paralysis Virus and is studied with analytical and computational techniques. We use well established methods for autonomous systems to study the stability of equilibria. Using computer simulations, we investigate whether the results of the autonomous case carry over to the case where the coefficients are functions of time. (Received September 20, 2016)

1125-34-2818 **Hem Raj Joshi\*** ([joshi@xavier.edu](mailto:joshi@xavier.edu)) and **Mark Miller**. *Modeling Harmful Algal Blooms in the Western Basin of Lake Erie and their Economic Impact*.

We propose a simple relationship between algal species and Zebra Mussel through a basic ecological predator-prey model: the Lotka-Volterra model. Through this model, we can learn what level of Zebra Mussel population will maximize water transparency through the elimination of algal populations with the most important result being the elimination of large-scale algal blooms. Then, the economic impact of the bloom will be assessed, as well as the necessary decrease in phosphorus loading needed to sufficiently reduce the risk of compromising the Toledo water supply. (Received September 20, 2016)

1125-34-3079 **Padmanabhan Seshaiyer** ([pseshaiy@gmu.edu](mailto:pseshaiy@gmu.edu)), 4400 University Drive, MS 3F2, George Mason University, Fairfax, VA 22030, and **Pradyuta Padmanabhan\*** ([pradyuta@gmail.com](mailto:pradyuta@gmail.com)), 4400 University Drive, MS 3F2, George Mason University, Fairfax, VA 22030. *Mathematical modeling, analysis and computation of the influence of preventive measures on the spread of the Zika Virus*. Preliminary report.

Recently, the Zika arbovirus transmitted through the *Aedes aegypti* mosquitoes has been shown to be transmitted to humans, not only through vector transmission, but also through sexual contact. While there is a lot of research currently being conducted to find vaccines for the treatment of the disease, other methods of prevention and eradication of the disease recommended by the Center for Disease Control and Prevention include using insecticide treated bed nets (ITN) and indoor residual spraying (IRS). The ITNs developed using chemicals

such as pyrethroids, can maintain effective levels of insecticide for a long time as well as repel mosquitoes. In this work, we investigate an enhanced mathematical model that incorporates ITNs and IRS as methods for eradication of Zika. In this work, we develop an ordinary differential equation system that builds on classical SEIR epidemiological models, with added constraints for the two preventive measures, namely ITN and IRS. We derive the basic reproduction number analytically and compute the final size for the epidemic for various conditions involving ITNs and IRS numerically. We present ranges for combination of compliance and efficacy for ITNs and IRS that can potentially eradicate the disease. (Received September 21, 2016)

1125-34-3097 **Daniel Allin Korytowski\*** (dkorytow@asu.edu) and **Hal Smith**. *Permanence and Stability of a Kill the Winner Model in Marine Ecology*.

We focus on the long term dynamics of "killing the winner" Lotka-Volterra models of marine communities consisting of bacteria, virus, and zooplankton. Under suitable conditions, it is shown that there is a unique equilibrium with all populations present which is stable, the system is permanent, and the limiting behavior of its solutions is strongly constrained. (Received September 21, 2016)

1125-34-3114 **Stephen M Steward\*** (stewards2@winthrop.edu), 416 Mariah St., Fort Mill, SC 29715, and **Joshua B Dasburg**. *Incorporating the Cancer Stem Cell Hypothesis into a Treatment Model of Glioblastoma Multiforme*. Preliminary report.

In this talk, we extend the work of Kronik, Kogan, Vainstein, and Agur (2008) by incorporating the cancer stem cell hypothesis into a treatment model for Glioblastoma Multiforme. Cancer Stem Cells (CSCs) are a specialized form of tumor cell with normal adult stem cell properties. CSCs are believed to be one of the primary reasons for cancer recurrence since they are more resilient to current treatment practices and are able to repopulate the tumor once their own population has regenerated. We present a system of nonlinear ordinary differential equations that describes the interaction between cancer stem cells, tumor cells, and alloreactive cytotoxic T-lymphocytes (CTLs). Under the assumption of constant treatment, we present sufficient conditions for a treatment threshold that ensures a cure state that is globally asymptotically stable. We also explore a more biologically accurate treatment schedule in which CTLs are injected periodically. We consider cases where treatment is applied continuously over varying time intervals, as well as treatment injections using the Dirac Delta function. We conclude with a discussion of biological implications. (Received September 21, 2016)

## 35 ► Partial differential equations

1125-35-11 **Gigliola Staffilani\***, Massachusetts Institute of Technology, Cambridge, MA 02139. *The many faces of dispersive and wave equations*.

The mathematical nature of dispersion is the starting point of a very rich mathematical activity that has seen incredible progress in the last twenty years, and that has involved many different branches of mathematics: Fourier and harmonic analysis, analytic number theory, differential and symplectic geometry, dynamical systems and probability. In this talk I will give examples of these diverse directions and related open problems. (Received May 22, 2016)

1125-35-36 **Xin Yang\*** (yangxin1@msu.edu), 3313 Trappers Cove Trail, Apt 2D, Lansing, MI 48910. *Blow-up problems for the heat equation with a local nonlinear Neumann boundary condition*.

This paper estimates the blow-up time for the heat equation  $u_t = \Delta u$  with a local nonlinear Neumann boundary condition: The normal derivative  $\partial u / \partial n = u^q$  on  $\Gamma_1$ , one piece of the boundary, while on the rest part of the boundary,  $\partial u / \partial n = 0$ . The motivation of the study is the partial damage to the insulation on the surface of space shuttles caused by high speed flying subjects. We will prove the finite time blow-up of the solution and estimate both upper and lower bounds of the blow-up time in terms of the area of  $\Gamma_1$ . In many other works, they need the convexity of the domain  $\Omega$  and only consider the problem with  $\Gamma_1 = \partial\Omega$ . In this paper, we remove the convexity condition and in addition, we deal with the local nonlinearity, namely  $\Gamma_1$  can be just part of  $\partial\Omega$ . (Received June 07, 2016)

1125-35-38 **Michael Victor Klivanov\*** (mklibanv@unc.edu), Department of Mathematics and Statistics, University of North Carolina at Charlotte, Charlotte, NC 28223, and **Dinh Liem Nguyen, Loc Hoang Nguyen and Aleksandr Egor Kolesov**. *Globally convergent numerical methods for coefficient and phaseless inverse problems*.

Coefficient inverse problems are both ill-posed and nonlinear. These two factors cause the well known phenomena of multiple local minima and ravines of corresponding least squares Tikhonov functional. The latter means in turn

that the convergence of any optimization to the exact solution cannot be rigorously proved unless a its starting point is located in a sufficiently small neighborhood of that solution. This is the so-called "local convergence". To the contrary, the group led by the first author has made a significant progress in the last eight years in the development of globally convergent numerical methods for these problems. In other words, theorems are proven which establish that each of these methods delivers some points in sufficiently small neighborhood of the exact solution without any a priori knowledge of this neighborhood. At least one of those methods will be presented, along with computational results. In addition, uniqueness theorems and reconstruction methods will be presented for coefficient inverse problems without the phase information. In fact, these problems are closely linked with the first topic. (Received June 10, 2016)

1125-35-91 **Ugur Abdulla, Montie Avery, Jian Du, Naba Mukhtar\*** (nhm005@bucknell.edu) and **Adam Prinkey**. *Qualitative Properties of Solutions to the Nonlinear Parabolic PDEs with Double-Degenerate Fast Diffusion.*

We consider the Cauchy problem for the double degenerate parabolic equation  $u_t = (|(u^m)_x|^{p-1}(u^m)_x)_x - bu^\beta$ ,  $x \in \mathbb{R}$ ,  $t > 0$ ,  $u(x, 0) = C(-x)_+^\alpha$ , where  $C, \alpha, m, p > 0, b \in \mathbb{R}$ . We prove the existence/non-existence of an interface for this problem and the explicit asymptotics of the solution near the interface or at infinity. Our research focuses on the open case of fast diffusion range:  $0 < mp < 1$ . The properties of the case when  $p = 1$  were fully classified in [U.G.Abdulla, *Nonlinear Analysis*, 50, 4(2002), 541-560]. Through scaling analysis we prove the interface behavior is determined by the competition between diffusion and reaction. For  $\beta < mp$ , the interface propagates with finite speed and expands or shrinks depending on the value of  $\alpha$ . We prove explicit bounds for the interface and the solution  $u$ . If  $\beta \geq mp$ , we prove that the interface expands with infinite speed and we derive explicit asymptotic formulas for the solution at infinity. The rigorous methods we apply include scaling, construction of super- and sub- solutions to the problem, and special comparison theorems in general domains. We also confirmed our results numerically with a WENO scheme and developed test problems for a new numerical interface tracking method. (Received July 20, 2016)

1125-35-92 **Ugur Abdulla, Lamees Alzaki and Robert Balkin\***, rbalkin@mymail.mines.edu, and **Jian Du and Elizabeth Schloss**. *Evolution of Free Boundaries for the Nonlinear Fokker-Planck Equation with Reaction.*

We investigate the problem on short-time behavior of free boundaries and local solutions near it in the following Cauchy problem for the nonlinear Fokker-Planck equation with reaction:

$$u_t = (u^m)_{xx} + a(u^\gamma)_x + bu^\beta, x \in \mathbb{R}, t > 0; u|_{t=0} = C(-x)_+^\alpha$$

where  $m > 1, \gamma > 0, \beta > 0$ . The problem of determining the short-time behavior of the free boundaries, or interfaces, is the Barenblatt problem. The full solution of this problem for the reaction-diffusion equation was given in [Abdulla and King, *SIAM J. Math. Anal.*, 32, 2(2000), 235-260] and [Abdulla, *Nonlinear Analysis*, 50, 4(2002), 541-560]. The goal of this project is to apply the methods of these papers to solve the open problem for the above equation.

By applying scaling method, we identify seven regions of the parameter space that correspond to different short-time behavior, and prove explicit asymptotic formula for the local solution near the interface in all cases. Based on these asymptotic results, and with further application of the method of super- and subsolutions, we prove the asymptotic formula for the short-time behavior of the interfaces. A WENO numerical scheme was applied to the problem and numerical results support our proved estimations. (Received July 20, 2016)

1125-35-93 **Ugur G. Abdulla, R. Beekie\*** (beeki001@umn.edu), **V. Bakshtynov, E. Cosgrove, J. Jones, S. Seif and N. Wubshet** (nwubshet13@ole.augie.edu). *Fréchet Differentiability in Optimal Control of the Stefan Problem.*

We prove Fréchet differentiability and derive the formula for the Fréchet gradient in the optimal control of the Stefan type free boundary problem for a linear second order parabolic PDE. The reaction coefficient, unknown free boundary, and boundary flux are components of the control vector. The cost functional consists of the  $L_2$ -declination of the trace of the temperature at the final moment, temperature at the fixed boundary and final position of the free boundary from available measurements. We follow a new variational formulation developed in *U. G. Abdulla, Inverse Problems and Imaging*, 7,2(2013),307-340.

In this project we use infinite dimensional calculus in Besov-Hilbert space framework. We introduce the adjoint problem as an infinite dimensional analogue of Lagrange multipliers. Through the use of sharp embedding theorems in fractional Sobolev-Besov spaces we derive the formula for the Fréchet gradient expressed in terms of the state vector and the solution of the adjoint problem. We use the Fréchet differentiability result and

necessary optimality condition for the implementation of a projective gradient method in Hilbert space setting for the numerical solution of the problem. (Received July 20, 2016)

1125-35-107 **Wenqiang Feng\*** ([wfeng1@utk.edu](mailto:wfeng1@utk.edu)), Department of Mathematics, The University of Tennessee, Knoxville, TN 37996, **Cheng Wang** ([cwang1@umassd.edu](mailto:cwang1@umassd.edu)), Department of Mathematics, The University of Massachusetts, North Dartmo, North Dartmouth, MA 02747, and **Steven Matthew Wise** ([swise1@utk.edu](mailto:swise1@utk.edu)), Department of Mathematics, The University of Tennessee, Knoxville, TN 37996. *An Energy Stable Finite-Difference Scheme for Functionalized Cahn-Hilliard Equation and its Convergence Analysis*. Preliminary report.

We present and analyze an unconditionally energy stable and convergent finite difference scheme for the Functionalized Cahn-Hilliard and the Cahn-Hilliard-Willmore equations. One key difficulty associated with the energy stability is based on the fact that, one nonlinear energy functional term in the expansion appears as non-convex, non-concave. To overcome this subtle difficulty, we add two auxiliary terms to make the combined term convex, which in turns yields a convex-concave decomposition of the physical energy. As a result, an application of the convex splitting methodology assures both the unique solvability and the unconditional energy stability of the proposed numerical scheme. To deal with a 4-Laplacian solver in an  $H^{-1}$  gradient flow at each time step, we apply an efficient preconditioned steepest descent algorithm to solve the corresponding nonlinear systems. In addition, a global in time  $H_{\text{per}}^2$  stability of the numerical scheme is established at a theoretical level, which in turn ensures the full order convergence analysis of the scheme. A few numerical results are presented, which confirm the stability and accuracy of the proposed numerical scheme. (Received July 29, 2016)

1125-35-109 **Oleksandr Misiats\*** ([om25@nyu.edu](mailto:om25@nyu.edu)), 15 Washington pl, 4M, New York, NY 10003, and **Robert Kohn** and **Stefan Muller**. *Analysis of microstructures in martensite-austenite phase transitions using sharp variational bounds*.

We consider a singularly perturbed variational problem of minimizing the the surface and elastic energies of the order parameter  $u$  in a two-dimensional rectangular domain. This model, originally suggested by Kohn and Muller, comes from martensitic phase transitions, in which two distinct phases of the martensite correspond to  $u_y(x, y) = 1$  and  $u_y(x, y) = -1$ . It was observed that minimizers develop self-similar branched microstructures in the case when the boundary condition is not compatible with either of the phases, and may have zigzag microstructures otherwise. In my talk, I will describe several patterns of the behavior of minimizers depending on the choice of boundary conditions, derive sharp global and local energy bounds, and discuss the applications to 2D and 3D linear elasticity models. (Received July 30, 2016)

1125-35-140 **Aviv Gibali\*** ([avivg@braude.ac.il](mailto:avivg@braude.ac.il)), Department of Mathematics, ORT Braude College, P.O. Box 78, Karmiel 2161002, Israel, 2161002 Karmiel, Israel. *Gradient and Extragradient methods for the inverse problem of tumor identification*.

In this talk we consider an inverse problem of parameter identification in linear incompressible elasticity with an application to tumor identification within the interior of the human body. We present several gradient-based and extragradient methods for the solution of the elastography inverse problem arising in the identification of cancerous tumors. From a mathematical standpoint, this inverse problem requires the identification of a variable parameter in a system of incompressible elasticity. We use an equation error approach to formulate the inverse problem as a convex optimization problem.

Joint work with B. Jadamba, A. A. Khan, F. Raciti and B. Winkler. (Received August 03, 2016)

1125-35-149 **Dipendra Regmi\*** ([regmid@farmingdale.edu](mailto:regmid@farmingdale.edu)), Farmingdale State College, SUNY, 2350 Broadhollow Rd, Farmingdale, NY 11735. *Global regularity for the 2D magneto-micropolar equations with partial dissipation*.

We study the global existence and regularity of classical solutions to the 2D incompressible magneto-micropolar equations with partial dissipation. The magneto-micropolar equations model the motion of electrically conducting micropolar fluids in the presence of a magnetic field. When there is only partial dissipation, the global regularity problem can be quite difficult. We are able to single out three special partial dissipation cases and establish the global regularity for each case. As special consequences, the 2D Navier-Stokes equations, the 2D magnetohydrodynamic equations, and the 2D micropolar equations with several types of partial dissipation always possess global classical solutions. This is a joint work with J. Wu. (Received August 04, 2016)

1125-35-186 **Mourad Sini\***, Altenbergersstrasse 69, Linz, Austria. *Groeger-Meyers's estimate and justification of the enclosure method for the Maxwell system.*

The enclosure method is a direct reconstruction method designed to recover the shape of an inclusion from its response to probing waves of the form of geometric optics. Compared to the well known reconstruction methods, linearity is not necessary. However, as it uses geometric optics solutions as probing signals, it provides only 'high frequency' features of the shapes as the support function, the distance function, etc. This method is proposed by M. Ikehata and then developed and refined by several other authors as G. Nakamura, J. N. Wang, G. Uhlmann and many others. To justify it, some a priori geometrical conditions are imposed to the shapes. In addition, its extension to the full Maxwell system was an open issue.

In this talk, we explain how we can remove these geometrical conditions and extend the method to the full Maxwell system. One of the basic tools we use is the so called Groeger-Meyers's estimate which means, in particular, that the solution operator is an isomorphism in the  $L^p$ -spaces with a range of the power  $p$  depending on the contrast of the materials defining the shape. We derive such an estimate for the full Maxwell system and use it to justify the enclosure method avoiding any geometrical condition. (Received August 10, 2016)

1125-35-204 **Yongzhi Steve Xu\*** (ysxu0001@louisville.edu), Department of Mathematics, University of Louisville, Louisville, KY 40292. *Study of some inverse problems raised from a free boundary model of ductal carcinoma in situ.*

In this talk we discuss some inverse problems raised from a free boundary model of ductal carcinoma in situ (DCIS). In [1], we introduced a number of inverse problems of free boundary model of DCIS. We will present some recent results related to the inverse problems.

We use some transformation and heat potential theory to establish the integral form of solution and proved the existence and uniqueness of solution to the related problem. Then we consider the inverse problem of determining the potential function of model from moving boundary information, which related to the mammography screening of DCIS. Algorithm and numerical simulation are presented to demonstrate the validity and applicability of solutions. (Received August 11, 2016)

1125-35-280 **Wen-Xiu Ma\*** (mawx@cas.usf.edu), Department of Mathematics and Statistics, Tampa, FL 33620. *Integrable Hamiltonian equations from matrix loop algebras.*

We will talk about a zero curvature formulation to integrable Hamiltonian equations associated with matrix loop algebras, both semisimple and non-semisimple. Hamiltonian structures and Liouville integrability will be established by either the trace identity or the variational identity. The two real three dimensional Lie algebras,  $\mathfrak{sl}(2, \mathbb{R})$  and  $\mathfrak{so}(3, \mathbb{R})$ , will be used to show illustrative examples of integrable Hamiltonian equations. (Received August 22, 2016)

1125-35-296 **Carlos E Kenig\*** (cek@math.uchicago.edu), University of Chicago, Chicago, IL 60637. *Overview: The focusing energy critical wave equation.* Preliminary report.

The theory of nonlinear dispersive equations has seen a spectacular development in the last 35 years. The initial works studied the behavior of special solutions such as solitons/traveling waves. Then, there was a systematic study of the well-posedness theory, using extensively tools from harmonic analysis. The last 25 years have seen a lot of interest in the study of the long-time behavior of solutions, for large data. Issues like blow-up, global existence, scattering and long-time asymptotic behavior have come to the forefront, especially in critical problems. We will concentrate the discussion on the energy critical wave equation, in the focusing case. In the defocusing case, it was shown (1990-2000) that all (large) data in the energy space yield global in time solutions which scatter. The focusing case is very different since one can have finite time blow-up, even for solutions which remain bounded in the energy norm, and solutions which exist for all time and are bounded in the energy norm, but do not scatter, for instance traveling waves. In this lecture I will give an overview of the progress in the last 10 years in the program of obtaining a complete understanding of the global dynamics of solutions which remain bounded in the energy space. (Received August 24, 2016)

1125-35-297 **Carlos E Kenig\*** (cek@math.uchicago.edu), University of Chicago, Chicago, IL 60637. *The focusing energy critical wave equation: the radial case in 3 space dimensions.* Preliminary report.

In this second lecture we will concentrate on the classification, in terms of their asymptotic behavior, of radial solutions of the focusing energy critical wave equation in the radial three dimensional case. This will include a discussion of the proof of soliton resolution, for solutions bounded in the energy norm (Duyckaerts-Kenig-Merle, 2013). (Received August 24, 2016)

1125-35-298 **Carlos E Kenig\*** ([cek@math.uchicago.edu](mailto:cek@math.uchicago.edu)), University of Chicago, Chicago, IL 60637.  
*The focusing energy critical wave equation: the non-radial case.* Preliminary report.

In this third lecture we will give an outline of the proof of soliton resolution, along a well-chosen sequence of times, for general (non-radial) solutions of the energy critical wave equation which remain bounded in the energy norm (for dimensions 3,4,5,6). This is work of Duyckaerts-Jia-Kenig-Merle from 2016. (Received August 24, 2016)

1125-35-326 **Marian Bocea\*** ([mbocea@luc.edu](mailto:mbocea@luc.edu)), Loyola University Chicago, Department of Mathematics and Statistics, 1032 W. Sheridan Road, Chicago, IL 60660. *On a family of inhomogeneous torsional creep problems.*

The asymptotic behavior of solutions to a family of inhomogeneous PDEs in divergence form is studied in an Orlicz-Sobolev setting. Solutions are shown to converge uniformly to the distance function to the boundary of the domain. One consequence is that a well-known result in the analysis of problems modeling torsional creep continues to hold under much more general constitutive assumptions on the stress. Joint work with M. Mihailescu. (Received August 26, 2016)

1125-35-333 **Dimitrios Roxanas\*** ([droxanas@math.ubc.ca](mailto:droxanas@math.ubc.ca)), Vancouver. *Long-time dynamics of solutions to the focusing energy-critical heat equation.*

We study the focusing energy-critical nonlinear heat equation  $u_t - \Delta u - |u|^2 u = 0$ , in  $\mathbb{R}^4$ . We prove that solutions emanating from initial data with energy and kinetic energy below those of the stationary solution are global and decay to zero. We show that global solutions dissipate to zero building on a refined small data theory and  $L^2$ -dissipation, expanding on ideas that have previously been applied to the Navier-Stokes system. To rule out the possibility of blow-up we argue by resorting to the “concentration-compactness plus rigidity” approach of Kenig and Merle for dispersive equations. We exploit the dissipation but our proof does not rely on maximum/comparison principles. The above result extends to all dimensions  $d \geq 3$ . This is joint work with Stephen Gustafson. (Received September 18, 2016)

1125-35-342 **Tetyana Malysheva\*** ([malyshe@uwgb.edu](mailto:malyshe@uwgb.edu)) and **Luther W. White** ([lwhite@ou.edu](mailto:lwhite@ou.edu)).  
*Weak well-posedness of a fully coupled model of chemical thermo-poroelasticity arising in petroleum rock mechanics.*

A fully coupled system of partial differential equations constituting the general model of chemical thermo-poroelasticity for a fluid-saturated porous media will be considered. The model consists of parabolic equations representing thermal, solute, and fluid diffusions coupled with Navier-type elliptic equations with time as a parameter describing elastic deformation of porous medium with incorporated thermal, solute, and fluid flow effects.

The well-posedness theory for the initial-boundary value problem arising from petroleum rock mechanics applications is developed. It is shown that, subject to some natural assumptions imposed on rock/fluid parameters, there exists a unique weak solution to the problem and this solution depends continuously on the initial and boundary data. The proof is based on the proposed pseudo-decoupling technique and the standard Galerkin method with the refined element-wise approach to energy estimates. Numerical experiments confirm the applicability of the obtained well-posedness results for models with real data parameters. (Received August 28, 2016)

1125-35-390 **Huajun Gong** and **Tao Huang\***, NYU-ECNU Institute of Mathematical Sciences, at NYU Shanghai, and **Jinkai Li**. *Nonuniqueness of weak solutions to the nematic liquid crystal flows.*

For suitable initial data, we construct infinitely many weak solutions to the nematic liquid crystal flows in dimension three. All the solutions are in the axisymmetric class with “backward bubbling” at a large time and bounded energy for any finite time. (Received August 30, 2016)

1125-35-456 **Qingshan Chen\*** ([qsc@cllemson.edu](mailto:qsc@cllemson.edu)), Department of Mathematical Sciences, Martin O-210, Clemson University, Clemson, SC 29631. *The quasi-geostrophic equations for large-scale geophysical flows with a free surface.* Preliminary report.

When the length scale of the flow is on the same order of the Rossby deformation radius, the classical rigid-lid assumption is no longer valid, the impact of the free surface deformation on the vorticity field is no longer negligible, and therefore it has to be accounted for in the model. In this talk, we present some new results concerning the well-posedness of the barotropic quasi-geostrophic equation under a free surface. Both simply connected domains and domains with holes representing above-surface islands will be considered. The connection

of the QG model to other more complex and more realistic models will also be discussed. (Received September 02, 2016)

1125-35-471 **Yuri Latushkin** and **Alim Sukhtayev\*** (alimsukh@iu.edu). *The Maslov index, spectral flow, Hadamard formula: recent advances.*

In this talk we will address recent results on relations between the Morse and Maslov indices for differential operators on a finite interval, full line, and in multidimensional domains. We will also discuss a Hadamard-type formula relating the rate of change of the eigenvalues of the differential operators when the domain is changing and the value of the (Maslov) crossing form. (Received September 02, 2016)

1125-35-479 **Pei Pei\*** (ppe@otterbein.edu), **Mohammad Rammaha** and **Daniel Toundykov**. *Weak solutions and blow-up for wave equations of  $p$ -Laplacian type with supercritical sources.*

This paper investigates a quasilinear wave equation with Kelvin-Voigt damping,  $u_{tt} - \Delta_p u - \Delta u_t = f(u)$ , in a bounded domain  $\Omega \subset \mathbb{R}^3$  and subject to Dirichlet boundary conditions. The operator  $\Delta_p$ ,  $2 < p < 3$ , denotes the classical  $p$ -Laplacian. The nonlinear term  $f(u)$  is a source feedback that is allowed to have a supercritical exponent, in the sense that the associated Nemytskii operator is not locally Lipschitz from  $W_0^{1,p}(\Omega)$  into  $L^2(\Omega)$ . Under suitable assumptions on the parameters, we prove existence of local weak solutions, which can be extended globally provided the damping term dominates the source in an appropriate sense. Moreover, a blow-up result is proved for solutions with negative initial total energy. This is joint work with Dr. Mohammad A. Rammaha and Dr. Daniel Toundykov. (Received September 03, 2016)

1125-35-495 **Ramjee P Sharma\***, University of North Georgia, Oakwood, GA 30566, and **Jiahong Wu**, Oklahoma State University, Stillwater, OK 74074. *Numerical computations of 2D Boussinesq equations with fractional dissipation.* Preliminary report.

In this talk we will talk about our recent results from the numerical computation of the following 2D Boussinesq equations with fractional dissipation:

$$\begin{aligned}\partial_t \omega + u \cdot \nabla \omega &= -\nu(-\Delta)^\alpha \omega + \partial_{x_1} \theta, & \nu \geq 0, \quad 0 < \alpha \leq 1, \\ \partial_t \theta + u \cdot \nabla \theta &= -\kappa(-\Delta)^\beta \theta, & 0 < \beta \leq 1, \\ \nabla \cdot u &= 0.\end{aligned}$$

Here  $u$  is the velocity field,  $\omega$  is the vorticity field and  $\theta$  is a scalar. We have used parallel pseudospectral method to compute the solutions. These equations model geophysical flows such as atmospheric fronts and ocean circulation. (Received September 04, 2016)

1125-35-515 **Michael Sever\*** (sever@math.huji.ac.il), Department of Mathematics, The Hebrew University, Givat Ram, Jerusalem, Israel. *An admissibility condition for weak solutions of multidimensional nonlinear systems of conservation laws.*

An admissibility condition for weak solutions of systems of conservation laws is presented, which is valid in any number of dimensions and does not require hyperbolicity of the underlying system. For the Cauchy problem for scalar conservation laws or for hyperbolic systems in one space dimension, criteria similar to familiar entropy conditions on the discontinuities are obtained. In higher dimensions, however, admissibility for weak solutions of the Cauchy problem for hyperbolic systems generically fails. In very special cases, admissibility is recovered for weak solutions of reduced systems corresponding to stationary or self-similar systems. (Received September 05, 2016)

1125-35-518 **Melvin Lipka\*** (melvin.lipka@cameron.edu), 2800 W Gore Blvd, Lawton, OK 73505, and **Narayan Thapa** (nthapa@cameron.edu), 2800 West Gore Blvd, Lawton, OK 73505. *Identification of time dependent control parameter through finite difference method in parabolic partial differential equation.*

Inverse problems are the oldest most important problems in science and engineering. Because of their applications in medical imaging, underground prospecting, nondestructive testing, astronomical imaging, image processing, remote sensing, and data mining, Business, Industry, and Government (BIG) sectors are very interested in computational inverse problems. Because of massive increase in computing power and development of powerful numerical techniques, the field of inverse problems has undergone rapid development recently. In this work, we consider a parabolic partial differential equation with time dependent control parameter. Forward Time Central Space, Backward Time Central Space, and Crank-Nicolson method are used to identify the control parameter. Numerical experiments are presented and the stability of the solution is discussed. (Received September 05, 2016)

1125-35-548 **Barbara Abraham-Shrauner\*** ([bas@wustl.edu](mailto:bas@wustl.edu)), Electrical and Systems Engineering, Washington University, St. Louis, MO 63130. *Analytic Lie symmetric solutions of nonlinear partial differential equations*. Preliminary report.

Nonlinear partial differential equations with independent variables invariant under translations are analyzed for exact analytic solutions. Conditions for certain traveling wave solutions of nonlinear partial differential equations (NLPDEs) include the power index and net number of derivatives of each term. Hyperbolic tanh and sech functions and Jacobian elliptic functions sn, cn and dn are considered. Requirements for the  $G'/G$  method are discussed. Examples include the generalized KdV equations and Zakharov equations. (Received September 06, 2016)

1125-35-555 **Michael E. Taylor\*** ([met@math.unc.edu](mailto:met@math.unc.edu)), Math. Dept., UNC, Chapel Hill, NC 27599. *Toeplitz operators on uniformly rectifiable domains*.

We consider Hardy spaces of solutions to certain first-order elliptic systems of PDE on uniformly rectifiable domains, singular integral operators that yield projections onto the space of their boundary values, and Toeplitz operators associated with these projections. We discuss results on the index of such Toeplitz operators, when they are Fredholm. This talk covers work with D. Mitrea, I. Mitrea and M. Mitrea, following work with S. Hofmann and M. Mitrea. (Received September 06, 2016)

1125-35-579 **Steven Derochers\*** ([sjderoch@ncsu.edu](mailto:sjderoch@ncsu.edu)). *On the Semigroup Generator for the Total Linearization of a Hydro-Elasticity Model*. Preliminary report.

In this talk, we consider a fluid structure system involving coupled fluid flow. In the examination of the well-posedness of this system, we utilize a nonstandard means to eliminate the pressure term. This process involves specific extensions into the solid domain. Unlike the standard model, the well-posedness of this system depends on the fluid's viscosity and new terms on the interface which involve, among other things, the curvature of the boundary. Furthermore we implement a finite element scheme for approximating solutions of a prototype system to numerically investigate the dependence of the discretized model on the "new" terms not present in the standard system. (Received September 06, 2016)

1125-35-580 **Irena Lasiecka and Justin T Webster\*** ([websterj@cofc.edu](mailto:websterj@cofc.edu)), 66 George St., Charleston, SC 29424. *Exponential Attractors for Fluttering Plate Models*.

Recent studies of flutter (an instability occurring when a flexible structure is immersed in an inviscid fluid flow) have led to interesting problems in the study of long-time behavior of coupled systems. The panel flutter model involves a nonlinear (von Karman or Berger) plate embedded in the plane, coupled to a perturbed wave equation on the upper half-space. In certain regimes (e.g., for large flow velocities) this model can be reduced to a plate equation with a delay potential, or simply a non-dissipative plate equation. The latter is referred to as a piston-theoretic plate.

In this talk we discuss the reduction of the flow-plate (fluttering panel) model to a piston-theoretic plate equation. We present recent results on the existence of smooth and finite dimensional global attractors for this flutter model in the absence of imposed damping. We utilize the recent quasistability theory, and discuss how it yields a fractal exponential attractor for the dynamics. For Berger's plate model, we show that "large" interior damping actually results in smooth exponential attracting via a novel decomposition of the nonlinear dynamics and the transitivity property of exponential attraction. (Received September 06, 2016)

1125-35-581 **Justin T Webster\*** ([websterj@cofc.edu](mailto:websterj@cofc.edu)), 66 George Street, Charleston, SC 29424. *Modeling Challenges for the Flutter of a Cantilevered Structure in a Flow*.

Flutter is a (bounded response) instability brought about by the interaction of an elastic structure with a surrounding fluid flow. Here, we describe the difficult problem of modeling the phenomenon for thin structures where a portion of the boundary is free. Beyond the obvious applications in aerospace, flutter arises in: (i) biomedical applications (sleep apnea), and (ii) sustainable energy (providing a low-cost power generation). Modeling, predicting, controlling, and preventing flutter has been a foremost engineering problem engineering for nearly 70 years. Mathematically, there is a lack of well-posedness and long-time analysis.

We discuss recent results for mathematical models of panel flutter; this simpler situation involves a nonlinear structure coupled to a perturbed wave equation with the entire structural boundary restricted. We then discuss the ways in which the analysis breaks down when a portion of the structure's boundary is free. The challenges in the analysis can be viewed as reflections of the difficulty in modeling the physics of the problem. Various recent results will be discussed for multiple models which address in/stability, along with recent numerical simulations. (Received September 06, 2016)

1125-35-652 **Kun Zhao\*** (kzhao@tulane.edu). *Analysis of a dissipative hyperbolic system arising from biology.*

In this talk, I will present a group of recent results concerning the rigorous analysis of a nonlinear PDE model arising from biosciences, which has primary applications in chemotaxis research. In particular, global well-posedness, long-time asymptotic behavior, diffusion limits, and boundary layer formation of classical solutions will be discussed. (Received September 08, 2016)

1125-35-662 **Kun Zhao\*** (kzhao@tulane.edu). *Global dynamics of Cahn-Hilliard-Brinkman equations in critical space.*

The Cahn-Hilliard-Brinkman equations is a coupled PDE system describing phase separation of binary fluids in porous media. A heuristic argument shows that the global well-posedness is critical in  $\mathbb{R}^4$  in the sense of scaling-invariance against free energy, while the problem is sub-critical in  $\mathbb{R}^d$  for  $d = 1, 2, 3$ . In this talk, I will present some recent results concerning the global well-posedness and long-time behavior of classical solutions to an initial-boundary value problem of the model in  $\mathbb{R}^4$ . (Received September 08, 2016)

1125-35-675 **Zhivko S. Athanasov\*** (zhivko@math.bas.bg), Institute of Mathematics, Bulgarian Academy of Sciences, G. Bonchev Str. 8, 1113 Sofia, Bulgaria. *Operator Differential Equations in Hilbert Spaces.* Preliminary report.

In this talk, the theory of dissipative linear operators in a Hilbert space developed by R. S. Phillips (1953) is applied to the study of the operator differential equation (E)  $\dot{x}(t) + A(t)x(t) = f(t), x(0) = x_0$ , where  $A(t), t \in [0, T]$ , is a family of unbounded linear operators whose domains and ranges are in a Hilbert space  $H$  and  $f$  is a measurable function on  $[0, T]$  with values in  $H$ . The prototype of (E) is a partial differential equation or a system of such equations, where  $A(t)$  is the differential operator with respect to space variables in some region and  $t$  corresponds to the time variable. Under conditions on  $A(t)$  similar to those of  $T$ . Kato (1953), we first show the existence of a weak solution of (E), and then, that the weak solution is a unique strong solutions which is the limit of a sequence of classical solutions. An application is considered to a time-dependent hyperbolic system of partial differential equations. (Received September 09, 2016)

1125-35-753 **Lingju Kong\*** (lingju-kong@utc.edu). *Weak solutions for nonlinear Neumann boundary value problems with  $p(x)$ -Laplacian operators.*

We study the nonlinear Neumann boundary value problem with a  $p(x)$ -Laplacian operator

$$\begin{cases} \Delta_{p(x)} u + a(x)|u|^{p(x)-2}u = f(x, u) & \text{in } \Omega, \\ |\nabla u|^{p(x)-2} \frac{\partial u}{\partial \nu} = |u|^{q(x)-2}u + \lambda|u|^{w(x)-2}u & \text{on } \partial\Omega, \end{cases}$$

where  $\Omega \subset \mathbb{R}^N$ , with  $N \geq 2$ , is a bounded domain with smooth boundary and  $q(x)$  is critical in the context of variable exponent  $p_*(x) = (N-1)p(x)/(N-p(x))$ . Using the variational method and a version of the concentration-compactness principle for the Sobolev trace immersion with variable exponents, we establish the existence and multiplicity of weak solutions for the above problem. (Received September 10, 2016)

1125-35-772 **Michele Coti Zelati\*** (micotize@umd.edu). *Deterministic and stochastic aspects of fluid mixing.*

The process of mixing of a scalar quantity into a homogenous fluid is a familiar physical phenomenon that we experience daily. In applied mathematics, it is also relevant to the theory of hydrodynamic stability at high Reynolds numbers - a theory that dates back to the 1830's and yet only recently developed in a rigorous mathematical setting. In this context, mixing acts to enhance, in certain senses, the dissipative forces. Moreover, there is also a transfer of information from large length-scales to small length-scales vaguely analogous to, but much simpler than, that which occurs in turbulence. In this talk, we focus on the study of the implications of these fundamental processes in linear settings, with particular emphasis on the long-time dynamics of deterministic systems (in terms of sharp decay estimates) and their stochastic perturbations (in terms of invariant measures). (Received September 11, 2016)

1125-35-775 **Mihaela Ifrim\*** (ifrim@math.berkeley.edu) and **Daniel I Tatarutataru@berkeley.edu**. *Long time dynamics for several water waves related models.*

The aim of the talk is toward better understanding the long time dynamics of solutions for several water waves related model problems. We will first discuss local well posedness and then revisit the (almost) global existence questions for small localized data. (Received September 11, 2016)

1125-35-786 **Elisabeth MM Brown\*** ([embrown5@ncsu.edu](mailto:embrown5@ncsu.edu)) and **Michael Shearer**. *A Dual-Flux Conservation Law for Plume Migration in Carbon Sequestration.*

A quasi-linear hyperbolic partial differential equation with a discontinuous flux models geologic carbon dioxide (CO<sub>2</sub>) migration and storage. Dual flux curves emerge in this model, giving rise to flux discontinuities. One flux describes the invasion of the plume into pore space, and the other captures the flow as the plume leaves CO<sub>2</sub> bubbles behind, which are then trapped in the pore space. Flux functions with discontinuities in space have been previously studied; the flux in this model depends on how the unknown is changing at any position and time. We prove the existence of an entropy solution of the Cauchy problem for any initial CO<sub>2</sub> plume using wave-front tracking. During our analysis, we introduce a new construction with *cross-hatch characteristics* in regions of the characteristic plane where the solution is constant, and the characteristic speed depends on which flux is invoked. We present a computer simulation that tracks CO<sub>2</sub> plume migration in the characteristic plane. Some wave interactions yield novel phenomena due to the dual flux, such as shock-rarefaction interactions that would persist for all time with a single flux, here are completed in finite time. (Received September 11, 2016)

1125-35-810 **Huijing Du\*** ([hdu5@unl.edu](mailto:hdu5@unl.edu)), 304 Avery Hall, University of Nebraska, Lincoln, Lincoln, NE 68588. *Multiscale modeling for epidermis layer formation and homeostasis.*

The skin epidermis is a stratified epithelium composed of multiple layers of epithelial cells with distinct boundaries separating each other. In epidermis, different cells including stem cells form stratified layers with appropriate size and proportions. How the epidermis develops from a single layer of stem cells to form a complex multilayered structure of several different cell types remains elusive. We construct stochastic, three-dimensional, and multiscale models consisting of a lineage of four cell types. The simulations show a balance between proliferation and differentiation during lineage progression is crucial for the development and maintenance of epithelial tissue. It also found that selective cell adhesion of different cell types is critical to the boundary sharpening between layers and to the highly ordered structure. The long-range morphogen gradients provides additional feedback regulations on the robustness of overall layer formation. Our model is built upon a previous experimental finding that revealed the role of *Ovol* transcriptional factors in regulating epidermal specification, proliferation and differentiation. Direct comparison of experimental phenotypes and simulation perturbations was carried out, showing consistence with each other. (Received September 12, 2016)

1125-35-811 **Saleh Tanveer** and **Charis Tsikkou\*** ([tsikkou@math.wvu.edu](mailto:tsikkou@math.wvu.edu)). *Analysis of 2+1 Diffusive-Dispersive PDE Arising in River Braiding.*

In the context of a weakly nonlinear study of bar instabilities in a river carrying sediment, P. Hall introduced an evolution equation for the deposited depth which is dispersive in one spatial direction, while being diffusive in the other. In this talk, we present local existence and uniqueness results using a contraction mapping argument in a Bourgain-type space. We also show that the energy and cumulative dissipation are globally controlled in time. (Received September 12, 2016)

1125-35-812 **Charis Tsikkou\*** ([tsikkou@math.wvu.edu](mailto:tsikkou@math.wvu.edu)). *Singular Shocks in a Chromatography Model.*

We consider a system of two equations that can be used to describe nonlinear chromatography and produce a coherent explanation and description of the unbounded solutions (singular shocks) that appear in Mazzotti's model. We use the methods of Geometric Singular Perturbation Theory, to show existence of a viscous solution to Dafermos-DiPerna regularization. (Received September 12, 2016)

1125-35-822 **Adelaide Akers\*** ([ahqfgc@mail.missouri.edu](mailto:ahqfgc@mail.missouri.edu)), 202 Mathematical Sciences Building, University of Missouri-Columbia, Columbia, MO 65203. *Existence and symmetry of small-amplitude solitary water waves with discontinuous vorticity.* Preliminary report.

We consider a two-dimensional body of water with constant density which lies below a vacuum. The ocean bed is assumed to be impenetrable, while the boundary which separates the fluid and the vacuum is assumed to be a free boundary. Following the work of M.D. Groves and E. Wahlen (2008), we use the Hamiltonian structure of the system coupled with center manifold reduction techniques to establish the existence of small-amplitude solitary water waves with potentially discontinuous vorticity. Furthermore, we utilize a modified moving planes method to show that such solitary waves possess even symmetry. (Received September 12, 2016)

1125-35-834 **Bin Xu\*** ([xu@math.utah.edu](mailto:xu@math.utah.edu)), 155 S 1400 E, ROOM 233, Salt Lake City, UT 84102, and **Paul Bressloff** ([bressloff@math.utah.edu](mailto:bressloff@math.utah.edu)), 155 S 1400 E, ROOM233, Salt Lake City, UT 84102. *A theory of synchrony for active compartments with delayed coupled through bulk diffusion.*

We consider two well-mixed, identical compartments located at either end of a bounded, one-dimensional domain. The compartments can exchange signaling molecules with the bulk domain where the signaling molecules undergo

diffusion. The concentration of signaling molecules within each compartment is modeled by a delay differential equation (DDE), while the concentration in the bulk medium is modeled by a partial differential equation (PDE) for diffusion. Coupling in the resulting PDE-DDE system is via flux terms at the boundaries. One interpretation of the model is in terms of a single rod-like cell such as fission yeast, with each compartment a dynamically active membrane at the ends of the cell and the bulk domain representing the cytoplasm. Using linear stability analysis, numerical simulations and bifurcation analysis, we investigate the effect of diffusion on the onset of a supercritical Hopf bifurcation. The direction of the Hopf bifurcation is determined by numerical simulations and a winding number argument. Near a Hopf bifurcation point, we find that there are synchronized oscillations with two possible modes: in-phase and anti-phase. Our numerical result suggests that the selection of the in-phase or anti-phase oscillation is sensitive to delayed feedback. (Received September 12, 2016)

1125-35-885 **Dehua Wang\*** ([dwang@math.pitt.edu](mailto:dwang@math.pitt.edu)), Department of Mathematics, University of Pittsburgh, Pittsburgh, PA 15260. *Hyperbolic Conservation Laws for Isometric Immersions.*

Isometric immersion is an important problem in geometry. The Gauss-Codazzi equations for isometric immersions of surfaces will be considered. Recent results on global solutions for the hyperbolic problem with negative Gauss curvatures will be presented. In particular, the global smooth isometric immersion of surfaces with slow decay rates for the curvatures will be discussed. (Received September 12, 2016)

1125-35-888 **Ami Radunskaya\***, [aer04747@pomona.edu](mailto:aer04747@pomona.edu). *Getting to the target on time.*

Many treatments are time-sensitive, and getting the treatment to the right spot at the right time is crucial to its effectiveness. For example, when treating certain diseases of the brain such as Parkinson's, we need to deliver bursts of drugs to specific spots in the brain, and we need to get these drugs through the blood-brain-barrier. When treating a tumor, we need to keep toxic drugs away from normal tissue, and we need to give immunotherapies in conjunction with drugs that inhibit immune-suppression by the tumor. Mathematically, we would like to model new drug delivery technologies, such as nanoparticles loaded with drugs, decorated with homing devices, and delivered in time-released media. We then need to formulate and solve a complicated, multi-objective optimization problem, and gauge the uncertainty in our predictions: uncertainties that could be life-threatening.

In this talk we will present several models that attack these specific problems, highlighting the challenges and the promises (Received September 12, 2016)

1125-35-899 **Josselin Garnier\*** ([josselin.garnier@polytechnique.edu](mailto:josselin.garnier@polytechnique.edu)), Department of Applied Mathematics, Ecole Polytechnique, 91128 Palaiseau, France. *Imaging with intensity cross correlations and application to ghost imaging.*

In sensor array imaging the data recorded by an array of receivers and emitted by an array of sources are processed to produce an image of the object that is illuminated. In this talk we analyze an imaging method called ghost imaging that can produce an image of an object by correlating the intensities measured by a high-resolution (multi-pixel) detector that does not view the object and a low-resolution (single-pixel) detector that does view the object. This gives the principle of a one-pixel camera. We clarify the roles of the partial coherence of the source that illuminates the object and of the scattering properties of the medium through which the waves propagate. (Received September 13, 2016)

1125-35-911 **Atanas Stefanov\*** ([stefanov@ku.edu](mailto:stefanov@ku.edu)) and **Mats Ehrnström**. *On solitary waves for the Whitham equation.*

We consider the Whitham equation, defined on the whole line. Due to the smoothing nature of the linear operator, the question for existence of traveling wave solutions has been open till recently. In 2012, Ehrnström-Groves-Wahlén have constructed such waves  $\varphi(x - \mu t)$ , but only for values of  $\mu$  slightly bigger than one, even though the admissible range of wave speeds is  $\mu \in (1, 2)$ . The approach in EGW consists of a tour de force calculus of variations, supplemented by a bifurcation argument from the small KdV waves.

In this work (joint with M. Ehrnström), we construct a one parameter family of such waves, including  $\mu$  close to one, that is in the EGW range, as well as waves with wave speeds  $\mu$  close to two. In fact, we believe that the construction yields waves for the entire interval  $\mu \in (1, 2)$ , but such claim hinges upon a technical detail that is unavailable at the moment. The argument uses calculus of variation construction, very different than the one employed by EGW. In fact, it is based on constraints in appropriately selected Orlicz spaces. Finally, all our traveling waves are shown to be bell-shaped, confirming the available numerical evidence. (Received September 13, 2016)

1125-35-922 **Qingtian Zhang\*** (qzhang@math.ucdavis.edu). *Global wellposedness of cubic Camassa-Holm equations.*

We will talk about the Cauchy problem of cubic Camassa-Holm equation. This equation is derived from a model of shallow water dynamics. We prove the global existence of entropy weak solution to this problem in space  $H^1$  with its derivative in BV. The stability and uniqueness of entropy weak solution are obtained in  $W^{1,1}$ . (Received September 13, 2016)

1125-35-927 **Atanas Stefanov\*** (stefanov@ku.edu). *On the ground states for the generalized Hartree problem.*

We consider standing wave solutions  $e^{i\omega t}\Phi(x)$  of the generalized Hartree problem - with fractional dispersion and arbitrary power non-linearity, in any dimension  $d \geq 1$ . We establish the existence of ground states, under appropriate constraints on the parameters. The uniqueness of such ground states is an outstanding (and exciting) open problem in the field, with only a few results known in the classical cases (standard Laplacian,  $p = 2$  and  $d = 3, 4, 5$ ), most notably by Lieb'77 and recently by Ma-Zhao'10 and Wang'15,16. Stability of such solutions (without knowing uniqueness) is an interesting open problem as well. We report on a new development in this direction. (Received September 13, 2016)

1125-35-976 **Lydia R. Bieri\*** (lbieri@umich.edu), University of Michigan, Department of Mathematics, East Hall, Ann Arbor, MI 48109. *Black hole formation and stability: a mathematical investigation.*

The dynamics of the Einstein equations feature the formation of black holes. The latter are related to the presence of trapped surfaces in the spacetime manifold. The mathematical study of these phenomena has gained momentum since D. Christodoulou's breakthrough result proving that in the regime of pure general relativity trapped surfaces form through the focusing of gravitational waves. (The latter were observed for the first time last year by LIGO.) The proof combines new ideas from geometric analysis and nonlinear partial differential equations (pde) as well as it introduces new methods to solve large data problems. These methods have many applications beyond general relativity. D. Christodoulou's result was generalized by S. Klainerman and I. Rodnianski. In this talk, we investigate the dynamics of the Einstein equations, focusing on these works. Moreover, we address the question of stability of black holes and what has been known so far, involving recent works of many contributors. (Received September 13, 2016)

1125-35-1031 **Sergei Avdonin\*** (s.avdonin@alaska.edu), 2534 Bridlewood Lane, Atlanta, GA 30339, and **Fahed Al-Musallam**. *Inverse Problems for Krein's String.*

We consider boundary inverse problems for a nonhomogeneous string with masses attached to some interior points. We demonstrate that the density of the string, masses and their coordinates can be recovered using the dynamical or spectral Neumann-to-Dirichlet map associated to a boundary point of the string. Our identification algorithm is based on the exact controllability of the string in a sharp time interval with respect to a nonsymmetric Sobolev space with the regularity increasing at each 'mass' point. (Received September 14, 2016)

1125-35-1040 **Sergei Avdonin** (s.avdonin@alaska.edu), 2534 Bridlewood Lane, Atlanta, GA 30339, and **Luz de Teresa\*** (deteresa@gmail.com). *The Kalman condition for the boundary controllability of coupled 1-d wave equations.*

In this talk, we prove the exact controllability of a system of  $N$  one-dimensional wave equations when the control is exerted on a part of the boundary by means of one control. We give a Kalman condition (necessary and sufficient) and give a description, non optimal in general, of the attainable set. (Received September 18, 2016)

1125-35-1074 **Minh Kha\*** (kha@math.tamu.edu), **Peter Kuchment** and **Yehuda Pinchover**. *On Riemann-Roch-Liouville theorem for elliptic PDEs on abelian coverings.* Preliminary report.

Taking a motivation from classical analysis rather than from algebraic geometry, Gromov and Shubin were able to generalize the classical Riemann-Roch theorem to solutions of general elliptic operators with point singularities and beyond. Their Riemann-Roch formula is a connection between two dimensions of two spaces that are dual in some sense. These spaces contain solutions that are allowed to have certain singularities and are required to have prescribed zeros such that the multiplicities are controlled. Meanwhile, the classical Liouville theorem states that the space of solutions of any given polynomial growth is finite dimensional. For periodic elliptic operators on abelian coverings, a Liouville type result was established by Kuchment and Pinchover. The dimensions of such spaces are also calculated explicitly in many cases. In this talk, we will present some results combining Riemann-Roch and Liouville theorems for elliptic operators on abelian coverings. This is joint work with Peter Kuchment and Yehuda Pinchover. (Received September 14, 2016)

- 1125-35-1078 **Jerome Goddard II, Quinn Morris\*** (qamorris@uncg.edu), **Catherine Payne, Jordan Price and R. Shivaji.** *Analysis of steady states for classes of reaction-diffusion equations with U-shaped density dependent dispersal on the boundary.*

We consider positive solutions to equations of the form

$$\begin{cases} -\Delta u = \lambda u(1-u), & x \in \Omega, \\ \frac{\partial u}{\partial \eta} + \gamma \sqrt{\lambda}(u-A)^2 u = 0, & x \in \partial\Omega, \end{cases}$$

where  $\lambda > 0, \gamma > 0, A \in (0, 1)$  are parameters,  $\Omega$  is a bounded domain in  $\mathbb{R}^n$ ;  $n \geq 1$  with smooth boundary  $\partial\Omega$  and  $\frac{\partial u}{\partial \eta}$  is the outward normal derivative. Such models arise in the study of population dynamics in a habitat  $\Omega$  when the population exhibits U-shaped density dependent dispersal on the boundary. We analyze the persistence of the population (existence, non-existence, uniqueness and multiplicity of positive solutions) as the patch size ( $\lambda$ ) and the hostility of the outside matrix ( $\gamma$ ) vary. We obtain results when  $\Omega = (0, 1)$  via a quadrature method, and when  $\Omega$  is any bounded domain in  $\mathbb{R}^n$ ;  $n > 1$  by the method of sub-super solutions. (Received September 14, 2016)

- 1125-35-1081 **Ryan C. Thompson\*** (ryan.thompson@uncg.edu), Department of Mathematics, University of North Georgia, 82 College Circle, Dahlonega, GA 30597, and **John Holmes** (holmes.782@osu.edu), Department of Mathematics, The Ohio State University, 231 West 18th Avenue, Columbus, OH 43210. *Well-posedness and Continuity Properties of the Fornberg-Whitham Equation in Besov Spaces.*

In this presentation, we will exhibit well-posedness of the Fornberg-Whitham equation in Besov spaces  $B_{2,r}^s$  in both the periodic and non-periodic cases. This will imply the existence and uniqueness of solutions in the aforementioned spaces along with the continuity of the data-to-solution map provided that the initial data belongs to  $B_{2,r}^s$ . We will also provide a result on the sharpness of continuity on the data-to-solution map by showing that it is not uniformly continuous from any bounded subset of  $B_{2,r}^s$  to  $C([-T, T]; B_{2,r}^s)$ . If time permits, we will observe a Cauchy-Kowalevski type theorem for this equation that establishes the existence and uniqueness of real analytic solutions and also provide blow-up criterion for solutions. (Received September 14, 2016)

- 1125-35-1089 **Vincent R. Martinez\*** (vmartin6@tulane.edu), Tulane University, Mathematics Department, 424 Gibson Hall, New Orleans, LA 70118, and **Animikh Biswas, Michael S. Jolly, Eric J. Olson and Edriss S. Titi.** *Analytical studies for a Data Assimilation Algorithm: Surface data, Higher-order synchronization, and Time-averaged measurements.*

This talk will discuss recent studies in a data assimilation algorithm proposed by Azouani, Olson, and Titi. The algorithm exploits the finite-dimensionality of the dynamics of certain dissipative equations for which "determining quantities" exist, e.g. knowledge of sufficiently many Fourier modes of the solution for all time determines all higher modes asymptotically in time. By collecting a sufficient amount of such quantities, one can then define an algorithm to produce an approximating solution that converges to the reference solution corresponding to the collected data asymptotically in time and at an exponential rate. In this talk, we will discuss several joint works with A. Biswas, M.S. Jolly, E.J. Olson, and E.S. Titi in which we investigate the topologies that the synchronization takes place, its ability to accommodate the more physical case of time-averaged observables, and studies that support the idea that for vertically-constrained flows, one need only assimilate data collected at the boundary to synchronize the approximating flow with the reference flow in the domain's interior. In these works, we use the 2D Navier-Stokes and 2D subcritical surface quasi-geostrophic equations as our main examples. (Received September 14, 2016)

- 1125-35-1092 **Ratnasingham Shivaji, Inbo Sim and Byungjae Son\*** (b\_son@uncg.edu), 830 West Market Street, Apt 623, Greensboro, NC 27401. *Uniqueness results for classes of semipositone p-Laplacian problems.*

We consider steady state reaction diffusion equations on the exterior of a ball, namely, boundary value problems of the form :

$$\begin{cases} -\Delta_p u = \lambda K(|x|)f(u) & \text{in } \Omega_E, \\ Bu = 0 & \text{on } |x| = r_0, \\ u \rightarrow 0 & \text{when } |x| \rightarrow \infty, \end{cases}$$

where  $\Delta_p z := \operatorname{div}(|\nabla z|^{p-2} \nabla z)$ ,  $1 < p < n$ ,  $\lambda > 0$ ,  $\Omega_E := \{x \in \mathbb{R}^n \mid |x| > r_0 > 0\}$ , and  $Bu \equiv u$  or  $Bu \equiv \frac{\partial u}{\partial \eta} + c(u)u$  where  $c \in C((0, \infty), (0, \infty))$  and  $\frac{\partial u}{\partial \eta}$  is the outward normal derivative of  $u$  on  $|x| = r_0$ . Here  $K \in C^1([r_0, \infty), (0, \infty))$  satisfies  $\lim_{r \rightarrow \infty} K(r) = 0$ , and  $f \in C^1[0, \infty)$  is strictly increasing and satisfies  $f(0) < 0$ ,  $\lim_{s \rightarrow \infty} f(s) = \infty$ ,  $\lim_{s \rightarrow \infty} \frac{f(s)}{s^{p-1}} = 0$  and  $\frac{f(s)}{s^q}$  is nonincreasing on  $[a, \infty)$  for some  $a > 0$  and  $q \in (0, p-1)$ . We establish uniqueness results for positive radial solutions for  $\lambda \gg 1$ . (Received September 14, 2016)

- 1125-35-1100 **Felipe Linares, Gustavo Ponce\*** ([ponce@math.ucsb.edu](mailto:ponce@math.ucsb.edu)) and **Thomas C. Sideris**. *Special properties of solutions to the IVP associated to the Camassa-Holm equation on the line.*

We shall study special properties of solutions to the IVP associated to the Camassa-Holm equation on the line related to the regularity and the decay of solutions. The first aim is to show how the regularity on the initial data is transferred to the corresponding solution in a class containing the peak on solutions. Using this we shall show that the decay results obtained Himonas-Misiolek-Ponce-Zhou extend to this class of solutions. (Received September 14, 2016)

- 1125-35-1105 **Dionyssios Mantzavinos\*** ([dmantzavinos@umass.edu](mailto:dmantzavinos@umass.edu)), Department of Mathematics and Statistics, University of Massachusetts Amherst, Amherst, MA 01003, and **Alex Himonas** ([himonas.1@nd.edu](mailto:himonas.1@nd.edu)), Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556. *On the analysis of integrable evolution equations.*

The first PDE with a soliton solution was derived by Boussinesq in 1872. Several other equations followed since, including the Korteweg-de Vries, nonlinear Schrödinger and Camassa-Holm equations. All of those models are completely integrable systems and possess a variety of traveling wave solutions. During the last fifty years, the study of these equations has been intensified from analytic, geometric and algebraic points of view. In this talk, we shall discuss earlier results and recent developments on some of these equations. (Received September 14, 2016)

- 1125-35-1106 **Alex Himonas\*** ([himonas.1@nd.edu](mailto:himonas.1@nd.edu)), Department of Mathematics, University of Notre Dame, Notre Dame, IN 46556, and **Dionyssios Mantzavinos** ([dmantzavinos@umass.edu](mailto:dmantzavinos@umass.edu)), Department of Mathematics and Statistics, University of Massachusetts Amherst, Amherst, MA 01003. *Well-posedness of nonlinear dispersive PDEs via the unified transform method.*

The unified transform method was introduced in late nineties as the analogue of the inverse scattering transform machinery for integrable nonlinear equations on the half-line. It was later understood that it also has significant implications for linear initial-boundary value problems. In this talk, this method is employed in a new direction, namely for showing well-posedness of nonlinear dispersive equations on the half-line with data in Sobolev spaces. (Received September 14, 2016)

- 1125-35-1110 **Tao Huang** ([th79@nyu.edu](mailto:th79@nyu.edu)), 3663 Zhongshan Road North, Shanghai, Peoples Rep of China, **Yuanzhen Shao** ([shao92@purdue.edu](mailto:shao92@purdue.edu)), 150 N. University Street, West Lafayette, IN 47907, and **Changyou Wang\*** ([wang2482@purdue.edu](mailto:wang2482@purdue.edu)), 150 N. University Street, West Lafayette, IN 47907. *Harmonic map flow in 2 dimension under weak anchoring boundary condition.* Preliminary report.

In this talk, I will discuss the heat flow of harmonic maps under weak anchoring boundary condition, a new boundary condition arising from the Landau-De Gennes models of nematic liquid crystals. In dimension two, we are able to establish the existence of a unique, global "almost regular" solution, whose asymptotic behavior near the point defects will also be described. This is a joint work with Tao Huang and Yuanzhen Shao. (Received September 14, 2016)

- 1125-35-1162 **Liang Kong\*** ([lkong9@uis.edu](mailto:lkong9@uis.edu)), One University Plaza, MS WUIS 13, Mathematical Sciences, UIS, Springfield, IL 62703. *Asymptotic Dynamics of Two Species Competition Systems.* Preliminary report.

In this study, we investigate how the localized spatial variation affects the asymptotic dynamics of two species competition systems. We obtain the existence of spreading speeds and linear determinacy for two species competition systems with nonlocal dispersal in periodic habitats with localized spatial variation. (Received September 15, 2016)

- 1125-35-1206 **Irena Lasiecka, Michael Pokojovy** and **Xiang Wan\*** ([xw5he@virginia.edu](mailto:xw5he@virginia.edu)). *Global Wellposedness and Uniform Stability of a Quasilinear Thermo-elastic PDE system.*

We consider a nonlinear thermoelastic system defined on a bounded domain  $\Omega \subset \mathbb{R}^n$ ,  $n = 2$  or  $3$  with the boundary conditions imposed on  $\Gamma = \partial\Omega$  corresponding to the simply supported plate. The main goal of this talk is to discuss the wellposedness and long term behavior of suitable solutions of the system.

I will first introduce the background of this model, and then briefly talk about the work on the case of a Euler-Bernoulli plate. Our main challenge is to consider the case of Kirchoff plate, of which the system is of hyperbolic-parabolic type. From a mathematical point of view, the most important message is that the *analyticity* and *maximal regularity* of the associated linear system are *gone*. We will show how to choose suitable topologies to overcome this difficulty.

This is a joint work with Irena Lasiecka, University of Memphis, and Michael Pokojovy, University of Konstanz. Germany. (Received September 15, 2016)

1125-35-1268 **Elodie Pozzi\*** ([elodie.pozzi@math.u-bordeaux.fr](mailto:elodie.pozzi@math.u-bordeaux.fr)), Institut Mathématiques de Bordeaux, France. *Hardy Smirnov spaces of pseudo-analytic functions.*

Let  $\Omega \subset \mathbb{C}$  be a domain bounded by a rectifiable Jordan curve,  $\phi: \mathbb{D} \rightarrow \Omega$  a conformal map and  $1 < p < \infty$ . We study a class of functions that are solutions in the distributional sense of  $\bar{\partial}w = \alpha\bar{w}$  with  $\alpha \in L^2(\Omega)$  satisfying

$$\sup_{0 < \rho < 1} \int_{\Gamma_\rho} |w(z)|^p |dz| < \infty,$$

where  $\Gamma_\rho = \phi(\mathbb{T}_\rho)$ . In this case, we say that  $w$  belong to  $\mathcal{F}_\alpha^p(\Omega)$ . For  $\alpha = 0$ , such functions belong to the (analytic) Smirnov space  $E^p(\Omega)$ . We will give some properties of  $\mathcal{F}_\alpha^p$ -functions and will give the definition of the trace of  $w$  denoted  $w_{\partial\Omega}$ . For  $\psi \in L^p_{\mathbb{R}}(\partial\Omega)$ , we will prove that there exists  $w \in \mathcal{F}_\alpha^p(\Omega)$  such that  $\text{Re } w_{\partial\Omega} = \psi$ . This result will permit us to solve the Dirichlet problem for  $\text{div}(\sigma\nabla u) = 0$  for  $\log(\sigma) \in W^{1,2}(\Omega)$  with boundary data  $\psi \in L^p_{\mathbb{R}}(\partial\Omega)$  and under some assumptions on  $\Omega$ . This talk is based on joint work with L. Baratchart and E. Russ. (Received September 15, 2016)

1125-35-1289 **Chenjie Fan\*** ([cjfan@math.mit.edu](mailto:cjfan@math.mit.edu)). *The  $L^2$  weak sequential convergence of radial mass critical NLS solutions with mass above the ground state.*

We will explore the dynamic of non-scattering  $L^2$  solution  $u$  to the radial mass critical nonlinear Schrödinger equation with mass just above the ground state, and show that there exists a time sequence  $\{t_n\}_n$ , such that  $u(t_n)$  weakly converges to the ground state  $Q$  up to scaling and phase transformation. (Received September 17, 2016)

1125-35-1293 **Abhinandan Chowdhury\*** ([chowdhury@savannahstate.edu](mailto:chowdhury@savannahstate.edu)), Savannah, GA. *Analysis of Memory Effects for the Heat Conductivity of Random Suspensions of Spheres by Using the Method of Random-point Approximation.*

Method of Stochastic functional expansions with random-point basis function is applied to the case of heat conduction of a particulate medium (suspension) subjected to a time-dependent spatially constant temperature gradient. It is argued that the basis function which is appropriate as the basis in modelling the chaotic behavior of nonlinear dynamical systems, e.g. turbulence, is not so well suited for composite materials. It is shown that within the first order of approximation with respect to the concentration, the equation for the kernel of the 1st -order functional integral is the equation of the disturbance introduced by a single spherical inclusion (filler) in a matrix subjected to a time varying spatially constant gradient at infinity. After solving the resulting initial-boundary value problem, the effective connection between the heat flux and temperature gradient is established. It turns out that the effective law involves a retardation (memory integral) of the temperature gradient. Approximate expression for the memory kernel is found by employing a method based on infinite series expansion. An interesting limiting case of filler material with infinite conductivity is discussed where memory integral becomes the Riemann-Liouville half integral. (Received September 16, 2016)

1125-35-1331 **Vera Mikyoung Hur, Mathew A. Johnson and Ashish Kumar Pandey\*** ([akpande2@illinois.edu](mailto:akpande2@illinois.edu)). *Transverse instability in Kadomtsev-Petviashvili (KP), rotation-modified KP and full-dispersion KP equations.*

The instability caused by perturbing a 1D-wave by a perturbation in transverse direction is called transverse instability. We study this phenomenon for KdV periodic waves as a 1D-wave of KP equation. For KP-I equation, we provide a complete picture of transverse instability with respect to periodic as well as non-periodic perturbations in transverse direction. We extend our analysis to rotation-modified KP and full-dispersion KP models and obtain some new transverse instabilities in these models. (Received September 16, 2016)

1125-35-1344 **Keng Deng\*** ([deng@louisiana.edu](mailto:deng@louisiana.edu)), 1401 Johnston Street, Lafayette, LA 70504, and **Yixiang Wu**. *Extinction and uniform strong persistence of a size-structured population model.*

In this paper, we study the long-time behavior of a size-structured population model. We define a basic reproduction number  $\mathcal{R}$  and show that the population dies out in the long run if  $\mathcal{R} < 1$ . If  $\mathcal{R} > 1$ , the model has a unique positive equilibrium, and the total population is uniformly strongly persistent. Most importantly, we show that there exists a subsequence of the total population converging to the positive equilibrium. (Received September 16, 2016)

1125-35-1394 **Suleyman Ulusoy\*** ([ulusoy77@gmail.com](mailto:ulusoy77@gmail.com)), Gulluk Mah, Koru Sok, no 7, 54100 Sakarya, Turkey. *On Nonlocal Keller-Segel Type Equations.*

We analyze an equation that is gradient flow of a functional related to Hardy-Littlewood-Sobolev inequality in whole Euclidean space  $\mathbb{R}^d$ ,  $d \geq 3$ . Under the hypothesis of integrable initial data with finite second moment and energy, we show local-in-time existence for any mass of “free-energy solutions”, namely weak solutions with some free energy estimates. We exhibit that the qualitative behavior of solutions is decided by a critical value. The motivation for this work is to generalize Keller-Segel model to higher dimensions. (Received September 16, 2016)

1125-35-1517 **Hussein Awala\*** ([hussein.awala@temple.edu](mailto:hussein.awala@temple.edu)), Department of Mathematics, Temple University, Wachman Hall/1805 N Broad st, Philadelphia, PA 19122. *Mellin Transform Techniques for the Mixed Problem in Two Dimensions.*

In this talk I will discuss the boundary value problem with mixed Dirichlet and Neumann boundary conditions for the Laplacian and the Lamé system in infinite sectors in two dimensions. Using a potential theory approach the problem is reduced to inverting a singular integral operator (SIO) naturally associated with the problem on appropriate function spaces. Mellin transform techniques are then employed in the study of the spectrum of the aforementioned SIO. (Received September 17, 2016)

1125-35-1520 **Hussein Awala\*** ([hussein.awala@temple.edu](mailto:hussein.awala@temple.edu)), Department of Mathematics, Temple University, Wachman Hall/ 1805 N Broad st, Philadelphia, PA 19122. *Validated Numerics Methods for Mixed Boundary Value Problem for the System of Elastostatics.*

Elliptic boundary value problems with mixed Dirichlet and Neumann type boundary conditions arise naturally in connection with physical phenomena such as conductivity, heat transfer, elastic deformations, and electrostatics. In my talk I will discuss recent well-posedness results for the mixed boundary problem for the system of elastostatics in infinite sectors in two dimensions. These results are obtained through a blend of Calderon-Zygmund theory methods, Mellin transform techniques, and validated numerics. This work is part of an ongoing collaboration project with Irina Mitrea and Warwick Tucker. (Received September 17, 2016)

1125-35-1539 **Milena Stanislavova\*** ([stanis@ku.edu](mailto:stanis@ku.edu)), Department of Mathematics, 405, Snow Hall, 1460 Jayhawk Blvd., Lawrence, KS 66045, and **Sevdzhan Hakkaev** and **Atanas Stefanov**. *Spectral Stability for Classical Periodic Waves of the Ostrovsky and Short Pulse Models.* Preliminary report.

We consider the Ostrovsky and short pulse models in a symmetric spatial interval, subject to periodic boundary conditions. For the Ostrovsky case, we revisit the classical periodic traveling waves and for the short pulse model, we explicitly construct traveling waves in terms of Jacobi elliptic functions. For both examples, we show spectral stability, for all values of the parameters. This is achieved by studying the non-standard eigenvalue problems in the form  $Lu = \lambda u'$ , where  $L$  is a Hill operator. (Received September 17, 2016)

1125-35-1568 **Jill C Pipher\*** ([jill\\_pipher@brown.edu](mailto:jill_pipher@brown.edu)). *Boundary value problems for second order parabolic and elliptic operators.*

We discuss recent progress in variable coefficient elliptic and parabolic pde boundary value problems, Dirichlet and Neumann. (Received September 18, 2016)

1125-35-1587 **John C Schotland\*** ([schotland@umich.edu](mailto:schotland@umich.edu)). *Inverse Transport and Acousto-Optic Imaging.*

We consider the inverse problem of recovering the optical properties of a highly-scattering medium from acousto-optic measurements. Using such measurements, we show that the scattering and absorption coefficients of the radiative transport equation can be reconstructed with Lipschitz stability by means of algebraic inversion formulas. This is joint work with Francis Chung. (Received September 18, 2016)

1125-35-1631 **Curtis A Holliman\*** ([holliman@cua.edu](mailto:holliman@cua.edu)), Curtis Holliman, Department of Mathematics, Aquinas Hall 116, Washington, DC 20064, and **Alex Himonas** and **Carlos Kenig**. *Norm-Inflation and nonuniqueness results for Novikov's equation.* Preliminary report.

Novikov's equation is an integrable equation that can be thought of as a cubic analogue to the well-known Camassa-Holm equation. We prove that when one takes initial data in Sobolev spaces with exponents less than  $3/2$  the data-to-solution map becomes discontinuous in the sense of norm-inflation. Additionally, if the Sobolev exponent is less than  $5/4$ , it is possible to construct non unique solutions. This is a joint work with Alex Himonas and Carlos Kenig. (Received September 18, 2016)

1125-35-1641 **David Michael Harper\*** (dharpo12@fiu.edu). *PDEs and hypercomplex differentiable functions.*

Hypercomplex numbers are unital algebras over the real numbers. We show that differentiable functions over various hypercomplex number systems can be useful in the characterization and study of generalizations of the Laplace equation. Each class of differentiable functions enjoys a rich connection with a particular partial differential equation in much the same way as the complex differentiable functions connect with the Laplace equation. (Received September 18, 2016)

1125-35-1668 **M Burak Erdogan\*** (berdogan@math.uiuc.edu) and **Nikolaos Tzirakis.** *Regularity properties of the cubic NLS equation and the Zakharov system on the half-line.*

We discuss the local and global wellposedness of the cubic NLS equation and the Zakharov system on the half-line with rough initial and boundary data. We will also present a smoothing property that states that the difference of the solution and the associated linear solution is smoother than the initial data. (Received September 18, 2016)

1125-35-1694 **Geng Chen\*** (gengchen@ku.edu), Department of Mathematics, University of Kansas, Lawrence, KS 66044. *Global well-posedness for nonlinear wave equations with applications in nematic liquid crystals.*

In this talk, we will discuss our recent progress for the global well-posedness of Holder continuous weak solutions for some wave systems modeling nematic liquid crystals. (Received September 18, 2016)

1125-35-1709 **Rowan Killip, Jason Murphy\*** (murphy@math.berkeley.edu), **Monica Visan** and **Jiqiang Zheng.** *The focusing nonlinear Schrödinger equation with inverse square potential.*

For the focusing cubic nonlinear Schrödinger equation (NLS) in three space dimensions, Duyckaerts, Holmer, and Roudenko have established a sharp criterion for blowup versus scattering for solutions below the ground state. In this talk, we will adapt their approach to prove an analogous result for the case of NLS with an inverse square potential. To prove our result, we need to understand the sense in which the original equation embeds into the equation with potential in certain limiting regimes. This is joint work with R. Killip, M. Visan, and J. Zheng. (Received September 18, 2016)

1125-35-1715 **Rowan Killip, Satoshi Masaki, Jason Murphy\*** (murphy@math.berkeley.edu) and **Monica Visan.** *Conditional scattering for the mass-subcritical NLS.*

Many recent works on NLS and NLW have proven conditional scattering results for equations that lack a conserved quantity at the level of the critical regularity. Typically, one proves scattering under the assumption of *a priori* bounds in the critical Sobolev space. We prove an analogous result in the setting of the mass-subcritical NLS, for which the critical regularity  $s_c$  is negative. Instead of working with Sobolev norms, we use a scale-invariant weighted norm that is natural in this setting. Our result covers all short-range nonlinearities for which  $|s_c| < 1$ , in all dimensions  $d \geq 1$ . Our arguments apply equally well in the defocusing and focusing settings. This is joint work with R. Killip, S. Masaki, and M. Visan. (Received September 19, 2016)

1125-35-1727 **Hao Jia\*** (jiahao@math.ias.edu). *Dynamics of energy critical wave equations.*

We will survey recent results in the study of dynamics of energy critical wave equations and progresses in the soliton resolution conjecture in the nonradial case. (Received September 19, 2016)

1125-35-1729 **Hao JIA\*** (jiahao@math.ias.edu). *Dynamics of nonlinear wave equations.*

We will survey recent results in the study of energy critical wave equations in connection with the soliton resolution conjecture. Several model equations will be discussed. In one model, we can demonstrate rigorously the important role played by stability property of solitons in the global dynamics. (Received September 19, 2016)

1125-35-1737 **Michael E Filippakis\*** (mfilip@unipi.gr), University of Piraeus Research Center, 122 Grigoriou Labraki str. GR-18532 Piraeus, Greece, 18532 Piraeus, Greece, Greece. *Nodal solutions for indefinite Robin problems*

We consider a semilinear Robin problem driven by the negative Laplacian plus an indefinite, unbounded potential. The reaction term is a Caratheodory function of arbitrary structure outside an interval  $[-c, c]$  ( $c > 0$ ), odd on  $[-c, c]$  and concave near zero. Using a variant of the symmetric mountain pass theorem, together with truncation, perturbation and comparison techniques, we show that the problem has a whole sequence  $\{u_n\}_{n \geq 1}$  of distinct nodal solutions converging to zero in  $C^1(\bar{\Omega})$ .

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(Received September 19, 2016)

1125-35-1741 **Rachidi Bolaji Salako\*** (rbs0016@auburn.edu) and **Wenxian Shen**. *Global existence and asymptotic behavior of classical solutions to a parabolic-elliptic chemotaxis system with logistic source on  $R^N$ .*

Keller-Segel equations are used to illustrate the time evolution of mobile species toward the gradient of a chemical substance. Such systems are referred to as chemotaxis systems. Considering the classical Keller-Segel Parabolic-Elliptic/Parabolic chemotaxis systems, in the absence of logistic source, it is known that finite-time blow up of nonnegative solutions can occur when the spatial dimension is greater or equal 2, but never occurs in the case that the spatial dimension is equal to 1. In this talk, we consider a Parabolic-Elliptic chemotaxis system with logistic source and prove : (i) Local /Global existence of classical solution for various nonnegative initial functions, (ii) Asymptotic behavior of classical solutions. We shall also discuss about some on going works about the existence of front propagation and spreading speeds . (Received September 19, 2016)

1125-35-1754 **Gigliola Staffilani\*** (gigliola@mit.edu). *The benefits of randomizing initial data when proving well-posedness for certain dispersive equations.*

In the last twenty years, enormous progress has been made in settling fundamental questions concerning dispersive type equations: on existence of solutions, as well as their long time behavior, singularity formation and interactions. This body of work has focused primarily on deterministic aspects of wave phenomena that have been studied with sophisticated tools from harmonic analysis, nonlinear Fourier analysis, analytic number theory and geometry. More recently, though, a growing interest has been shown by the community in incorporating a non deterministic point of view in the field of dispersive PDE. In fact, as a more detailed picture of these kinds of waves emerges, and as the questions asked become yet more challenging, it also becomes clear that even if a certain property is not true for all solutions of a certain equation, one can still prove it is generically true. In this talk I will show few examples of this recent tread. (Received September 19, 2016)

1125-35-1761 **Benjamin Harrop-Griffiths\*** (benjamin.harrop-griffiths@cims.nyu.edu), **Mihaela Ifrim** and **Daniel Tataru**. *Finite depth gravity water waves in holomorphic coordinates.*

We consider irrotational gravity water waves with finite bottom in 2d. We discuss the local well-posedness of this problem in holomorphic coordinates and establish cubic lifespan bounds for small initial data. This is joint work with Mihaela Ifrim and Daniel Tataru. (Received September 19, 2016)

1125-35-1782 **Ademir Fernando Pazoto\***, Instituto de Matemática, Cidade Universitária - Ilha do Fundão, Rio de Janeiro, RJ 21941-909, Brazil. *Controllability of the Kuramoto-Sivashinsky equation on star-shaped trees.*

In this work we treat two control problems for the linear Kuramoto-Sivashinsky equation on a network. More precisely, the equation is considered on a star-shaped tree with Dirichlet and Neumann boundary conditions. By using the moment theory we can derive null controllability properties with boundary controls acting in the external vertices. In particular, the controllability holds if the anti-diffusion parameter of the equation does not belong to a countable critical set of real numbers. (Received September 19, 2016)

1125-35-1818 **Xiaoyi Zhang\*** (xiaoyi-zhang@uiowa.edu), 14 Maclean Hall, Iowa City, IA, 52246, Iowa City, IA 52246. *The symplectic non-squeezing for nonlinear Schrodinger equations.*

In this talk, we will discuss our recent work on the symplectic nonsqueezing for cubic nonlinear schrodinger equations. This talk is based on the joint work with R. Killip and M. Visan. (Received September 19, 2016)

1125-35-1823 **Ugur G. Abdulla\*** (abdulla@fit.edu), Melbourne, FL 32901. *Recent Advances on Inverse Parabolic Free Boundary Problems.*

This talk presents recent advances on inverse Stefan type parabolic free boundary problems, where some of the coefficients of the PDE or some boundary data on the fixed boundary is missing and must be found along with the temperature and free boundary. We discuss both one-phase and multiphase cases. In one-phase case optimal control framework is employed where missing data and free boundary are components of the control vector. Multiphase optimal Stefan problem is reduced to optimal control problem for quasilinear PDE with

discontinuous coefficient. We prove the well-posedness of the optimal control problem and the convergence of the sequence of the discrete optimal control problems to the original problem both with respect to cost functional and optimal control. We prove the Frechet differentiability in Besov spaces, necessary condition for optimality, Pontryagin type maximum principle under minimal regularity assumptions on the data. (Received September 19, 2016)

1125-35-1837 **Feride Tiglay\***, 1179 University Drive, Newark, OH 43055. *The Cauchy problem for Euler-Poisson equation and nonuniform dependence*. Preliminary report.

We consider the Cauchy problem for an integrable Euler-Poisson equation which describes the fluctuations in the ion density of a two-component plasma of positively charged ions and negatively charged electrons. We review the well posedness in Sobolev spaces and show that the solution map is not uniformly continuous. (Received September 19, 2016)

1125-35-1848 **Josiah Park\*** ([jpark685@gatech.edu](mailto:jpark685@gatech.edu)), School of Mathematics, Georgia Tech, Atlanta, GA 30332. *Steklov Problems with Non-Smooth Boundary: Bounds and Asymptotics on Eigenvalues*. Preliminary report.

In this talk, bounds on the counting function for Steklov spectrum on euclidean domains with non-smooth boundary are presented. On boxes in  $\mathbb{R}^d$ , we give lower estimates on the leading terms for an expression involving the spectral counting function for eigenvalues of the Dirichlet-to-Neumann operator. (Received September 20, 2016)

1125-35-1880 **Stephen Anco\*** ([sanco@brocku.ca](mailto:sanco@brocku.ca)), Department of Mathematics & Statistics, Brock University, St Catharines, Ontario L2S3A1, Canada. *Integrable multi-component peakon equations from a modified AKNS scheme*.

The standard AKNS scheme for generating integrable evolution systems is modified to obtain integrable peakon systems. In the simplest case, the modified scheme yields a large family of integrable multi-component peakon equations, together with their recursion operators, symmetries, conservation laws, and bi-Hamiltonian structure. (Received September 19, 2016)

1125-35-1900 **Benjamin Harrop-Griffiths\*** ([benjamin.harrop-griffiths@cims.nyu.edu](mailto:benjamin.harrop-griffiths@cims.nyu.edu)). *Long time behavior of solutions to the mKdV*.

We discuss the long time behavior of solutions to the modified Korteweg-de Vries equation on the real line. For sufficiently small, smooth, decaying data we prove global existence and derive modified asymptotics without relying on complete integrability. (Received September 19, 2016)

1125-35-1907 **Jingwei Hu\***, Purdue University, West Lafayette, IN 47907. *A fast spectral method for the Boltzmann collision operator with general collision kernels*.

We propose a simple fast spectral method for the Boltzmann collision operator with general collision kernels. In contrast to the direct spectral method (Pareschi and Russo, 2000; Gamba and Tharkabhushanam 2009) which requires  $O(N^6)$  memory to store precomputed weights and has  $O(N^6)$  numerical complexity, the new method has complexity  $O(MN^4 \log N)$ , where  $N$  is the number of discretization points in each of the three velocity dimensions and  $M$  is the total number of discretization points on the sphere and  $M \ll N^2$ . Furthermore, it requires no precomputation for the variable hard sphere (VHS) model and only  $O(MN^4)$  memory to store precomputed functions for more general collision kernels. Although a faster spectral method is available (Mouhot and Pareschi 2006) (with complexity  $O(MN^3 \log N)$ ), it works only for hard sphere molecules, thus limiting its use for practical problems. Our new method, on the other hand, can apply to arbitrary collision kernels. A series of numerical tests is performed to illustrate the efficiency and accuracy of the proposed method.

Joint work with I. Gamba, J. Haack, and C. Hauck. (Received September 19, 2016)

1125-35-1917 **Lorena Bociu\*** ([lvbociu@ncsu.edu](mailto:lvbociu@ncsu.edu)). *The role of structural viscosity in poro-visco-elastic models*.

Fluid flow through porous elastic or viscoelastic structures is relevant for many applications in biology and medicine, like tissue perfusion in the body. There is strong evidence that disruption of fluid flow inside the pores can damage the medium. We consider a system of PDEs that models fluid flow through poro-(visco)-elastic material, with permeability depending nonlinearly on the solid strain, and with volumetric and boundary forcing terms. We investigate the existence of weak solutions and the influence of viscoelasticity on the regularity of solution and forcing terms. We provide numerical evidence that sudden changes in volumetric and/or boundary sources of linear momentum may lead to uncontrolled fluid-dynamical responses if the solid component of the medium is not viscoelastic. This finding could have tremendous consequences on the understanding of many

pathological conditions, such as glaucoma. [L. Bociu, G. Guidoboni, R. Sacco, and J. Webster, Analysis of nonlinear poro-elastic and poro-visco-elastic models, *Archive for Rational Mechanics and Analysis*, 222 (2016), 1445-1519]. (Received September 19, 2016)

1125-35-1935 **Zihua Guo** (zihua.guo@monash.edu), School of Mathematical Sciences, Monash Unive, VICTORIA, 3800, Australia, **Zaher Hani\*** (hani@math.gatech.edu), School of Math., GeorgiaTech, 686 Cherry Street, Atlanta, GA , and **Kenji Nakanishi** (nakanishi@ist.osaka-u.ac.jp), Department of Pure and Applied Mathematics, Suita, Osaka, Suita, Osaka, 565-0871, Japan. *Scattering for the 3D Gross-Pitaevskii equation.*

We study the Cauchy problem for the 3D Gross-Pitaevskii equation. The global well-posedness in the natural energy space was proved by Gerard. In this paper we prove scattering for small data in the same space with some additional angular regularity, and in particular in the radial case we obtain small-energy scattering. (Received September 19, 2016)

1125-35-1939 **Joyce R. McLaughlin\*** (mclauj@rpi.edu), 110 8th Street, Troy, NY 12180-3590. *Unique Continuation, Uniqueness and Optimization for Viscoelastic Models.* Preliminary report.

In Biomechanical Imaging of tissue and Imaging in geophysics, viscoelastic models are used in order for the mathematical models: (1) to accurately predict the data; and (2) given the data, to enable the imaging functional to accurately compute biomechanical properties of tissue of physical properties of the earth. The mathematical structure of these integro-differential operators, in the time/space domain or in the frequency domain, have new properties. We present unique continuation and uniqueness results in the time/space domain and optimization results in the frequency domain. (Received September 19, 2016)

1125-35-1949 **Irina Mitrea\*** (imitrea@temple.edu), 640 Wachman Hall, 1805 N. Broad St., Philadelphia, PA 19122. *The Dirichlet problem for elliptic systems with data in Köthe function spaces.*

In this talk I will discuss well-posedness results for the Dirichlet problem for second-order, homogeneous, elliptic systems, with constant complex coefficients, in the upper half space, with boundary data from Lebesgue spaces, variable exponent Lebesgue spaces, Lorentz spaces, Zygmund spaces, as well as their weighted versions. A key tool in this analysis is establishing boundedness of the Hardy-Littlewood maximal operator on appropriate Köthe function spaces. This is joint work with Dorina Mitrea, Marius Mitrea and Jose Maria Martell. (Received September 19, 2016)

1125-35-1972 **Walter Craig\*** (craig@math.mcmaster.ca), Department of Mathematics & Statistics, McMaster University, 1280 Main Street West, Hamilton, Ontario L8S 4K1, Canada. *Birkhoff normal form for nonlinear wave equations.*

Wave equations can be considered as Hamiltonian PDEs, that is, partial differential equations that can be considered in the form of a Hamiltonian system. Many theorems on global existence of small amplitude solutions of nonlinear wave equations in  $\mathbb{R}^n$  depend upon a competition between the time decay of solutions and the degree of the nonlinearity. Decay estimates are more effective when inessential nonlinear terms are able to be removed through a well-chosen transformation. In this talk, we construct Birkhoff normal forms transformations for the class of wave equations which are Hamiltonian PDEs and null forms, giving a new proof via canonical transformations of the global existence theorems for null form wave equations of S. Klainerman and J. Shatah in space dimensions  $n \geq 3$ . The critical case  $n = 2$  is also under consideration. These results are work in collaboration with A. French and C.-R. Yang (Received September 19, 2016)

1125-35-2019 **Mathew Baxter\*** (mabaxter@fgcu.edu), Florida Gulf Coast University, 10501 FGCU Blvd. S., Fort Myers, FL 33993, **Mangalagama Dewasurendra**, University of Central Florida, 4393 Andromeda Loop N., Orlando, FL 32816, and **Kuppalapalle Vajravelu**, University of Central Florida, 4393 Andromeda Loop N., Orlando, FL 32816. *A Method of Directly Defining the Inverse Mapping for Solutions of Coupled Systems of Nonlinear Differential Equations.*

Recently, Liao introduced a new method for finding analytical solutions to nonlinear differential equations. In this paper, we extend this idea to nonlinear systems. We study the system of nonlinear differential equations that governs nonlinear convective heat transfer at a porous flat plate, and find functions that approximate the solutions by extending Liao's Method of Directly Defining the Inverse Mapping (MDDiM). (Received September 19, 2016)

1125-35-2031 **Justin L Taylor\*** (jtaylor52@murraystate.edu), **Russell M Brown** and **Seick Kim**.  
*The Heat Kernel for Systems of Linear Elasticity.*

We consider the elliptic system of linear elasticity with bounded measurable coefficients in a domain where the second Korn inequality holds. We construct the heat kernel of the system subject to mixed boundary conditions under the assumption that weak solutions of the elliptic system are Hölder continuous in the interior. Moreover, we show that if weak solutions of the mixed problem are Hölder continuous up to the boundary, then the corresponding heat kernel has a Gaussian bound. As an application, we construct the Green's function for the elliptic mixed problem. (Received September 19, 2016)

1125-35-2034 **Jacek Jendrej\*** (jacek@math.uchicago.edu). *On pure two-bubbles for focusing energy-critical dispersive equations.*

We consider some focusing energy-critical dispersive equations with radially symmetric data. These equations admit stationary solutions called "bubbles". Due to the scaling symmetry, the stationary states form a non-compact one-parameter family. We study solutions which converge in the energy space to a sum of two bubbles. These bubbles are decoupled in the sense that they are concentrated at scales whose ratio converges to zero. In the radially symmetric case, such solutions are the simplest purely non-dispersive objects other than the stationary states. (Received September 19, 2016)

1125-35-2052 **Zhiwu Lin\*** (zlin@math.gatech.edu), **Jiayin Jin** and **Shasha Liao**. *Nonlinear modulational instability of dispersive wave models.*

Modulational instability (also called side band instability, Benjamin-Feir instability) is an important instability mechanism in lots of physical models, including 2D water waves and model equations such as KDV, BBM, and Whitham equations. It leads to the breakdown of periodic traveling wave pattern in these modes and the formation of stable structures such as envelope solitons. In the literature, such instability had been studied a lot from the linearized equation, i.e., the spectra of the linearized operator. With Shasha Liao and Jiayin Jin, we prove nonlinear modulational instability for lots of dispersive models including nonlinear Schrodinger equation, BBM, and KDV type equations (KDV, Benjamin-Ono, Whitham etc). The nonlinear instability is proved for both periodic and localized perturbations. The two main ingredients in the proof are: for the linear step, the semigroup estimates are obtained by using the Hamiltonian structures of the linearized PDEs; for the nonlinear step, the loss of derivative in the nonlinear term is overcome by the construction of higher order approximation solutions. (Received September 19, 2016)

1125-35-2071 **Casey Douglas\*** (cjdouglas@smcm.edu), 18952 E. Fisher Rd., St. Mary's City, MD 20686.  
*Non-Euclidean Photography: The World of Mirrors.*

Optical devices consisting of cameras and mirrors are called catadioptric sensors, whose design lies at the intersection of both pure and applied mathematics. Its primary goal is to construct mirror surfaces to photograph the world in informative ways. In particular, mirrors are designed so that the catadioptric sensor preserves certain geometric properties, thereby establishing deep and original connections between differential geometry and optical engineering. Some examples include, wide-angle mirrors with no distortion and mirrors mimicking classical map projections. We will give multiple examples of accessible yet open problems in mirror design. (Received September 20, 2016)

1125-35-2080 **Farhod Abdullayev\*** (fabdullayev@wpi.edu), Department of Mathematical Sciences, 100 Institute Road, Worcester, MA 01609. *On quasi-static limits of one-dimensional dynamic cohesive fracture.* Preliminary report.

Quasi-static models are based on the assumption that whatever is driving the motion, for example loading or Dirichlet conditions, varies slowly in time compared to the elastic wave speed of the material. We analyze a one-dimensional model of dynamic cohesive fracture with varying Dirichlet condition, and take the limit as the speed with which the condition changes goes to zero. We then study the question of whether the usual model for quasi-static cohesive fracture is the limit of dynamic cohesive fracture. (Received September 19, 2016)

1125-35-2091 **Shijun Zheng\*** (szheng@georgiasouthern.edu), Department of Mathematical Sciences, Georgia Southern University, Statesboro, GA 30460-8093. *Minimal mass-energy dynamics for rotating BEC.*

I will review some recent progress concerning long time behaviors (existence, blowup and excited states) for the Gross-Pitaevskii equation with rotation. The GPE belongs in the type of magnetic nonlinear Schrodinger systems (mNLS) with a Lorentz gauge, which arises in modeling dilute, trapped boson gases with integer-spin in ultra-cold temperature.

Such system may exhibit interesting symmetries as well as stationary wave phenomenon, accompanied by spinor and quantized vortex, a remarkable signature for the superfluidity of Bose-Einstein condensation (BEC). I will discuss certain sharp criterion in terms of the mass and energy for the ground state in two and three dimensions. (Received September 19, 2016)

1125-35-2095 **Andres Contreras\***, New Mexico State University, Department of Mathematical Sciences, 1290 Frenger Mall, Las Cruces, NM 88003. *Uniaxiality vs Biaxiality in the Q-tensor model.*

In joint work with Xavier Lamy we show that in the low temperature limit, nematics always take advantage of the extra degrees of freedom and avoid isotropic melting thus proving rigorously biaxial escape takes place (Received September 19, 2016)

1125-35-2097 **Xiaoming Wang\***, Department of Mathematics, Florida State University, 1017 Academic Way, Tallahassee, FL 32306. *Coupling of free flow with porous media flow.*

Coupled free flow and porous media flow is of great importance in science and engineering applications. However, the interface boundary conditions are not very well understood, especially when the interface is curved or when the Reynolds number is large. We present a few recent results on the derivation of the interface boundary conditions based on variational principle at the low Reynolds number regime. We also show that two well-known competing interface boundary conditions at high Reynolds number are consistent in the sense that they lead to the same leading order behavior at small Darcy number. (Received September 19, 2016)

1125-35-2098 **Casey P Rodriguez\*** (c-rod216@math.uchicago.edu) and **Andrew Lawrie** (alawrie@mit.edu). *Solitons and scattering for a semi-linear Skyrme equation.*

In this talk we consider a generalization of energy super-critical wave maps which were introduced by Adkins and Nappi as an alternative to Skyrme wave maps. These are corotational maps from 1+3 dimensional Minkowski space into the 3-sphere which satisfy a certain semi-linear geometric wave equation. Each finite energy Adkins-Nappi wave map has a fixed topological degree  $n$  which is an integer. We will discuss recent work in which we prove that for each  $n \in \mathbb{N} \cup \{0\}$  there exists a unique, nonlinearly stable Adkins-Nappi harmonic map  $Q_n$  (a static solution) with degree  $n$ , and we have the following conditional large data result: any Adkins-Nappi wave map of degree  $n$  whose critical norm is bounded on its interval of existence must be global and scatter to  $Q_n$  as  $t \rightarrow \pm\infty$ . (Received September 20, 2016)

1125-35-2112 **Jiayin Jin\*** (jin@math.gatech.edu), School of Mathematics, Georgia Institute of Technology, 686 Cherry ST, Atlanta, GA 30332, and **Zhiwu Lin** and **Chongchun Zeng**. *Dynamical Structures near the solitary waves of the supercritical gKDV equation.*

It is well known that the solitary waves of the supercritical gKDV equation are linearly unstable. By using the Hamiltonian structure of the linearized gKDV equation, one can show that at each solitary wave, the energy space can be linearly decomposed into unstable space, stable space and center space. Based on the linear decomposition, we construct stable manifolds, unstable manifolds, center-stable manifolds, center-unstable manifolds and center manifolds of the orbits of the whole family of solitary waves. The existence of unstable manifolds implies the nonlinear instability of solitary waves. Beyond instability and stability analysis, these invariant manifolds give a clear picture of the complex dynamics near solitary waves. The construction of these invariant manifolds is nontrivial, because the nonlinearity contains a loss of derivative. (Received September 19, 2016)

1125-35-2134 **David S Torain II\*** (david.torain@montgomerycollege.edu). *Using Torain's Equations as a Predictable Model in the Sciences.*

Torain's equations are essentially a set of parametric non-linear differential equations that use an analytical tool as a mathematical model. These equations offer a global science view that computer calculations cannot provide. Since only numerical methods have been previously developed in the sciences to investigate this problem, this research proposes a new analytical approach. This approach is based on the notion that when one of the sixteen parameters of Torain's equations is sent to infinity, the general solution, involving the remaining fifteen parameters can be expressed in terms of elementary functions. This is made possible because, in this limit besides the time invariance group, an exact invariance-scaling group exists. This approach uses the science field to study the resulting mathematical model. (Received September 19, 2016)

1125-35-2142 **Gui-Qiang Chen, Mikhail Feldman\*** (feldman@math.wisc.edu) and **Wei Xiang**. *Uniqueness for shock reflection problem.*

We discuss shock reflection problem for compressible gas dynamics, von Neumann conjectures on transition between regular and Mach reflections, and existence of regular reflection solutions for potential flow equation.

Then we will talk about recent results on uniqueness of regular reflection solutions for potential flow equation, in a natural class of self-similar solutions. (Received September 19, 2016)

1125-35-2144 **Guillaume Bal\***, gb2030@columbia.edu. *Some inverse problems and boundary controls for phase-space transport equations.*

Consider a prescribed solution to a diffusion equation in a domain embedded in a larger one. Can one (approximately) control such a solution from the boundary of the larger domain? The answer is positive and this form of a Runge approximation is a corollary to the unique continuation property (UCP) that holds for such equations. Now consider a (phase space) transport equation, which models a large class of scattering phenomena, and whose vanishing mean free path limit is the above diffusion model. This talk will present positive as well as negative results on the control of transport solutions from the boundary. In particular, we will show that internal transport solutions can indeed be controlled from the boundary of a larger domain under sufficient convexity conditions. Such results are not based on a UCP. In fact, UCP does not hold for any positive mean free path even though it does apply in the (diffusion) limit of vanishing mean free path. Such controls find applications in inverse problems that model a large class of coupled-physics medical imaging modalities.

This is joint work with Alexandre Jollivet (Received September 19, 2016)

1125-35-2146 **Stephen Pankavich\*** (pankavic@mines.edu), 1500 Illinois Street, Golden, CO 80401. *Global Classical Solutions of the Vlasov-Maxwell-Fokker-Planck System.*

The Vlasov-Maxwell system is a fundamental kinetic model of plasma dynamics. When one includes effects due to collisions with a fixed background of particles, the result is the Vlasov-Maxwell-Fokker-Planck system. The first Lorentz-invariant model, which considers relativistic velocities, was recently derived by Calogero in 2010. Here, we shall discuss well-posedness results for global-in-time classical solutions of this system and its non-relativistic analogue, each posed in a variety of dimensional settings. Our methods utilize a gain in regularity stemming from the diffusion to arrive at smooth solutions stemming from initial data which lack even weak differentiability. (Received September 19, 2016)

1125-35-2153 **Francisco Marcos Lopez-Garcia\*** (marcos.lopez@im.unam.mx), Av. Universidad s/n. Col. Lomas de Chamilpa, CP 62210 Cuernavaca, Morelos Mexico, Apartado Postal 273. Admon. De correos #3, Cuernavaca, Morelos., Mexico, and **Alberto Mercado** and **Luz De Teresa**. *Uniform controllability of a 1-D transport equation with a fourth degree viscosity term.* Preliminary report.

We study the cost of the uniform null controllability of a 1-D transport equation with a fourth degree viscosity term, with the control acting on the left boundary. (Received September 19, 2016)

1125-35-2171 **Casey Jao\*** (cjao@math.berkeley.edu). *Microlocal dispersive estimates and the energy-critical NLS on perturbations of  $\mathbf{R}^3$ .*

It is well known that solutions to the linear Schrödinger equation in  $\mathbf{R}^d$  decay in time at a rate of  $t^{-d/2}$ . This is in general false on a curved background due to refocusing of geodesics. However, as shown by Burq, Gérard, and Tzvetkov, it holds microlocally provided one restricts to times less than a sufficiently small multiple of the semiclassical parameter, essentially stopping the geodesic flow before refocusing can occur. I will discuss long-time refinements of this estimate which exhibit weaker but still nontrivial decay. Such weak dispersive estimates play an essential role in the study of energy-critical NLS on manifolds. Time permitting, I will specifically discuss the quintic NLS on small perturbations of  $\mathbf{R}^3$ . (Received September 19, 2016)

1125-35-2180 **Fabio Pusateri\***, fabiop@math.princeton.edu. *On global solutions of some fluids equations.*

We will discuss some recent results on the global regularity and asymptotic behavior of some fluids models. (Received September 19, 2016)

1125-35-2201 **Sung-Jin Oh\*** (sjoh@kias.re.kr) and **Daniel Tataru**. *On global regularity and scattering for geometric wave equations.*

I will survey and present recent progress concerning the question of global regularity and scattering for geometric wave equations, with particular emphasis on large data results in energy critical dimensions. Examples of such equations include the wave map, Maxwell-Klein-Gordon, and Yang-Mills equations. (Received September 19, 2016)

1125-35-2226 **Adam Louis Prinkey\*** ([aprinkey2009@my.fit.edu](mailto:aprinkey2009@my.fit.edu)), 3620 Misty Oak Drive, Apt 1507, Melbourne, FL 32901, and **Ugur G Abdulla** ([abdulla@fit.edu](mailto:abdulla@fit.edu)), 150 West University Blvd, Melbourne, FL 32901. *Qualitative Properties of Solutions to Nonlinear Parabolic PDEs with Double Degenerate Fast Diffusion.*

We consider the problem of interface development and local behavior of solutions near the interface in the following Cauchy problem for the nonlinear double degenerate parabolic PDE with reaction and fast diffusion:

$$u_t = \left( |(u^m)_x|^{p-1} (u^m)_x \right)_x + bu^\beta, \quad x \in \mathbb{R}, t > 0; \quad u(x, 0) = C(-x)_+^\alpha$$

The problem arises in applications involving turbulent filtration of material through a porous media. The interface behavior is determined by the competition between the diffusion and the reaction terms. The full solution for the reaction-diffusion equation ( $p = 1$ ) was given in 2002 [U.G. Abdulla, *Nonlinear Analysis: Theory, Methods and Applications*, 4, 2002, 541-560]. Our aim is to apply the methods of this paper to give a full classification for double degenerate reaction-diffusion equations with fast diffusion ( $0 < mp < 1; m, p > 0$ ). First we apply the nonlinear scaling method to identify which term dominates in the various regions of the  $(\alpha, \beta)$ -parameter space. We then construct super/subsolutions and apply special comparison theorems in irregular domains to prove explicit formulas for the interface and local solution, with precise estimations up to constant coefficients. (Received September 20, 2016)

1125-35-2234 **Dat Cao\***, Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX 79409, **Tuoc Phan**, Department of Mathematics, University of Tennessee, Knoxville, TN 37996, and **Tadele Mengesha**, Department of Mathematics, University of Tennessee, Knoxville, TN 37996. *Weighted estimates of solutions to degenerate and singular elliptic equations.*

We study the weight- $W^{1,p}$  estimates of weak solutions to the equation

$$\operatorname{div}[\mathcal{A}(x)\nabla u] = \operatorname{div}[\mathbf{F}] \quad \text{in } \Omega,$$

where  $\Omega \subset \mathbb{R}^n$  is an open bounded domain,  $\mathbf{F} : \Omega \rightarrow \mathbb{R}^n$  is a given vector field, and the coefficient matrix  $\mathcal{A}$  satisfies the degenerate elliptic condition

$$\Lambda\mu(x)|\xi|^2 \leq \langle \mathcal{A}(x)\xi, \xi \rangle \leq \Lambda^{-1}\mu(x)|\xi|^2, \quad \forall \xi \in \mathbb{R}^n, \quad \text{a.e. } x \in \mathbb{R}^n,$$

with  $\Lambda > 0$ , and the weight  $\mu$  to be in some Muckenhoupt class. (Received September 20, 2016)

1125-35-2247 **Daniele Garrisi\*** ([daniele.garrisi@inha.ac.kr](mailto:daniele.garrisi@inha.ac.kr)), West Building, Office No. 5W443, Inha University, 253 Yonghyun-Dong, Nam-Gu, Incheon, 402751, South Korea, and **Vladimir Georgiev**, Dipartimento di Matematica, Universita' di Pisa, Largo Bruno Pontecorvo n. 5, 56127 Pisa, Italy. *Non-degeneracy and uniqueness of  $H^1$  symmetric minima for combined power-type non-linearities.*

The orbital stability of standing-wave solutions to the non-linear Schrödinger equation

$$i\partial_t \varphi(t, x) + \Delta_x \varphi(t, x) - G(\varphi(t, x)) = 0$$

follows from properties of the set of minima of the functional

$$E(u) = \frac{1}{2} \int_{\mathbb{R}^n} |\nabla u(x)|^2 dx + \int_{\mathbb{R}^n} G(u(x)) dx$$

on the constraint  $S = \{u \in H_{r,+}^1(\mathbb{R}^n) \mid \|u\|_{L^2}^2 = 1\}$ . The functions  $H_{r,+}^1(\mathbb{R}^n)$  are radially symmetric (with respect to the origin),  $H^1$  and positive. Stability holds when this set is finite. It is known from Kwong (ARMA, 1989) and Berestycki and Lions (ARMA 1983) that if  $G(s) = -a|s|^p$ , there is exactly one minimum. We show that this uniqueness holds for combined power-type non-linearities,  $G(s) = -a|s|^p + b|s|^q$  and spatial dimension  $n = 1$ . We also provide a condition on  $G$  ensuring that minima of  $E$  on  $S$  are non-degenerate. As a consequence, there are only finitely many of them. (Received September 20, 2016)

1125-35-2266 **Yaxin Zhang\*** ([yz7211@rit.edu](mailto:yz7211@rit.edu)), School of Mathematical Sciences, Rochester Institute of Technology, 85 Lomb Memorial Drive, Rochester, NY 14623, and **Baasansuren Jadamba**. *Parameter Estimation in a Mathematical Model of Tumor Growth.* Preliminary report.

We consider a minimization problem with PDE constraints arising from mathematical modeling of an avascular, spheric tumor growth. The model is based on the diffusion based growth of tumor and consists of a system of coupled partial differential equations (PDEs). The problem is solved numerically and the results are compared to the model data and biological parameters that are available. (Received September 20, 2016)

1125-35-2294 **George Avalos\*** ([gavalos@math.unl.edu](mailto:gavalos@math.unl.edu)), Department of Mathematics, University of Nebraska-Lincoln, Lincoln, NE 68588. *Semigroup Wellposedness of a Compressible Fluid-Structure PDE Interactive Model.*

In this talk, we consider a partial differential equation (PDE) system which models a fluid-structure interaction of current interest within the mathematical literature. The coupled PDE model under discussion involves a Stokes system, which evolves on a three dimensional domain, interacting with a Lamé system of elasticity which evolves on a flat portion of said fluid domain. Moreover there is an additional coupling PDE which determines the associated pressure variable of the fluid-structure system. In addition, because of the presence of an “ambient flow” vector field, the coupled PDE is not dissipative. This is joint work with Pelin Güven Geredeli of Hacettepe University (Ankara, Turkey) and Justin Webster of The College of Charleston. (Received September 20, 2016)

1125-35-2320 **Ugur G. Abdulla and Roqia A. Jeli\*** ([rjeli2011@my.fit.edu](mailto:rjeli2011@my.fit.edu)). *Evolution of Interfaces for the Nonlinear Parabolic  $p$ -Laplacian Type Reaction-Diffusion Equations.*

We present a full classification of the short-time behaviour of the interfaces and local solutions to the nonlinear parabolic  $p$ -Laplacian type reaction-diffusion equation of non-Newtonian elastic filtration

$$u_t - \left( |u_x|^{p-2} u_x \right)_x + bu^\beta = 0, p > 2, \beta > 0$$

The interface may expand, shrink, or remain stationary as a result of the competition of the diffusion and reaction terms near the interface, expressed in terms of the parameters  $p, \beta, \text{sign } b$ , and asymptotics of the initial function near its support. In all cases, we prove the explicit formula for the interface and the local solution with accuracy up to constant coefficients. The methods of the proof are based on nonlinear scaling laws, and a barrier technique using special comparison theorems in irregular domains with characteristic boundary curves. (Received September 20, 2016)

1125-35-2382 **Narayan Thapa\*** ([nthapa@cameron.edu](mailto:nthapa@cameron.edu)), 2800 W Gore Blvd, Lawton, OK 73505. *On the Numerical Solution of Second Order Hyperbolic Nonlinear Partial Differential Equation.* Preliminary report.

Numerical methods are crucial in estimating parameters in nonlinear partial differential equations. In this work, we present a computational algorithm for approximate solutions of the adjoint system to estimate optimal parameters. We also discuss error at different noise levels and we present results from numerical experiments. (Received September 20, 2016)

1125-35-2401 **Dhanapati Adhikari\*** ([dadhikari@marywood.edu](mailto:dadhikari@marywood.edu)), Marywood Univeristy, 2300 Adams Avenue, Scranton, PA 18509. *On the global regularity of two-dimensional incompressible Boussinesq equations with mixed partial dissipation.*

The Boussinesq equations concerned here model geophysical flows such as atmospheric fronts and oceanic circulation. Mathematically the 2D Boussinesq equations serve as a lower dimensional model of the 3D hydrodynamics equations. In fact, the 2D Boussinesq equations retain some key features of the 3D Euler and Navier-Stokes equations such as the vortex stretching mechanism. The issue of global regularity of two-dimensional Boussinesq equations can be difficult when there is only partial dissipation or no dissipation at all. This talk presents a recent result on the global (in time) regularity of classical solutions of the 2D incompressible Boussinesq equations with mixed partial dissipation. (Received September 20, 2016)

1125-35-2467 **Yuan Zhou\*** ([zhouy@mail.usf.edu](mailto:zhouy@mail.usf.edu)), Department of Mathematics and Statistics, University of South Florida, 4202 East Fowler Ave, Tampa, FL 33620, and **Wen-Xiu Ma.** *Quasi-period solutions to reduced DKN hierarchy.*

In our recent paper, we studied the reduced D-Kaup–Newell spectral problems from  $sl(2, \mathbb{R})$ , and established a hierarchy of commuting bi-Hamiltonian soliton equations by zero curvature equations and we also explicitly computed their hereditary recursion operators.

In this paper, we discuss a new spectral problem:  $\phi_x = U(\lambda, p, q)\phi$  which possesses the same soliton hierarchy as the reduced D-Kaup–Newell spectral problems. With the aid of the Lax matrix, we introduce an hyperelliptic curve of arithmetic genus  $n$ :  $\mathcal{K}_n = \{(\lambda, y) : y^2 - R(\lambda) = 0\}$ , where  $R$  is a monic polynomial of degree  $2n + 2$ .

In order to formulate algebro-geometric solutions to the soliton hierarchies in terms of the Riemann theta functions, we develop a scheme to determine Dubrovin type equations for zeros and poles of meromorphic functions. We straighten out all flows in soliton hierarchies under the Abel-Jacobi coordinates associated with Lax pairs, and study the asymptotic behaviors of the Baker-Akhiezer functions. Finally, we succeed in finding the theta function representations of the potentials  $p$  and  $q$ . (Received September 20, 2016)

- 1125-35-2479 **Shijun Zheng\*** ([szheng@georgiasouthern.edu](mailto:szheng@georgiasouthern.edu)), Department of Mathematical Sciences, Georgia Southern University, Statesboro, GA 30460. *Orbital Stability of Standing Waves for Dispersive Models with Nonlocal Nonlinearity*. Preliminary report.

We consider orbital stability for several dispersive type PDEs including the nonlinear fractional Schrödinger equation with singular and unbounded potentials. We show in some cases the instability by constructing counterexamples. This reports our recent joint works with Shihui Zhu, Nyla Basharat, Hichem Hajaiej and Yi Hu. (Received September 20, 2016)

- 1125-35-2485 **Magdalena Czubak\*** ([magda.czubak@colorado.edu](mailto:magda.czubak@colorado.edu)). *Almost sure wellposedness for 2D wave equations with null forms. Part I*.

The null condition was introduced by Klainerman. The nonlinearities exhibiting null structure appear in many systems including Wave Maps, Yang Mills, Maxwell-Klein-Gordon and the space-time Monopole equation. Two dimensions create obstacles towards low regularity wellposedness that are not present in higher dimensions. In this talk we describe recent progress on improving the wellposedness results by suitably randomizing the initial data. We discuss both a periodic and nonperiodic setting. (Received September 20, 2016)

- 1125-35-2499 **James Dilts\*** ([jdilts@ucsd.edu](mailto:jdilts@ucsd.edu)), **Michael Holst**, **Tamara Kozareva** and **David Maxwell**. *Numerical Bifurcation Analysis of a Coupled Elliptic System*.

The Einstein constraint equations are a coupled system of elliptic differential equations that, in general relativity, restrain the allowed choices of initial data. A major goal is the parameterization of all solutions to these equations. While each equation is individually well understood and well behaved, the coupled system is more complicated. When the mean curvature parameter (a function which will be the mean curvature of the initial data in the resulting spacetime) is constant or nearly constant, freely given seed data leads to a unique solution of the equations. However, when the mean curvature is far from constant, the equations can have multiple or no solutions. In this talk we will discuss our mostly numerical results showing nonuniqueness and nonexistence using the numerical bifurcation tool AUTO. (Received September 20, 2016)

- 1125-35-2509 **C. MaLyn Lawhorn\*** ([lawhornc2@winthrop.edu](mailto:lawhornc2@winthrop.edu)), 251 E Jones St, Blacksburg, SC 29702, and **Zachary Abernathy** and **Kristen Abernathy**. *Using Partial Differential Equations to Model Tumor Growth*.

Malignant tumors are a collection of cancerous cells that form in various parts of the body. Because a tumor can grow almost anywhere in the body and become particularly problematic after angiogenesis occurs, it is of interest to understand the spatial growth of the tumor. Using an existing PDE model by McGillan et al., we incorporate a stem cell population and explore the behavior of the tumor cells and cancer stem cells as they invade healthy tissue. We also investigate the stability of stationary solutions and numerically demonstrate four different tumor states. (Received September 20, 2016)

- 1125-35-2573 **Yongjin Lu\*** ([y1u@vsu.edu](mailto:y1u@vsu.edu)), Department of Mathematics and Economics, PO Box 9068, Virginia State University, Petersburg, VA 23838. *Uniform stability near a non-trivial equilibrium of a nonlinear fluid-viscoelastic structure interaction*. Preliminary report.

We consider uniform stability near a non-trivial equilibrium of a nonlinear fluid-structure interaction, where the elastic body exhibits small but rapid oscillations and the coupled dynamics is governed by a PDE system coupling Navier-Stokes equation with a wave equation. FSI considered here could model the dynamics of a structure submerged or surrounding viscous non-compressible fluid and has wide applications ranging from aerospace engineering, civil engineering, medicine and environmental sciences, etc. Uniform stability near a non-trivial equilibrium of FSI is obtained by implementing a viscoelastic damping on the structure and a fully supported interior feedback on the fluid. (Received September 20, 2016)

- 1125-35-2583 **Ryan Honan Pellico\*** ([ryan.pellico@trincoll.edu](mailto:ryan.pellico@trincoll.edu)), Department of Mathematics, 300 Summit Street, Hartford, CT 06106. *Periodic Oscillations in a Nonlinear Suspension Bridge Model*.

A nonlinear system of partial differential equations is presented to model the vertical motions of the roadbed and main cable of a suspension bridge. The model couples a beam equation and wave equation via a “one-sided” Hooke’s Law, which accounts for the fact that the suspension cables may lose tension if the roadbed rises high enough. This behavior was observed before the collapse of the Tacoma Narrows Bridge. Using numerical continuation and stability analysis, we provide numerical evidence and bifurcation diagrams demonstrating the existence of multiple stable periodic responses to the same periodic forcing. Separable solutions are investigated using methods for ordinary differential equations while more general solutions are investigated using a finite

difference scheme and an implicit-explicit initial value solver. The method of steepest descent is also used to find entirely new branches of periodic solutions. (Received September 20, 2016)

1125-35-2626 **U. G. Abdulla** ([abdulla@fit.edu](mailto:abdulla@fit.edu)), 150 W. University Blvd, Mathematical Sciences Department, Melbourne, FL 32901, and **J. G. Goldfarb\*** ([jgoldfar@fit.edu](mailto:jgoldfar@fit.edu)), 150 W. University Blvd., Mathematical Sciences Department, Melbourne, FL 32901. *Frechet Differentiability in Besov Spaces in the Optimal Control of Parabolic Free Boundary Problems.*

We consider the inverse Stefan type free boundary problem, where information on the boundary heat flux and density of the sources are missing and must be found along with the temperature and the free boundary. We pursue optimal control framework where boundary heat flux, density of sources, and free boundary are components of the control vector. We prove the Frechet differentiability in Besov spaces, and derive the formula for the Frechet differential under minimal regularity assumptions on the data. The result implies a necessary condition for optimal control and opens the way to the application of projective gradient methods in Besov spaces for the numerical solution of the inverse Stefan problem. (Received September 20, 2016)

1125-35-2648 **Nyla Basharat** and **Yi Hu\*** ([yihu@georgiasouthern.edu](mailto:yihu@georgiasouthern.edu)), Department of Mathematical Sciences, Georgia Southern University, Statesboro, GA 30460, and **Shijun Zheng**. *Some blow-up rates of solutions to nonlinear Schrödinger equations with rotations.* Preliminary report.

In this talk we consider the nonlinear Schrödinger equation with rotation  $iu_t = -\frac{1}{2}\Delta u + V(x)u + u|u|^{p-1} - \Omega \cdot Lu$  and introduce some recent progress of the blow up rate. In the mass super critical and energy subcritical range, for radially symmetric initial data, we give a universal upper bound on the blow up rate. In the mass critical case, assuming some spectral property, we give limiting profiles of blow-up solutions. This is a joint work with Nyla Basharat and Shijun Zheng. (Received September 20, 2016)

1125-35-2675 **Dana Mendelson\***, [dana@math.uchicago.edu](mailto:dana@math.uchicago.edu). *Almost sure wellposedness for 2D wave equations with null forms (Part II).*

The null condition was introduced by Klainerman. The nonlinearities exhibiting null structure appear in many systems including Wave Maps, Yang Mills, Maxwell-Klein-Gordon and the space-time Monopole equation. Two dimensions create obstacles towards low regularity wellposedness that are not present in higher dimensions. In this talk we describe recent progress on improving the wellposedness results by suitably randomizing the initial data. We discuss both a periodic and nonperiodic setting. (Received September 20, 2016)

1125-35-2688 **Dong Li\*** ([dli@math.ubc.ca](mailto:dli@math.ubc.ca)), Vancouver, BC V6T1Z2, Canada. *Instabilities in fluid models.*

I will discuss some recent developments concerning the instabilities of some fluid PDE models. (Received September 20, 2016)

1125-35-2693 **Dong Li\*** ([dli@math.ubc.ca](mailto:dli@math.ubc.ca)). *Some new results on Kato-Ponce type inequalities.*

I will discuss some new results on Kato-Ponce type inequalities and generalizations of Kenig-Ponce-Vega type estimates, and applications to PDE models. (Received September 20, 2016)

1125-35-2717 **Jeff Morgan** ([jjmorgan@central.uh.edu](mailto:jjmorgan@central.uh.edu)) and **Vandana Sharma\*** ([vandan@asu.edu](mailto:vandan@asu.edu)). *Global Existence for Volume Surface Reaction Diffusion Systems with Dynamic Boundary Conditions.*

We consider reaction diffusion systems where some components react and diffuse on the surface, and others diffuse inside the domain and react with surface components through dynamic boundary conditions. Under reasonable hypotheses, we prove the existence of component wise non-negative global solutions. (Received September 20, 2016)

1125-35-2737 **Ashok Aryal\*** ([aaryal@ksu.edu](mailto:aaryal@ksu.edu)), Department of Mathematics, 138 Cardwell Hall, Kansas State University, Manhattan, KS 66506. *Geometry of underlying set in Mean Value Theorem for general divergence form elliptic operators.*

In his Fermi Lectures, Caffarelli gave an alternative proof of the mean value theorem(MVT) for the Laplacian where the functions do not need twice differentiability. In the same lecture he mentioned that one can also extend it to cover the case of general divergence form elliptic operators. Ivan Blank and Zheng Hao proved the MVT for general divergence form elliptic operators over a set which is comparable to balls. The topology and geometry of such sets, which we will call hereafter “the mean value balls”, are mostly unknown. I will discuss

some recent results that we (me and my adviser Ivan Blank) have proved on the geometry of the mean value balls and some related problems. (Received September 20, 2016)

1125-35-2778 **Svetlana Roudenko\*** (roudenko@gwu.edu), Washington, DC 20052. *Instability of solitary waves in the KdV-type equations.* Preliminary report.

We revisit the instability of solitons in the critical and supercritical cases of the KdV equation, and investigate a similar phenomenon in the higher dimensional generalizations of the KdV equation. (Received September 20, 2016)

1125-35-2781 **Danny Arrigo\***, Department of Mathematics, University of Central Arkansas, Conway, AR 72035, and **Brandon Ashley, Seth Bloomberg** and **Thomas Deatherage.** *Nonclassical Symmetries of a Nonlinear Diffusion Equation and a System Equivalent.*

It is generally known that classical point and potential Lie symmetries of differential equations (the latter calculated as point symmetries of an equivalent system) can be different. We question whether this is true when the symmetries are extended to nonclassical symmetries. In this talk we consider a nonlinear diffusion equation where we will show that the majority of the nonclassical point symmetries are inclusive to the nonclassical potential symmetries. We highlight a special case where the opposite is true. (Received September 20, 2016)

1125-35-2784 **Svetlana Roudenko\*** (roudenko@gwu.edu), Washington, DC 20052. *Blow up solutions in the semilinear Schroedinger equation.*

We discuss the behavior of the blow up solutions in the focusing nonlinear Schroedinger equation. (Received September 20, 2016)

1125-35-2816 **Xuwen Chen** and **Justin Holmer\*** (justin\_holmer@brown.edu), Brown University, Box 1917, 151 Thayer St, Providence, RI 02912. *The rigorous derivation of the 2D cubic focusing NLS from quantum many-body evolution.*

We consider a 2D time-dependent quantum system of  $N$ -bosons with harmonic external confining and attractive interparticle interaction in the Gross-Pitaevskii scaling. We derive stability of matter type estimates proving that the  $k$ -th power of the energy controls the  $H^1$  Sobolev norm of the solution over  $k$  particles. This estimate is new and more difficult for attractive interactions than repulsive interactions. For the proof, we use a version of the finite-dimensional quantum de Finetti theorem. This *a priori* bound allows us to prove that the corresponding BBGKY hierarchy converges to the GP limit as was done in many previous works treating the case of repulsive interactions. As a result, we obtain that the *focusing* nonlinear Schrödinger equation is the mean-field limit of the 2D time-dependent quantum many-body system with attractive interatomic interaction and asymptotically factorized initial data. An assumption on the size of the  $L^1$ -norm of the interatomic interaction potential is needed that corresponds to the sharp constant in the 2D Gagliardo-Nirenberg inequality. (Received September 20, 2016)

1125-35-2822 **Milana Pavic-Colic** and **Maja Taskovic\***, taskovic@math.upenn.edu. *Exponential moments for the homogeneous Kac equation.*

The Kac equation is a model for a one-dimensional rarefied gas in which particle collisions conserve mass and energy but not the momentum. It can be viewed as a simplified model of the Boltzmann equation for Maxwell molecules. Like the Boltzmann equation, it is an integro-differential equation with a bilinear integral operator. The angular kernel of the integral operator often has a non-integrable angular singularity (non-cutoff regime) which is sometimes cut off to simplify the analysis of the equation. The non-integrable singularity though is known to have a smoothing effect.

In 1993 Desvillettes proved that the homogeneous Kac equation with an angular cutoff propagates exponential moments of order 2 and of order 1. We apply recent techniques of Mittag-Leffler moments to show propagation of exponential moments of all orders between 0 and 2 (not necessarily integers). Using the same technique we are also able to treat the non-cutoff Kac equation, in which case the order of the exponential moment depends on the singularity rate of the angular kernel. We also consider the Boltzmann equation for Maxwell molecules, both with and without an angular cutoff. This is a joint work with Milana Pavic-Colic. (Received September 20, 2016)

1125-35-2824 **Jonathan Luk** and **Sung-Jin Oh\*** (sjoh@kias.re.kr). *Instability of Cauchy horizon under spherically symmetric perturbations.*

The strong cosmic censorship conjecture of Penrose predicts that the Cauchy horizon inside a rotating (Kerr) or charged (Reissner-Nordstrom) black hole is unstable under generic perturbations. In this talk, I will present

recent results concerning the instability of the Cauchy horizon of the Reissner-Nordström (subextermal, nonzero charge) spacetime in the class of spherically symmetric scalar perturbations. (Received September 20, 2016)

1125-35-2854 **Fagner D. Araruna\*** (fagner@mat.ufpb.br), Universidade Federal da Paraíba, Departamento de Matemática, João Pessoa, Paraíba 58051-900, Brazil. *Stackelberg-Nash exact controllability for parabolic equations.*

We will apply the concept of Stackelberg-Nash strategies to control parabolic systems. We will assume that we can act on the equation through a hierarchy of controls. A first control (the leader) is assumed to choose the policy. Then a Nash equilibrium pair (corresponding to a noncooperative multiple-objective optimization strategy) is found; this governs the action of the other controls (the followers). We will obtain the exact controllability to a prescribed (but arbitrary) trajectory. (Received September 20, 2016)

1125-35-2915 **Sedar Ngoma\*** (nzb0015@auburn.edu), 221 Parker Hall, Auburn, AL 36849, **Dmitry Glotov**, 221 Parker Hall, Auburn, AL 36849, **A. J. Meir**, 209 D Clements Hall, Dallas, TX 75275, and **Willis E. Hames**, 210 Petrie Hall, Auburn, AL 36849. *Recovering a time-dependent diffusion coefficient in a parabolic PDE with applications in geochronology.*

We consider a problem arising in geochronology, a branch of geology in which the dating of rock formations and geological events is studied. In particular, we investigate the reconstruction of temperature histories of rocks, that amounts to solving a time-dependent inverse diffusion coefficient problem with an integral overspecification. Using the Banach fixed point theorem, we show the existence and uniqueness of classical solutions. We describe a numerical scheme used to approximate the solutions of the inverse problem by means of a finite element discretization for space and the backward implicit Euler method in time. Numerical experiments illustrate the accuracy and efficiency of the proposed scheme. (Received September 20, 2016)

1125-35-2956 **Jared C Bronski\***, 1409 W Green St, Urbana, IL 61801. *Stability of quasiperiodic solutions to nonlinear Schrödinger type equations.*

We consider the stability of quasi-periodic solutions to standing and traveling wave solutions to nonlinear dispersive equations of nonlinear Schrödinger type. We prove a result concerning the spectrum of the linearized operator in the neighborhood of the origin in the spectral plane that can be considered to be a rigorous form of the Whitham modulation theory. (Received September 20, 2016)

1125-35-2982 **Thomas Beck\*** (tdbeck@mit.edu). *Level sets of ground state eigenfunctions.*

We will consider the ground state eigenfunction for a class of Schrödinger operators on a convex two dimensional domain. The assumptions on the potential will ensure that the level sets of the eigenfunctions are convex. By constructing length scales and an orientation of the domain, depending on the eigenvalues of associated ordinary differential operators, we can determine the shape of the intermediate level sets. We will also discuss questions concerning the infinitesimal shape of the level sets near to the maximum of the eigenfunction. (Received September 20, 2016)

1125-35-3036 **Tristan Buckmaster\*** (buckmaster@cims.nyu.edu). *Onsager's conjecture and non-uniqueness in fluid equations.*

In this talk I will discuss new results related to Onsager's conjecture and non-uniqueness to fluid equations. (Received September 20, 2016)

1125-35-3045 **Jerry Bona\*** (bona@math.uic.edu). *Propagation of water waves.* Preliminary report.

This talk will address wellposedness and illposedness for certain water wave models that include regularized long-wave (BBM) equation. (Received September 20, 2016)

1125-35-3068 **Sung-Jin Oh\*** (sjoh@math.berkeley.edu). *Small data global existence and decay for relativistic Chern-Simons equations.* Preliminary report.

We establish a general small data global existence and decay theorem for Chern-Simons theories with a general gauge group, coupled with a massive relativistic field of spin 0 or 1/2. (Received September 20, 2016)

1125-35-3133 **Alexandru Ionescu\*** (aionescu@math.princeton.edu), Fine Hall, Washington Road, Princeton, NJ 08544. *Bilinear estimates in quasilinear evolution problems.*

I will discuss the role of bilinear estimates in proving long-term regularity for certain quasilinear evolution equations. The main examples are water-wave systems, plasma models, and certain problems in General Relativity. (Received September 21, 2016)

1125-35-3135 **Alexandru Tamasan\*** ([tamasan@math.ucf.edu](mailto:tamasan@math.ucf.edu)), Department of Mathematics, 4000 Central Florida Blvd., Orlando, FL 32816. *Stability in an hybrid inverse conductivity problems from minimal data*. Preliminary report.

We consider the hybrid inverse problem of reconstructing the electrical conductivity inside a body from minimal interior knowledge of a current density field inside. Two structural stability results will be presented, one of which constitutes a preliminary report. (Received September 21, 2016)

1125-35-3142 **C Flores\***, One University Drive, California State University Channel Islands, BTE 2762, Camarillo, CA 93012. *Projects for Undergraduates in the construction of solutions to truncated Moment Problems and applications to PDEs*. Preliminary report.

This presentation outlines an investigation of the synergy between truncated moment problems and their wide applications in the theory of partial differential equations (PDEs). For instance, some PDEs display a special time evolution of the moments up to a finite order, such as in the case of the Benjamin-Ono equation, a model for long internal gravity waves. As part of this project, undergraduate student participants study how to derive useful measures with prescribed moment properties. In this talk, we discuss the motivation of the project, the undergraduate student involvement, as well as the mathematical results. (Received September 21, 2016)

1125-35-3146 **Robert Strain\*** ([strain@math.upenn.edu](mailto:strain@math.upenn.edu)), Department of Mathematics University of Penns, David Rittenhouse Lab, 209 South 33rd Street, Philadelphia, PA 19104. *On the Musket problem*.

The Muskat problem models the dynamics of an interface between two in-compressible immiscible fluids with different characteristics, in porous media. The phenomena have been described using the experimental Darcy's law. Saffman and Taylor (1958) related this problem with the evolution of an interface in a Hele-Shaw cell since both physical scenarios can be modeled analogously. In this talk we will discuss existence results, singularity results, and long time decay behavior of the Muskat problem in 2D and in 3D. (Received September 21, 2016)

1125-35-3148 **Andrew Lawrie\*** ([alawrie@mit.edu](mailto:alawrie@mit.edu)). *Energy subcritical nonlinear wave equations*. Preliminary report.

We discuss some recent work with Dodson, and Dodson, Mendelson, Murphy on the energy subcritical nonlinear wave equation. We prove that if the critical norm of a solution stays bounded on the maximal time of existence, then the solution must be globally defined and scattering. The main new technical ingredient of the proof is a novel version of the so called double Duhamel trick, which allows us to access conserved quantities like the energy for solutions with pre-compact trajectories in the critical space. (Received September 21, 2016)

1125-35-3152 **Yajie Zhang\*** ([wonderfulzyj@gmail.com](mailto:wonderfulzyj@gmail.com)), 715 W Cherry Ln, Apt 2, State College, PA 16803. *A Regularity Analysis of Parabolic Transmission Problem on Polygonal Domain*.

We study theoretical and practical issues for second order linear parabolic equation with jump discontinuities in its coefficients on a polygonal domain that may have cracks or vertices that touch the boundary. We consider in particular a linear parabolic equation with appropriate initial condition and mixed boundary/interface conditions, where the matrix  $A$  has variable, piecewise smooth coefficients. We establish some regularity results and, under some additional conditions, we also establish well-posedness in weighted Sobolev spaces in the cases when there are no Neumann boundary conditions imposed on adjacent sides of the polygonal domain. When Neumann boundary conditions are imposed on adjacent sides, we fail to have well-posedness according to the Lumer-Phillips Theorem. (Received September 21, 2016)

1125-35-3156 **Yuliya Gorb\*** ([gorb@math.uh.edu](mailto:gorb@math.uh.edu)), University of Southern California. *A Robust Preconditioner for High-Contrast Problems*.

This talk concerns robust numerical treatment of an elliptic PDE with high contrast coefficients. A finite-element discretization of such an equation yields the linear system whose conditioning worsens as the variations in the values of PDE coefficients becomes large. A saddle point description with a semi-positive definite matrix of the corresponding discrete problem is introduced and a robust preconditioner for the Lanczos method of minimized iterations used for its solution is proposed. Numerical examples demonstrate effectiveness and robustness of the proposed class of preconditioners that yield the number of iterations independent of the contrast. (Received September 23, 2016)

1125-35-3162 **Avner Friedman\*** ([afriedman@math.ohio-state.edu](mailto:afriedman@math.ohio-state.edu)), Ohio State University. *Free Boundary Problems Arising in Biology*

In a free boundary problem one seeks to solve a system of PDEs in a domain  $G$  whose boundary, or a part of it, is unknown, and to also determine the free boundary. Classical free boundary problems include contact problems in

elasticity, melting of ice, propagation of jets, and cavitation flows. In recent years new free boundary problems arose in the context of biological or biomedical processes. Examples include the healing of a wound, the growth of a tumor, the formation of a plaque in the artery (atherosclerosis) which leads to a heart attack or a stroke, the development of granulomas in tuberculosis, abdominal aorta aneurysm, and biofilms. In this talk I will briefly describe these biological problems, introduce their mathematical models, and display simulations of the models and their biological significance. Finally, I will review rigorous mathematical results for the PDE models, and state open problems. (Received October 03, 2016)

1125-35-3163 **George Karniadakis\*** ([george\\_karniadakis@brown.edu](mailto:george_karniadakis@brown.edu)), Brown University. *Multiscale Modeling of Blood Clotting*

We propose a new multiscale framework that seamlessly integrates four key components of blood clotting namely, blood rheology, cell mechanics, coagulation kinetics and transport of species and platelet adhesive dynamics. We use transport dissipative particle dynamics (tDPD), which is the extended form of original DPD, as the base solver, while a coarsegrained representation of blood cell's membrane accounts for its mechanics. Our results show the dominant effect of blood flow and high Peclet numbers on the reactive transport of the chemical species signifying the importance of membrane bound reactions on the surface of adhered platelets. This new multiscale particle-based methodology helps us probe synergistic mechanisms of thrombus formation, and can open new directions in addressing other biological processes from sub-cellular to macroscopic scales. (Received October 03, 2016)

1125-35-3164 **Mihaela Ignatova\*** ([ignatova@math.princeton.edu](mailto:ignatova@math.princeton.edu)), Princeton University. *Global existence for fluid-structure models*

We address a fluid-structure system coupling the incompressible Navier-Stokes equation and a linear elasticity equation with interior damping. The interaction takes place at a common interface and it is described by the transmission boundary conditions matching the velocities and the stress forces at the interface. We prove the global existence and exponential decay of smooth solutions for small initial data. This is a joint work with Igor Kukavica, Irena Lasiecka, and Amjad Tuffaha. (Received October 03, 2016)

1125-35-3165 **Nathaniel Trask\*** ([natrask@sandia.gov](mailto:natrask@sandia.gov)), Sandia National Laboratory. *Compatible meshfree discretization with applications to electrophoretic suspension flows.*

Meshless methods promise an effective means of discretizing Lagrangian hydrodynamics and interfacial flows by avoiding the computational expense of maintaining high quality deforming meshes at each timestep. By abandoning a mesh however, we lose the exterior calculus framework that forms the foundation for mesh-based compatible discretizations. In this work, we present a new discretization generalizing classical staggered discretization on primal/dual meshes to an epsilon-ball graph constructed from particle locations. This meshless discretization enjoys high-order convergence and stability properties typical of compatible mesh-based methods. For the div-grad model problem, equal order L2 and H1 convergence is obtained for smooth diffusivities, while nearly-monotone fluxes are maintained for the discontinuous case. For the steady Stokes problem, equal-order convergence is obtained for both pressure and velocity. For both of these problems, a sparse discretization is obtained that can be efficiently preconditioned using standard AMG techniques. We present fundamental approximation properties of these schemes along with results that use this approach as a foundation to develop monolithic solvers to study problems in dense suspension flows driven by electrokinetic effects. (Received October 03, 2016)

1125-35-3166 **Josip Tambaca\*** ([tambaca@math.hr](mailto:tambaca@math.hr)), University of Zagreb. *Mathematical modeling of stents as a net of 1D elastic curved rods*

Stents are metallic mesh-like tubes inserted within the inner-most layer of vascular wall to prop the arteries open. As such the stents are modeled within the elasticity theory, usually three-dimensional. The struts of the mesh are thin and therefore the numerical simulations turn out to be very time and space consuming. Therefore a simple models are plead for. In this talk we will present the model based on the one-dimensional model used for stent struts. Since the stent struts are thin model gives a very good approximation of the three-dimensional model. Simple structure of the model implies very efficient numerical algorithms that can be performed in real time on a personal computer. This allows quick analysis and testing of different stent designs. Numerical and analytical analysis of the model will be presented and supported by simulations. Further, the stent model will be coupled with the two-dimensional shell equations for the model of interaction of the stent and vessel. Colaborators: S. Canic (University of Houston), L. Grubisic, M. Ljulj, B. Zucek (University of Zagreb) (Received October 03, 2016)

1125-35-3167 **Yuliya Gorb\*** ([gorb@math.uh.edu](mailto:gorb@math.uh.edu)), University of Houston. *Asymptotic Analysis for High Contrast Problems*

The motivation behind the studies described in this talk is driven by the applications in which processes occur in media that contain multiple scales and whose constituents have vastly different mechanical properties. Such media are referred to as high contrast composites interest in which within the mathematical community has greatly grown in recent years. A mathematical formulation of the corresponding problem is given by PDEs whose coefficients have high variations in their values within the given domain. A few asymptotic techniques will be presented that are developed by taking into account details of the complex geometry of the domain. Both scalar and vectorial formulations will be explored. (Received October 03, 2016)

## 37 ► *Dynamical systems and ergodic theory*

1125-37-52 **Heewon Kim\*** ([khw9421@naver.com](mailto:khw9421@naver.com)), Department of mathematics, Science complex A-dong #320, Ewha womans university, Seoul, Seoul 03760, South Korea, **Jiwon Seo** ([ijaydoubleu@gmail.com](mailto:ijaydoubleu@gmail.com)), Department of mathematics, Science complex A-dong #320, Ewha womans university, Seoul, Seoul 03760, South Korea, **Bora Jeong** ([nddrigo@naver.com](mailto:nddrigo@naver.com)), Department of mathematics, Science complex A-dong #320, Ewha womans university, Seoul, Seoul 03760, South Korea, and **Chohong Min**. *An Experiment of the Malkus-Lorenz Waterwheel and Its Measurement by Image Processing*.

We introduce a simple and efficient experiment setup for the Malkus-Lorenz waterwheel. Through a series of image processing techniques, our work is listed as one of the few experiments that measure not only the angular velocity but also the mass distribution. Our experiment is to observe qualitative changes on the waterwheel as the leakage rate changes, while the other physical parameters are fixed. We perform a bifurcation analysis for the qualitative changes, and the phase portraits from experiments are validated by the bifurcation analysis. (Received June 24, 2016)

1125-37-59 **Ali Allahem\*** ([a.allahem@qu.edu.sa](mailto:a.allahem@qu.edu.sa)), Maths Department, Sciences College, Qassim University, Buraydah, Qassim 81999, Saudi Arabia. *The concept of the dividing surface in collinear Hydrogen exchange reaction*.

Transition state theory (TST) describes the elementary chemical reaction rate. There are three main regions in the reaction: reactant, product and the transition state (TS). The transition state must have two properties to make the transition state theory exact: all reactive trajectories must cross the TS (dividing surface) and the reactive trajectories cross it only once. Dynamical effects recrossing is possible from coupling in kinetic energy where TST provides upper bound of the exact reaction rate. Historically, (Wigner 1938) developed the reaction rate theory and extended the idea from configuration space to phase space. (Pollak et al 1978) found the structure of the dividing surface in the collinear  $H_2 + H$  reaction. It is well-known as unstable periodic orbit dividing surface (PODS). We are going to talk about the reactivity on the dividing surface of this reaction. (Received June 30, 2016)

1125-37-65 **Maleafisha Joseph Pekwa Stephen Tladi\*** ([stephen.tladi@ul.ac.za](mailto:stephen.tladi@ul.ac.za)), Department of Mathematics & Applied Mathemati, University of Limpopo, Sovenga, Limpopo 0727, South Africa. *Adiabatic Chaos and Transport in Mesoscale Vortices and Vortex Rings*.

Advances in our understanding of the dynamics of mesoscale eddies and vortex rings of the Agulhas Current Retroflection, and the Gulf Stream in the Middle Atlantic Bight have been based on modons which are Rossby solitary waves of quasi-geostrophic equations. This article focuses primarily on geometric singular perturbation (GSP) theory and Melnikov techniques to address the problem of transport for translating and rotating modons to the quasi-geostrophic potential vorticity system. A very general question is what hypotheses on the equations and singular solutions guarantee that the solutions approximate some solutions for the perturbed quasi-geostrophic potential vorticity system. We present a geometric approach to the problem which gives more refined a priori energy type estimates on the position of the invariant manifold and its tangent planes as the manifold passes close to a normally hyperbolic piece of a slow manifold. We apply Melnikov technique to show that the Poincare map associated with modon equations has transverse heteroclinic orbits. We appeal to the Smale-Birkhoff Homoclinic Theorem and assert the existence of an invariant hyperbolic set which contains a countable infinity of unstable periodic orbits, a dense orbit and infinitely many heteroclinic orbits. (Received July 10, 2016)

- 1125-37-90 **Ugur G Abdulla, Rashad U Abdulla\*** (abdulla@fit.edu), **Emily Ribando-Gros, Muhammad U Abdulla and Naveed Iqbal.** *Second Minimal Orbits, Sharkovski Ordering, and Universality in Chaos.*

We prove the outstanding conjecture on the number of second minimal odd periodic orbits of the continuous endomorphisms on the real line. An  $n$  periodic orbit of the map is called a second minimal if  $n$  is the successor of the minimal orbit of the map in Sharkovski ordering. It is proved that for any integer  $k \geq 3$  there are  $4k - 3$  types of second minimal  $2k + 1$ -orbits with accuracy up to inverses. Our proof presents fine classification of the second minimal odd orbits in terms of cyclic permutations and directed graphs. The result is applied to the problem on the distribution of superstable periodic windows within the chaotic regime of the bifurcation diagram of the one-parameter family of unimodal maps. It is revealed that by fixing the maximum number of appearances of the periodic windows there is a universal pattern of distribution. In particular, the first appearance of all the orbits is always a minimal Stefan orbit, while the second appearance is always a second minimal orbit with precisely Type 1 digraph according to our classification. The reason for the relevance of the Type 1 minimal orbit is the fact that the topological structure of the unimodal map with single maximum is equivalent to the structure of the Type 1 piecewise monotonic endomorphism. (Received July 20, 2016)

- 1125-37-119 **Matthew Foreman\*** (mforeman@math.uci.edu). *Applications of descriptive set theory to the classification of measure preserving diffeomorphisms of the torus.*

The isomorphism problem in ergodic theory was formulated by von Neumann in 1932 in his pioneering paper *Zur Operatorenmethode in der klassischen Mechanik*. The problem has been solved for some special classes of transformations; Halmos and von Neumann completely characterize ergodic MPTs with pure point spectrum and Bernoulli shifts were classified by Ornstein using the notion of entropy. Many properties of ergodic MPTs were studied over the years in connection with this problem. In 2011, Foreman, Rudolph and Weiss showed that isomorphism between ergodic transformations is not Borel, and hence is inaccessible to countable methods that use countable amounts of information. The technique did not extend to show an anti-classification theorem for  $C^\infty$ -diffeomorphisms of manifolds. Here we prove:

**Theorem:** If  $M$  the torus  $\mathbb{T}^2$  then the measure-isomorphism relation among pairs  $(S, T)$  of ergodic Lebesgue measure preserving  $C^\infty$ -diffeomorphisms of  $M$  is not a Borel. Moreover the isomorphism relation is strictly more complex than any equivalence relation induced by an  $S^\infty$ -action. The proof gives examples of many new isomorphism classes of ergodic diffeomorphisms, such as distal transformations of infinite height.

This is joint work with B. Weiss. (Received September 20, 2016)

- 1125-37-136 **Nirja Dave\***, Department of Mathematics, 301 Thackeray Hall, University of Pittsburgh, Pittsburgh, PA 15260, and **Andre Lesartre**, Dept. of Applied Mathematics & Statistics, Colorado School of Mines, Golden, CO 80401. *Control of Cardiac Tissue Using Feedback-Based Synchronization.*

Cardiac arrhythmias are a type of heart disease caused by the irregular propagation of electrical signals in the heart. The resultant waves can have complex patterns including one or more spiral waves. We assess the effectiveness of synchronization to control complex spiral-wave dynamics in two-dimensional cardiac tissue using three different reaction-diffusion models, each consisting of a voltage variable and a recovery variable. Our approach involves synchronizing two unidirectionally coupled systems by applying a feedback term that is proportional to the difference in the voltage variable averaged locally over uniformly spaced sensors to one of the systems, with the quality of synchronization measured in the voltage. Our study extends previous work by Berg et al. through the use of additional cardiac-specific models, a time-discrete feedback term, different initial conditions for the response system, and different model parameters for the driver and response systems. We vary model and synchronization parameters to determine the range of conditions for which synchronization is possible. Our findings show that synchronization in the recovery variable is key to synchronization success in the voltage variable. (Received August 03, 2016)

- 1125-37-137 **Jamshaid Shahir\***, Department of Mathematics and Statistics, 1000 Hilltop Circle, University of Maryland, Baltimore County, Baltimore, MD 21250, and **Kimberly Truong**, Division of Applied Mathematics, 182 George St, Brown University, Providence, RI 02912. *Using Delay Differential Equations to Study Calcium Alternans in a Model of Intracellular Calcium Cycling.*

Cardiac arrhythmias are irregular beatings of the heart caused by disruptions in the electrical activity that triggers contraction. One mechanism that can give rise to arrhythmias is calcium alternans, a dynamical state characterized by alternating large and small intracellular calcium concentrations in response to periodic stimuli. Despite the need to understand mechanisms for calcium-driven cardiac alternans, however, many ordinary

differential equation models of intracellular calcium cycling do not produce alternans, thus restricting the scope of such models for studying alternans behavior. Delay differential equations (DDEs), which in many contexts produce complex dynamics, may be a promising tool for promoting alternans in cardiac models. We introduce DDEs in the equations for the calcium current gating variables, currents, and the release function in a model of intracellular calcium cycling. After suppressing alternans in the original model, we show that alternans can be induced by DDEs in certain compartments of the cell. We analyze the changes in the calcium concentrations, currents, and gating variables in response to these DDEs and discuss the mathematical and physiological implications of our findings. (Received August 03, 2016)

1125-37-161 **Aminur Rahman\*** (ar276@njit.edu), Culimore Hall, University Avenue, Department of Mathematical Sciences, New Jersey Institute of Technology, Newark, NJ 07103. *The Chaotic Ballet of Walking Droplets.*

Pilot wave theory in quantum mechanics, specifically the works of de Broglie and later Bohm, initially showed great potential, but was later abandoned in favor of the Copenhagen interpretation. However, in recent years the idea of particle trajectories driving the statistics seen in quantum-like experiments has made a resurgence due to the study of quantum analogs in hydrodynamic pilot wave theory through the works of Couder *et al.* and Bush *et al.*

In 2010, Bush and collaborators developed hydrodynamic models for a bouncing droplet experiencing propulsion from its own waves. They observed indications of chaos, however due to the complexity of the models, mathematical analysis proved to be extremely difficult. Recently, Gilet developed a model for straight-line walking with the droplets exciting a single eigenmode at each impact. He observed by numerical means, evidence of a Neimark-Sacker bifurcations (N-S) and chaotic dynamics.

This talk discusses the pioneering experiments of Couder and Bush, and some of the early models. It is shown how the reduced models were developed and possible connections with previous integro-differential model and experiments. The main results in the talk prove the existence of (N-S) and a new global bifurcation leading to chaos. (Received August 05, 2016)

1125-37-184 **Ian Jordan\*** (idj2@njit.edu), New Jersey Institute of Technology, and **Aminur Rahman** (ar276@njit.edu), Culimore Hall, University Avenue, Department of Mathematical Sciences, New Jersey Institute of Technology, Newark, NJ 07103. *Discrete dynamical modeling and experimental investigation of chaotic NOR gates and set/reset flip-flops.*

It has been observed through experiments and *SPICE* simulations that logical circuits based upon Chua's circuit exhibit complex dynamical behavior. Some of these circuits have been modeled as systems of ordinary differential equations. However, as the number of components in newer circuits increases so does the complexity. This renders continuous dynamical systems models impractical and necessitates new modeling techniques. In recent years some discrete dynamical models have been developed using various simplifying assumptions. To create a robust modeling framework for chaotic logical circuits, we developed both deterministic and stochastic discrete dynamical models, which exploit the natural recurrence behavior, for two chaotic NOR gates and a chaotic set/reset flip-flop (RSFF). This work presents a complete applied mathematical investigation of logical circuits. Experiments on our own designs of the above circuits are modeled and the models are rigorously analyzed and simulated showing surprisingly close qualitative agreement with the experiments. Furthermore, the models are designed to accommodate dynamics of similarly designed circuits. This will allow researchers to develop ever more complex chaotic logical circuits with a simple modeling framework. (Received August 10, 2016)

1125-37-275 **Pamela B. Pyzza\*** (pbpyzza@owu.edu), 61 S. Sandusky St., Delaware, OH 43015, and **Gregor Kovacic** and **David Cai**. *Idealized Models of Insect Olfaction.*

When a locust detects an odor, the stimulus triggers a specific sequence of network dynamics of the neurons in its antennal lobe. The odor response begins with a series of synchronous oscillations, followed by a short quiescent period, with a transition to slow patterning of the neuronal firing rates, before the system finally returns to a background level of activity. We begin modeling this behavior using an integrate-and-fire neuronal network, composed of excitatory and inhibitory neurons, each of which has fast-excitatory, and fast- and slow-inhibitory conductance responses. We further derive a firing-rate model for each (excitatory and inhibitory) neuronal population, which allows for more detailed analysis of and insight into the plausible olfaction mechanisms seen in experiments, prior models, and our numerical model. We conclude that the transition of the network dynamics through fast oscillations, a pause in network activity, and the slow modulation of firing rates can be described by a system which has a limit cycle of the fast variables, slowly passes through a saddle-node-on-a-circle bifurcation

eliminating the oscillations, and, eventually, slowly passes again through the bifurcation point, producing a new limit cycle with a slower period. (Received August 22, 2016)

1125-37-317 **Hiroki Sumi\*** ([sumi@math.sci.osaka-u.ac.jp](mailto:sumi@math.sci.osaka-u.ac.jp)), Department of Mathematics, Graduate School of Science, Osaka University, 1-1, Machikaneyama, Toyonaka, Osaka 5600043, Japan. *Finding Roots of Any Polynomials by Random Relaxed Newton's Methods*. Preliminary report.

We develop the theory of random complex dynamical systems and apply the results to finding roots of any complex polynomials by random relaxed Newton's methods. More precisely, for any complex polynomial  $f$ , let  $N_{f,\lambda}(z) = z - \lambda \frac{f(z)}{f'(z)}$ , where  $z$  is a point in the Riemann sphere and  $\lambda \in \{\mathbb{C} \mid |\lambda - 1| < r\}$  ( $\frac{1}{2} < r < 1$ ), and we consider the random dynamical system on the Riemann sphere such that at every step we choose  $\lambda \in \{\lambda \in \mathbb{C} \mid |\lambda - 1| < r\}$  according to the uniform distribution, and map the point under  $N_{f,\lambda}$ . We show that for any polynomial  $f$ , for any initial value  $z$  in the complex plane which is not a root of  $f'$ , the random orbit starting with  $z$  tends to a root of  $f$  almost surely, which is **the virtue of the effect of randomness**. In fact, such a statement **cannot hold in the deterministic relaxed Newton's method** and any other deterministic complex analytic iterative schemes to find roots of polynomials. Thus the above result deals with a **randomness-induced phenomenon**. For the preprint, see <https://arxiv.org/abs/1608.05230>. (Received August 26, 2016)

1125-37-395 **Erkan Nane and Yinan Ni\*** ([yzn0005@auburn.edu](mailto:yzn0005@auburn.edu)). *Stability of the solution of stochastic differential equation driven by time-changed Lévy noise*.

This paper studies stabilities of the solution of stochastic differential equation (SDE) driven by time-changed Lévy noise in both probability and moment sense. This provides more flexibility in modeling schemes in application areas including physics, biology, engineering, finance and hydrology. Necessary conditions for solution of time-changed SDE to be stable in different senses will be established. Connection between stability of solution to time-changed SDE and that to corresponding original SDE will be disclosed. Examples related to different stabilities will be given. We study SDEs with time-changed Lévy noise, where the time-change processes are inverse of general Lévy subordinators. These results are important generalization of the results in Wu's "Stability of stochastic differential equation with respect to time-changed Brownian motion, 2016". (Received August 31, 2016)

1125-37-406 **Jerome Buzzi, Sylvain Crovisier and Todd Fisher\*** ([tfisher@mathematics.byu.edu](mailto:tfisher@mathematics.byu.edu)). *Entropy of  $C^1$  diffeomorphisms without a dominated splitting*.

A classical construction due to Newhouse creates horseshoes from hyperbolic periodic orbits with large period and weak domination through local perturbations. Our main theorem shows that, when one works in the  $C^1$  topology, the entropy of such horseshoes can be made arbitrarily close to an upper bound deriving from Ruelle's inequality, i.e., the sum of the positive Lyapunov exponents (or the same for the inverse diffeomorphism, whichever is smaller). Adapting classical techniques, we use perturbations that are local and can be chosen to preserve volume or symplectic form or a homoclinic connection.

This optimal entropy creation yields a number of consequences for  $C^1$ -generic diffeomorphisms, especially in the absence of a dominated splitting. For instance, in the conservative settings, we find formulas for the topological entropy, deduce that the topological entropy is continuous but not locally constant at the generic diffeomorphism and we prove that these generic diffeomorphisms have no measure of maximum entropy. In the dissipative setting, we show the locally generic existence of infinitely many homoclinic classes with entropy bounded away from zero. (Received August 31, 2016)

1125-37-425 **Diana Davis and W. Patrick Hooper\*** ([whooper@ccny.cuny.edu](mailto:whooper@ccny.cuny.edu)), 160 Convent Ave, New York, NY 10032. *Periodicity and ergodicity in the trihexagonal tiling*.

We consider the dynamics of light rays in the trihexagonal tiling where triangles and hexagons are transparent and have equal but opposite indices of refraction. We find that almost every ray of light is dense in a region of a particular form: the region has infinite area and consists of the plane with a periodic family of triangles removed. We also completely describe initial conditions for periodic and drift-periodic light rays. (Received September 01, 2016)

1125-37-465 **Diana Davis\*** ([07djd@williams.edu](mailto:07djd@williams.edu)), Williams College, Department of Mathematics and Statistics, 18 Hoxsey Street, Williamstown, MA 01267. *Periodic billiard paths on the pentagon*.

On the square billiard table, saddle connections are given by relatively prime integer vectors  $[a, b]$ , and a trajectory in this direction has period  $2(a + b)$ ; this is the only shape for which these results were known. We find the

analogous results for the regular pentagon billiard table. We do this by relating the double pentagon to the golden L, and creating a tree structure on the set of periodic directions. The periodic paths turn out to be very beautiful, and I will show many pictures that we created with Sage. This is joint work with Samuel Lelièvre. (Received September 02, 2016)

1125-37-588 **Scott R. Kaschner\*** (skaschne@butler.edu), Butler University, Dept. of Mathematics and Actuarial Science, 4600 Sunset Ave., JH270, Indianapolis, IN 46208, and **Roland K. W. Roeder**, 402 N. Blackford Rd., LD270, Indianapolis, IN 46202. *Dynamical Degrees and the Projective Heat Map Acting on Pentagons.*

In a recent monograph, Schwartz provided a nearly complete description of the dynamics of the projective heat map  $H$ , a rational map of  $\mathbb{R}^2$  that maps any pentagon  $P$  to the pentagon whose vertices are the projective midpoints of the edges of  $P$ . We place Schwartz's work on the real dynamics of  $H$  into the complex perspective by computing its first dynamical degree and presenting some corollaries about the dynamics of  $H$ . (Received September 07, 2016)

1125-37-691 **Kelly B. Yancey\*** (kbyancey1@gmail.com), **Austin J. Parker** and **Matthew P. Yancey**. *Distances between regular languages using entropy.* Preliminary report.

A problem that has emerged in computer science is determining the similarity between regular languages. We will represent a regular language by a deterministic finite automata (a directed graph with some marked data) and then use ideas from symbolic dynamics to develop a metric between the languages. (Received September 09, 2016)

1125-37-693 **Jon Fickenscher** and **Kelly B. Yancey\*** (kbyancey1@gmail.com). *Structure of Rigidity Sequences for Substitution Dynamical Systems.* Preliminary report.

A special class of dynamical systems that we will focus on are substitutions. This class of systems provides a variety of ergodic theoretic behavior and is connected to self-similar interval exchange transformations. During this talk we will explore rigidity sequences for these systems. A sequence  $(n_m)$  is a rigidity sequence for the dynamical system  $(X, T, \mu)$  if  $\mu(T^{n_m} A \cap A) \rightarrow \mu(A)$  for all positive measure sets  $A$ . We will discuss the structure of rigidity sequences for substitutions that are rank-one and substitutions that have constant length. (Received September 09, 2016)

1125-37-714 **Konrad Schöbel\*** (konrad.schoebel@gmail.com), Jena, Germany. *An Algebraic Geometric Approach to the Classification of Superintegrable Systems and Hypergeometric Orthogonal Polynomials.*

Special functions are of widespread use virtually everywhere in science and technology. Yet there is no theory which would explain, for a reasonably wide class of special functions, their properties, interrelations, symmetries and other structures behind the façade of seemingly endless formulae in rows, tabulated in thick multi-volume compendiums and huge online databases.

A prolific source of special functions are superintegrable systems. We argue that their classification has always been considered in the wrong category, that of sets, and not in its natural category, that of projective varieties. We substantiate our claim by showing that the set of (2nd order) superintegrable systems in the Euclidean plane carries a natural structure of a projective variety with a linear isometry group action, which simplifies the known classification radically and enriches it with a formerly unknown algebraic and geometric structure.

Based on this proof of concept, we outline an approach to accomplish the classification of (2nd order) superintegrable systems in higher dimensions. In dimension two this leads to a projective variety underlying the Askey scheme of hypergeometric orthogonal polynomials: the basis for a unified treatment of their properties, interrelations and symmetries. (Received September 09, 2016)

1125-37-758 **Jane M. Hawkins\*** (jmh@math.unc.edu), Mathematics Department, CB #3250, Univ. of N. Carolina at Chapel Hill, Chapel Hill, NC 27599. *Benford's Law: Detecting Fraud in Business Using Ergodic Theory.* Preliminary report.

Benford's Law is the observation that in many collections of numbers like mathematical tables or real-life data, the leading significant digits are not uniformly distributed, as might be expected, but are heavily skewed toward the smaller digits. Tax auditors and insurance companies use this to detect fraud. We discuss, using basic ideas in ergodic theory, what is behind Benford's Law, and give some applications to mathematical (and other) settings. (Received September 10, 2016)

1125-37-759 **Dogan Comez** ([dogan.comez@ndsu.edu](mailto:dogan.comez@ndsu.edu)) and **Mrinal K Roychowdhury\***, School of Mathematical and Stat Sciences, University of Texas Rio Grande Valley, 1201 West University Drive, Edinburg, TX 78539. *Optimal Quantization*.

Consider a Borel probability measure  $P$  on  $\mathbb{R}^d$  and a natural number  $n \in \mathbb{N}$ . Then, the  $n$ th quantization error for  $P$  is defined by:

$$V_n := V_n(P) = \inf \left\{ \int \min_{a \in \alpha} \|x - a\|^2 dP(x) : \alpha \subset \mathbb{R}^d, \text{card}(\alpha) \leq n \right\},$$

where  $\|\cdot\|$  denotes the Euclidean norm on  $\mathbb{R}^d$ . A set  $\alpha$  for which the infimum is achieved and does not contain more than  $n$  points is called an *optimal set of  $n$ -means* for the probability measure  $P$ . Recently, we have determined the optimal sets of  $n$ -means and the  $n$ th quantization error for different fractal probability measures. I will talk about it. (Received September 11, 2016)

1125-37-774 **Terrence M Adams\*** ([tma4321@gmail.com](mailto:tma4321@gmail.com)). *Dorothy Maharam's "On orbits under ergodic measure preserving transformations"*.

In 1965, the Transactions of the AMS published the article "On orbits under ergodic measure preserving transformations" by Dorothy Maharam. In this paper, Dr. Maharam introduced the notions of D-sequence and E-sequence. She also showed that an E-sequence on the unit interval determines a unique ergodic Lebesgue measure preserving transformation (e.m.p.t.), and that almost every orbit of an e.m.p.t. is an E-sequence. This raises the question of how well we can learn an e.m.p.t., or in statistical terms, a stationary ergodic process from observing its samples. This continues to be a difficult problem with increasing relevance in a number of scientific disciplines. We will give a survey of some recent results in this area while drawing connections to Dr. Maharam's original result. (Received September 11, 2016)

1125-37-809 **Lorelei Koss\*** ([koss@dickinson.edu](mailto:koss@dickinson.edu)), Dickinson College, Dept. of Mathematics and Computer Science, P.O. Box 1773, Carlisle, PA 17013. *Elliptic functions with disconnected Julia sets*.

Quadratic polynomials and rational maps have Julia sets that are either Cantor or connected. Higher degree polynomials or rational maps, on the other hand, can have Julia sets that are disconnected but not Cantor. For elliptic functions, a complete classification of the connectivity of the Julia set has been completed for only a few families. In some cases, all of the elliptic functions in the family have connected Julia sets. In other cases, all of the elliptic functions in the family have Julia sets that are either Cantor or connected. In this talk, we present examples of elliptic functions with Julia sets that are disconnected but not Cantor. (Received September 12, 2016)

1125-37-898 **Johannes Jaerisch\*** ([jaerisch@riko.shimane-u.ac.jp](mailto:jaerisch@riko.shimane-u.ac.jp)), **Marc Kesseböhmer** and **Sara Munday**. *A multifractal analysis for cuspidal windings on hyperbolic surfaces*.

We introduce the cusp winding process of the geodesic flow on a hyperbolic surface  $\mathbb{H}/G$ , for non-elementary, finitely generated, free Fuchsian groups with parabolic elements. We investigate the multifractal decomposition of the limit set of  $G$  with respect to the mean cusp winding number. We will completely determine its multifractal spectrum in terms of an associated topological pressure function. This extends previous results for continued fraction expansions on the modular surface  $\mathbb{H}/\text{PSL}(2, \mathbb{Z})$ . (Received September 13, 2016)

1125-37-909 **John Franks\*** ([j-franks@northwestern.edu](mailto:j-franks@northwestern.edu)), Dept. of Mathematics, Northwestern University, 2033 Sheridan Road, Evanston, IL 60208-2730, and **Bryna Kra** and **Van Cyr**. *The spacetime of a shift automorphism*. Preliminary report.

This talk represents joint work with Van Cyr and Bryna Kra. The automorphism group of a one dimensional shift space over a finite alphabet, is known typically to contain a rich collection of subgroups. We study constraints which exist on this countable group and their relationship to the dynamics of the shift. We start by considering a single automorphism, studying the naturally associated two dimensional shift system and its dynamical properties such as nonexpansive subspaces. We then use these structural results to study the algebraic structure of the group of all automorphisms of the shift. We show that the complexity and the entropy of the shift system place nontrivial structure on the group of automorphisms. (Received September 13, 2016)

1125-37-913 **Sarah Day\*** ([sday@math.wm.edu](mailto:sday@math.wm.edu)), Department of Mathematics, College of William & Mary, P.O. Box 8795, Williamsburg, VA 23187, and **Rafael Frongillo** ([raf@colorado.edu](mailto:raf@colorado.edu)), 430 UCB, Boulder, CO 80309. *Sofic shifts via Conley index theory: computing lower bounds on recurrent dynamics for maps*.

Recent work has demonstrated the effectiveness of computational Conley index techniques in extracting dynamics from discrete-time systems governed by maps. I will discuss an automated approach that builds on earlier work

and uses Conley index information to construct sofic shifts that are topologically semi-conjugate to the system under study. This allows for the uncovering and recording of increasingly complicated dynamics in chaotic systems. As illustration, we present results for the two-dimensional Hénon map and the infinite-dimensional Kot-Schaffer map. (Received September 13, 2016)

1125-37-930 **Olusegun Michael Otunuga\*** (otunuga@marshall.edu), Department of Mathematics, Marshall University, One John Marshall Drive, Huntington, WV 25755. *Parameter Identification For The SEIRS Epidemic Model: Case Study Influenza*. Preliminary report. A stochastic system of differential equations for disease spread is developed. The stochasticity appears due to fluctuations in the transmission rate of the disease. The local lagged adapted generalized method of moments (LLGMM) is explained and used to identify the time-dependent transmission rate and time-dependent noise for the stochastic susceptible, exposed, infectious, temporarily immune, susceptible disease model (SEIRS) with vital rates. The method is applied to US influenza data from the 2005-2006 through 2015-2016 influenza seasons. Confidence intervals are given for possible future infectious levels. (Received September 13, 2016)

1125-37-942 **Sarah Bailey Frick\*** (sarah.frick@furman.edu), 3300 Poinsett Hwy, Greenville, SC 29613, **Karl Petersen**, Chapel Hill, NC, and **Sandra Shields**, Charleston, SC. A *condition for topological conjugacy to an odometer*.

We give a necessary and sufficient condition for any adic system to be topologically conjugate to an odometer and use this condition to determine the probability that a random order on a fixed diagram, or a diagram constructed at random in some way, is topologically conjugate to an odometer. (Received September 13, 2016)

1125-37-972 **Benjamin Z Webb\*** (bwebb@mathematics.byu.edu), 308 TMCB, Brigham Young University, Provo, UT 84602. *Intrinsic Stability of Dynamical Networks*.

The dynamics of real networks are resilient to changes in their environment that cause time-delays. In this talk we introduce a new notion of stability that can be used to describe how a network maintains stability under such changes. We refer to this stronger form of stability as intrinsic stability and show how intrinsic stability is preserved under changes to the network's structure that correspond to time-delays. This type of stability is of potential importance in applications, especially network design, since time-delays often have a destabilizing effect on a network's stability. (Received September 13, 2016)

1125-37-981 **Mark Embree, Jake Fillman and May Mei\*** (meim@denison.edu). *Continuum Fibonacci Schrödinger Operators*.

Discrete Schrödinger operators with potentials generated by aperiodic subshifts over a finite alphabet have been studied since the mid 1980's. More specifically, the Fibonacci Hamiltonian has been a particularly well-studied example. Several continuum analogues have also been considered. In this talk, we discuss spectral properties of one such operator in which each letter of the subshift sequence is replaced with a function. (Received September 13, 2016)

1125-37-1025 **Natalie Priebe Frank\*** (nafrank@vassar.edu), Box 248, Vassar College, Poughkeepsie, NY 12604, and **E. Arthur Robinson, Jr., Michael Baake and Uwe Grimm**. *Towards spectral analysis of self-similar tilings via a renormalization approach*. Preliminary report.

We examine the example of a tiling of the line given by the non-Pisot substitution rule  $a \rightarrow abbb$ ,  $b \rightarrow a$ . We describe how, when natural tile lengths are used, the pair correlation functions that determine the diffraction spectrum satisfy easily computable renormalization equations. These equations carry through to the autocorrelation and diffraction measures and therefore can be useful for computing the spectral type of these measures. In our example, the spectral type is known to contain a continuous part and the renormalization approach provides clues to whether this part can be absolutely or singularly continuous with respect to Lebesgue measure. (Received September 14, 2016)

1125-37-1076 **Christian P Wolf\*** (cwolf@ccny.cuny.edu), 160 Convent Ave, New York, NY 10031. *Are rotation sets and localized entropies computable?* Preliminary report.

Given a continuous dynamical system  $f : X \rightarrow X$  on a compact metric space  $X$  and a  $m$ -dimensional continuous potential  $\Phi = (\phi_1, \dots, \phi_m) : X \rightarrow \mathbb{R}^m$ , the generalized rotation set  $Rot(\Phi)$  is defined as the set of all  $\mu$ -integrals of  $\Phi$ , where  $\mu$  runs over all invariant probability measures. Analogously to the classical entropy of  $f$ , one can associate to each  $w \in Rot(\Phi)$  the localized entropy  $H(w)$  at  $w$ . In this talk, we consider the question about the computability of rotation sets and localized entropies. We present positive results for subshifts of finite type and interior points of the rotation set. We also show that the situation is more complicated when dealing with points at the boundary of the rotation set. (Received September 14, 2016)

1125-37-1077 **Johann D. Gaebler\*** (jgaebler@college.harvard.edu), 187 Pforzheimer Mail Ctr., 56 Linnaean St., Cambridge, MA 02138, and **Cesar E. Silva, Xiaoyu Xu and Zirui Zhou.** *Partially Bounded Transformations have Trivial Centralizer.*

We prove that for infinite rank-one transformations satisfying a certain property called “partial boundedness,” the only commuting transformations are powers of the original transformation. This shows that a large class of measure-preserving rank-one transformations with bounded cuts have trivial centralizer. We also extend our result to analogous classes of rank-one  $\mathbb{Z}^d$ -actions and flows. (Received September 14, 2016)

1125-37-1203 **Fabien Durand** (fabien.durand@u-picardie.fr), **Nicholas Ormes\*** (nic.ormes@du.edu) and **Samuel Petite** (samuel.petite@u-picardie.fr). *Self-induced systems.*

In this talk we will discuss self-induced minimal Cantor systems. A minimal Cantor system is said to be self-induced whenever it is conjugate to one of its induced systems. Substitution subshifts and some odometers are classical examples, and these are the only examples in the equicontinuous or expansive case. Nevertheless, we exhibit a zero entropy self-induced system that is neither equicontinuous nor expansive. We also provide non-uniquely ergodic self-induced systems with infinite entropy. Moreover, we give a characterization of self-induced minimal Cantor systems in terms of substitutions on finite or infinite alphabets. (Received September 15, 2016)

1125-37-1211 **Rich Stankewitz\*** (rstankewitz@bsu.edu), **Toshiyuki Sugawa** and **Hiroki Sumi.** *Hereditarily Non Uniformly Perfect Sets of Full Dimension.*

We introduce the concept of hereditarily non uniformly perfect (HNUP) sets, compact sets for which no compact subset is uniformly perfect, and compare the following properties: HNUP sets, Hausdorff dimension zero sets, logarithmic capacity zero sets, Lebesgue 2-dimensional measure zero sets, and porous sets. In particular, we give a random method for generating HNUP sets of positive Hausdorff dimension. Also, we construct a set of full Hausdorff dimension in the plane which is HNUP. Lastly, we state two related open questions. (Received September 15, 2016)

1125-37-1256 **Jean-Jacques Kengwong-Keumo\*** (jkengwou@cameron.edu), Department of Mathematical Sciences, Cameron University, Lawton, OK 73505. *Competition between a nonallelopathic phytoplankton and an allelopathic phytoplankton species under predation.*

We propose a model of two-species competition in the chemostat for a single growth-limiting, nonreproducing resource that extends that of Roy (2009). The model allows allelopathic effects of one toxin-producing species, both on itself (autotoxicity) and on its nontoxic competitor (phytotoxicity). We show that a stable coexistence equilibrium exists as long as (a) there are allelopathic effects and (b) the input nutrient concentration is above a critical value. The model is reconsidered under predation and instantaneous nutrient recycling. Each of the models is analyzed for boundedness, equilibria, stability, and uniform persistence (or permanence). Each model structure fits very well with some harmful algal bloom observations where the phytoplankton assemblage can be envisioned in two compartments, toxin producing and non-toxic. The *Prymnesium Parvum* literature, where the suppressing effects of allelochemicals are quite pronounced, is a classic example. This work advances knowledge in an area of research becoming ever more important, which is understanding the functioning of allelopathy in food webs. (Received September 15, 2016)

1125-37-1421 **Joanna Furno\*** (jfurno@iupui.edu). *Radon-Nikodym derivatives for rational  $p$ -adic translations.*

For a fixed prime  $p$ , the group of  $p$ -adic integers contains the integers and rational numbers without any factors of  $p$  in the denominator. The set of  $p$ -adic integers can also inherit Bernoulli product measures from the product space on  $p$  symbols. This talk will discuss when an ergodic translation by a rational number is nonsingular for a Bernoulli product measure on the  $p$ -adic integers. For these, the Radon-Nikodym derivatives and orbit equivalence classes will be given. (Received September 16, 2016)

1125-37-1439 **Ryan Broderick\*** (broderir@uci.edu). *Finite orbits in random subshifts of finite type.*

In this talk we will discuss an important class of dynamical systems called subshifts of finite type (SFTs), which are used to model other dynamical systems and also extensively studied in their own right. We will describe a method of defining a random SFT, which depends on a parameter  $\alpha$ , and discuss a dichotomy in the behavior of a “randomly chosen” SFT for different values of the parameter.

More precisely, given a finite alphabet  $A$ , a positive integer  $n$ , and a real number  $0 < \alpha < 1$ , we define a random subset  $\omega \subset A^n$  by independently including each element with probability  $\alpha$ . We define the associated random SFT  $X_\omega$  to be the subshift having  $A^n \setminus \omega$  as its set of forbidden words. It is known that for  $\alpha > 1/|A|$ , as  $n \rightarrow \infty$  the probability that  $X_\omega$  has positive entropy tends to 1. We discuss a recent result that complements this: For  $\alpha < 1/|A|$ , the probability that  $X_\omega$  consists of a single finite orbit tends to 1 as  $n \rightarrow \infty$ . Thus, there

is a critical value dividing the parameter space into values for which a random SFT displays high complexity (positive entropy) with high probability and values for which it displays very low complexity (finiteness) with high probability. (Received September 16, 2016)

1125-37-1541 **James E Reid\*** ([jamesreid2@my.unt.edu](mailto:jamesreid2@my.unt.edu)), Denton, TX 76201. *Packing measure of super separated iterated function systems.*

In the context of fractal geometry, the natural extension of volume in  $\mathbb{R}^d$  to include fractal sets is given by Hausdorff and packing measures. If  $J$  is the limit set of an iterated function system (IFS) in  $\mathbb{R}^d$  satisfying the open set condition, then  $J$  is often a fractal set. It is well known that the  $h$ -dimensional packing measure of  $J$  is positive and finite when  $h$  is given by Hutchinson's formula. Feng was able to find exact formulas for the  $h$ -dimensional packing measure for a large class of Cantor sets in the interval  $[0, 1]$ .

In this talk, we will introduce the super separation condition for an IFS. We use super separation to reduce the problem of computing the  $h$ -dimensional packing measure to checking densities of a finite number of balls around each point in the limit set. We then use this fact to find formulas for the packing measure of a class of Cantor sets in  $\mathbb{R}$  (extending Feng's result), a class of fractals based on regular convex polygons in  $\mathbb{R}^2$ , and a class of fractals based on regular simplexes in  $\mathbb{R}^d$  for  $d \geq 3$ . (Received September 17, 2016)

1125-37-1608 **Lori Alvin**, [lalvin@bradley.edu](mailto:lalvin@bradley.edu), **Drew D Ash\***, [drash@davidson.edu](mailto:drash@davidson.edu), and **Nic Ormes**, [normes@du.edu](mailto:normes@du.edu). *Bounded Topological Speedups.*

Given a dynamical system  $(X, T)$  one can define a speedup of  $(X, T)$  as another dynamical system  $S : X \rightarrow X$  where  $S = T^{p(\cdot)}$  for some  $p : X \rightarrow \mathbb{Z}^+$ . In 2015, the speaker gave necessary and sufficient conditions for a minimal Cantor system to be a topological speedup of another minimal Cantor system. Through this theorem, one can show speedups need not stay in the same topological orbit equivalence class of the original system. In this talk, we will focus on recent joint work with Lori Alvin and Nic Ormes on a strengthening of topological speedups, namely bounded topological speedups. In this case, we require that our "jump function"  $p$  be bounded and hence continuous. Here the motivating question is: what, if anything, can be preserved with the added structure of  $p$  being bounded? We will highlight two theorems which yield positive answers to this question. Specifically, a bounded speedup of an odometer is a conjugate odometer and a bounded speedup of an aperiodic, primitive substitution is again an aperiodic, primitive substitution, though is never conjugate to the original substitution system. (Received September 18, 2016)

1125-37-1734 **Gheorghe Craciun\*** ([craciun@math.wisc.edu](mailto:craciun@math.wisc.edu)). *Toric Differential Inclusions and a Proof of the Global Attractor Conjecture.*

The Global Attractor Conjecture says that a large class of polynomial dynamical systems, called *toric dynamical systems*, have a globally attracting point within each linear invariant space. In particular, these polynomial dynamical systems never exhibit multistability, oscillations or chaotic dynamics.

The conjecture was formulated by Fritz Horn in the early 1970s, and is related to convergence results for the Boltzmann equation.

We describe the history of this problem, including the relevance of this problem to the study of homeostasis in biological interaction networks. Then, we introduce *toric differential inclusions*, and discuss about how they can be used to prove this conjecture in full generality. (Received September 19, 2016)

1125-37-1834 **Daniel Cuzzocreo\*** ([dcuzz@math.northwestern.edu](mailto:dcuzz@math.northwestern.edu)). *Parameter space structures for complex rational maps.*

We discuss some results characterizing the qualitative structure of the bifurcation loci for some families of complex rational maps of the Riemann sphere. (Received September 19, 2016)

1125-37-1846 **David Constantine**, **Jean-Francois Lafont** and **Daniel J. Thompson\*** ([thompson@math.osu.edu](mailto:thompson@math.osu.edu)). *Thermodynamic formalism for geodesic flow on locally CAT(-1) spaces.*

Locally CAT(-1) spaces are geodesic metric spaces satisfying a certain metric notion of negative curvature. These spaces are not necessarily manifolds, covering examples such as graphs equipped with an interior metric, yet they still have a geodesic flow defined on them. While there are analogies with the dynamics of the geodesic flow on a negative curvature Riemannian manifold, the full power of uniformly hyperbolic dynamics is not currently available in this setting: there is still a coding of the geodesic flow by a suspension flow over a shift of finite type, but rather than being a conjugacy, it is via an orbit semi-equivalence.

In general, orbit equivalence is too weak a relationship to preserve any refined dynamical information. However, we are able to use a geometric argument to show that the geodesic flow inherits the weak specification

property from the suspension flow. We use the specification property directly on the geodesic flow for CAT(-1) spaces, obtaining thermodynamic results analogous to the negative curvature Riemannian setting. In particular, we prove uniqueness of equilibrium states for Holder potential functions, the large deviations principle, and the equidistribution of weighted periodic orbits. (Received September 19, 2016)

1125-37-1876 **Steve Kass** and **Kathleen Madden\*** ([kmadden2@csu.edu](mailto:kmadden2@csu.edu)). *Generalized Shift Spaces and Shear*. Preliminary report.

Generalized shift spaces are higher dimensional symbolic dynamical systems that generalize free products. We'll look at examples of these systems and discuss their place in the class of systems with a property we call shear. (Received September 19, 2016)

1125-37-1896 **Yvonne Niyonzima\*** ([ynyonzima@smith.edu](mailto:ynyonzima@smith.edu)), **Angelica M Rosario-Santos** ([arosariosantos@smith.edu](mailto:arosariosantos@smith.edu)) and **Marisa Bloom** ([mbloom@smith.edu](mailto:mbloom@smith.edu)). *Further results on KAM tori appearing in the Charged Isosceles three-body Problem*. Preliminary report.

The complete 3D flow is bounded by the collision manifold, a compact smooth sphere, and it circulates around the vertical axis. We'll discuss further numerical results concerning the observed KAM tori. (Received September 19, 2016)

1125-37-1984 **Lien-Yung Kao\*** ([lkao@nd.edu](mailto:lkao@nd.edu)), University of Notre Dame, Department of Mathematics, 255 Hurley Hall, Notre Dame, IN 46556. *Pressure Type Metrics on Spaces of Metric Graphs*.

We will discuss two Riemannian metrics on a moduli space of metric graphs. Both of them are constructed via thermodynamical formalism and could be thought of as analogues of the Weil-Petersson metric for the space of metric graphs. In particular, we will discuss and compare geometric features of these two metrics with the "classic" Weil-Petersson metric in Teichmüller theory. This work is motivated by an earlier work of Pollicott and Sharp. (Received September 19, 2016)

1125-37-2000 **Matthew A Morena\*** ([mamorena@yhc.edu](mailto:mamorena@yhc.edu)) and **Kevin M Short**. *Fundamental Cupolets of Chaotic Systems*.

Cupolets represent highly accurate approximations to the unstable periodic orbits of chaotic systems and large numbers can be efficiently generated via a particular control method. Cupolets exhibit the interesting property that a given set of controls will uniquely identify a cupolet regardless of the initial state of the parent chaotic system. Recently, we demonstrated that this property allows for controlled transitions between nearly any two cupolets. Now, we discuss how this result can be used to classify cupolets according to their reducibility: a cupolet is classified as fundamental if its orbit cannot be decomposed into the orbits of simpler cupolets and is called composite when a decomposition is possible. Our work introduces a new way to generate higher order cupolets simply by amalgamating fundamental cupolets via sequences of controlled transitions. This allows for large collections of cupolets to be collapsed onto smaller subsets of fundamental cupolets without losing any dynamical information. We also discuss the potential for analyzing a chaotic system through its set of cupolets according to the framework that has been established through unstable periodic orbits. (Received September 19, 2016)

1125-37-2007 **Ugur G Abdulla** ([abdulla@fit.edu](mailto:abdulla@fit.edu)), Department of Mathematics, Florida Institute of Technology, 150W University Blvd, Melbourne, FL 32901, **Naveed H Iqbal\*** ([nchaudhr@my.fit.edu](mailto:nchaudhr@my.fit.edu)), Department of Mathematics, Florida Institute of Technology, 150W University Blvd, Melbourne, FL 32901, and **Rashad U Abdulla** and **Muhammad U Abdulla**. *On the fine classification of Second Minimal Odd Periodic Orbits of the Continuous Endomorphisms on the Real Line*.

An  $n$ -periodic orbit is second minimal if it is the immediate successor of a minimal period in the Sharkovskii ordering. We prove the conjecture forwarded in (Abdulla, U.G., Iqbal, N., Abdulla, R.U., Abdulla, M.U., and Turnquist, A. (2016). "On the Fine Classification of Periodic Orbits of the Continuous Endomorphisms on the Real Line and Universality in Chaos," *Joint Mathematics Meetings (JMM) 2016*, , (1116-39-970), Seattle, Washington, January 4-January 9, 2016.) regarding the fine classification of second minimal odd  $2k + 1$ -periodic,  $k \geq 3$  orbits and show there are only  $4k - 3$  such orbits with accuracy up to inverses. Furthermore, we demonstrate that each of these periodic orbits can have only one of 6 possible topological structures. (Received September 20, 2016)

1125-37-2016 **Gareth E Roberts\*** (groberts@holycross.edu), 1 College Street, Worcester, MA 01610. *Using Morse Theory in the Planar  $N$ -Vortex Problem.* Preliminary report.

The planar  $n$ -vortex problem is a Hamiltonian system of differential equations that effectively approximates vorticity evolution in fluid dynamics. Because collisions between point vortices must be excluded, the topology of the underlying configuration space is non-trivial and Morse theory can be applied to yield interesting results. We identify an explicit connection between the Morse index of a critical point of the Hamiltonian restricted to a level surface of the angular impulse and the eigenvalues of the corresponding relative equilibrium periodic solution (a rigidly rotating configuration). The Morse inequalities are then used to classify the stability (and instability) of all relative equilibria in some particular cases of the four-vortex problem. (Received September 19, 2016)

1125-37-2187 **Isaac Loh and Cesar E Silva\*** (csilva@williams.edu). *On conditions implied by ergodic cartesian square for nonsingular ergodic actions.*

We study conditions weaker than ergodic cartesian square for nonsingular transformations and actions such as weak double ergodic and ergodic with isometric coefficients. We give examples of rank-one transformations that are weak doubly ergodic and rigid (so all their cartesian products are conservative), but their 2-fold cartesian product is not ergodic. We also show that a weak doubly ergodic nonsingular group action is ergodic with isometric coefficients, and consider other examples. (Received September 20, 2016)

1125-37-2205 **Hongying Shu\*** (hshu@tongji.edu.cn), Shanghai, 200092, Peoples Rep of China, and **Yuming Chen and Lin Wang.** *Joint impacts of the cell-free and cell-to-cell infection modes on viral dynamics.*

Virus can disseminate between uninfected target cells via two modes, namely, the diffusion-limited cell-free viral spread and the direct cell-to-cell transfer using virological synapses. To examine how these two viral infection modes impact the viral dynamics, in this talk, we propose and analyze a general viral infection model that incorporates these two viral infection modes. Under some reasonable assumptions, we show that the numbers of secondly infected cells through the cell-free infection mode and the cell-to-cell infection mode both contribute to the basic reproduction number, which determines a global threshold dynamics. When the underlying assumptions are not satisfied, oscillations via global Hopf bifurcation can be observed. Two-parameter bifurcation analyses are carried out to explore the joint impacts on viral dynamics for the interplay between nonlinear target-cell dynamics and intracellular delays and between the two infection modes. (Received September 19, 2016)

1125-37-2213 **Madeleine Elyze, Alexander S Kastner\*** (ask2@williams.edu), **Juan Ortiz Rhoton, Vadim Semenov and Cesar E Silva** (csilva@williams.edu). *On the absence of an ergodic multiplier property in infinite measure.*

It is known from the work of Brunel and Aaronson, Lin, and Weiss that if  $T$  is a conservative ergodic nonsingular transformation without a finite invariant measure, then there exists a conservative ergodic infinite measure-preserving  $K$ -automorphism  $S$  such that  $T \times S$  is not conservative. It is known that conservative ergodic infinite measure-preserving  $K$ -automorphism cannot be rigid. We prove that for any countable collection of nonsingular transformation  $\{T_n\}$  with no equivalent finite invariant measure, there exists a rank-one (hence conservative ergodic) transformation  $S$  such that  $T_n \times S$  is not conservative for each  $n$ , and that moreover  $S$  can be chosen to be rigid or have infinite ergodic index. We also study related questions for infinite measure-preserving  $\mathbb{Z}^d$ -actions and flows. (Received September 20, 2016)

1125-37-2217 **Joseph Auslander\*** (jna@math.umd.edu). *Proximal and distal equivalence of minimal flows.*

This is joint work with Gernot Greschonig. A central problem in topological dynamics is the classification of minimal flows. An important invariant is the Ellis group of a flow, a closed subgroup of the automorphism group  $G$  of the universal minimal flow  $(M; T)$ . The Ellis groups classify minimal flows up to proximal equivalence. That is, two minimal flows  $(X; T)$  and  $(Y; T)$  have the same Ellis group if and only if there is a minimal flow  $(Z; T)$  and proximal extensions (homomorphisms)  $\alpha : Z \rightarrow X$  and  $\beta : Z \rightarrow Y$ . The remaining problem is the classification of minimal flows in the same proximal class, equivalently with the same Ellis group. This is accomplished by distal equivalence, the existence of a minimal flow  $(Z; T)$  and distal homomorphisms  $\alpha : Z \rightarrow X$  and  $\beta : Z \rightarrow Y$ . Invariants for distal equivalence are obtained in terms of minimal idempotents in the enveloping semigroup. Two minimal flows are isomorphic if and only if they are both proximally and distally equivalent. A minimal flow is determined by an "icer" (invariant closed equivalence relation) on  $M$ , which in turn is determined by a closed subgroup of  $G$  and certain subsets of the proximal relation of  $M$ . Necessary and sufficient conditions are obtained for the latter. (Received September 19, 2016)

1125-37-2223 **Remus Radu\*** ([remus.radu@stonybrook.edu](mailto:remus.radu@stonybrook.edu)), Stony Brook University, Institute for Mathematical Sciences, Stony Brook, NY 11794-3660. *Dynamics of neutral dissipative holomorphic germs of  $(\mathbb{C}^2, 0)$ .*

In this talk we discuss the local geometry and dynamics of holomorphic germs of  $(\mathbb{C}^2, 0)$  with a semi-neutral fixed point at the origin. This is joint work with Misha Lyubich and Raluca Tanase. (Received September 19, 2016)

1125-37-2275 **E. Arthur Robinson, Joseph Rosenblatt and Ayşe A. Şahin\*** ([ayse.sahin@wright.edu](mailto:ayse.sahin@wright.edu)). *New developments in directional dynamics.* Preliminary report. Directional dynamics, introduced by Milnor in the 70's, is a classification tool for actions of higher rank groups. In this talk we introduce some new developments in the study of directional dynamics for  $\mathbb{Z}^d$ ,  $\mathbb{R}^d$ , and other group actions. (Received September 20, 2016)

1125-37-2337 **Eduardo Dueñez\*** ([eduardo.duenez@utsa.edu](mailto:eduardo.duenez@utsa.edu)), Mathematics Department, The University of Texas at San Antonio, San Antonio, TX 78249, and **José N Iovino.** *Polynomial ergodic descent and uniformly metastable convergence.*

We formulate the Mean Ergodic Theorem (MET) for polynomial actions of an abelian group on a Hilbert space as a statement about classes of metric structures that are axiomatizable in a suitable first-order-like logic (Henson's logic of "approximate satisfaction" of positive bounded formulas). This formulation allows for a very clean proof of MET. As a byproduct of the first-order axiomatization, we show that ergodic averages admit uniformly metastable rates of convergence (in Tao's sense) using the compactness principle in logic.

(Research supported by NSF grant DMS-1500615.) (Received September 20, 2016)

1125-37-2341 **Vaughn Climenhaga\*** ([climenha@math.uh.edu](mailto:climenha@math.uh.edu)). *Tower constructions in smooth and symbolic dynamics.*

Existence, uniqueness, and statistical properties of invariant measures for dynamical systems with hyperbolic behavior can be studied by constructing a certain "tower" that represents the system as a suspension over the full shift on a countable alphabet. I will discuss recent results that permit this construction to be carried out under relatively weak hypotheses. (Received September 20, 2016)

1125-37-2600 **Cesar E Silva\*** ([csilva@williams.edu](mailto:csilva@williams.edu)). *The mathematical work of Dorothy Maharam in ergodic theory and measure theory.* Preliminary report.

Dorothy Maharam (1917-2014) made several important contributions to measure theory, ergodic theory and category algebras. We will discuss some of her work with emphasis on her work in ergodic theory and measure theory. (Received September 20, 2016)

1125-37-2618 **Bernd Sing\*** ([bernd.sing@cavehill.uwi.edu](mailto:bernd.sing@cavehill.uwi.edu)), Department of Mathematics, The University of the West Indies, Cave Hill, P.O. Box 64, Bridgetown, BB00000, Barbados, and **Dirk Frettlöh**, Technische Fakultät, Universität Bielefeld, Postfach 100131, 33501 Bielefeld, Germany. *Invertible substitutions and continued fractions.* Preliminary report.

We are interested in symbolic dynamical systems generated by bi-infinite fixed-points of primitive substitutions over an alphabet with two letters that are *invertible*, i.e., that are also automorphisms of the free group with two generators. Such substitutions are also known as *Sturmian* substitutions, i.e., the language generated by such a substitution has exactly  $n + 1$  subwords of length  $n$  for every natural number  $n$ .

It is useful to consider the *substitution matrix* (also known as *incidence matrix* or *Abelianization matrix*) associated to such substitutions, namely, the matrix the  $ij$ -entry of which counts the number of occurrences of the letter  $i$  in the substitute of the letter  $j$ . Here, this substitution matrix is an element of  $GL(2, \mathbb{Z})$ . Using the theory of continued fractions, we first calculate eigenvalues and eigenvectors, which yield basic combinatorial information like the letter frequencies in the bi-infinite fixed-points, and then also consider the question which of these dynamical systems are topologically conjugate. (Received September 20, 2016)

1125-37-2875 **Lauren Lazarus\*** ([llazarus@hmc.edu](mailto:llazarus@hmc.edu)), **Matthew Davidow** and **Richard Rand.** *Periodic forcing of a first-order delay limit cycle oscillator.*

The first-order delay limit cycle oscillator  $x'(t) = -x(t - T) - x^3(t)$  has been shown to exhibit many similarities to oscillator models given by second-order ordinary differential equations. In this talk, we discuss its response to periodic external forcing.

The forced system exhibits quasiperiodic motion outside of a resonance region, where it has periodic motion at the frequency of the forcer. By perturbation methods and bifurcation theory, we show that the resonance region is asymmetric in the frequency detuning, and that there are regions where stable periodic and quasiperiodic

motions coexist. Some bifurcation behaviors of the system are directly comparable to the forced van der Pol oscillator. (Received September 20, 2016)

1125-37-2903 **Quang-Nhat Le\*** (qnhatle@math.brown.edu). *A family of discrete dynamical systems in real projective geometry.*

Polygon iterations provide an abundance of interesting discrete dynamical systems in geometry, especially in Euclidean and affine geometries. Recently, the advance of computers has allowed the study of polygon iterations in projective geometry, which was previously limited by the high computational complexity of the associated rational maps, to take off. Notable examples are the pentagram map and the projective midpoint map, both first studied by Richard Schwartz as potential analogues of the classical midpoint map.

In this talk, we will discuss recent work on a one-parameter family of projectively natural polygon iterations that includes both the pentagram map and the projective midpoint map. They can be regarded as discrete dynamical systems on the space of polygons in the real projective plane, modulo projective transformations. Except for 2 parameters, corresponding to the pentagram map and its inverse, these polygon iterations are observed to possess a single globally attracting fixed point, which allows us to define their Julia sets. Coincidentally, when observing the varying Julia sets, we discovered that this family contains two projective analogues of Varignon's theorem for quadrilaterals. (Received September 20, 2016)

1125-37-2944 **Scott Cook\*** (scook@tarleton.edu), Tarleton State University, Dept of Mathematics, Box-0470, Stephenville, TX 76402. *Thermodynamics and Thermophoresis in Random Billiard Dynamical Systems.*

We discuss a class of random billiard dynamical systems as a framework to study thermodynamics and diffusion of gas particles interacting with a rough surface. We first introduce a notion of temperature via perturbation of the usual billiard reflection law. This allows for heat reservoirs and the study of thermodynamical properties like heat flow and entropy production. Though we begin with single particle systems in one and two dimensions, we will focus on three dimensional systems with many interacting particles. We will distinguish one "Brownian" particle whose motion we wish to control using thermal gradients. This serves as a mathematical model of the process called thermophoresis in chemical engineering applications where an airborne "pollutant" (aerosol) is driven toward a target site by a temperature differential. If time permits, we will discuss techniques for simulating such systems using parallel algorithms on a GPU. (Received September 20, 2016)

1125-37-3150 **Talitha M. Washington\*** (talitha.washington@howard.edu), Department of Mathematics, Academic Support Building B, Room 221, 2441 Sixth Street, NW, Washington, DC 20059. *Nonstandard Finite Difference Schemes: the Mathematics of Ronald Mickens.* Preliminary report.

In the late 1980's, Ronald Mickens of Clark Atlanta University introduced the concept of a "Nonstandard Finite Difference Scheme (NFDS)" as a methodology which would best approximate solutions to systems of differential equations. In this talk, we give a survey of NFDS's as used in various problems from mathematical physics and mathematical biology. We also discuss the lasting legacy of Ronald Mickens and his impact on the mathematical sciences. (Received September 21, 2016)

1125-37-3151 **Dong Chen\*** (dxc360@psu.edu), Department of Mathematics, 109 McAllister Bldg, University Park, PA 16802. *Two types of KAM-nondegenerate nearly integrable systems with positive metric entropy.*

The celebrated KAM Theory says that if one makes a small perturbation of a non-degenerate completely integrable system, we still have a huge measure of invariant tori with quasi-periodic dynamics in the perturbed system. These invariant tori are known as KAM tori. What happens outside KAM tori draws lots of attention. In this talk I will present two types of  $C^\infty$  small Lagrangian perturbation of the geodesic flow on a flat torus. Both resulting flows have positive metric entropy. From this result we get positive metric entropy outside some KAM tori. What is special in the second type is that positive metric entropy comes from an arbitrarily small tubular neighborhood of one trajectory. This is a joint work with D. Burago and S. Ivanov. (Received September 21, 2016)

## 39 ► *Difference and functional equations*

1125-39-401 **Oluwaseye Adekanye\*** (seyeadekanye@gmail.com), Howard University, Department of Mathematics, 2241 6th Street NW, Washington, DC 20059, and **Talitha Washington** (talitha.washington@howard.edu), Howard University, Department of Mathematics, 2441 6th Street, NW, Washington, DC 20059. *Creating Non-standard Finite Difference Schemes : Why Bother?* Preliminary report.

Many real world phenomena can be modeled by dynamical systems that describe the evolution of phenomena over time. For example, the growth and decay equation models how a quantity changes over time. The transport equation with a flux term models the flow of a particle through a given medium. The Airy equation models the diffraction of light. Using the growth and decay equation, we can develop the foundation for an exact non-standard finite difference scheme (NSFD) which can preserve properties of the dynamical system into its discretization. Some equations require the NSFD scheme to adhere to time and space step size constraints. In this talk, we will show how to construct NSFD schemes that outperform the traditional standard finite difference schemes. (Received August 31, 2016)

1125-39-485 **Ronald E. Mickens\*** (rmickens@caw.edu), Department of Physics, Clark Atlanta University, Atlanta, GA 30314. *What is a NSFD Scheme?*

The main purpose of this presentation is to provide an answer to the question asked in the title. After giving the genesis of the NSFD methodology [1], we define the two features critical to the construction of NSFD schemes for differential equations: (i) a new formulation of the discrete first-derivative which introduces the concept of denominator functions [2]; and (ii) the nonlocal discrete representation of functions of the dependent variables. The implementation of the NSFD methodology has its basis in the concept of dynamic consistency [3], i.e., the exact incorporation of particular properties of the differential equations into the finite difference models of these equations. Finally, we consider several explicit examples of how to construct NSFD schemes and discuss a number of currently unresolved issues.

### References

[1] R.E. Mickens, *NSFD Models of Differential Equations* (World Scientific, London, 1994). [2] R.E. Mickens, "Calculation of denominator functions", *Numer. Methods PDE*, Vol. 23 (2007), 672-691. [3] R.E. Mickens, "Dynamic consistency", *J. Difference Eqs. Appl.*, Vol. 11 (2005), 645-653. (Received September 03, 2016)

1125-39-496 **Candace M. Kent\*** (cmkent@vcu.edu), 3510 Hanover Avenue, Richmond, VA 23221, and **Stevo Stevic**. *A Proposal for an Application for a Max-Type Difference Equation*. Preliminary report.

Difference Equations with the maximum function, unlike differential equations with the maximum function, have no known applications. We propose, for the sake of dialogue, that the particular nonautonomous max-type difference equation

$$x_{n+1} = \max \left\{ \frac{A_n^{(0)}}{x_n}, \frac{A_n^{(1)}}{x_{n-1}}, \dots, \frac{A_n^{(k)}}{x_{n-k}} \right\}, \quad n = 0, 1, \dots,$$

where the  $A_n^{(i)}$  are positive periodic sequences and the initial conditions are positive, may serve as a model of the onset and offset of a seizure as is seen, for example, in mesial temporal lobe epilepsy. (Received September 04, 2016)

1125-39-513 **Sabrina Heike Streipert\*** (shsbrf@mst.edu), Madison, WI 53719, and **Martin Bohner**. *Susceptible-Infected-Susceptible Model on Time Scales*.

We introduce the epidemic model following the hypothesis of the disease flow *Susceptible*  $\rightarrow$  *Infected*  $\rightarrow$  *Susceptible*, short *SIS*, on time scales. After a brief introduction of time scales, we give a formulation of the SIS-model on time scales for the case of constant population size and varying total population. We present the derivation of the solutions to both cases and discuss the stability of the steady-states. Examples on the discrete and the quantum space are presented to illustrate the results. (Received September 04, 2016)

1125-39-656 **Vlajko L Kocic\*** (vkocic@xula.edu), Mathematics Department, Xavier University of Louisiana, 1 Drexel Dr., New Orleans, LA 70125. *Dynamics of certain classes of nonlinear discontinuous difference equations*. Preliminary report.

Dynamics of nonlinear difference equations of the form

$$x_{n+1} = f(x_n), n = 0, 1, \dots$$

where the function  $f$  has jump discontinuities has been studied. In particular we address the existence of invariant intervals, oscillation with semicycles, existence and stability of periodic orbits. (Received September 08, 2016)

1125-39-715 **Rudi Weikard\***, Department of Mathematics, University of Alabama at Birmingham, Birmingham, AL 35226. *Stability for the inverse resonance problem for the CMV operator.*

For the class of unitary CMV operators with super-exponentially decaying Verblunsky coefficients we treat the inverse resonance problem of reconstructing the operator from its resonances - the zeros of the Jost function. We establish a stability result for the inverse resonance problem that shows continuous dependence of the operator coefficients on the location of the resonances.

This joint work with Roman Shterenberg and Maxim Zinchenko. (Received September 09, 2016)

1125-39-768 **Kevin Ahrendt\*** ([kahrendt@huskers.unl.edu](mailto:kahrendt@huskers.unl.edu)). *Unique Solutions to Nonlinear Boundary Value Problems with a Fractional Self-Adjoint Difference Equation.* Preliminary report.

We will present a quick overview of some results regarding difference equations using the nabla fractional calculus on a discrete domain. In particular, we will give an introduction to the nabla fractional calculus along with an overview of results regarding a self-adjoint difference equation containing a Caputo fractional difference. Finally, we will use the Contraction Mapping Theorem to prove there exists unique solutions to certain nonlinear self-adjoint boundary value problems. (Received September 11, 2016)

1125-39-957 **Talitha M Washington\*** ([talitha.washington@howard.edu](mailto:talitha.washington@howard.edu)), 4119 Postgate Terrace Apt 303, Silver Spring, MD 20906. *Employing Conservation Laws to Construct Nonstandard Finite Difference Schemes.*

Often in modeling with differential equations, conservation laws arise such as in population dynamics. In this talk, we discuss how various laws lead to the construction of nonstandard finite difference (NSFD) schemes. Our schemes will be formulated within the NSFD methodology of Ronald Mickens. By using a number of well-known population models, we will illustrate the details of our procedures by constructing appropriate NSFD discretizations. We will show that by incorporating the conservation laws into the scheme, we can achieve a better approximation to the solution. (Received September 13, 2016)

1125-39-1024 **Mustafa R. S. Kulenovic\*** ([mkulenovic@uri.edu](mailto:mkulenovic@uri.edu)), Department of Mathematics, University of Rhode Island, Kingston, RI 02881. *Invariant Curves for Planar Competitive and Cooperative Maps.* Preliminary report.

We present some results on the existence of invariant curves for planar monotone maps that are linearly ordered in either south-east or north-east ordering. Some of these curves are the stable or unstable manifolds of hyperbolic fixed points (saddle points) or non-hyperbolic fixed points of both stable and unstable types, and are also the boundary of basins of attraction of such points. (Received September 14, 2016)

1125-39-1075 **S N Elaydi\*** ([selaydi@trinity.edu](mailto:selaydi@trinity.edu)), One Trinity Place, San Antonio, TX 78212, and **E C Balreira** and **R Luis**. *Monotone Discrete Dynamical systems.*

In this talk we will present the latest results on the global stability of monotone maps on finite-dimensional Euclidean spaces. Applications to biological models such as the n-dimensional Leslie-Gower and the Ricker models will be given. Related open problems and conjectures will be presented. (Received September 14, 2016)

1125-39-1084 **Eddy A Kwessi\*** ([ekwessi@trinity.edu](mailto:ekwessi@trinity.edu)), San Antonio, TX 78212, **Saber N Elaydi**, San Antonio, and **Brian Dennis** and **George Livadiotis**. *Non-Standard Discretization of Models with the Allee Effect.* Preliminary report.

In this talk, we will apply the non-standard discretization technique to continuous models in ecology and epidemiology. In particular, we focus our attention on the discretization of multi-species models with and without the Allee effect. (Received September 14, 2016)

1125-39-1246 **J. M. Cushing\*** ([cushing@math.arizona.edu](mailto:cushing@math.arizona.edu)), Department of Mathematics, 617 N Santa Rita, University of Arizona, Tucson, AZ 85721. *Difference equation models in population dynamics whose coefficients are subject to Darwinian evolution.*

Difference equations used to define discrete time dynamical systems have widespread use in population dynamics. Constant coefficients give rise to autonomous equations. Equations with periodic coefficients are motivated by periodically oscillating environments or vital rates. Equations with stochastic coefficients give rise to nonlinear stochastic processes. Another reason model coefficients can change in time is that they might be subject to Darwinian evolution. I will talk about evolutionary versions of difference equations (or systems of difference equations) for population dynamics in which coefficients can depend on a vector of phenotypic traits (strategies)

subject to evolutionary selection. I will describe a general theorem that addresses questions concerning extinction versus survival, the bifurcation of survival (positive) equilibria, evolutionary stable strategies, and the role that the direction of bifurcation plays with regard to stability, strong Allee effects, tipping points, and other phenomena. Collaborators are Filipe Martins and Alberto Pinto, University of Porto, and Amy Veprauskas, University of Louisiana at Lafayette. (Received September 15, 2016)

1125-39-1360 **Allan C Peterson\*** (apeterson1@math.unl.edu), Dr. Allan Peterson, 6650 Blue Ridge Lane, Lincoln, NE 68516. *A Monotonicity Theorem for the Quantum Calculus.*

We give results concern monotonicity of functions whose domain is a quantum set. Also comparison theorems and asymptotic properties of solutions for fractional equations on a quantum set will be addressed. (Received September 16, 2016)

1125-39-1636 **H Sedaghat\***, Department of Mathematics, Richmond, VA 23284-2014, and **N Lazaryan.** *Implications of inter-stage interactions for extinction and the Allee region in an age-structured population model.*

In an age-structured population (e.g. adults and juveniles) with a Ricker fitness function and time-dependent vital rates, we examine conditions for the convergence of orbits to the origin (extinction) in the presence of the Allee effect. When stages (or generations) interact, we find that extinction need not occur in the absence of positive fixed points, a situation that is impossible without inter-stage interactions. We also examine the shift in the Allee equilibrium caused by the occurrence of interactions between stages. We find that this shift away from the origin leads to an expected enlargement of the extinction region but surprisingly the enlargement is not the maximum possible allowed by the shift. (Received September 18, 2016)

1125-39-1671 **Najat Ziyadi\*** (najat.ziyadi@morgan.edu), Department of Mathematics, Morgan State University, 1700 East Cold Spring Lane, Baltimore, MD 21251, and **Abdul-Aziz Yakubu.** *A Discrete-Time Nutrients-Phytoplankton-Oysters Model of a Bay Ecosystem.* Preliminary report.

In this talk, we will introduce a discrete-time NPO model that describes the interactions of nutrients (N), phytoplankton (P) and oysters (O) in a bay ecosystem. The NPO model is based on the Nicholson-Bailey model with constant recruitment for the nutrients and constant survival rates phytoplankton and oysters. Using the NPO model, we will derive verifiable conditions for the persistence and extinction of phytoplankton and oysters in a bay ecosystem. (Received September 18, 2016)

1125-39-2056 **Eze R Nwaeze\*** (enwaeze@mytu.tuskegee.edu). *A new weighted Ostrowski type inequality on arbitrary time scale.*

In this talk, a new weighted generalized Montgomery identity is discussed. Using this identity, we obtain a weighted Ostrowski type inequality for parameter function on an arbitrary time scale. In addition, the real, discrete and quantum cases are considered. (Received September 19, 2016)

1125-39-2284 **Ross A. Chiquet\*** (rchiquet@louisiana.edu), **A. S. Ackleh, B. A. Ma, T. Tang, A. Veprauskas, H. Caswell and N. Sidorovskaia.** *Analysis of Lethal and Sublethal Impacts of Environmental Disasters on Sperm Whales Using Stochastic Modeling.*

We develop autonomous and nonautonomous matrix population models to study lethal and sublethal impacts of environmental disasters, such as oil spills, on the Gulf of Mexico sperm whales. We investigate how reductions in the survival probabilities and in fecundity affect the sperm whale population. We then investigate the long term effect of an environmental disaster on the population by assuming that the lethal and sublethal effects are for a fixed period of time after which the vital rates recover to their original values. We also inspect the effects of demographic stochasticity on the recovery probabilities and the recovery time of the population and develop formulas to calculate the sensitivity of the recovery time to changes in the initial population distribution, reduction proportion and reduction time. (Received September 20, 2016)

1125-39-2357 **Rajendra B Dahal\***, rdahal@coastal.edu, and **Christopher S Goodrich,** cgood@prep.creighton.edu. *An Almost Sharp Monotonicity Result for Discrete Sequential Fractional Delta Differences.*

We consider a monotonicity-type result for functions  $f : \mathbb{N}_a \rightarrow \mathbb{R}$  satisfying the sequential fractional difference inequality

$$\Delta_{1+a-\mu}^{\nu} \Delta_a^{\mu} f(t) \geq 0,$$

for  $t \in \mathbb{N}_{2+a-\mu-\nu}$ , where  $0 < \mu < 1$ ,  $0 < \nu < 1$ , and  $1 < \mu + \nu < 2$ . A comparison between the sequential and non-sequential settings is provided, and we note that nontrivial dissimilarities exist between the two settings.

We demonstrate, in addition, that, in a certain sense, our results are almost sharp. Finally, some numerical examples are provided in order to clarify our results. (Received September 20, 2016)

1125-39-2387 **Valmir Bucaj\*** ([vb11@rice.edu](mailto:vb11@rice.edu)), 6100 Main St. Ms-136, Houston, TX 77005. *On the Kunz-Souillard approach to localization for the discrete one dimensional generalized Anderson model.*

We prove dynamical and spectral localization at all energies for the discrete *generalized Anderson model* via the Kunz-Souillard approach to localization. This is an extension of the original Kunz-Souillard approach to localization for Schrödinger operators, to the case where a single random variable determines the potential on a block of an arbitrary, but fixed, size  $\alpha$ . For this model, we also prove positivity of the Lyapunov exponents at all energies. In fact, we prove a stronger statement where we also allow finitely supported distributions. We also show that for any size  $\alpha$  *generalized Anderson model*, there exists some finitely supported distribution  $\nu$  for which the Lyapunov exponent will vanish for at least one energy. Moreover, restricting to the special case  $\alpha = 1$ , we describe a pleasant consequence of this modified technique to the original Kunz-Souillard approach to localization. In particular, we demonstrate that actually the single operator  $T_1$  is a strict contraction in  $L^2(\mathbb{R})$ , whereas before it was only shown that the second iterate of  $T_1$  is a strict contraction. (Received September 20, 2016)

1125-39-2608 **Carlos E. Arreche\*** ([cearrech@math.ncsu.edu](mailto:cearrech@math.ncsu.edu)), Mathematics Department, North Carolina State University, Raleigh, NC 27695. *Projectively integrable linear difference equations and their Galois groups.*

A linear difference equation is integrable if its solutions also satisfy a linear differential system of the same size. The difference equation is projectively integrable if it becomes integrable "after moding out by scalars". Based on recent results of R. Schäfke and M. Singer, we show that when the system has coefficients in  $\mathbb{C}(x)$  and  $\sigma$  is either a shift,  $q$ -dilation, or Mahler operator, the difference-differential Galois group  $G$  attached to an integrable or projectively integrable difference equation has a very special form. As an application of these results, one can prove that certain combinatorially-defined generating functions do not satisfy any polynomial differential equations. This is joint work with Michael Singer. (Received September 20, 2016)

## 40 ► Sequences, series, summability

1125-40-430 **Aaron Berger\*** ([aaron.berger@yale.edu](mailto:aaron.berger@yale.edu)), 18 Park Hill Terrace, West Windsor, NJ 08550. *The Maximum Length of  $k$ -bounded,  $t$ -avoiding Zero-sum Sequences over  $\mathbb{Z}$ .*

Let  $\mathcal{S}$  be a multiset of integers. We say  $\mathcal{S}$  is a *zero-sum sequence* if the sum of its elements is 0. We study zero-sum sequences whose elements lie in the interval  $[-k, k]$  such that no subsequence of length  $t$  is also zero-sum. Augspurger, Minter, Shoukry, Sissokho, and Voss show that there are arbitrarily long zero-sum sequences with these restrictions unless  $t$  is divisible by  $\text{LCM}(2, 3, 4, \dots, 2k - 1)$ . We confirm a conjecture of these authors that for  $k$  and  $t$  such that this divisibility condition holds, every zero-sum sequence of length at least  $t + k^2 - k$  contains a zero-sum subsequence of length  $t$ , and that this is the minimal length for which this property holds. (Received September 01, 2016)

1125-40-698 **Ke Jin\*** ([kejin@udel.edu](mailto:kejin@udel.edu)). *The Length of the Longest Common Subsequences of Two Independent Mallows Permutations.* Preliminary report.

The Mallows measure is a probability measure on  $S_n$  where the probability of a permutation  $\pi$  is proportional to  $q^{l(\pi)}$  with  $q > 0$  being a parameter and  $l(\pi)$  the number of inversions in  $\pi$ . We prove a weak law of large numbers for the length of the longest common subsequences of two independent random permutations drawn from the Mallows measure, when  $q$  is a function of  $n$  and  $n(1 - q)$  has limit in  $\mathbb{R}$  as  $n \rightarrow \infty$ . (Received September 09, 2016)

1125-40-2461 **Kathleen Lee\*** (klee6@poets.whittier.edu), **Michelle Haver** (m-haver.1@onu.edu), **William McDermott** (willm97@vt.edu) and **Alex Wilson** (wils1256@msu.edu). *Classification of numerical sequences originating from recursive polynomial sequences.* Preliminary report.

In this talk, we classify the asymptotic behavior of sequences which can be generated from polynomials which satisfy the following recurrence relation:

$$\begin{cases} M_n(x, y) = xM_{n-1}(x, y) + yM_{n-2}(x, y) & n \geq 2 \\ M_0 = a \\ M_1 = bx + cy + d \end{cases}$$

where  $a, b, c$ , and  $d$  are real constants. We present a Binet formula for  $M_n$ , which allows us to classify these sequences using a triangle in  $\mathbb{R}^2$ . The sequence  $M_n(x, y)$  evaluated inside the triangle converges to zero, while  $M_n(x, y)$  evaluated outside the triangle diverges. We also discuss subtle behaviors on the boundary such as periodicity and convergence to a constant. Moreover, we present a finite sum expression for  $M_n$ , which can be used to generate sequences that we interpret combinatorially. One example we provide a combinatorial proof for is

$$\sum_{k=0}^{\lfloor \frac{n+1}{2} \rfloor} \left[ \binom{n-k}{k} + (w-1) \binom{n-k-1}{k} \right] w^k (w-1)^k = w^n$$

Finally, we explain how the first derivative of  $M_n$  with respect to  $y$  under certain conditions generates sequences that are typically found by convolution of two numerical sequences. (Received September 20, 2016)

## 41 ► *Approximations and expansions*

1125-41-172 **Sergey Denisov\*** (denissov@math.wisc.edu). *Solution to Steklov's problem and related questions.*

We will discuss some recent developments around the solution to Steklov's problem in the theory of orthogonal polynomials. In particular, we will present some new methods to estimate the size of monic orthogonal polynomials given a weight from certain class. (Received August 08, 2016)

1125-41-220 **Maxim L Yattselev\*** (maxyatts@iupui.edu), 402 North Blackford Street, LD 251, Department of Mathematical Sciences, IUPUI, Indianapolis, IN 46202, and **Alexander Aptekarev** and **Alexander Bogolubsky**. *Szegő-type asymptotics for ray sequences of Frobenius-Padé approximants.*

Let  $\hat{\sigma}$  be a Cauchy transform of a possibly complex-valued Borel measure  $\sigma$  and  $\{p_n\}$  be a system of orthonormal polynomials with respect to a measure  $\mu$ ,  $\text{supp}(\mu) \cap \text{supp}(\sigma) = \emptyset$ . An  $(m, n)$ -th Frobenius-Padé approximant to  $\hat{\sigma}$  is a rational function  $P/Q$ ,  $\deg(P) \leq m$ ,  $\deg(Q) \leq n$ , such that the first  $m+n+1$  Fourier coefficients of the linear form  $Q\hat{\sigma} - P$  vanish when the form is developed into a series with respect to the polynomials  $p_n$ . Asymptotics of the Frobenius-Padé approximants to  $\hat{\sigma}$  along ray sequences  $\frac{n}{n+m+1} \rightarrow c > 0$ ,  $n-1 \leq m$ , is presented when  $\mu$  and  $\sigma$  are supported on intervals on the real line and their Radon-Nikodym derivatives with respect to the arcsine distribution of the respective interval are holomorphic functions. (Received August 13, 2016)

1125-41-257 **Vira Babenko\*** (vbabenko@ithaca.edu), Ithaca College, Department of Mathematics, 953 Danby Road, Ithaca, NY 14850. *Approximation of functions with values in general and specific L-spaces.*

In this talk, we show that classical approximation operators such as Bernstein, Schoenberg, and other positive linear operators can be adapted to functions with values in L-spaces (which are generalizations of set-valued and fuzzy-valued functions). We obtain error estimation of approximation for functions with values in L-spaces by such operators, as well as error estimations for some formulas of approximate integration. The results are used to develop algorithms for the solution of integral equations involving functions with values in L-spaces. We also discuss problems of approximation by generalized polynomials and splines for functions whose values lie in a specific L-space, namely a space of sets. Because the structures of spaces of sets are richer than the structure of general L-spaces, we have additional tools in the former space (e.g., the support function of a set), which allows us to obtain deeper results for the approximation and interpolation of set-valued functions. (Received August 19, 2016)

1125-41-292 **Roza Aceska\***, 2000 W University Ave, Muncie, IN 47305, and **Yeon Hyang Kim**.  
*Scalable frames generated by actions of iterative operators.*

In a finite dimensional Hilbert space  $H$ , with a fixed finite set of vectors  $G \subset H$ , we study the iterative actions of an operator  $A$  on  $H$ . Under certain conditions on  $A$  and  $G$ , the set of iterations  $F_G(A) = \{A^j g \mid g \in G, 0 \leq j \leq L(g)\}$  is a frame for  $H$ . Frames of type  $F_G(A)$  have special properties; for instance, the canonical dual frame of  $F_G(A)$  has an iterative set structure as well.

We state the relations between  $A$ ,  $G$  and the number of iterations  $L$ , which ensure that the system  $F_G(A)$  is a tight or a scalable frame. We study more closely the special case when  $A$  is Hermitian, that is, we exploit its unitary diagonalization. In addition, we answer the question of frame scalability and full spark frames for several special cases. (Received August 24, 2016)

1125-41-466 **Erwin Miña-Díaz\*** ([minadiaz@olemiss.edu](mailto:minadiaz@olemiss.edu)), University of Mississippi, Department of Mathematics, Hume Hall 305, P.O. Box 1848, University, MS 38677-1848. *Some asymptotic results for multi-orthogonal polynomials on arcs of the unit circle.* Preliminary report.

We will present some new results on the asymptotic behavior of multi-orthogonal polynomials of type II with respect to an Angelesco system of two measures supported on symmetric arcs of the unit circle. (Received September 02, 2016)

1125-41-473 **ulises fidalgo\*** ([uxf6@case.edu](mailto:uxf6@case.edu)), Yost Hall (Math Department), 2049 Martin Luther King Jr. Drive, Cleveland, OH 44106-7058. *Some sufficient conditions of convergence in interpolatory quadrature rules.*

We use a connection between interpolatory quadrature formulas and Fourier series to find a class of convergent schemes of interpolatory quadrature rules. (Received September 02, 2016)

1125-41-484 **Abey López-García\*** ([lopezgarci@southalabama.edu](mailto:lopezgarci@southalabama.edu)), University of South Alabama, Department of Mathematics and Statistics, ILB 325, 411 University Blvd North, Mobile, AL 36688, and **Guillermo López Lagomasino** and **Erwin Miña-Díaz**. *Nikishin systems on star-like sets and associated multi-orthogonal polynomials.*

Nikishin systems of functions and measures were introduced by E.M. Nikishin in 1980 in a very influential paper in approximation theory. These systems provided for the first time a wide class of functions that possess convergent Hermite-Padé approximants. They also give rise to interesting families of multi-orthogonal polynomials. These polynomials have been investigated with interest on the real line for many years, and in this context many properties are now known. In this talk I will describe algebraic and asymptotic properties of such polynomials in the new context of star-like sets in the complex plane. (Received September 03, 2016)

1125-41-500 **Edward B Saff\*** ([edward.b.saff@vanderbilt.edu](mailto:edward.b.saff@vanderbilt.edu)), Department of Mathematics, Stevenson Center Building #1, Vanderbilt University, Nashville, TN 37240, and **Nikos Stylianopoulos**. *Relative Asymptotics of Orthogonal Polynomials for Perturbed Measures.*

In the context of orthogonal polynomials in the plane we introduce the notion of a polynomially small (PS) perturbation of a measure. Namely, if  $\mu_0 \geq \mu_1$  and  $\{p_n(\mu_j, z)\}_{n=0}^\infty, j = 0, 1$ , are the associated orthonormal polynomial sequences, then  $\mu_0$  a PS perturbation of  $\mu_1$  if  $\|p_n(\mu_1, \cdot)\|_{L_2(\mu_0 - \mu_1)} \rightarrow 0$ , as  $n \rightarrow \infty$ . In such a case we establish relative asymptotic results for the two sequences of orthonormal polynomials. We also provide results dealing with the behaviour of the zeros of PS perturbations of area orthogonal (Bergman) polynomials. (Received September 04, 2016)

1125-41-627 **Rujie Yin\*** ([rujie.yin@duke.edu](mailto:rujie.yin@duke.edu)), 120 Science Drive, P.O. box 90320, Durham, NC 27708-0320, and **Tingran Gao**, **Yue M. Lu** and **Ingrid Daubechies**. *A tale of two bases: local-nonlocal regularization on image patches with convolution framelets.*

We propose an image representation scheme combining the local and nonlocal characterization of patches in an image. Our representation scheme is shown to be equivalent to a tight frame constructed from convolving local bases (e.g. wavelet frames, discrete cosine transforms, etc.) with nonlocal bases (e.g. spectral basis induced by nonlinear dimension reduction on patches), and we call the resulting frame elements *convolution framelets*. Insight gained from analyzing the proposed representation leads to a novel interpretation of a recent high-performance patch-based image inpainting algorithm using Point Integral Method (PIM) and Low Dimension Manifold Model (LDMM). In particular, we show that LDMM is a weighted  $\ell_2$ -regularization on the coefficients obtained by decomposing images into linear combinations of convolution framelets; we extend the original LDMM to a reweighted version that yields further improved inpainting results. Our framework can be potentially generalized to interpret more complex image processing algorithms. (Received September 08, 2016)

1125-41-723 **Mohsen Razzaghi\*** ([razzaghi@math.msstate.edu](mailto:razzaghi@math.msstate.edu)), Department of Mathematics and Statistics, Mississippi State University, Mississippi State, MS 39762. *Solution of fractional differential equations by polynomial series.*

Fractional differential equations (FDEs) are generalizations of ordinary differential equations to an arbitrary (non integer) order. FDEs have attracted considerable interest because of their ability to model complex phenomena. In this talk, a numerical method for solving the initial and boundary-value problems for FDEs is presented. The method is based upon the fractional polynomial series approximations. The operational matrix for the fractional polynomial series is given. This matrix is then utilized to reduce the solution of the FDEs to a system of algebraic equations. The method is computationally very attractive and gives very accurate results. The numerical solutions are compared with available exact or approximate solutions in order to assess the accuracy of the proposed method. (Received September 09, 2016)

1125-41-893 **Yang Wang\*** ([yangwang@ust.hk](mailto:yangwang@ust.hk)), Department of Math, Hong Kong Univ. of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. *On Generalized Phase Retrieval.*

In many applications such as X-ray Crystallography, imaging, communication and others one must construct a function/signal from only the magnitude of the measurements. These measurements can be, for example, the Fourier transform of the density function. While it is well known that we can recover a function from its Fourier transform, the classical phase retrieval problem asks whether we can recover a function from only the magnitude of its Fourier transform. The phase retrieval problem has since been extended to a much broader class of settings, referring to the reconstruction of a signal from only the magnitude of its linear measurements or more generally, from quadratic measurements. In this talk we consider the problem in the more general setting, asking whether a signal can be reconstructed from its quadratic measurements. The problem turns out to be quite challenging. Many fundamental theoretical problems remain unresolved. Equally interesting is the connections to some classical problems on the embedding of projective spaces into Euclidean spaces and nonsingular bilinear forms. In this talk I'll give a brief overview and discuss some of the recent progresses. (Received September 13, 2016)

1125-41-912 **Radu Balan** ([rvbalan@math.umd.edu](mailto:rvbalan@math.umd.edu)), **Maneesh Singh** ([maneesh.singh@verisk.com](mailto:maneesh.singh@verisk.com)) and **Dongmian Zou\*** ([zou@math.umd.edu](mailto:zou@math.umd.edu)). *Lipschitz Properties for Deep Convolutional Networks.*

In this talk we review robustness properties of deep convolutional networks. Convolutional neural networks are feature extractors widely used in machine learning. We discuss the use of families of nonlinear functions and their analytic properties. We start from the standard scattering network and then discuss more general networks. (Received September 13, 2016)

1125-41-967 **Michael C. Northington\*** ([mcnv3@gatech.edu](mailto:mcnv3@gatech.edu)), **Doug Hardin**, **Alexander Powell** and **Shahaf Nitzan**. *Balian-Low Type Theorems for Shift-Invariant Spaces.*

When shift-invariant spaces and Gabor systems are used as approximation spaces, it is advantageous for the generators of such spaces to be localized and for the spaces to be representative of a large class of functions. However, the celebrated Balian-Low Theorem shows that if a Gabor system generated by a function forms an orthonormal basis for  $L^2(\mathbb{R})$ , then the function must be poorly localized in either time or frequency. In this talk, I will discuss similar restrictions on the generators of finitely-generated shift-invariant spaces. In particular, I will show that if the integer translates of a well-localized function,  $f \in L^2(\mathbb{R}^d)$ , form certain types of bases for the shift-invariant space generated by  $f$ , then this space cannot be invariant under any non-integer shift. (Received September 13, 2016)

1125-41-1050 **Keaton Hamm\*** ([keaton.hamm@vanderbilt.edu](mailto:keaton.hamm@vanderbilt.edu)). *Interpolation in Shifted Function Spaces.*

We consider the problem of interpolating functions from shift-invariant spaces and more general function spaces of the form

$$V(\psi, \mathcal{X}) := \left\{ \sum_{j \in \mathbb{Z}} c_j \psi(\cdot - x_j) : (c_j) \in \ell_2(\mathbb{Z}) \right\}$$

where the interpolants themselves lie in a similar space of translates of a given kernel. We discuss conditions on the shift kernel  $\psi$  such that the sampling problem at certain nonuniform point-sets  $\mathcal{X} \subset \mathbb{R}$  is well-defined, and additionally give sufficient conditions on a family of kernels  $(\phi_\alpha)_{\alpha \in A}$  such that one can recover  $f \in V(\psi, \mathcal{X})$  from interpolants  $I_\alpha f \in V(\phi_\alpha, \mathcal{Y})$  without necessarily requiring that  $\mathcal{X} = \mathcal{Y}$ . (Received September 14, 2016)

1125-41-1710 **Erik I Verriest\*** (erik.verriest@ece.gatech.edu), School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0250. *An extension of Lagrange-Burman Inversion with Application to Positioning via Delayed Signals.*

The problem of inversion of implicit equations  $\tau(t) = Bx(t) + Cx(t - \tau(t))$ , with  $\tau : \mathbb{R} \rightarrow \mathbb{R}^n$  known and  $x : \mathbb{R} \rightarrow \mathbb{R}^n$  is addressed. This generalizes the Lagrange-Burman inversion. With  $\dot{x} = Ax$ , conditions for observability, extending the PBH-tests are derived. Various scenarios for the problem of positioning of mobile units (MU) in the ocean are solved with finite speed ( $c$ ) signalling. It is assumed that the emitted signals provide delay information,  $\{\tau_i(t)\}$ , and that the positions  $\{R_i\}$  of stationary platforms are known. In a first scenario, each mobile unit (MU) transmits a signal, which is received after transmission delay by the stationary platforms. In the second scenario, the MU's are passive and only scatter sonar signals transmitted by the platforms. The problem lies in the fact that the transmission delay  $\tau$  depends on the line-of-sight distance the mobile units had  $\tau$  seconds ago. This leads to an implicit nonlinear relation between position and observed delays. For a single MU, these implicit equations take the form  $c\tau(t) = \|r(t - \tau(t)) - R_i\|$  and  $c\tau(t) = \|r(t) - R_i\| + \|r(t - \tau(t)) - R_i\|$  respectively, where  $r(t)$  is the position vector of the MU. (Received September 18, 2016)

1125-41-2633 **Peter M Luthy\*** (peter.luthy@mountsaintvincent.edu). *A structure theorem for the class of Parseval frame MRA wavelets for  $L^2(\mathbb{R})$ .*

A classical theorem attributed to Naimark states that Parseval frames are, more or less, orthogonal projections of orthonormal bases. More precisely, given a Parseval frame  $\mathcal{B}$  in a Hilbert space  $H$ , one can embed  $H$  in a larger Hilbert space  $K$  so that the image of  $\mathcal{B}$  is the orthogonal projection of an orthonormal basis for  $K$ . While this is a nice way to think about Parseval frames, the space  $K$  could be rather abstract. In this talk, we will discuss Parseval frame MRA wavelets for  $L^2(\mathbb{R})$  and see that a certain analogue of Naimark's theorem holds: the scaling functions for Parseval frame MRA wavelets are the orthogonal projections of "much nicer" such scaling functions. For the class of semiorthogonal Parseval frame MRAs, the "much nicer" scaling functions are simply scaling functions for orthonormal MRA wavelets. One major advantage that this analogue of Naimark's theorem has is that one never needs to leave  $L^2(\mathbb{R})$ . (Received September 20, 2016)

1125-41-2748 **Joel A Rosenfeld\*** (joelar@ufl.edu), **Spencer A Rosenfeld** and **Warren E Dixon**. *A Mesh-free Approach to Estimating the Fractional Laplacian via Radial Basis Functions.*

This paper investigates the use of radial basis function (RBF) interpolants to estimate a function's fractional Laplacian of a given order. The mesh-free approach yields an algorithm that can be implemented in high dimensional settings without adjustment. Moreover, the fractional Laplacian is defined in terms of the Fourier transform, and the symmetry of RBFs can be exploited to simplify the estimation problem. Convergence results are established for compactly supported RBFs when the function itself is compactly supported. Further results demonstrate convergence when a function is in the native space for a Wendland RBF (i.e. a Sobolev space) and satisfies a certain  $L^1$  condition. Numerical experiments demonstrate the developed method by estimating the fractional Laplacian of several functions and by numerically solving a fractional Poisson equation with extended Dirichlet condition. (Received September 20, 2016)

1125-41-2904 **Dylan Airey\*** (dylan.airey@utexas.edu), **Steve Jackson** and **Bill Mance**. *Some complexity results in the theory of normal numbers.*

Let  $\mathcal{N}(b)$  be the set of real numbers which are normal to base  $b$ . A well-known result of Ki and Linton is that  $\mathcal{N}(b)$  is  $\Pi_3^0$ -complete. We show that the set  $\mathcal{N}^\perp(b)$  of reals  $y$  which preserve  $\mathcal{N}(b)$  under addition is also  $\Pi_3^0$ -complete. We use the characterization of  $\mathcal{N}^\perp(b)$  given by Rauzy in terms of an entropy-like quantity called the noise. It follows from our results that no further characterization theorems could result in a still better bound on the complexity of  $\mathcal{N}^\perp(b)$ . We compute the exact descriptive complexity of other naturally occurring sets associated with noise. One of these is complete at the  $\Pi_4^0$  level. Finally, we get upper and lower bounds on the Hausdorff dimension of the level sets associated to the noise. (Received September 20, 2016)

1125-41-3034 **Huibin Cang** and **Stefano Marchesini\*** (smarchesini@lbl.gov), MS 2-400, 1, Cyclotron rd, Berkeley, CA 94720-819, and **Anne Sakdinawat**, Stanford Linear Accelerator Center, 2575 Sand Hill Rd, Menlo Park, CA 94025. *Lens design for X-ray imaging*. Preliminary report.

X-ray lenses fabricated by lithographic methods are the most popular imaging forming optics because of their versatility, cost, and efficiency. The binary lithographic pattern can be modified to generate structured illumination to improve throughput, resolution, dynamic range and depth of field information. Lithography generates binary structures, known as Fresnel zone plates, which diffract at different harmonic orders and are affected by aliasing artifacts. We investigate the nonlinear problem whereby a binary structure provides the optimal

illumination for a blind phase retrieval experiment with limited dynamic range detectors. (Received September 20, 2016)

1125-41-3095 **Hussain Y. Al-Hammali\*** ([alhammah@math.oregonstate.edu](mailto:alhammah@math.oregonstate.edu)), 1500 SW Jefferson St., Kidder Hall office 368, Corvallis, OR 97331. *The Reconstruction of The Band-limited Functions of Polynomial Growth with minimal Oversampling.*

In this talk, we will consider the reconstruction of functions that are band-limited in a distributional sense and have polynomial growth when restricted to the real line. A sampling series with an oversampling by a finite number of points will be derived. Oversampling by adding a few additional points can also be used to generate a faster decay of the sampling functions which causes more rapid convergence in the sampling series. Also, we will derive an approximate reconstruction by multiplying the function to be reconstructed by a smooth cut-off function and use of slight ratio-type oversampling. By ratio-type oversampling we mean a sampling set of higher density, e.g., using  $\lambda\mathbb{Z}$ , with  $0 < \lambda < 1$  instead of  $\mathbb{Z}$ . Finally, a numerical example will be given to compare three sampling series for a function in  $\mathcal{B}_\pi^\infty$ , the space of band-limited functions that are bounded when restricted to the real line. (Received September 21, 2016)

1125-41-3121 **Frank Stenger\*** ([sinc\\_f@msn.com](mailto:sinc_f@msn.com)), School of Computing, University of Utah, Salt Lake City, UT 84103. *Ultrasonic tomography inversion of the Helmholtz equation via indefinite convolution.*

A novel procedure is presented for obtaining the solution to the 3-d Helmholtz equation. The procedure uses novel indefinite convolution. It is extremely fast and accurate, and it enables an efficient method of inversion, for ultrasonic tomography. (Received September 21, 2016)

## 42 ► *Fourier analysis*

1125-42-66 **John J. Benedetto\*** ([jjb@math.umd.edu](mailto:jjb@math.umd.edu)) and **Weilin Li**. *Super-resolution by means of Beurling minimal extrapolation.* Preliminary report.

We address the super-resolution question: Given spectral data defined on a finite set of d-dimensional multi-integers; of all complex Radon measures on the d-dimensional torus, whose Fourier transform equals this data, does there exist exactly one with minimal total variation? We first note that this is a mathematical formulation of a large class of super-resolution problems that arise in image processing. We prove a theorem that has quantitative implications about the possibility and impossibility of constructing such a unique measure. Our method introduces the notion of an admissibility range that fundamentally connects Beurling's theory of minimal extrapolation with the Candes and Fernandez-Granda theory of super-resolution. The method is also well-suited for the construction of explicit examples. (Received July 04, 2016)

1125-42-114 **Michael T Lacey\*** ([lacey@math.gatech.edu](mailto:lacey@math.gatech.edu)), School of Mathematics, Georgia Tech, Atlanta, GA 30332. *Recent Results in Sparse Domination.* Preliminary report.

The Hilbert transform is a subtle, non-positive operator, which subtle non local effects. A Sparse operator is a positive operator, a sum of local averages. Surprising, the Hilbert transform, applied to a fixed function, can be dominated by a sparse operator, which depends upon the function. This surprising principle yields very deep information about the Hilbert transform. The principle has many extensions, and we will survey some of these, with an eye towards those topics which would have been very attractive to Cora Sadosky. (Received August 01, 2016)

1125-42-214 **Brian Preskitt, Aditya Viswanathan\*** ([aditya@math.msu.edu](mailto:aditya@math.msu.edu)), **Mark Iwen** and **Ryan Saab**. *Robust and Fast Phase Retrieval from Local Correlation Measurements.*

Certain imaging applications such as x-ray crystallography and ptychography require the recovery of a signal from phaseless (or magnitude-only) measurements - a problem commonly referred to as Phase Retrieval. This is a challenging problem since the phase encapsulates a significant amount of structure in the underlying signal. In this talk, we discuss a framework for solving the discrete phase retrieval problem using (in many cases, deterministic) local correlation measurements. We develop an essentially linear-time phase retrieval algorithm and present theoretical recovery guarantees as well as numerical results demonstrating its speed, accuracy and robustness. (Received August 12, 2016)

1125-42-224 **Matthew Fickus\*** ([matthew.fickus@afit.edu](mailto:matthew.fickus@afit.edu)), Department of Mathematics & Statistics, Air Force Institute of Technology, 2950 Hobson Way, Wright-Patterson AFB, OH 45433. *Some recent advances on equiangular tight frames.*

An equiangular tight frame (ETF) is a type of optimal packing of lines in Euclidean space. They arise in several applications, including compressed sensing and coding theory. ETFs are difficult to construct, and all known infinite families of them involve some type of combinatorial design. We discuss several contributions to the field we've made in the past year, including some new infinite families of ETFs related to balanced incomplete block designs. (Received August 15, 2016)

1125-42-286 **Albert Fannjiang\*** ([fannjiang@math.ucdavis.edu](mailto:fannjiang@math.ucdavis.edu)) and **Pengwen Chen.** *Fixed Point Algorithms for Phase Retrieval.*

We discuss the convergence properties of a general class of fixed point algorithms for phase retrieval and ptychography. In particular, we present the local geometric convergence theorem for algorithms related to the classical Douglas-Rachford algorithm and demonstrate the global nature of convergence in practice. (Received August 23, 2016)

1125-42-350 **Alex Iosevich\*** ([iosevich@gmail.com](mailto:iosevich@gmail.com)), 145 Dunrovin Lane, Rochester, NY 14618. *On some recent developments related to the Fuglede conjecture.*

We will describe some recent work on orthogonal exponential bases and Riesz bases in vector spaces over finite fields. (Received August 29, 2016)

1125-42-360 **Stefanie Petermichl\*** ([stefanie.petermichl@gmail.com](mailto:stefanie.petermichl@gmail.com)), Universite Paul Sabatier, 118 Route de Narbonne, 31062 Toulouse, France. *Higher order Journe commutators and multi-parameter BMO.*

We discuss an endpoint of work begun by Sarah Ferguson and Cora Sadosky - the characterisation of multi-parameter BMO spaces via boundedness of commutators.

The classical result by Nehari implies that the commutator of the Hilbert transform with symbol multiplication is bounded if and only if the symbol belongs to BMO. Already the real variable analog involving Riesz transforms required a different set of techniques (due to Coifman-Rochberg-Weiss). When passing to the two-parameter setting, natural extensions include Cotlar-Sadosky little BMO or Chang-Fefferman product BMO. The little BMO case corresponds to a commutator with tensor product Hilbert transforms (due to Ferguson-Sadosky using Toeplitz forms) while the product BMO case was an important open question settled by Ferguson-Lacey, involving so-called higher order commutators. Mixing these cases provides a characterisation of all multi-parameter BMO spaces. The Toeplitz forms in the real variable situation lead to a surprising difficulty, solved through Journe operators. (Received August 29, 2016)

1125-42-503 **Demetrio Labate\*** ([dlabate@math.uh.edu](mailto:dlabate@math.uh.edu)), University of Houston, Department of Mathematics, 651 Philip Guthrie Hoffman Hall, Houston, TX 77204-3008, and **Kanghui Guo** ([kanghuiguo@missouristate.edu](mailto:kanghuiguo@missouristate.edu)), Missouri State University, Department of Mathematics, Springfield, MO 65804. *Detection of singularities by discrete multiscale representations.*

One of the most remarkable properties of the continuous curvelet and shearlet transforms is their sensitivity to the directional regularity of functions and distributions. As a consequence of this property, these transforms can be used to characterize the geometry of edge singularities of functions and distributions by their asymptotic decay at fine scales. This ability is a major extension of the capability of the conventional continuous wavelet transform which can only describe pointwise regularity properties. However, while in the case of wavelets it is relatively easy to relate the asymptotic properties of the continuous transform to properties of discrete wavelet coefficients, this problem is surprisingly challenging in the case of discrete curvelets and shearlets. In this talk, we present novel non-asymptotic estimates showing that discrete shearlet and curvelet coefficients can detect, in a precise sense, the location and orientation of curvilinear edges. We show the connection and implications of this result to sparse approximation properties and other applications. (Received September 04, 2016)

1125-42-540 **Amin Boumenir\*** ([boumenir@westga.edu](mailto:boumenir@westga.edu)), Department of Mathematics, 1601 maple st, UWG, Carrollton, GA 30118. *Sampling for multi-spectral theory of Sturm-Liouville systems.*

We extend the sampling method, [2], which usually helps compute eigenvalues of one dimensional Sturm-Liouville operators, to deal with the computation of multi parameter eigenvalues of Sturm-Liouville systems. To do so, we use the 2-dimensional version of the Whitaker-Shannon-Kotelnikov sampling theorem [3], to find a representation

for eigencurves [1] whose intersections are precisely the eigenvalues. These are easily approximated by computing a finite square matrix of size  $n$  and we show that the truncation error is of order  $\ln(n)/\sqrt{\{n\}}$ , [1].

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2. A. Boumenir: Computing Eigenvalues of periodic Sturm-Liouville problems by the Shannon-Whittaker sampling theorem, *Math. Comp*, 68, (1999) 1057–1066
3. A. J. Jerri: The Shannon sampling theorem: Its various extensions and applications: A tutorial review. *Proceedings of the IEEE*, 65 (11) (1977), 1565–1596. (Received September 06, 2016)

1125-42-547 **Brian Simanek\*** ([brian\\_simanek@baylor.edu](mailto:brian_simanek@baylor.edu)), Baylor Math Department, One Bear Place #97328, Waco, TX 76798. *New Universality Results for Polynomial Reproducing Kernels.*

Given a positive, finite, and compactly supported measure on the complex plane with infinitely many points in its support, let  $K_n(x, y)$  denote the reproducing kernel for polynomials of degree at most  $n$  in the space  $L^2(\mu)$ . We are interested in understanding the behavior of  $d_n K_n(x + ac_n, x + bc_n)$  as  $n \rightarrow \infty$ , where  $\{c_n\}$  and  $\{d_n\}$  are appropriate sequences of real numbers,  $a$  and  $b$  are complex numbers, and  $x$  is in the support of the measure. Such asymptotics are known to exist for a wide variety of measures and in many cases the limit is stable under certain perturbations of the measure (hence the name *universality*). We will discuss new results that demonstrate the existence of this limit for new classes of measures, including measures on the unit circle that have a Fisher-Hartwig type singularity and area type measures on a certain disconnected polynomial lemniscate. (Received September 06, 2016)

1125-42-575 **Cong Q Hoang\*** ([cqhoang@crimson.ua.edu](mailto:cqhoang@crimson.ua.edu)) and **Kabe Moen** ([kabe.moen@ua.edu](mailto:kabe.moen@ua.edu)). *Muckenhoupt-Wheeden conjectures for sparse operators.*

We provide an example of a pair of weights  $(u, v)$  for which the Hardy-Littlewood maximal function is bounded from  $L^p(v)$  to  $L^p(u)$  and from  $L^{p'}(u^{1-p'})$  to  $L^{p'}(v^{1-p'})$  while a dyadic sparse operator is not bounded on the same domain and range. Our construction also provides an example of a single weight for which the weak-type endpoint does not hold for sparse operators. (Received September 06, 2016)

1125-42-615 **Kasso A Okoudjou\***, 2111 Mathematics Building, College Park, MD 20742. *The HRT conjecture for real-valued functions.* Preliminary report.

In this talk, we introduce an extension principle to investigate the Heil-Ramanathan-Topiwala (HRT) conjecture. More specifically, knowing that the Conjecture holds for a given function  $g \in L^2(\mathbb{R})$  and a given set  $\Lambda = \{(a_k, b_k)\}_{k=1}^N \subset \mathbb{R}^2$ , we seek the set of all (new) points  $(a, b) \in \mathbb{R}^2 \setminus \Lambda$  such that the conjecture remains true for the same function  $g$  and the new set  $\Lambda_1 = \Lambda \cup \{(a, b)\}$ . We demonstrate the merit of this approach by giving a new proof of the HRT conjecture for 3 points and by establishing the conjecture for certain sets of 4 points when the generator  $g$  is real-valued. (Received September 07, 2016)

1125-42-620 **Alex Iosevich\*** ([iosevich@math.rochester.edu](mailto:iosevich@math.rochester.edu)), 145 Dunrovin Ln, Rochester, NY 14618. *Falconer conjecture and its variants.*

We shall discuss some recent developments pertaining to the Falconer distance conjecture and its variants. (Received September 08, 2016)

1125-42-894 **Rongrong Wang\*** ([rongwang@math.ubc.ca](mailto:rongwang@math.ubc.ca)), 1984 mathematics road, Vancouver, BC V6T1Z2, Canada. *Frequency extrapolation from band-limited data.*

Frequency extrapolation from band-limited data is an important problem in the field of exploration seismology. Due to physical constraints, the frequency range of seismic data are approximately 5Hz-180Hz depending on the specific setup of the survey. This band is often considered severely limiting in both the low and high frequency extent for the purposes of deducing large-scale structures as well as fine features of the underlying subterranean geology. In particular, the missing low frequencies from 0Hz to 5Hz are known to be extremely essential for an accurate inversion procedure. In this talk, we study possible approaches to solve the frequency extrapolation problem from mid-band data. In particular, we provide conditions as well as theoretical guarantees for an accurate spectrum reconstruction. The results are further supported by both stylized numerical experiments as well as a simulation for seismic inversion. (Received September 13, 2016)

1125-42-988 **Darrin Speegle\*** ([speegled@slu.edu](mailto:speegled@slu.edu)) and **Robert Steward**. *Tiling the line by affine shifts of a prototile.* Preliminary report.

We present conditions on a set  $\Gamma = \{(x(t), y(t))\}$ , where  $x$  and  $y$  are continuous, such that there exists a sampling  $\{t_n\}_{n=1}^\infty$  and a set  $E$  such that

$$\{x(t_n)^{-1}(E + y(t_n)) : 1 \leq n < \infty\}$$

is a measurable tiling of the line. The relationship between this problem and the existence of wave packet frames will also be discussed. (Received September 13, 2016)

1125-42-1004 **Azita Mayeli\*** ([amayeli@gc.cuny.edu](mailto:amayeli@gc.cuny.edu)). *Fuglede Conjecture in finite vector spaces over prime fields.*

The equivalence relation between tiling and spectral property of a set has its root in the Fuglede Conjecture a.k.a Spectral Set Conjecture in  $\mathbb{R}^d$ ,  $d \geq 1$ . In 1974, Fuglede asserted that a Lebesgue measurable set  $\Omega \subset \mathbb{R}^d$ , with positive and finite measure, tiles  $\mathbb{R}^d$  by its translations if and only if the Hilbert space  $L^2(\Omega)$  possesses an orthogonal basis of exponentials. A variety of results were proved for establishing connection between tiling and spectral property for some special cases of  $\Omega \subset \mathbb{R}^d$ . However, the conjecture is false in general for dimensions 3 and higher.

Let  $E \subseteq \mathbb{F}_q^2$ , where  $q$  is a prime and  $\mathbb{F}_q^2$  is the vector space over the prime field  $\mathbb{F}_q$ . In this talk we shall show that every function  $f : E \rightarrow \mathbb{C}$  can be expanded as a linear combination of characters orthogonal in  $L^2(E)$  if and only if  $E$  tiles  $\mathbb{F}_q^2$  by translations. In other words, we prove that the Fuglede Conjecture holds for  $\mathbb{F}_q^2$ . We will also discuss the background and the history of this problem in a variety of settings. (Received September 13, 2016)

1125-42-1060 **Akram Aldroubi\*** ([akram.aldroubi@vanderbilt.edu](mailto:akram.aldroubi@vanderbilt.edu)), Vanderbilt University, SC 1520, Nashville, TN 37240, and **Ilya Krishtal** and **Sui Tang**. *Reconstruction of signals from phaseless samples of evolutionary systems.*

Assume  $f \in \mathbb{R}^n$  is an unknown function evolving under the action of an operator  $A$  on  $\mathbb{R}^n$  such that at time  $n$  the signal evolves to  $f_n = A^n f$ . Let  $\Omega \subset \{1, 2, \dots, n\}$ . We consider the problem of finding conditions on  $A, \Omega$  and  $L_i$  such that any  $f \in \mathbb{R}^n$  can be uniquely determined up to a sign from the unsigned samples

$$Y = \{|f(i)|, \dots, |A^{L_i-1} f(i)| : i \in \Omega\}.$$

(Received September 14, 2016)

1125-42-1221 **Brett D. Wick\*** ([wick@math.wustl.edu](mailto:wick@math.wustl.edu)), Department of Mathematics, Washington University - St. Louis, St. Louis, MO 63130. *Commutators and BMO Associated to the Bessel Operator.*

The Bessel operator is the differential operator  $-\frac{d^2}{dx^2} - \frac{2\lambda}{x} \frac{d}{dx}$ . Akin to how the Riesz transforms and *BMO* are connected to the usual Laplacian, there are analogous operators and function spaces associated to the Bessel operator. In this talk we will discuss some results connected to commutators related to the Riesz transform associated to the Bessel operator and the characterization of boundedness of these commutators in terms of *BMO* functions associated to the Bessel operator.

This talk is based on joint work with Ji Li, Xuan Duong, Dongyong Yang, and Yumeng Ou. (Received September 15, 2016)

1125-42-1235 **John E Herr\*** ([jeherr@butler.edu](mailto:jeherr@butler.edu)). *de Branges-Rovnyak Spaces and the Kaczmarz Algorithm.*

We demonstrate that for  $b$  an inner function and  $\mathcal{H}(b)$  its associated de Branges-Rovnyak space, the Taylor series of functions in  $\mathcal{H}(b)$  come precisely from applying the Kaczmarz algorithm to their  $L^2(\mu)$  boundary functions, where  $\mu$  is the singular measure associated to  $b$  via the Herglotz representation theorem. In particular, we demonstrate that the normalized Cauchy transform from  $L^2(\mu)$  to  $\mathcal{H}(b)$  can be expressed using a Parseval frame in  $L^2(\mu)$ . (Received September 15, 2016)

1125-42-1292 **Alexander M Powell\*** ([alexander.m.powell@vanderbilt.edu](mailto:alexander.m.powell@vanderbilt.edu)) and **Xuemei Chen**. *Error moments and optimal sampling distributions for the randomized Kaczmarz algorithm.*

The randomized Kaczmarz algorithm is a simple iterative method for recovering a signal from a collection of linear measurements. We analyze error moments of the Kaczmarz algorithm when linear measurements are made using a random frame or random fusion frame, and we address the question of finding optimal sampling distributions. (Received September 16, 2016)

1125-42-1326 **Calvin Hotchkiss\*** ([hotchkis@iastate.edu](mailto:hotchkis@iastate.edu)) and **Eric S Weber**. *Fourier Bases on the "Skewed Sierpinski Gasket".*

We consider a certain iterated function system, whose invariant set is a skewed Sierpinski gasket,  $\mathcal{S}$ . The set  $\mathcal{S}$  has the standard middle-thirds Cantor set as its trace on both the  $X$  and  $Y$  axes. We show the existence of several sequences of exponentials which form an orthonormal basis on  $L^2(\mathcal{S})$ . Results on  $\mathcal{S}$  cast light on the problem of finding a Fourier frame for that Cantor set. (Received September 16, 2016)

1125-42-1477 **Willi Freeden\*** ([freeden@rhrk.uni-kl.de](mailto:freeden@rhrk.uni-kl.de)), University of Kaiserslautern, Geomathematics Group, MPI-Building, Paul-Ehrlich-Strasse 26, 67663 Kaiserslautern, Palatinate, Germany, and **M Zuhair Nashed** ([m.nashed@ucf.edu](mailto:m.nashed@ucf.edu)), Department of Mathematics, University of Central Florida, Orlando, FL 32816-1. *Shannon-Type Sampling, Paley-Wiener Reproducing Hilbert Space Structure, and Multi-Variate Antenna Problem.*

The famous Shannon sampling theorem gives an answer to the question of how a one-dimensional time-dependent bandlimited signal can be reconstructed from discrete values in lattice points. In this talk, we are concerned with multi-variate space-dependent Shannon-type sampling theorems. An answer is given to the problem of how a signal bandlimited to a regular region allows a reconstruction from discrete values in the lattice points of a (general) multi-dimensional lattice. Explicit characterizations of over- and undersampling are studied, thereby specifying not only the occurrence, but also the type of aliasing in a thorough mathematical description. As a consequence, multi-variate Shannon-type lattice sampling becomes available in a standard and Gaussian summability context. Some aspects of constructive approximation in a resulting Paley–Wiener framework are indicated, such as the recovery of a finite set of lost samples, the Paley-Wiener reproducing Hilbert space context of spline interpolation. Finally, a solution of the inverse multivariate antenna problem is investigated within the Paley-Wiener framework. (Received September 17, 2016)

1125-42-1680 **Naomi Feldheim, Ohad Feldheim and Shahaf Nitzan\*** ([shahaf.nitzan@math.gatech.edu](mailto:shahaf.nitzan@math.gatech.edu)). *Persistence as a spectral property.*

A Gaussian stationary sequence is a random function  $f : \mathbb{Z} \mapsto \mathbb{R}$ , for which any vector  $(f(x_1), \dots, f(x_n))$  has a centered multi-normal distribution and whose distribution is invariant to shifts. Persistence is the event of such a random function to remain positive on a long interval  $[0, N]$ . Estimating the probability of this event has important implications in engineering, physics, and probability. However, though active efforts to understand persistence were made in the last 50 years, until recently, only specific examples and very general bounds were obtained. In the last few years, a new point of view simplifies the study of persistence, namely - relating it to the spectral measure of the process. In this work we use this point of view to develop new spectral and analytical methods in order to study the persistence in cases where the spectral measure is 'small', or 'big', near zero. (Received September 18, 2016)

1125-42-2340 **Azita Mayeli\*** ([amayeli@gc.cuny.edu](mailto:amayeli@gc.cuny.edu)). *Wavelet Sets in Vector Spaces over Cyclic Groups of Prime Order.*

Let  $q$  be an odd prime and  $\mathbb{F}_q^d$ ,  $d \geq 1$ , be the vector space over the cyclic space of prime order,  $\mathbb{F}_q$ . In this talk we shall introduce tight wavelet sets in  $\mathbb{F}_q^d$  and characterize them in terms of multiplicative and translation tilings. Then we shall use a number theoretic approach to provide a constructive method for obtaining such sets when  $d \geq 2$  and  $q \equiv 3 \pmod{4}$ . (Received September 20, 2016)

1125-42-2406 **Jade Larriva-Latt, Angela Morrison\*** ([arm14@albion.edu](mailto:arm14@albion.edu)), **Alison Radgowski** and **Joseph Tobin**. *Living on the Edge: Improved Reconstruction of Fourier Series using Jump Information with Applications to MRI.*

Magnetic Resonance Imaging (MRI) is a critical non-invasive tool used by medical professionals to take images of the human body. MRI machines work by returning the Fourier Coefficients corresponding to the patient being imaged which are then used to reconstruct a picture of the patient. The imaging process is error prone due to, e.g., instrumentation limitations as well as motion by the patient during the scanning process. Additionally, due to the presence of multiple tissues and organs in patients' bodies, the underlying images tend to have a piecewise-smooth structure, resulting in imaging errors that distort the boundaries between tissues due to the Gibbs Phenomenon. We propose a highly effective method of detecting edges from Fourier data in order to produce more accurate reconstructions by mitigating Gibbs artifacts. We describe several advanced sampling and reconstruction methods supported by numerical results that produce quicker and more accurate reconstructions relative to the modern standard. (Received September 20, 2016)

1125-42-2763 **Lechao Xiao\*** ([xle@math.upenn.edu](mailto:xle@math.upenn.edu)), 209 South 33rd Street, David Rittenhouse Lab., Philadelphia, PA 19104. *Endpoint estimates for one-dimensional oscillator integral operators.*

The one-dimensional oscillatory integral operator associated to the real analytic phase  $S$  is given by

$$T_\lambda f(x) = \int_{-\infty}^{\infty} e^{i\lambda S(x,y)} \chi(x,y) f(y) dy.$$

In this talk, we will provide a complete characterization for the mapping properties of  $T_\lambda$  on  $L^p(\mathbb{R})$  in terms of the Newton polyhedron of  $S$ . More precisely,  $\|T_\lambda\|_p \lesssim |\lambda|^{-\alpha} \|f\|_p$  holds for some  $\alpha > 0$  if and only if the point

$(\frac{1}{\alpha p}, \frac{1}{\alpha p'})$  lies in the reduced Newton polyhedron of  $S$ , and this estimate is sharp if and only if it lies on the reduced Newton diagram. (Received September 20, 2016)

## 43 ► *Abstract harmonic analysis*

1125-43-69 **Travis D Andrews, John J Benedetto\*** (jbb@math.umd.edu) and **Jeffrey J Donatelli**. *Group frames and the theory of frame multiplication*. Preliminary report.

We formulate a mathematical theory of frame multiplication, in which two essential algebraic operations can be made compatible in a natural way. The motivation comes from our approach to defining vector-valued ambiguity functions, that in turn are formulated to provide realistic modelling of multi-sensor environments in which a useful time-frequency analysis is required.

The technology underlying frame multiplication theory is the theory of frames, short time Fourier transforms (STFTs), and the representation theory of finite groups. The functions whose ambiguity function we wish to define have a given finite group  $G$  as their domain, e.g.,  $G = \mathbb{Z}/N\mathbb{Z}$ . Our results have the following form: i. if frame multiplication exists in the context of the aforementioned operations, then the vector-valued ambiguity function is well-defined; ii. frame multiplication exists if and only if the finite frames that arise in the theory are of a certain type, e.g., harmonic frames or, more generally, group frames. (Received July 08, 2016)

1125-43-542 **Azita Mayeli\*** (amayeli@gc.cuny.edu), NYC, NY. *Structure of cyclic shift-invariant spaces on the Heisenberg group*.

In signal processing and applications, signals and images are considered as functions of finite energy in a subspaces which are shift-invariant. The structure of these spaces and their bases has been studied using range function approach by M. Bownik in the Euclidean frame theory and by the author and her collaborators in non-commutative settings. An alternative approach using *bracket map* for the study of the cyclic shift-invariant (or principle shift-invariant) subspaces and their bases on locally compact abelian groups has been introduced by E. Hernandez, H. Sikic, G. Weiss and E. Wilson. In this talk, we shall introduce an extension of the bracket map for the non-commutative Lie group, the Heisenberg group, and use it to establish conditions on the generator of a cyclic space under which such space has an orthonormal basis, Riesz basis or a frame. As an application of our theory we show how the translation frames for cyclic subspaces on the Heisenberg group can be associated to orthonormal Gabor systems on the Euclidean space. We will conclude this talk by addressing few interesting questions with regard to the properties of the generators in terms of the bracket map. (Received September 06, 2016)

1125-43-601 **Sigurdur Helgason\*** (helgason@mit.edu). *Eisenstein Integrals and C functions*.

In Harish-Chandra's work on semisimple  $G$  his  $c$ -function expressed the density of the Plancherel formula for the spherical transform on  $G$ . Two individual factors in the formula for  $c$  express analytic results for the symmetric space  $G/K$  and the horocycle space  $G/MN$ . Each irreducible representation of  $K$  gives rise to an Eisenstein Integral and accompanying  $C$  functions. Here we describe some results and conjectures for these  $C$  and relate these to work of L. Cohn. (Received September 15, 2016)

1125-43-687 **Peter G. Casazza\*** (casazzap@missouri.edu), 814 Hulen Drive, Columbia, MO 65203, **John Haas** (terraformthedreamscape@gmail.com), Department of Mathematics, University of Missouri, Columbia, MO 65211, **Amineh Farzannia** (afwwc@mail.missouri.edu), Department of Mathematics, University of Missouri, Columbia, MO 65211, and **Tin T. Tran** (tttr29@mail.missouri.edu), Department of Mathematics, University of Missouri, Columbia, MO 65211. *Biangular Harmonic Frames*.

Equiangular tight frames (ETFs) and biangular tight frames (BTFs) - sets of unit vectors with basis-like properties whose pairwise absolute inner products admit exactly one or two values, respectively - are useful for a wide range of applications. It is known that harmonic ETFs are characterized by combinatorial objects called difference sets.

This work is dedicated to the study of the underlying combinatorial structures that generate harmonic BTFs. We show that if a harmonic frame is generated by a divisible difference set, a partial difference set or by a special structure with certain Gauss summing properties - all three of which are generalizations of difference sets that fall under the umbrella term "bidifference sets" - then it is either a BTF or an ETF. However, we also show that the relationship between harmonic BTFs and bidifference sets is not as straightforward as the correspondence between harmonic ETFs and difference sets, as there are examples of bidifference sets that do not generate harmonic BTFs. In addition, we study another class of combinatorial structures, the nested divisible

difference sets, which yields an example of a harmonic BTF that is not generated by a bidifference set. (Received September 09, 2016)

1125-43-694 **Fulton B Gonzalez\*** ([fulton.gonzalez@tufts.edu](mailto:fulton.gonzalez@tufts.edu)), Department of Mathematics, Tufts University, Medford, MA 02155, **Jens Christensen** ([jchristensen@colgate.edu](mailto:jchristensen@colgate.edu)), Department of Mathematics, Colgate University, Hamilton, NY 13346, and **Tomoyuki Kakehi** ([kakehi@math.okayama-u.ac.jp](mailto:kakehi@math.okayama-u.ac.jp)), Department of Mathematics, Faculty of Science, Okayama University, 3-1-1 Tsushima-naka, Kita-ku, Okayama, 700-8530, Japan. *Surjectivity of Mean Value Operators on Noncompact Symmetric Spaces.*

Let  $X = G/K$  be a symmetric space of the non-compact type. For  $f \in \mathcal{E}(X)$  and a fixed point  $y \in X$ , the mean value  $M^y f$  is the function on  $X$  given by

$$M^y f(gK) = \int_K f(gk \cdot y) dk \quad g \in G.$$

We show that the mean value operator  $f \mapsto M^y f$  is surjective on  $\mathcal{E}(X)$  if  $X$  is either complex or of rank one. For general higher rank symmetric spaces  $X$  we show that  $M^y$  is surjective if  $y$  is  $K$ -conjugate to a point in an appropriate Weyl subchamber. For a fixed  $K$ -invariant distribution  $\mu \in \mathcal{E}'(X)$  we also show that Ehrenpreis' slow decrease condition on its spherical Fourier transform is a sufficient condition for convolution with  $\mu$  to be surjective on  $\mathcal{E}(X)$ . (Received September 09, 2016)

1125-43-736 **Tomasz Przebinda\*** ([tprzebinda@gmail.com](mailto:tprzebinda@gmail.com)), Department of Mathematics, University of Oklahoma, Norman, OK 73019. *The character and wave front set correspondence in the stable range.*

We relate the distribution characters and the wave front sets of unitary representation for real reductive dual pairs of type I in the stable range. (Received September 10, 2016)

1125-43-1132 **E. K. Narayanan** ([ek.narayanan@gmail.com](mailto:ek.narayanan@gmail.com)), Department of Mathematics, Indian Institute of Science, Bangalore, 560012, India, and **Angela Pasquale\*** ([angela.pasquale@univ-lorraine.fr](mailto:angela.pasquale@univ-lorraine.fr)), Institut Elie Cartan de Lorraine (IECL), UMR CNRS 7502, Bâtiment A, Ile du Saulcy, Université de Lorraine, Metz, France. *Radial parts of differential operators and a one-parameter family of hypergeometric functions of type BC.*

Heckman-Opdam's hypergeometric functions associated with root systems generalize the restrictions to a maximal Cartan subspace of Harish-Chandra's spherical functions on Riemannian symmetric spaces. Likewise, the restrictions of Harish-Chandra's spherical functions on line bundles over Hermitian symmetric spaces admit generalizations to one-parameter families of hypergeometric functions on root systems of type BC. In this talk, we revisit and complete the work on these functions independently started by Shimeno and Heckman in the 90ies and continued by Ho and Ólafsson more recently: the commutative family of differential operators to which they are associated, the corresponding family of Dunkl-Cherednik operators, asymptotics, estimates and boundedness properties. (Received September 15, 2016)

1125-43-1170 **Mishko Mitkovski\*** ([mmitkov@clemson.edu](mailto:mmitkov@clemson.edu)). *Toeplitz operators and sampling sets.* Preliminary report.

It is well known that Toeplitz operators can be used to describe sampling and interpolation sets in the Paley-Wiener space. In this talk I will show how general Toeplitz operators can be used to study sampling sets in a much wider class of function spaces. (Received September 15, 2016)

1125-43-1297 **Irina Holmes\*** ([irina.holmes@email.wustl.edu](mailto:irina.holmes@email.wustl.edu)), St. Louis, MO, and **Brett D. Wick** and **Stefanie Petermichl**. *Weighted Inequalities for Little BMO.* Preliminary report.

We discuss two-weight inequalities for commutators with the double Hilbert transform in the multiparameter context of little BMO. Specifically, we discuss  $A_p$  weighted bounds for  $[b, H_1 H_2]$ , where  $b$  is a function in little BMO and  $H_i$  is the Hilbert transform in each variable. (Received September 16, 2016)

1125-43-1340 **James Rosado\*** ([rosado42@rowan.edu](mailto:rosado42@rowan.edu)), Department of Mathematics, Rowan University, 201 Mullica Hill Road, Glassboro, NJ 08028, **Hieu Nguyen** ([nguyen@rowan.edu](mailto:nguyen@rowan.edu)), Department of Mathematics, Rowan University, 201 Mullica Hill Road, Glassboro, NJ 08028, and **Lei Cao** ([leicaomath@gmail.com](mailto:leicaomath@gmail.com)), Department of Mathematics, Georgian Court University, 900 Lakewood Avenue, Lakewood, NJ 08701. *Partitions of Equiangular Tight Frames.*

We prove an algorithm to partition a special class of equiangular tight frames (ETFs) according to a theorem by Marcus, Spielman, and Srivastava (MSS), which as a corollary yields a positive solution to the Kadison-Singer

problem. The proof of our algorithm also leads to a refinement of the bound described by the MSS theorem for those ETFs generated by recursive skew-symmetric conference matrices. (Received September 16, 2016)

1125-43-1832 **Jeff Geronimo\*** ([geronimo@math.gatech.edu](mailto:geronimo@math.gatech.edu)), School of Mathematics, Georgia Tech, Atlanta, GA 30332. *Bivariate Bernstein-Szego measures on the Square*. Preliminary report. I will discuss various conditions that imply when a non-degenerate bivariate measure supported on the square is absolutely continuous with respect to Lebesgue measure with a density that is the reciprocal of a positive polynomial. Such measures are called Bernstein-Szego measures. This is joint work with Plamen Iliev. (Received September 19, 2016)

1125-43-2576 **Joseph W Iverson** ([jiverson@math.umd.edu](mailto:jiverson@math.umd.edu)), **John D Jasper\*** ([john.jasper@uc.edu](mailto:john.jasper@uc.edu)) and **Dustin G Mixon** ([dustin.mixon@afit.edu](mailto:dustin.mixon@afit.edu)). *Equiangular tight frames from association schemes*.

An equiangular tight frame (ETF) is a set of unit vectors whose coherence achieves the Welch bound. Though they arise in many applications, there are only a few known methods for constructing ETFs. One of the most popular classes of ETFs, called harmonic ETFs, is constructed using the structure of finite abelian groups. In this talk we will discuss a broad generalization of harmonic ETFs. This generalization allows us to construct ETFs using many different structures in the place of abelian groups, including nonabelian groups, Gelfand pairs of finite groups, and more. (Received September 20, 2016)

1125-43-2694 **Ishwari J Kunwar\*** ([ikunwar3@math.gatech.edu](mailto:ikunwar3@math.gatech.edu)). *Weighted estimates for multilinear dyadic operators and their commutators*.

Multilinear dyadic paraproducts and Haar multipliers, which arise naturally through the decomposition of the pointwise product of two or more functions, serve as the discrete dyadic models of multilinear Calderón-Zygmund operators. In this talk, I will present weighted estimates for these multilinear dyadic operators, and their commutators with dyadic BMO functions. (Received September 20, 2016)

## 44 ► *Integral transforms, operational calculus*

1125-44-108 **Boris Rubin\*** ([borisr@math.lsu.edu](mailto:borisr@math.lsu.edu)), Department of Mathematics, 303 Lockett Hall, Louisiana State University, Baton Rouge, LA 70803. *New Inversion Formulas for the Horospherical Transform*.

We obtain new inversion formulas of two kinds for the horospherical transform in the real hyperbolic space. This transform is a hyperbolic analogue of the classical hyperplane Radon transform. The formulas of the first kind employ mean value operators and are applicable to functions in Lebesgue spaces. The formulas of the second kind work on smooth functions and rely on the properties of hyperbolic potentials. The latter can be explicitly inverted by polynomials of the Beltrami-Laplace operator. (Received July 29, 2016)

1125-44-141 **Fatma Terzioglu\*** ([fatma@math.tamu.edu](mailto:fatma@math.tamu.edu)) and **Peter Kuchment** ([kuchment@math.tamu.edu](mailto:kuchment@math.tamu.edu)). *Image Reconstruction from Compton Camera Data*.

In this presentation, we address analytically and numerically the inversion of the integral transform (*cone* or *Compton* transform) that maps a function on  $\mathbb{R}^n$  to its integrals over conical surfaces. It arises in a variety of imaging techniques, e.g. in astronomy, optical imaging, and homeland security imaging, especially when the so called Compton cameras are involved. We present several inversion formulas for the cone transform and the results of their numerical implementation in two and three dimensions. (Received September 01, 2016)

1125-44-1008 **Otmar Scherzer\*** ([otmar.scherzer@univie.ac.at](mailto:otmar.scherzer@univie.ac.at)), Computational Science Center, University of Vienna, Oskar-Morgenstern Platz 1, 1090 Vienna, Austria. *Parameter Estimation in Optical Coherence Tomography*. Preliminary report.

Optical Coherence Tomography (OCT) is an imaging technique producing high-resolution images of the inner structure of biological tissues. OCT is based on Low Coherence Interferometry (LCI) considering also the coherence properties of light. Standard OCT operates using broadband and continuous wave light and the images are obtained by measuring the time delay and the intensity of backscattered light from the sample.

We consider the inverse scattering problem of OCT to reconstruct the electric susceptibility. The problem is formulated in three dimensions and we approximate the scattered field as a solution of the Maxwell's equations under the second-order Born approximation, considering that the detection point is far enough from the sample. (Received September 14, 2016)

1125-44-1013 **Loc Xuan Tran\*** ([loc.tran@epfl.ch](mailto:loc.tran@epfl.ch)), EPFL SB MATHAA CAMA, MA C1 632 (Bâtiment MA), Station 8, CH-1015 Lausanne, Switzerland, and **Minh Hoai Nguyen** ([hoai-minh.nguyen@epfl.ch](mailto:hoai-minh.nguyen@epfl.ch)), EPFL SB MATHAA CAMA, MA B2 535 (Bâtiment MA), Station 8, 1015 Lausanne, Switzerland. *Approximate Cloaking for Maxwell equations via Transformation Optics: Non-resonant and Resonant Phenomena*.

In this talk, we discuss the approximate cloaking for Maxwell equations via transformation optics using the approximate scheme in the spirit of the one introduced by Kohn et al. (Inverse Problems 2008) in the quasistatic acoustic setting. Hence no lossy layer is required in the construction of the cloaking device. On one hand we show that, in the non-resonant case, cloaking is achieved and the energy of the field is finite. On another hand, in the resonant case, we reveals that cloaking might not be achieved and the energy can go to infinity. Our work extends the one of Nguyen (CPAM 2012) where the acoustic setting was investigated. (Received September 14, 2016)

1125-44-1851 **Lance Nielsen\*** ([lnielsen@creighton.edu](mailto:lnielsen@creighton.edu)), Department of Mathematics, Creighton University, Omaha, NE 68178. *Combining Continuous and Discrete Phenomena in Feynman's Operational Calculus in the Presence of a  $(C_0)$  Semigroup: Feynman-Kac Formulas with Lebesgue-Stieltjes Measures*. Preliminary report.

Using an evolution equation for Feynman's operational calculus with time-ordering measures having non-zero discrete parts, we illustrate some "Feynman-Kac formulas with Lebesgue-Stieltjes measures". These formulas arise from a direct application of the evolution equation when certain choices for the generator of the  $(C_0)$  semigroup, the operators and the associated time-ordering measures are made. (Received September 19, 2016)

1125-44-2358 **Hua Chen\*** ([chenhua@udel.edu](mailto:chenhua@udel.edu)), Ewing 115, Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, and **Udita N. Katugampola** ([udita@udel.edu](mailto:udita@udel.edu)), Ewing 311, Department of Mathematical Sciences, University of Delaware, Newark, DE 19716.  *$\rho$ -Laplace and  $\rho$ -Fourier Transforms of Katugampola fractional operators and their Applications*.

The *Katugampola fractional integrals and derivatives* are generalizations of both the Riemann–Liouville (RL) and the Hadamard fractional differintegrals. In this talk, the Laplace and Fourier transforms of Katugampola fractional operators are derived. It shows that the Laplace and Fourier transforms of the Katugampola fractional differintegrals are also the generalizations of that of RL fractional differintegrals. Then, we present some illustrative examples that use the proposed Laplace and Fourier transform to solve differential equations involving Katugampola fractional operators. Finally, we show some interesting connections between the Laplace and Mellin transforms when applying the idea to derive the transforms of the Hadamard fractional differintegrals. The Sumudu transforms, a variation of Laplace transform, of the operators in question are also derived. (Received September 20, 2016)

## 45 ► *Integral equations*

1125-45-1026 **Volker Michel\***, Geomathematics Group, Department of Mathematics, University of Siegen, 57068 Siegen, Germany, and **Sarah Orzowski**, Geomathematics Group, Department of Mathematics, University of Siegen, 57068 Siegen, Germany. *Orthonormal basis systems on the ball and their role in inverse problems*.

In Earth sciences and in medical imaging, some tomographic inverse problems occur where the domain of the unknown function is a ball. Some of these inverse problems are represented by integral equations or at least include integral operators in their modelling. Since several regularization methods for such problems (including some more modern methods like a wavelet analysis or matching-pursuit-type greedy algorithms) profit from the knowledge of a singular value decomposition of the operator corresponding to the inverse problem, it is essential to study the possibility how orthonormal basis systems on the ball can be constructed.

There is, indeed, a notable variability in the construction of such systems, which opens the door to study different integral operators for functions on the ball. For this and other reasons, several authors have investigated orthogonal functions on the ball, from the early 20th century until present.

In this presentation, some of the known results on orthonormal bases on the ball are summarized and a recently published generalized concept for their use in the singular value decomposition of a class of integral operators is presented. The applications occur, for example, in the inversion of electric, gravitational, and magnetic field data. (Received September 14, 2016)

1125-45-1888 **Ricardo J Alonso\***, Rua Gen Venancio Flores 343, Apto 801, Rio de Janeiro, 22441-090, Brazil. *One dimensional dissipative Boltzmann equation.*

In this talk some essential questions about the dissipative Boltzmann equation are addressed. In particular we cover: (1) optimal cooling rate of the model, (2) existence and uniqueness of measure solutions and, (3) existence of non-trivial self-similar profile. Interesting open questions will be addressed at the end. (Received September 19, 2016)

1125-45-1901 **Minh-Binh Tran\***, University of Wisconsin, Madison, WI 53706. *Quantum kinetic problems.*

After the production of the first BECs, there has been an explosion of research on the kinetic theory associated to BECs. Later, Gardinier, Zoller and collaborators derived a Master Quantum Kinetic Equation for BECs and introduced the terminology "Quantum Kinetic Theory". In 2012, Reichl and collaborators made a breakthrough in discovering a new collision operator, which had been missing in the previous works. The talk is devoted to our results in collaboration with: Reichl on the derivation of quantum Boltzmann equations; Alonso and Gamba on the existence, uniqueness and propagation of moments for the quantum Boltzmann equation at very low temperature regime; and Escobedo on the convergence to equilibrium. (Received September 19, 2016)

1125-45-2366 **Irene M. Gamba\*** ([gamba@math.utexas.edu](mailto:gamba@math.utexas.edu)), Department of Mathematics and ICES, 1 University Station C1200, Austin, TX 78712. *The Dynamics of Particle Systems by Boltzmann Type Models.*

We will discuss recent progress in analytical and numerical methods covering from initial and boundary value problems, long time dynamics and stability issues. Interacting particle transport or kinetic collisional modeling was introduced in the last quarter of the nineteenth century by L. Boltzmann and J.C. Maxwell, independently, giving birth to the area of mathematical Statistical Mechanics and Thermodynamics. These types of evolution models concern a class of non-local, and non-linear integro-differential problems whose rigorous mathematical treatment and approximations are still emerging in comparison to classical non-linear PDE theory. Their applications range from rarefied elastic and inelastic gas dynamics including very low temperature regimes for quantum interactions, collisional plasmas and electron transport in nanostructures, to self-organized or social interacting dynamics. Based on a Markovian framework of birth and death processes, under the regime of molecular chaos propagation, their evolution is described by equations of non-linear collisional Boltzmann type. We will discuss recent progress in analytical and numerical methods covering issues on initial and boundary value problems, long time dynamics and stability.

(Received September 20, 2016)

## 46 ► *Functional analysis*

1125-46-3 **Tepper L. Gill\*** ([tgill@howard.edu](mailto:tgill@howard.edu)), DC. *Non-uniqueness of the dual of a Banach space and its application.*

Let  $\mathcal{B}$  be a Banach space with a shrinking S-basis. All studies assume the properties of  $\mathcal{B}^*$ , the dual, are unique. I show that  $\mathcal{B}^*$  has three different representations  $\mathcal{B}_d^*$ ,  $\mathcal{B}_h^*$  and  $\mathcal{B}_s^*$  bijectively related to  $\mathcal{B}$ .  $\mathcal{B}_d^*$  is the set of duality maps and  $\mathcal{B}_h^* \leftrightarrow \mathcal{B}$  is a (conjugate) isometric isomorphism. If the S-basis is not shrinking,  $\mathcal{B}_h^*$  and  $\mathcal{B}_s^*$  are subspaces of  $\mathcal{B}^*$ . Banach asked if a separable Banach space had a Schauder or S-Basis. Mazur proved that, every separable Banach space always contains an infinite dimensional subspace with a S-basis. In 1972 Enflo showed the existence of a separable Banach space without an S-basis. Our first application shows that, there exists Hilbert spaces with  $\mathcal{H}_1 \subset \mathcal{B} \subset \mathcal{H}_2$  as dense continuous embeddings. Markushevich gave a weaker definition of a basis (M-basis) and proved that every separable Banach space  $\mathcal{B}$  always has one. We give a positive answer to the norm one (M-basis) problem. (Received September 08, 2016)

1125-46-336 **Huichi Huang\*** ([hhuichi@hotmail.com](mailto:hhuichi@hotmail.com)), College of mathematics and statistics, Chongqing University, Chongqing, Chongqing 401331, Peoples Rep of China. *Mean ergodic theorem for discrete amenable quantum groups.*

We prove a mean ergodic theorem for discrete amenable groups and use it to prove a Wiener-type theorem for compact metrizable groups. (Received August 27, 2016)

1125-46-359 **Ronald G Douglas\*** ([rdouglas@math.tamu.edu](mailto:rdouglas@math.tamu.edu)), Department of Mathematics, MS 3368, College Station, TX 77843, and **Yi Wang** ([yiwang@math.tamu.edu](mailto:yiwang@math.tamu.edu)), Department of Mathematics, MS 3368, College Station, TX 77843. *Analytic Grothendieck-Riemann-Roch Theorem.*

A few years ago, a new approach to the Arveson conjecture was introduced independently by Englis-Eschmeier and Douglas-Tang-Yu exploiting the structure of the zero variety when it is "nice". Earlier results on the essential normality of the closure of a polynomial ideal in the Drury-Arveson space had utilized mostly operator-theoretic methods. In this talk we will discuss related approaches to more general ideals developing new machinery from complex harmonic analysis. In addition to results already obtained, we will speculate on what might come next. (Received August 29, 2016)

1125-46-363 **Noah Snyder\*** ([nsnyder@math.indiana.edu](mailto:nsnyder@math.indiana.edu)). *Trivalent categories.*

If  $N \subset M$  is a 2-supertransitive subfactor, then the bimodule  ${}_N M_N$  splits up as  ${}_N N_N \oplus {}_N X_N$  for some simple bimodule  $X$ . This bimodule  $X$  has some nice properties, for example the multiplication map on  $M$  restricts to a map  $X \otimes X \rightarrow X$ . I'll discuss work with Scott Morrison and Emily Peters where we classify what other ways you can have a bimodule with such a multiplication map which don't come from a subfactor. The techniques are planar algebraic and involve the discharging argument used in the proof of the 4-color theorem. (Received August 29, 2016)

1125-46-414 **Albert Jeu-Liang Sheu\*** ([asheu@ku.edu](mailto:asheu@ku.edu)). *Vector Bundles over Multi-pullback Quantum Projective Spaces.* Preliminary report.

By realizing  $C(\mathbb{P}^n(\mathcal{T}))$  and  $C(\mathbb{S}_H^{2n+1})$  as groupoid C\*-algebras for the quantum complex projective spaces  $\mathbb{P}^n(\mathcal{T})$  constructed from the multi-pullback quantum spheres  $\mathbb{S}_H^{2n+1}$  introduced by Hajac and collaborators, we study the classification of the unitary equivalence classes of projections or equivalently the isomorphism classes of finitely generated projective modules over  $C(\mathbb{P}^n(\mathcal{T}))$  and  $C(\mathbb{S}_H^{2n+1})$ , and identify those quantum principal  $U(1)$ -bundles introduced by Hajac and collaborators among the projections classified. (Received September 01, 2016)

1125-46-488 **Konrad Aguilar\*** ([konrad.aguilar@du.edu](mailto:konrad.aguilar@du.edu)). *Convergence of Quotients of AF Algebras in Quantum Propinquity by Convergence of Ideals.*

We provide conditions for when quotients of AF algebras are quasi-Leibniz quantum compact metric spaces building from our previous work with F. Latremoliere. Given a C\*-algebra, the ideal space may be equipped with natural topologies. Next, we impart criteria for when convergence of ideals of an AF algebra can provide convergence of quotients in quantum propinquity, while introducing a metric on the ideal space of a C\*-algebra. We then apply these findings to a certain class of ideals of the Boca-Mundici AF algebra by providing a continuous map from this class of ideals equipped with various topologies including the Jacobson and Fell topologies to the space of quotients with the quantum propinquity topology. Lastly, we introduce new Leibniz Lip-norms on any unital AF algebra motivated by Rieffel's work on Leibniz seminorms and best approximations. (Received September 05, 2016)

1125-46-570 **Jan Lang\*** ([lang@math.osu.edu](mailto:lang@math.osu.edu)), 100 Math Tower, 231 West 18th Avenue, Columbus, OH 43210. *Spectral theory on Banach spaces.*

We consider a compact linear map  $T$  acting between Banach spaces both of which are uniformly convex and uniformly smooth; it is supposed that  $T$  has trivial kernel and range dense in the target space. We will try to find conditions under which the action of  $T$  is given by a series. This provides a Banach-space version of the well-known Hilbert-space result of E. Schmidt and leads to study of non-linear differential equations (like  $p$ -Laplacian). (Received September 06, 2016)

1125-46-624 **Debashish Goswami\*** ([debashish\\_goswami@yahoo.co.in](mailto:debashish_goswami@yahoo.co.in)), Stat-Math Unit, Indian Statistical Institute, 203, B. T. Road, Kolkata, W. Bengal 700108, India. *Quantum symmetry of classical spaces.*

I give an overview of work done by myself and others on compact quantum group symmetries (co-action) of classical spaces including smooth manifolds and general compact metric spaces. In particular, I would like to briefly describe the non-existence of genuine quantum isometric (co)-actions on compact connected Riemannian manifolds and also (time permitting) the existence and examples of quantum isometry groups (which can be genuine quantum groups) of compact but possibly non-smooth metric spaces. (Received September 08, 2016)

1125-46-669 **Upasana Kashyap\*** ([upasana.kashyap@regiscollege.edu](mailto:upasana.kashyap@regiscollege.edu)), Regis College, 235 Wellesley St., Weston, MA 02493. *Weak\*-rigged modules over dual operator algebras and the Picard group.*

Weak\*-rigged modules are generalization of  $W^*$ -modules (Hilbert  $C^*$ -modules over von Neumann algebras which are self dual) over nonselfadjoint dual operator algebras. We discuss some new results about weak\*-rigged modules and their tensor products. We also discuss the Picard group of weak\*-closed subalgebras of a commutative algebra. For example, we compute the weak Picard group of  $H^\infty(D)$ , and prove that for a weak\*-closed function algebra  $A$ , the weak Picard group  $Pic_w(A)$  is a semidirect product of the automorphism group of  $A$ , and subgroup of  $Pic_w(A)$  consisting of symmetric equivalence bimodules. (Received September 08, 2016)

1125-46-682 **Clifford A Bearden\***, [cabearde@math.uh.edu](mailto:cabearde@math.uh.edu). *Hilbert  $C^*$ -modules over  $\Sigma^*$ -algebras.*  
A  $\Sigma^*$ -algebra is a concrete  $C^*$ -algebra that is sequentially closed in the weak operator topology. We study an appropriate class of  $C^*$ -modules over  $\Sigma^*$ -algebras analogous to the class of  $W^*$ -modules (selfdual  $C^*$ -modules over  $W^*$ -algebras), and we are able to obtain  $\Sigma^*$ -versions of virtually all the results in the basic theory of  $C^*$ - and  $W^*$ -modules. We also study  $\Sigma^*$ -analogues of Morita equivalence and make connections with operator space theory analogous to David Blecher's results for  $C^*$ -modules and  $W^*$ -modules. (Received September 09, 2016)

1125-46-794 **Barry Simon\*** ([bsimon@caltech.edu](mailto:bsimon@caltech.edu)), Mathematics 253-37, Caltech, Pasadena, CA 91125. *Spectral Theory Sum Rules, Meromorphic Herglotz Functions and Large Deviations.*  
After defining the spectral theory of orthogonal polynomials on the unit circle (OPUC) and real line (OPRL), I'll describe Verblunsky's version of Szegő's as a sum rule for OPUC and the Killip-Simon sum rule for OPRL and their spectral consequences. Next I'll explain the original proof of Killip-Simon using representation theorems for meromorphic Herglotz functions. Finally I'll focus on recent work of Gambo, Nagel and Rouault who obtain the sum rules using large deviations for random matrices. (Received September 11, 2016)

1125-46-796 **Piotr M. Soltan\*** ([piotr.soltan@fuw.edu.pl](mailto:piotr.soltan@fuw.edu.pl)). *Integrability of Homomorphisms of Locally Compact Quantum Groups.*  
I will explain what is meant by a homomorphism of locally compact quantum groups. One way of expressing this concept is through a certain kind of action of one locally compact quantum group on another. I will present some results characterizing integrability of such an action and its relation to closed quantum subgroups. (Received September 12, 2016)

1125-46-817 **Angshuman Bhattacharya\*** ([angshu@uga.edu](mailto:angshu@uga.edu)), Department of Mathematics, 526 BOYD, University of Georgia, Athens, GA 30602, and **Shuzhou Wang**. *Kirchberg's factorisation property for discrete quantum groups.*  
In 1964, Takesaki showed that the product of the left and right regular representations of the reduced  $C^*$ -algebra of the free group on two generators is not continuous. Wasserman showed that the product representation is continuous if the full group algebra is considered instead of the reduced one. Later Kirchberg isolated this property for discrete groups and called it the factorization property. Not much is known about such groups beyond Kirchberg's work.

In this talk I will report my joint work with Shuzhou Wang on Kirchberg's factorisation property for discrete quantum groups. (Received September 12, 2016)

1125-46-835 **Daniel Freeman\*** ([dfreema7@slu.edu](mailto:dfreema7@slu.edu)) and **Darrin Speegle**. *The discretization problem for continuous frames and coherent states.*  
There is a long history of creating frames by sampling coherent states and continuous frames. For instance, Gabor frames are formed by sampling the short time Fourier transform at a lattice. Continuous frames often arise naturally in mathematics and physics, but the sampled frames are usually more useful in application. Using the results of Marcus-Spielman-Srivastava in their solution of the Kadison-Singer problem, we prove that every bounded continuous frame may be sampled to obtain a frame. (Received September 12, 2016)

1125-46-977 **Armenak Petrosyan\*** ([armenak.petrosyan@vanderbilt.edu](mailto:armenak.petrosyan@vanderbilt.edu)), 1326 Stevenson Center Vanderbilt University, Nashville, TN 37240. *Iterative actions of operators on a system of vectors.*

We consider systems of vectors of the form

$$\{A^n g : g \in \mathcal{G}, n = 0, 1, 2, \dots\}$$

where  $A$  is a bounded operator on a Hilbert space  $\mathcal{H}$  and  $\mathcal{G}$  is a countable set of vectors in  $\mathcal{H}$ , and try to find conditions under which the system is a frame, basis, complete Bessel system etc. Some of the recent results and open questions will be discussed during the presentation. The problem takes its origins from a sampling theory

problem, called dynamical sampling problem, where the unknown signal is recovered from its spatio-temporal samples. (Received September 13, 2016)

1125-46-1021 **Michael Brannan\*** ([mbrannan@math.tamu.edu](mailto:mbrannan@math.tamu.edu)) and **Benoit Collins**. *The dual Temperley-Lieb basis and quantum group integrals.*

The Temperley-Lieb algebras form a remarkable family of finite dimensional algebras, arising in a variety of contexts, including quantum groups, subfactors, knot theory, and topological quantum computation. In this talk, I will discuss an approach to computing the dual basis associated to the usual planar diagram basis in the Temperley-Lieb algebra. Our approach to this problem is to interpret this in terms of an equivalent problem of integrating polynomial functions with respect to the Haar measure over certain quantum groups. Using this new perspective, we derive a Laurent series expansion for the dual basis coefficients, and using this expansion we answer a question of Jones on the non-vanishing of such coefficients. We also deduce the non-vanishing of the coefficients of the Jones-Wenzl projections as a particular consequence. (Received September 14, 2016)

1125-46-1085 **Terje Høim\*** ([thoim@fau.edu](mailto:thoim@fau.edu)), Wilkes Honors College, Florida Atlantic Univ., Jupiter, FL 33458 and Univ. of Tartu, Tartu, Estonia, and **D. A. Robbins** ([david.robbins@trincoll.edu](mailto:david.robbins@trincoll.edu)), Trinity College, Hartford, CT 06106. *Cover topologies and algebraic structures for some spaces of vector-valued functions.* Preliminary report.

Let  $X$  be a completely regular Hausdorff space, and  $\mathcal{D}$  a cover of  $X$  by  $C_b$ -embedded sets. Mati Abel, J. Arhippainen, and J. Kauppi in [Mediterr. J. Math. 7 (2010) 271-282] and [Cent. Eur. J. Math. 10 (2012), 1060-1066] describe the cover topology on  $C_b(X, \mathcal{D})$ , the space of continuous scalar-valued functions on  $X$  which are bounded on each  $D \in \mathcal{D}$ , and investigate the ideal and quotient structures of  $C_b(X, \mathcal{D})$ . We use ideas from the theory of bundles of topological vector spaces (in particular, bundles of Banach algebras), as found in e.g. G. Gierz [Lect. Notes Math. 955 (Springer-Verlag, 1982)] and the present authors' papers [Acta. Comment. Univ. Tartuensis Math. 14 (2010), 75-90] and [Quaest. Math. 34 (2011), 361-376] to define analogous cover topologies on section spaces of such bundles, and to discuss density properties, ideals, and quotients of such spaces. This work is also related to the authors' paper *Cover-strict topologies, ideals, and quotients for some spaces of vector-valued functions*, Banach J. Math. Anal. (to appear). (Received September 14, 2016)

1125-46-1090 **Yunxiang Ren\*** ([yunxiang.ren@vanderbilt.edu](mailto:yunxiang.ren@vanderbilt.edu)), 1104 18th Ave S, Apt 9, Nashville, TN 37212. *From skein theory to presentation for Thompson group.*

Jones introduced unitary representations of Thompson group  $F$  starting from a given subfactor planar algebra, and all unoriented links arise as matrix coefficients of these representations. Moreover, all oriented links arise as matrix coefficients of a subgroup  $\vec{F}$  which is the stabilizer of a certain vector. Later Golan and Sapir determined the subgroup  $\vec{F}$  and showed many interesting properties. In this paper, we investigate into a large class of groups which arises as subgroups of Thompson group  $F$  and reveal the relation between the skein theory of the subfactor planar algebra and the presentation of subgroup related to the corresponding unitary representation. Specifically, we answer a question by Jones about the 3-colorable subgroup. (Received September 14, 2016)

1125-46-1197 **Ryan K Tully-Doyle\*** ([ryan.tullydoyle@hamptonu.edu](mailto:ryan.tullydoyle@hamptonu.edu)), Department of Mathematics, Hampton University, Hampton, VA 23668. *Boundary behavior of analytic functions on the bidisk via Hilbert spaces.*

The Schur functions in two variables are the holomorphic maps from the complex unit bidisk  $\mathbb{D}^2$  into the unit disk  $\mathbb{D}$ . We characterize the differential behavior of a Schur function  $\varphi$  at a boundary singularity  $\tau \in \mathbb{T}^2$  in terms of the structure of an associated contractive operator  $Y$  on a Hilbert space  $\mathcal{H}$  arising from an Agler model. (Received September 15, 2016)

1125-46-1445 **Marat V. Markin\*** ([mmarkin@csufresno.edu](mailto:mmarkin@csufresno.edu)), 5245 N. Backer Avenue, M/S PB 108, Fresno, CA 93740-8001. *On the Smoothness of Weak Solutions of an Abstract Evolution Equation with a Scalar Type Spectral Operator.* Preliminary report.

For the abstract evolution equation

$$y'(t) = Ay(t), \quad t \geq 0,$$

with a *scalar type spectral operator*  $A$  in a complex Banach space, we find conditions on  $A$  *necessary and sufficient* for all *weak solutions* of the equation, which a priori need not be differentiable, to be *infinite differentiable* or *Gevrey ultradifferentiable* (in particular, *analytic* or *entire*) on  $[0, \infty)$  or  $(0, \infty)$  and analyze certain effects of their *smoothness improvement*.

Due to the *scalar type spectrality* of  $A$ , all the foregoing characterizations are formulated exclusively in terms of the location of its *spectrum* and generalize their analogues obtained earlier for a *normal operator*  $A$  in a complex Hilbert space. (Received September 16, 2016)

1125-46-1507 **Alexander A Katz\*** ([katza@stjohns.edu](mailto:katza@stjohns.edu)), St. John's University, SJC, Department of Math & CS, 8000 Utopia Parkway, SJH-334-G, Queens, NY 11439. *On bounded homomorphisms of real  $C^*$ -algebras into essentially real Banach algebras.*

We extend a result of Tomita (1960) from the case of complex  $C^*$ -algebras to real  $C^*$ -algebras by establishing that the image of a bounded homomorphism from a real  $C^*$ -algebra to essentially real Banach algebra is topologically isomorphic to a real  $C^*$ -algebra. (Received September 17, 2016)

1125-46-1519 **Mehrdad Kalantar\***, University of Houston, Houston, TX 77204, and **Pawel Kasprzak, Adam Skalski** and **Piotr Soltan**. *Open quantum subgroups of locally compact quantum groups, and induced representation.*

We introduce the notion of an open quantum subgroup of a locally compact quantum group and give several equivalent characterizations in terms of group-like projections, inclusions of quantum group  $C^*$ -algebras and properties of respective quantum homogenous spaces (joint work with Pawel Kasprzak and Adam Skalski). We then show the process of inducing unitary representations from open quantum subgroups of locally compact quantum groups, as defined by Kustermans, and as as defined by Vaes are both equivalent to the classical construction of Rieffel for inducing representations from  $C^*$ -subalgebras (joint work with Pawel Kasprzak, Adam Skalski and Piotr Soltan). (Received September 17, 2016)

1125-46-1616 **David Kerr\***, Department of Mathematics, Texas A&M University, College Station, TX 77843-3368. *Actions of amenable groups on the Cantor set and their crossed products.*

Recent historic progress in the classification theory for simple separable nuclear  $C^*$ -algebras has reduced the problem of the  $K$ -theoretic classifiability of crossed products of actions of countable amenable groups to the simple question of when their nuclear dimension is finite. In particular, this has raised the striking possibility that large classes of actions of completely arbitrary countable amenable groups have classifiable crossed products. I will give one indication of how this general program might proceed by making use of the concept of  $\mathcal{Z}$ -stability. (Received September 18, 2016)

1125-46-1798 **Chase T Worley\*** ([cworley3@vols.utk.edu](mailto:cworley3@vols.utk.edu)), 1403 Circle Dr., Knoxville, TN 37996. *A Finiteness Result for Circulant Core Hadamard Matrices.* Preliminary report.

A complex Hadamard matrix is a matrix which has complex entries of absolute value 1, and mutually orthogonal rows. Complex Hadamard matrices arise in several fields of mathematics and physics, such as cryptography, operator algebras, harmonic analysis, and quantum information theory. While the definition of a complex Hadamard matrix is deceptively simple, it is rather difficult to provide examples, and there is no general classification. By restricting ourselves to size and type, we can prove partial classification results and finiteness results. Our main result looks at circulant core Hadamard matrices. We will show that there are only finitely many such matrices of size  $p+1$  with  $p$  a prime number. This is joint work with Remus Nicoara. (Received September 19, 2016)

1125-46-1890 **Laura De carli\***, [decarli@fiu.edu](mailto:decarli@fiu.edu), and **Pierluigi Vellucci**. *Stability results for the  $n$ -order hold models.* Preliminary report.

We prove stability results for a class of Gabor frames in  $L^2(\mathbb{R})$ . Our results can be used to describe the effect of the timing jitters in the  $p$ -order hold models of signal reconstruction that are used in electronics and communication theory (Received September 19, 2016)

1125-46-1918 **Brent Nelson\*** ([brent@math.berkeley.edu](mailto:brent@math.berkeley.edu)) and **Max Fathi**. *Free Stein kernels and an improvement of the free logarithmic Sobolev inequality.*

In their 2015 paper, Ledoux, Nourdin, and Peccati used Stein kernels and Stein discrepancies to improve the classical logarithmic Sobolev inequality (relative to a Gaussian distribution). Simply put, Stein discrepancy measures how far a probability distribution is from the Gaussian distribution by looking at how badly it violates the integration by parts formula. In free probability, free semicircular operators are known to satisfy a corresponding "integration by parts formula" by way of the free difference quotients. Using this fact, we define the non-commutative analogues of Stein kernels and Stein discrepancies and use them to produce an improvement of Biane and Speicher's free logarithmic Sobolev inequality from 2001. We will also see several examples of free Stein kernels which have interesting connections to free monotone transport. (Received September 19, 2016)

1125-46-2015 **Maria Cristina Pereyra\*** ([crisp@math.unm.edu](mailto:crisp@math.unm.edu)), Department of Mathematics and Statistics, MSC01 1115, 1 University of New Mexico, Albuquerque, NM 87131. *Equivalent definitions of product Hardy spaces on spaces of homogeneous type.* Preliminary report.

We discuss different characterization of product Hardy spaces on spaces of homogeneous type: atomic, based on square functions and Auscher-Hytonen wavelets, based on square functions and Haar functions over adjacent

families of dyadic cubes. This is joint work with Jongsheng Han, Anna Kairema, Ji Li, and Lesley Ward. (Received September 19, 2016)

1125-46-2039 **Piotr M. Hajac\*** (pmh@impan.pl), Ul. Sniadeckich 8, 00-656 Warsaw, Mazowsze, Poland, and **Ludwik Dabrowski** (dabrow@sissa.it), Via Bonomea, 265, 34136 Trieste, Italy. *Noncommutative Borsuk-Ulam-type conjecture revisited*. Preliminary report.

Let  $H$  be the  $C^*$ -algebra of a non-trivial compact quantum group acting freely on a unital  $C^*$ -algebra  $A$ . Baum, Dabrowski and Hajac conjectured that there does not exist an equivariant  $*$ -homomorphism from  $A$  to the equivariant noncommutative join  $C^*$ -algebra  $A * H$ . When  $A$  is the  $C^*$ -algebra of functions on a sphere, and  $H$  is the  $C^*$ -algebra of functions on  $Z/2Z$  acting antipodally on the sphere, then the conjecture becomes the celebrated Borsuk-Ulam Theorem. Recently, Chirvasitu and Passer proved the conjecture when  $H$  is commutative. In a simple way, we extend this result to a far more general setting assuming only that  $H$  admits a character different from the counit. In particular, our result implies the non-contractibility of  $q$ -deformed compact Lie groups. (Received September 19, 2016)

1125-46-2085 **Sofya S Masharipova\*** (sofya.masharipova@saumag.edu), Dept of Math and CS, College of Sciences and, Engineering, 100 E University St, Magnolia, AR 71753, and **Shukhrat M Usmanov** (shukhrat.usmanov@waldorf.edu), Dept of Mathematics, Waldorf University, 106 S 6th St, Forest City, IA 50436. *Positive cones and positive linear functionals on algebras of operators on Pontryagin space  $\Pi_1$  with indefinite metrics: non-degenerated cases*. Preliminary report.

In this work we study the positive linear mappings and positive linear functionals on symmetric algebras of bounded linear operators on Pontryagin space  $Pi_1$ . Study based on structure of cones of positive linear operators, described by authors. Cases of degenerated and non-degenerated cones are considered. Connection with V. S. Shulmans' types of such algebras is studied (see Mat. Sbornik, 1972, 89, No 2). (Received September 19, 2016)

1125-46-2185 **Scott A. Atkinson\*** (scott.a.atkinson@vanderbilt.edu). *Minimal faces and Schur's Lemma for embeddings into  $R^U$* .

As shown by N. Brown in 2011, for a separable  $\Pi_1$ -factor  $N$ , the invariant  $\text{Hom}(N, R^U)$  given by unitary equivalence classes of embeddings of  $N$  into  $R^U$ —an ultrapower of the separable hyperfinite  $\Pi_1$ -factor—takes on a convex structure. This provides a link between convex geometric notions and operator algebraic concepts; for instance, extreme points are precisely the embeddings with factorial relative commutant. The geometric nature of this invariant provides a familiar context in which natural curiosities become interesting new questions about the underlying operator algebras. For example, consider the following simple question. Can four extreme points have a planar convex hull?

In this talk we will generalize the characterization of extreme points by showing that given an embedding  $\pi : N \rightarrow R^U$ , the dimension of the minimal face containing the equivalence class  $[\pi]$  is one less than the dimension of the center of the relative commutant of  $\pi$ . At the same time, we will establish the “convex independence” of extreme points—providing a negative answer to the above question. Along the way we make use of a version of Schur's Lemma for this context. (Received September 19, 2016)

1125-46-2193 **Valerie N. Nelson\*** (vnelson75@gmail.com), 8107 Mandan Terrace, Greenbelt, MD 20770. *Existence Results for Some Higher-Order Abstract Differential Equations with Applications to PDEs*.

In this talk, we first introduce a new class of functions called  $C^{(n)}$ -pseudo-almost automorphic functions, which generalizes the concept of  $C^{(n)}$ -almost automorphy (respectively,  $C^{(n)}$ -almost periodicity. Next, we present some of their properties and study their generalization called Stepanov-like  $C^{(n)}$ -pseudo-almost automorphy. Our next task consists of examining the existence of such solutions to some non-autonomous higher-order systems of differential equations in  $C^{(r)}$ -pseudo-almost automorphic spaces and intermediate spaces.

Our third task consists of studying the existence of  $C^{(r)}$ -pseudo-almost automorphic and Stepanov-like  $C^{(r)}$ -pseudo-almost automorphic solutions in intermediate spaces to some general higher-order differential equations with operator coefficients. We consider cases when the forcing term is either  $C^{(m)}$ -pseudo-almost automorphic and Stepanov-like  $C^{(m)}$ -pseudo-almost automorphic.

To illustrate the above-mentioned results, we close by examining the existence of  $C^{(m+1)}$ -pseudo-almost automorphic solutions to some damped second-order abstract differential equations and then to some (second order in time and fourth order in space) PDEs. These results represent work advised by Toka Diagana. (Received September 19, 2016)

1125-46-2436 **Alexander A Katz** ([katza@stjohns.edu](mailto:katza@stjohns.edu)), St. John's University, SJC, Department of Math & CS, 8000 Utopia Parkway, SJH-334-G, Queens, NY 11439, and **Roman Kushnir\*** ([kushnir\\_roman@yahoo.com](mailto:kushnir_roman@yahoo.com)), University of South Africa, Department of Mathematical Sciences, P.O.Box 392 Unisa, Pretoria, 0003, South Africa. *On Measurable Bundles of real  $C^*$ -algebras.*

We introduce a real  $C^*$ -algebra over the ring of all real measurable functions as a real symmetric Banach-Kantorovich  $*$ -algebra whose vector norm satisfies real  $C^*$ -like conditions and show that such an algebra admits a unique up to a  $*$ -isometry representation as a measurable Banach bundle of real  $C^*$ -algebras with vector-valued lifting. (Received September 20, 2016)

1125-46-2728 **Jeramiah A Hocutt\*** ([jhocutt@uf1.edu](mailto:jhocutt@uf1.edu)), Department of Mathematics, 1400 Stadium Rd, University of Florida, Gainesville, FL 32611, and **Paul L Robinson**. *Twisted Duality for the Clifford von Neumann Algebra.*

We consider the von Neumann algebra  $A(V)$  that envelops the Clifford algebra  $C(V)$  over a real inner product space  $V$  in its tracial left regular representation, showing that if  $Z$  is a real subspace of  $V$  then the von Neumann subalgebra generated by  $Z^\perp$  coincides with the graded commutant of the von Neumann subalgebra generated by  $Z$ . (Received September 20, 2016)

1125-46-2774 **Sarah A Reznikoff\*** ([sarahrez@ksu.edu](mailto:sarahrez@ksu.edu)), 138 Cardwell Hall, Mathematics Department, Manhattan, KS 66506. *Recent advances in combinatorially-defined  $C^*$ -algebras.* Preliminary report.

The last decade has seen great progress in the field of combinatorially-defined  $C^*$ -algebras. In particular, new uniqueness theorems have been proved for graph algebras,  $k$ -graph algebras, and groupoids. In this talk we give an overview of this work and present results in several new directions. (Received September 20, 2016)

1125-46-2808 **Oleg Friedman\*** ([friedman001@yahoo.com](mailto:friedman001@yahoo.com)), Touro College / Lander College for Men, Department of Mathematics, 7531 150-th Street, Kew Gardens Hills, NY 11367, and **Alexander A. Katz** ([katza@stjohns.edu](mailto:katza@stjohns.edu)), St. John's University, SJC of LAS, Department of Mathematics & CS, 8000 Utopia Parkway, SJH-334-G, Queens, NY 11439. *On defining real locally  $C^*$ -algebras.*

Necessary and sufficient conditions are given for a complete real lmc $*$ -algebra to be  $*$ -isomorphic and homeomorphic to the projective limit of a projective family of real  $C^*$ -algebras. (Received September 20, 2016)

1125-46-2911 **Dawn E. Archey\*** ([archeyde@udmercy.edu](mailto:archeyde@udmercy.edu)), **Julian Buck** and **N. Christopher Phillips**. *Centrally Large Subalgebras and Tracial  $\mathcal{Z}$ -Absorption.*

Centrally large subalgebras, which are a generalization of Putnum's orbit breaking subalgebras will be defined and then used to obtain the following result about absorption of the Jiang-Su Algebra: Let  $A$  be a simple infinite dimensional stably finite unital  $C^*$ -algebra, and let  $B$  be a centrally large subalgebra of  $A$ . If  $A$  and  $B$  are also separable and nuclear, then  $A$  is  $\mathcal{Z}$ -absorbing if and only if  $B$  is  $\mathcal{Z}$ -absorbing. (Received September 20, 2016)

1125-46-2978 **Judith A. Packer\*** ([packer@euclid.colorado.edu](mailto:packer@euclid.colorado.edu)), Department of Mathematics, CB 395, University of Colorado, Boulder, Boulder, CO 80309-0395. *Wavelets associated to representations of higher-rank graph algebras.* Preliminary report.

Here we discuss notions of wavelets defined on  $L^2$ -spaces for fractal-like sets associated to certain representations of higher-rank graph  $C^*$ -algebras, where the graphs in question are finite and strongly connected. We generalize work of M. Marcolli and A. Paolucci for Cuntz-Krieger  $C^*$ -algebras and obtain the wavelets using the isometries and partial isometries that generate the  $C^*$ -algebras in question and discuss some related spectral triples. This work is joint with C. Farsi, E. Gillaspay, A. Julien, and S. Kang. (Received September 20, 2016)

1125-46-3028 **Keivan Hassani Monfared** and **Ehssan Khanmohammadi\*** ([ehssan@fandm.edu](mailto:ehssan@fandm.edu)). *An Inverse Spectrum Problem for Infinite Graphs and Applications.*

In this talk we present our extensions of some recent results on inverse eigenvalue problems of finite graphs to the infinite setting by means of functional analytic methods. We show that for a given infinite graph  $G$  on countably many vertices, and a compact, infinite set of real numbers  $\Lambda$  there is a real symmetric matrix  $A$  whose graph is  $G$  and its spectrum is  $\Lambda$ . We also show that any two such matrices constructed by our method are approximately unitarily equivalent. (Received September 20, 2016)

## 47 ► Operator theory

1125-47-95 **Torrey M Gallagher\*** (tmg012@bucknell.edu). *The demiclosedness principle for mean nonexpansive mappings.*

In 1967 and 1968, Opial and Browder (respectively) proved their well-known versions of the Demiclosedness Principle for nonexpansive mappings. We will present a generalization of these theorems, which is due to the present author and has recently appeared in the Journal of Math Analysis & Applications, to the class of mean nonexpansive mappings. As a corollary, we will also discuss a theorem regarding weak convergence of the sequence of iterates of a mean nonexpansive mapping in the presence of asymptotic regularity at a point. (Received July 21, 2016)

1125-47-157 **George A Anastassiou\*** (ganastss@memphis.edu), Dr. G. Anastassiou, Department of Mathematical Sciences, University of Memphis, Memphis, TN 38152. *Self Adjoint Operator Harmonic Chebyshev-Grüss Inequalities.*

We present here very general self adjoint operator harmonic Chebyshev-Grüss inequalities with applications. (Received August 05, 2016)

1125-47-158 **George A Anastassiou\*** (ganastss@memphis.edu), Dr. George Anastassiou, Department of Mathematical Sciences, University of Memphis, Memphis, TN 38152. *Fractional Self Adjoint Operator Poincaré and Sobolev type Inequalities.*

We present here many fractional self adjoint operator Poincaré and Sobolev type inequalities to various directions. Initially we give several fractional representation formulae in the self adjoint operator sense. Inequalities are based in the self adjoint operator order over a Hilbert space. (Received August 05, 2016)

1125-47-200 **Abdolaziz Abdollahi\*** (abdollahi@shirazu.ac.ir), Department of Mathematics, College of Sciences, Shiraz University, 7146713565 Shiraz, Iran. *On the numerical range of finite order elliptic automorphism composition operators.*

In this talk we show that the matrix representation of a finite order elliptic automorphism composition operator on the Hardy space with respect to the Guyker's basis is a block Toeplitz operator. By using this representation we present some numerical and experimental results connecting to the shape of the numerical ranges of such operators. We also consider the following conjecture, posed in 2000 by P. Bourdon and J. Shapiro: "The numerical range is not a disc", and we give a positive answer to this conjecture. (Received August 12, 2016)

1125-47-242 **Genady Ya. Grabarnik\*** (genadyg@hotmail.com), Math & CS, St John's University, Queens, NY 11439. *Notes on ergodic theorems in non-commutative symmetric spaces.*

We establish individual ergodic theorem for positive kernels (or so called Danford Shwartz (DS+) operators acting on non commutative symmetric spaces. (Received August 18, 2016)

1125-47-255 **Stephan Ramon Garcia\*** (stephan.garcia@pomona.edu), Department of Mathematics, Pomona College, 610 N College Ave, Claremont, CA 91711. *Operators and lattices.*

How do Toeplitz operators and frames relate to properties of lattices? We discuss recent results about the interplay between operator theory and lattice theory. (Received August 19, 2016)

1125-47-413 **George R Exner\***, Department of Mathematics, Bucknell University, Lewisburg, PA 17837. *Mixed Bram-Halmos and Agler-Embry conditions.* Preliminary report.

The subnormality of a Hilbert space operator may be characterized either by the Bram-Halmos conditions (positivity of certain operator matrices) or the Agler-Embry conditions (positivity of certain operator differences). We define and consider mixed conditions involving matrices of operator differences, thus yielding conditions whose extremes are the Bram-Halmos and Agler-Embry conditions. We present results on these conditions for weighted shifts and for general operators. (Received September 01, 2016)

1125-47-494 **Milivoje Lukic\*** (milivoje.lukic@rice.edu), 6100 Main Street, Mathematics MS-136, Houston, TX 77005. *Higher-order Szegő theorems.*

We study relations between probability measures  $\mu$  on the unit circle and their sequences of Verblunsky coefficients  $\alpha = \{\alpha_n\}_{n=0}^{\infty}$  (which are coefficients in the recurrence relation obeyed by orthogonal polynomials with respect to  $\mu$ ).

The Szegő theorem is a celebrated result giving a necessary and sufficient integral criterion for  $\mu$  to have  $\alpha \in \ell^2$ . Higher-order Szegő theorems are similar equivalence statements involving weaker decay, and bounded variation, conditions on  $\alpha$ . We will discuss results which test Simon's conjecture for the general form of these theorems, in the multifrequency regime and in the regime of very slow decay. (Received September 04, 2016)

1125-47-529 **Soumyadip Acharyya\***, acharyys@erau.edu, and **Zhijian Wu**, zhijian.wu@unlv.edu.  
*Difference of two weighted composition operators on Bergman spaces.*

Let  $\varphi$  be a holomorphic self-map of the open unit disc  $\mathbb{D}$  and  $u$  be a measurable (**not necessarily holomorphic**) complex-valued function on  $\mathbb{D}$ . The linear map  $uC_\varphi$  on  $H(\mathbb{D})$  defined by

$$(uC_\varphi)(f)(z) = u(z)(f \circ \varphi)(z), \quad f \in H(\mathbb{D}), z \in \mathbb{D},$$

is called the weighted composition operator with weight  $u$  and symbol  $\varphi$ .

The talk will begin with a brief survey of some earlier results about the difference of two unweighted composition operators, in particular their Schatten - 2 membership, compactness, and, boundedness. Our main results include equivalent expressions for the Schatten - 2 norm, essential norm and operator norm of two weighted composition operators, acting from weighted Bergman spaces to  $L^p$  spaces. Insight into the techniques to solve those type of problems will be given. (Received September 05, 2016)

1125-47-621 **Constanze Liaw\***, One Bear Place #97328, Waco, TX 76798, and **Sergei Treil**. *Finite rank unitary perturbations.*

The unitary perturbations of a given unitary operator by finite rank  $d$  operators can be parametrized by  $d \times d$  unitary matrices; this generalizes the rank  $d = 1$  setting, where the Clark family is parametrized by the scalars on the unit circle.

For finite rank perturbations we investigate the functional model of a related class of contractions, as well as a (unitary) Clark operator that realizes such a model representation for a particular contraction. We find a universal representation of the adjoint of the Clark operator, which features a matrix-valued Cauchy integral operator. By universal we simply mean that our formula is given in the coordinate free Nikolski–Vasyunin functional model.

We express the matrix-valued characteristic functions of the model (for the class of contractions). Unlike in the case of rank one perturbations, these characteristic functions do not seem to be related via a linear fractional transformation.

In the case of inner characteristic functions results suggest a generalization of the normalized Cauchy transform to the finite rank setting.

This presentation is based on joint work with Sergei Treil. (Received September 08, 2016)

1125-47-629 **Alexandru Aleman, Michael Hartz, John E. McCarthy and Stefan Richter\***  
 (richter@math.utk.edu). *Interpolation for multipliers between spaces.* Preliminary report.

Let  $X$  be a set. If  $k_\lambda(z)$  is a positive definite function on  $X \times X$ , then we write  $\mathcal{H}_k$  for the Hilbert space of complex-valued functions on  $X$  with reproducing kernel  $k$ , and we write  $\hat{k}_\lambda = k_\lambda / \|k_\lambda\|$  for the normalized reproducing kernel at  $\lambda$ . If  $s$  and  $k$  are two reproducing kernels on  $X$ , then  $M(\mathcal{H}_s, \mathcal{H}_k)$  denotes the multipliers from  $\mathcal{H}_s$  to  $\mathcal{H}_k$ . A sequence of distinct points  $\{\lambda_n\} \subseteq X$  is called interpolating for  $M(\mathcal{H}_s, \mathcal{H}_k)$ , if for each  $\{w_n\} \in \ell^\infty$ , there is a  $\varphi \in M(\mathcal{H}_s, \mathcal{H}_k)$  such that  $M_\varphi^* \hat{k}_{\lambda_n} = \bar{w}_n \hat{s}_{\lambda_n}$  for each  $n$ .

We characterize the  $M(\mathcal{H}_s, \mathcal{H}_k)$ -interpolating sequences for the case where  $s_{z_0} = 1$  for some  $z_0 \in X$ ,  $\mathcal{H}_s$  has the complete Pick property, and  $k$  is of the form  $k_\lambda(z) = s_\lambda(z)^t$  for some  $t \geq 1$ .

If  $t = 1$ , then in some cases our results reduce to known and classical results, but they are new even for the Drury-Arveson space of the unit ball of  $\mathbb{C}^d$ . (Received September 08, 2016)

1125-47-709 **Rufus Willett\*** (rufus@math.hawaii.edu), 2565 McCarthy Mall, Keller 401A, Honolulu, HI 96822. *Hulanicki's theorem fails for groupoids.*

An important theorem of Hulanicki from the 60s says that a locally compact group is amenable if and only if its maximal and reduced  $C^*$ -algebras are the same. For groupoids, Renault defined a notion of (topological) amenability around 1980, and showed that it implies that the associated maximal and reduced  $C^*$ -algebras are the same. I'll describe an example showing that the converse fails, and say a little about what this has to do with exactness (without assuming prior knowledge of groupoids, their  $C^*$ -algebras, or exactness). (Received September 09, 2016)

1125-47-733 **Cheng Chu and Dima Khavinson\*** (dkhavins@usf.edu). *Spectral area of Toeplitz operators.*

We show that for hyponormal Toeplitz operators, there exists a lower bound for the area of the spectrum. This extends the known estimate for the spectral area of Toeplitz operators with an analytic symbol. (Received September 10, 2016)

1125-47-780 **Stefan Richter\*** ([srichter@utk.edu](mailto:srichter@utk.edu)). *Cyclic vectors in the Drury-Arveson space.*  
Preliminary report.

The Drury-Arveson space  $H_d^2$  is the space of analytic functions on the unit ball  $\mathbb{B}_d$  of  $\mathbb{C}^d$  with reproducing kernel  $k_w(z) = \frac{1}{1-\langle z, w \rangle}$ . It is well-known to consist of all analytic functions  $f$  on  $\mathbb{B}_d$  such that  $R^k f \in L^2((1 - |z|^2)^{2k-d} dV)$  for each positive integer  $k$  with  $2k - d > -1$ . Here  $R = \sum_i z_i \frac{\partial}{\partial z_i}$  is the radial derivative operator and  $dV$  denotes Lebesgue measure on  $\mathbb{B}_d$ . A function  $f \in H_d^2$  is called cyclic, if there are polynomials  $p_n$  such that  $p_n f \rightarrow 1$  in  $H_d^2$ .

We will give an overview of our recent results on cyclic functions in  $H_d^2$ . Although in the multivariable situation we don't have analogues of the inner-outer factorization and cut-off techniques available, it turns out that some of our methods are motivated by work on the single variable Dirichlet space. (Received September 11, 2016)

1125-47-883 **Anna Skripka\*** ([askripka@unm.edu](mailto:askripka@unm.edu)). *Trace class bounds for Schur multipliers.*  
Preliminary report.

We derive lower bounds for trace class norms of multilinear Schur multipliers and apply them to show that an  $n$ th order operator Taylor remainder arising from a  $C^n$ -function is not necessarily in the trace class  $S^1$  when a perturbation is in the  $n$ th Schatten class  $S^n$ . This extends the known result that not every Lipschitz function is operator Lipschitz in  $S^1$  and complements results on nice behavior of Schatten  $p$ -norms,  $1 < p < \infty$ , of operator Taylor remainders. The talk is based on joint work with D. Potapov, F. Sukochev, and A. Tomskova. (Received September 12, 2016)

1125-47-949 **James Eldred Pascoe\*** ([pascoe@math.wustl.edu](mailto:pascoe@math.wustl.edu)). *Applications of model-realization theory to inverse problems in free probability.*

Classically, Nevanlinna showed that there was bijection between positive finite Borel measures on  $\mathbb{R}$  and analytic self-maps of the upper half plane which satisfy the asymptotic condition  $\lim_{s \rightarrow \infty} |sf(is)| < \infty$  via the *Cauchy transform*. More recently, analogous problems have been considered in free probability. That is, there should be a correspondence between noncommutative probability and function theory on a noncommutative upper half plane. We will discuss how to re-frame Agler model-realization theory developed on the upper half plane to completely understand the inverse problem in the free probabilistic context. This talk represents joint work with Benjamin Passer and Ryan Tully-Doyle. (Received September 13, 2016)

1125-47-963 **Faruk Yilmaz\*** ([yilmaz@math.utk.edu](mailto:yilmaz@math.utk.edu)), 227 Ayres Hall. 1403 Circle Drive., Knoxville, TN 37996-1320. *Approximation of Invariant Subspaces in Some Dirichlet-type Spaces.*

The Dirichlet-type space  $\mathcal{D}_2$  consists of all analytic functions  $f$  on the unit disc  $\mathbb{D}$  such that  $f'$  is in the Hardy Hilbert space  $\mathbf{H}^2$ . In this talk, we discuss the result that proves that every nonzero invariant subspace of the multiplication operator  $M_z$  on the  $\mathcal{D}_2$  space can be approximated by finite co-dimensional ones. For the Dirichlet space  $\mathbf{D}$  we have a partial analogue. (Received September 13, 2016)

1125-47-989 **Ben Hayes\*** ([benjamin.r.hayes@vanderbilt.edu](mailto:benjamin.r.hayes@vanderbilt.edu)), Department of Mathematics, Vanderbilt University, 1326 Stevenson Center Ln, Nashville, TN 37240. *1-bounded entropy with applications to von Neuman algebras.*

I will discuss 1-bounded entropy, which is a invariant of a tracial von Neumann algebra which measures "how many matricial approximations" it has. I will present discuss applications, including new nonisomorphism results in  $\text{II}_1$ -factors, as well as applications to regularity problems and type III algebras. (Received September 13, 2016)

1125-47-1018 **Sujan Pant\*** ([sujan-pant@uiowa.edu](mailto:sujan-pant@uiowa.edu)), Iowa City, IA 52246. *Structural results for von Neumann algebras arising from poly-hyperbolic groups and Burger-Mozes groups.*

Denote by  $\mathcal{C}$  the class of all non-amenable groups that are hyperbolic of non-trivial free products. For every positive integer  $n$  denote by  $\text{Quot}_n(\mathcal{C})$  the class of groups that can be realized as  $n$ -step extensions of groups in  $\mathcal{C}$  [?]. We show that the von Neumann algebras of these groups enjoy the following structural property: Let  $\Gamma \in \text{Quot}_n(\mathcal{C})$  and suppose  $A_1, A_2, \dots, A_k \subset L(\Gamma)$  are *arbitrary* commuting subalgebras with no amenable direct summands that generate together a finite index of  $L(\Gamma)$ . Then  $\Gamma$  is commensurable to a product  $\Lambda_1 \times \Lambda_2 \times \dots \times \Lambda_k$  with  $\Lambda_i \in \text{Quot}_{n_i}(\mathcal{C})$  and  $n_1 + n_2 + \dots + n_k = n$ . Also, up to corners,  $A_i \cong L(\Lambda_i)$ , for all  $i$ . In particular,  $L(\Gamma)$  is prime if and only if  $\Gamma$  is virtually indecomposable as a product over groups in  $\text{Quot}(\mathcal{C})$ . The same techniques also show Burger-Mozes groups gives rise to prime von Neumann algebras. This is the first occurrence of prime factors arising from simple groups. This is based on a joint work with Rolando de Santiago.

(Received September 14, 2016)

1125-47-1109 **Muhammad Akram\*** (dr.makram@gcu.edu.pk), 225 Strouse Ave, State College, PA 16803, and **Shamaila Waheed**. *Coincidence and Common Fixed Point Results for Generalized Nonlinear Contractions in Partial Metric Spaces.*

In the present paper we establish two Coincidence point and common fixed point theorems for generalized nonlinear contractions in partial metric spaces. Further, uniqueness of such coincidence point and common fixed point is proved. Our results generalize and extend some existing fixed point theorem in literature. (Received September 14, 2016)

1125-47-1135 **Alexander Gorokhovskiy\*** (gorokhov@colorado.edu), Department of Mathematics, University of Colorado, Boulder, CO 80309-0395, and **Henri Moscovici** (henri@math.ohio-state.edu), Department of Mathematics, The Ohio State University, 231 West 18th Avenue, Columbus, OH 43210-1174. *Primary and secondary invariants of algebras of pseudodifferential symbols.* Preliminary report.

We compare different constructions of cyclic cocycles for the algebra of complete symbols of pseudodifferential operators. We show that our result leads to interesting index-theoretic consequences and a construction of invariants of the algebraic  $K$ -theory of the algebra of pseudodifferential symbols. (Received September 15, 2016)

1125-47-1164 **Mishko Mitkovski\*** (mmitkov@clemson.edu). *Localized frames and their applications to operator theory.* Preliminary report.

In this talk I will present the concept of frame localization. This concept is useful for operator theory because many important classes of operators can be almost-diagonalized with respect to a suitably chosen localized frame. These include: Short Time Fourier Transform multipliers, Calderon-Toeplitz operators, Toeplitz operators on various functions spaces, Anti-Wick operators, some pseudodifferential operators, some Calderon-Zygmund operators, and many others. The goal of my talk will be to show how frame localization can be used in the study of several operator-theoretic questions about these classes of operators. (Received September 15, 2016)

1125-47-1167 **Robert T.W. Martin\*** (rtwmartin@gmail.com), Dept. of Mathematics and Applied Mathematics, University of Cape Town, Cape Town, South Africa. *A Gleason solution model for row contractions.*

We extend the deBranges-Rovnyak functional model for completely non-coisometric (c.n.c.) contractions to a large class of c.n.c. row contractions from several copies of a Hilbert space to itself. In particular, we show that any c.n.c. row partial isometry is unitarily equivalent to a contractive Gleason solution in a deBranges-Rovnyak space contractively contained in Drury-Arveson space. Drury-Arveson space is a canonical several variable generalization of the Hardy space of analytic functions in the unit disk, and a Gleason solution is the appropriate several-variable analogue of the restricted backward shift. We further extend this model to a large class of row contractive extensions of c.n.c. row partial isometries. (Received September 15, 2016)

1125-47-1261 **Vita Borovyk\*** (vita.borovyk@uc.edu) and **Konstantin Makarov**. *Box approximation for a pseudo-differential operator with an unbounded symbol.*

We study the attractor of a dynamical system  $r \rightarrow A_r$ , where  $A_r$  is the operator obtained by surrounding a truncated Wiener-Hopf operator with the multiplication by  $e^{\alpha/2|\cdot|}$ ,  $\alpha > 0$ , on both sides. In the case when the symbol is a rational function with two real zeros  $A_r$  is a symmetric operator with deficiency indices  $(2, 2)$  and not semi-bounded from below. We show that in this case the dynamical system  $r \rightarrow A_r$  possesses a limit-circle-type attractor. (Received September 15, 2016)

1125-47-1282 **William M. Higdon\*** (wmhigdon@butler.edu), Department of Mathematics and Statistics, 4600 Sunset Avenue, Indianapolis, IN 46208. *On The Numerical Ranges of Composition Operators Induced By Mappings With The Denjoy-Wolff Point On The Boundary.*

The numerical range of an operator  $T$  on a Hilbert space  $H$ , denoted  $W(T)$ , is the set,

$$W(T) = \{ \langle Tx, x \mid x \in H \text{ and } \|x\| = 1 \}.$$

Elementary properties of  $W(T)$  include that it is a bounded subset of  $\mathbf{C}$ , it is convex, it contains the eigenvalues of  $T$ , and, more generally, its closure includes the spectrum of  $T$ . The work presented here answers the question posed by Professors Paul S. Bourdon and Joel H. Shapiro in their paper ("When Is Zero In The Numerical Range Of A Composition Operator?", J. IEOT., 44 (2002), 410-441):

"Suppose the symbol  $\phi$  of the composition operator  $C_\phi$  on  $H^2(D)$  is univalent, not linear fractional, and is of parabolic nonautomorphism type. Is  $0 \in W(C_\phi)$ ?"

The answer has been, in general, unknown. One property of such a mapping is that it has derivative equal to 1 at its Denjoy-Wolff point (boundary fixed point). Using elementary properties of  $C_\phi$ -invariant subspaces,

the answer to the question is demonstrated. Together with the work of Bourdon and Shapiro, this provides a complete description of when 0 belongs to the numerical range of a composition operator  $C_\phi$  on  $H^2(D)$ . (Received September 15, 2016)

1125-47-1367 **Raul Quiroga-Barranco\*** ([quiroga@cimat.mx](mailto:quiroga@cimat.mx)), Cimat, Av. Jalisco S/N, Mineral de Valenciana, 36000 Guanajuato, Guanajuato, Mexico. *Multiplicity-free restrictions of holomorphic discrete series and Toeplitz operators.*

Bounded symmetric domains provide many rich objects in analysis. We are mainly interested in their holomorphic discrete series, more specifically the branching properties of these representations. In particular, the theory of multiplicity-free restrictions allows us to have a better understanding of the holomorphic discrete series. In this area, the work of Kobayashi, Ólafsson and Ørsted has been of fundamental importance.

On the other hand, the Bergman space realizations of the holomorphic discrete series are also the base for the so called Toeplitz operator. These are defined as a multiplication operator by a measurable function followed by the Bergman projection. A fairly recent discovery is the existence of many rich commutative  $C^*$ -algebras generated by Toeplitz operators. From the very start this behavior was found to be related to group theory. More recently, it was proved to be closely related to corresponding multiplicity-free restrictions.

In this talk we will briefly explain this application of representation theory to operator theory. (Received September 16, 2016)

1125-47-1469 **Eungil Ko** and **Hyun Kwon\***, [hkwon@ua.edu](mailto:hkwon@ua.edu), and **Ji Eun Lee**. *Characterization of binormal matrices.*

We give several characterizations of binormal matrices. In particular, an easy way of showing the binormality of Toeplitz matrices is presented. (Received September 17, 2016)

1125-47-1611 **Arkady K Kitover\*** ([akitover@rider.edu](mailto:akitover@rider.edu)), 12135 Academy Rd 61, Philadelphia, PA 19154. *Essential spectra of weighted composition operators induced by non-invertible transformations.*

We consider weighted composition operators on  $C(K)$  induced by non-invertible surjective maps. In the case of an invertible weight we obtain a criterion for  $\lambda I - T$ ,  $\lambda \in \sigma(T)$  to be a surjection on  $C(K)$ . We also describe essential spectra of weighted composition operators acting on  $C(S)$ , where  $S$ , is the unit circle, and in disk-algebra, and induced by some finite Blaschke products. (Received September 18, 2016)

1125-47-1831 **Ian L Charlesworth\*** ([ilc@math.ucla.edu](mailto:ilc@math.ucla.edu)). *Bi-free probability: a new characterisation and consequences.* Preliminary report.

Bi-free probability was introduced to study simultaneously non-commutative random variables acting from the left and from the right. We demonstrate a new necessary and sufficient condition for bi-free independence, which is an analogue of the “moments of alternating products of centred terms vanish” characterisation of free independence, and present some consequences. (Received September 19, 2016)

1125-47-1910 **Lauren Sager\*** ([lbq32@wildcats.unh.edu](mailto:lbq32@wildcats.unh.edu)), Kingsbury Hall, 33 Academic Way, Durham, NH 03824. *Invariant subspaces for triangular algebras in Schatten  $p$ -classes.*

In this talk, we seek to characterize subspaces of the Schatten  $p$ -classes on an infinite dimensional Hilbert space  $\mathcal{H}$ , which are invariant under lower triangular algebras. In doing so, we prove a Beurling-Blecher-Labuschagne theorem for  $H^\infty$ -invariant subspaces of  $L^p(\mathcal{M}, \tau)$  where  $\mathcal{M}$  is a von Neumann algebra with semifinite, faithful, normal tracial weight  $\tau$ ,  $0 < p \leq \infty$ , and  $H^\infty$  is a non-commutative Hardy space, similar to those defined by Arveson. As an application of the main result, we completely characterize all  $H^\infty$ -invariant subspaces of  $L^p(\mathcal{M} \rtimes_\alpha \mathbb{Z}, \tau)$  where  $\mathcal{M} \rtimes_\alpha \mathbb{Z}$  is the non-self-adjoint crossed product of a von Neumann algebra  $\mathcal{M}$  by an action  $\alpha$  on  $\mathcal{M}$ . Then, we are able to completely characterize all lower triangular subalgebra-invariant subspaces of the Schatten  $p$ -class for  $0 < p \leq \infty$ . Our result answered a question asked implicitly by McAsey, Muhly and Saito in 1979. (Received September 19, 2016)

1125-47-2024 **Waleed K. Al-Rawashdeh\*** ([walrawashdeh@mttech.edu](mailto:walrawashdeh@mttech.edu)), Montana Tech, West Park Street, Butte, MT 59701. *Essential Norm of Weighted Composition Operators on Bargmann-Fock Spaces.*

Let  $\varphi$  be an entire self-map of the  $n$ -dimensional Euclidean complex space  $\mathbb{C}^n$  and  $\psi$  be an entire function on  $\mathbb{C}^n$ . A weighted composition operator induced by  $\varphi$  with weight  $\psi$  is given by  $(W_{\psi, \varphi} f)(z) = \psi(z)f(\varphi(z))$ , for  $z \in \mathbb{C}^n$  and  $f$  is entire function on  $\mathbb{C}^n$ . For any  $p > 0$  and  $\alpha > 0$ , the Bargmann-Fock  $\mathcal{F}_\alpha^p(\mathbb{C}^n)$  consists of all entire functions  $f$  on  $\mathbb{C}^n$  such that  $\|f\|_{p, \alpha}^p = \int_{\mathbb{C}^n} |f(z)|^p e^{-\frac{\alpha p}{2}|z|^2} dv(z)$  is finite. In this talk, we study weighted composition operators between Bargmann-Fock spaces  $\mathcal{F}_\alpha^p(\mathbb{C}^n)$  and  $\mathcal{F}_\alpha^q(\mathbb{C}^n)$  for  $0 < p, q < \infty$ . In particular,

we characterize the boundedness and compactness of these operators, when  $0 < p, q < \infty$ . We also present an estimate of the essential norm of these operators, when  $1 < p \leq q < \infty$ . (Received September 20, 2016)

1125-47-2037 **Rachael M. Norton\*** (rachael-norton@uiowa.edu). *A comparison of Nevanlinna-Pick theorems.*

Since the original proof of the Nevanlinna-Pick theorem in 1915, there have been a variety of generalizations to operator theory, all but two of which may be recovered by Muhly and Solel's result from 2004. Muhly and Solel think of Nevanlinna-Pick interpolation as an instance of commutant lifting. Constantinescu, Johnson, and Popescu, on the other hand, use the displacement equation to prove results which are fundamentally different from Muhly and Solel's. In this talk, we address the differences and discuss circumstances under which the theorems are equivalent. (Received September 19, 2016)

1125-47-2318 **Nazife Erkursun Ozcan\*** (erkursun.ozcan@hacettepe.edu.tr), Hacettepe University, Department of, Mathematics, Beytepe, 06800 Ankara, Turkey. *Ergodic Properties of Markov operator sequences on KB-spaces.* Preliminary report.

To characterize the asymptotic behavior of Markov operators, the concept of an attractor was used often. In this talk, a positive LR-sequence on KB-spaces with an attractor is investigated. Moreover it is shown that the positive LR-sequence on KB-spaces is strongly convergent (mean ergodic) if it has a weakly compact attractor. Moreover if the weakly compact attractor is an order interval, then a Markovian LR-sequence converges strongly to the finite dimensional fixed space. As a consequence we investigate also stability of LR-sequences of positive operators and existence of lower bound functions on KB-spaces.

Mathematics Subject Classification (2010): 47A35, 47B42, 47B65, 47D03, 47D06.

Keywords: Markovian LR-sequences, KB-space, strong convergence, attractor.

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2) Erkursun Ozcan, N. Stability and lower-bound functions of Markov semigroups on KB-spaces. (Received September 20, 2016)

1125-47-2408 **Jaedeok Kim\*** (jkim@jsu.edu) and **Youngmi Kim** (ykim@jsu.edu). *Numerical Range of Partial Isometries.*

Let  $S$  be a partial isometry on a Hilbert space  $\mathcal{H}$  with the initial space  $N$  and the final space  $M$ . A classification can be made for partial isometries into a few different types in terms of the geometric position of two subspaces  $M$  and  $N$ . The numerical range of partial isometry  $S$ ,  $W(S) = \{\langle S\xi, \xi \rangle : \xi \in \mathcal{H}, \|\xi\| = 1\}$ , will be described based on the classification of partial isometries. (Received September 20, 2016)

1125-47-2451 **Dhruba R. Adhikari\*** (dadhikar@kennesaw.edu), 1100 S. Marietta Pkwy, Math Building - Bldg. D, Marietta, GA 30060. *On the Uniqueness of Topological Degrees for Densely Defined Mappings Involving Variants of  $(S_+)$  Operators.* Preliminary report.

Let  $X$  be a separable reflexive Banach space,  $G$  a bounded open subset of  $X$ , and  $L$  a dense linear subspace of  $X$ . The uniqueness of the topological degree,  $d(A, G, 0)$ , for mappings  $A : X \supset D(A) \rightarrow X^*$  satisfying Condition  $(S_+)_L$  and invariant under certain homotopy is established. The existence of such a topological degree is first established by Kartsatos and Skrypnik, and later, by Berkovits by using a different approach to construct the degree. (Received September 20, 2016)

1125-47-2453 **Yuliya Babenko\*** (ybabenko@kennesaw.edu), Kennesaw State University, Math Building - Bldg. D, 1100 South Marietta Pkwy, MD # 9085, Marietta, GA 30060, and **Vladyslav Babenko** and **Nadiia Kriachko**. *Inequalities of Hardy-Littlewood-Polya type for functions of operators and their applications.*

In this talk, we present a generalized multiplicative Hardy-Littlewood-Polya type inequality, as well as several related additive inequalities, for functions of operators in Hilbert spaces. In addition, we find the modulus of continuity of a function of an operator on a class of elements defined with the help of another function of the operator. We then apply the results to solve the following problems: (i) the problem of approximating a function of an unbounded self-adjoint operator by bounded operators, (ii) the problem of best approximation of a certain class of elements from a Hilbert space by another class, and (iii) the problem of optimal recovery of an operator on a class of elements given with an error. (Received September 20, 2016)

1125-47-2455 **Dale Frymark\*** ([dale\\_frymark@baylor.edu](mailto:dale_frymark@baylor.edu)), Department of Mathematics, Baylor University, One Bear Place #97328, Waco, TX 76706-7328, and **Constanze Liaw** ([constanze\\_liaw@baylor.edu](mailto:constanze_liaw@baylor.edu)), Department of Mathematics, Baylor University, One Bear Place #97328, Waco, TX 76706-7328. *Boundary Conditions associated with the Left-Definite Theory for Differential Operators*. Preliminary report.

In the early 2000's, Everitt, Littlejohn and Wellman developed a general left-definite theory for certain self-adjoint operators which explicitly determined their domains. However, the description of these domains do not contain boundary conditions. We present characterizations of these domains given by the left-definite theory for all operators which possess a complete system of orthogonal eigenvectors, in terms of classical boundary conditions. (Received September 20, 2016)

1125-47-2520 **Gokul R Kadel\*** ([gkadel@cameron.edu](mailto:gkadel@cameron.edu)), Cameron University, 2800 W Gore Blvd, Lawton, OK 73505. *Spectrum of hypercyclic operators*.

An operator  $T : H \rightarrow H$ , where  $H$  is an infinite-dimensional separable Hilbert space, is said to be hypercyclic if there is a vector  $h \in H$  such that  $\{h, Th, T^2h, \dots\}$  is dense in  $H$ . The vector  $h$  is called the hypercyclic vector for the operator  $T$ . The spectrum of an operator  $T$ , denoted  $\sigma(T)$ , is defined as the set of complex numbers  $\lambda$  such that  $T - \lambda I$ , where  $I$  is the identity operator, is not invertible. The talk will provide a description of the spectral properties of operators that are hypercyclic. (Received September 20, 2016)

1125-47-2697 **Scott M LaLonde\*** ([slalonde@uttyler.edu](mailto:slalonde@uttyler.edu)), 3900 University Boulevard, Department of Mathematics, Tyler, TX 75799. *Permanence Properties of Exact Groupoids*. Preliminary report.

A locally compact groupoid is said to be exact if its associated reduced crossed product functor is exact. In this talk, we will investigate permanence properties of exact groupoids, some of which generalize known results for exact groups. In particular, exactness descends to certain types of closed subgroupoids, and any action of an exact groupoid on a locally compact Hausdorff space yields an exact transformation groupoid. We will also discuss a partial converse to the latter result, which is related to the notion of amenability at infinity. If time permits, we may also present some results about Fell bundles over exact groupoids. (Received September 20, 2016)

1125-47-2734 **Masayoshi Kaneda\*** ([mkaneda@uci.edu](mailto:mkaneda@uci.edu)), Department of Mathematics & Natural Sciences, College of Arts and Sciences, American University of Kuwait, P.O. Box 3323, Safat13034 Kuwait. *Core  $C^*$ -algebras of an operator space*.

First we show that the unit ball of an injective operator space has an extreme point. Next we discuss  $C^*$ -algebras that can be contained in a given operator space  $X$  as subalgebras when  $X$  is algebrized by contractive quasi-multipliers of  $X$ . We equip the contractive quasi-multipliers of  $X$  with a partial order, and show that the algebrization corresponding to a maximal contractive quasi-multiplier provides a maximal  $C^*$ -algebra contained in  $X$  which we call a core  $C^*$ -algebra of  $X$ . (Received September 20, 2016)

1125-47-2783 **Rolando de Santiago\*** ([rolando-desantiago@uiowa.edu](mailto:rolando-desantiago@uiowa.edu)), **Ionut Chifan**, **Sujan Pant** and **Thomas Sinclair**. *Product Rigidity for the von Neumann algebras of hyperbolic groups*.

Let  $\Gamma_1, \dots, \Gamma_n$  be hyperbolic ICC groups and denote by  $\Gamma = \Gamma_1 \times \dots \times \Gamma_n$ . We show whenever  $\Lambda$  is an arbitrary discrete group such that  $L(\Gamma) \cong L(\Lambda)$  then  $\Lambda = \Lambda_1 \times \dots \times \Lambda_n$  and up to amplifications  $L(\Gamma_i) \cong L(\Lambda_i)$  for all  $i$ ; in other words the von Neumann algebra  $L(\Gamma)$  completely remembers the product structure of the underlying group.

In addition, we will show that some of the techniques used to prove this product rigidity result can also be successfully applied to produce new examples of prime factors. In particular, we significantly generalize the primeness results obtained earlier by I. Chifan, Y. Kida and S. Pant for the factors arising poly-hyperbolic and surface braid groups.

These are joint works with I. Chifan and T. Sinclair, and S. Pant, respectively. (Received September 20, 2016)

1125-47-2860 **Debendra P Banjade\*** ([dpbanjade@coastal.edu](mailto:dpbanjade@coastal.edu)), Coastal Carolina University, Department of Mathematics and Statistics, P. O. Box. 261954, Conway, SC 29528. *Estimates for the Corona Theorem on  $H_{\mathbb{I}}^{\infty}(\mathbb{D})$* .

Let  $\mathbb{I}$  be a proper ideal of  $H^{\infty}(\mathbb{D})$ . We prove the corona theorem for infinitely many generators on the subalgebra  $H_{\mathbb{I}}^{\infty}(\mathbb{D})$ , in which the corona theorem for finitely many functions is known to hold, for example in [2]. This settles the conjecture of Ryle [1]. Moreover, we prove a generalized Wolff's Ideal Theorem for this subalgebra.

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1125-47-3015 **Michael G Dabkowski\*** (mdabkowsk@1tu.edu), 11403 Waverly, Plymouth, MI 48170, and **John H. Clifford**. *On The Numerical Range of the  $C_\phi C_\psi^*$* . Preliminary report.

We consider the numerical range the product of a composition operator  $C_\phi$  and the adjoint of a composition operator,  $C_\psi^*$ , both acting on the Hardy space  $H^2$ . For certain choices of  $\phi$  and  $\psi$  with small  $L^\infty$  norm, we show that the numerical range is a non-tangential approach region on the disc and explicitly compute the aperture at the boundary point of this region. We relate this problem to the problem of finding the spectrum of a certain class of Jacobi matrices. (Received September 20, 2016)

## 49 ► *Calculus of variations and optimal control; optimization*

1125-49-757 **Deepmala\*** (dmrai23@gmail.com), Indian Statistical Institute, 203 B. T. Road, Kolkata, W. Bengal 700108, India, and **R. N. Mohapatra** (ram.mohapatra@ucf.edu), Mathematics Department, 4000 Central Florida Blvd., Orlando, FL 32816. *Mathematical Programming Approach Applied to Solve Aircraft Recovery Problems*.

The airline industry is one of the most successful examples of applying operations research methods and tools for the planning and scheduling of resources. Flight irregularity is a serious and widespread problem all over the world which involves significant cost to airlines, passengers and the society. Due to the associated costs, the use of efficient and accurate recovery process is of importance to the airline industry. Aircraft Recovery Problem (ARP) arises when an existing flight schedule has been disrupted due to some unforeseen events impacting the scheduled operational performance. Disruption results in a significant increase in an airline's operational costs related to crew overtime and increased fuel usage. Optimization theory plays an important role to get feasible, cost minimizing plans that allows the airlines to recover from the disruptions and their associated delays. Aircraft recovery is a challenging problem in both industrial and academic fields, and advanced mathematical programming methods are important in this context. In this talk, we give a brief survey and propose a model for the aircraft recovery problem. Finally, we indicate the future course of research in this context (Received September 10, 2016)

1125-49-795 **Mahdi Soltanolkotabi\***, 3740 McClintock Avenue, Los Angeles, CA 90089. *Breaking sample complexity barriers via nonconvex optimization?*

In the past decade there has been significant progress in understanding when convex relaxations are effective for finding low complexity models from a near minimal number of data samples (e.g. sparse/low rank recovery from a few linear measurements). Despite such advances convex optimization techniques are often prohibitive in practice due to computational/memory constraints. Furthermore, in some cases convex programs are suboptimal in terms of sample complexity and provably require significantly more data samples than what is required to uniquely specify the low complexity model of interest. In fact for many such problems certain sample complexity "barriers" have emerged so that there are no known computationally tractable algorithm that can beat the sample complexity achieved by such convex relaxations. Motivated by a problem in imaging, In this talk I will discuss my recent results towards breaking such barriers via natural nonconvex optimization techniques. (Received September 11, 2016)

1125-49-807 **Frank Morgan\*** (fmorgan@williams.edu). *Least-perimeter Tiles of the Hyperbolic Plane*. In 2000 Thomas Hales proved that a regular hexagon is the least-perimeter tile of the Euclidean plane for given area. In the hyperbolic plane, what is the least-perimeter tile of given area? Since there is no scaling in the hyperbolic plane, for each area this is a different problem. There are regular n-gonal tiles for all n, but none have been proved to minimize perimeter. The optimal triangular tile cannot always be equilateral; is it isosceles? Perhaps for some areas the solution is not unique?

There has been some progress by my 2016 Williams College NSF SMALL undergraduate research Geometry Group—Leo Di Giosia, Jay Habib, Lea Kenigsberg, Dylanger Pittman, and especially Weitao Zhu <wz1@williams.edu>, who is talking about it at this meeting. (Received September 12, 2016)

1125-49-1345 **Tiziana Giorgi\*** (tgiorgi@nmsu.edu). *Analysis of a one-dimensional Landau-de Gennes model for bent-core liquid crystals.*

We consider an energy functional used in the physics literature to model the switching mechanism seen in a columnar phase of bent-core molecule liquid crystals. In this phase, it is possible to reorient the spontaneous polarization by applying an electric field, the reorientation can be achieved by either a rotation around the smectic cone or the molecular axis, or a combination of both. Building on previous results, we derive the closed form of the  $\Gamma$ -limit in the large column regime. (Received September 16, 2016)

1125-49-1481 **Viktoria Taroudaki\*** (victtar@uw.edu), 1959 NE Pacific St., Seattle, WA 98195, **Costas Smaragdakis** (kesmarag@gmail.com), Voutes University Campus, 70013 Heraklion, Crete, Greece, and **Michael Taroudakis** (taroud@uoc.gr), Voutes University Campus, 70013 Heraklion, Crete, Greece. *Statistical Near-Optimal Filtering Method with Application to Underwater Acoustics.* Preliminary report.

The signal characterization method suggested by Taroudakis et al (J.Acoust. Soc. Am. 119, 1396-1405 (2006)) based on the statistics of its 1-D wavelet transform coefficients and successfully applied for inverting acoustic signals in applications of acoustical oceanography, is sensitive to noise contamination of the signal but still, it provides good inversion results if an appropriate denoising strategy is applied. In this work the statistical signal characterization is applied to signals which are both blurred and noise contaminated. Deblurring of the signal is achieved by means of a technique introduced by Taroudaki and O' Leary (SIAM J. Sci. Comput. 37-6 A2947-A2968 (2015)) for image deblurring, and it is based on a statistical near optimal spectral filtering technique that takes advantage of the singular values of the approximated blurring matrix and the Picard Parameter of the signal that allows for estimation of the additive noise properties and estimation of the error. The study is extended to cases when no accurate knowledge of the blurring mechanism is available. It is shown by typical simulated experimental data, that the combination of deblurring and simple denoising strategies provide good results with respect to both signal characterization and subsequent inversions. (Received September 19, 2016)

1125-49-1486 **Michael McAsey\*** (mcasey@bradley.edu), 1501 W Bradley Ave, Bradley University, Peoria, IL 61625, and **Libin Mou**. *Tax Policy to Minimize the Gini Index.* Preliminary report.

The Gini index is a simple measure of inequity in the distribution of income (or other attributes) in a society. The index is a number between 0 and 1 measuring the area in the  $1 \times 1$  square between the line  $y = x$  (perfect equity) and the Lorenz curve of the society. The Lorenz curve  $L(p)$  is the fraction of the total income that the holders of the lowest  $p$ th fraction of income possess. The goal is to find a tax scheme to minimize the Gini index,  $G = 2 \int_0^1 [p - L(p)] dp$ , by raising the Lorenz curve. The re-distribution function  $q(x)$  of incomes normalized on  $[0, 1]$  gives the after-tax income; it satisfies (1)  $Bx \leq q(x) \leq Ax$ , (2)  $q'(x) \geq r \geq 0$ , and (3)  $q(x)/x$  is decreasing. The optimal  $q$  is a piecewise linear function that allows lower incomes to be most preserved ( $q(x) = Ax$  for  $x$  small), higher incomes to be least preserved ( $q(x) = Bx$  for  $x$  large) and has a linear transition between the two. (Received September 17, 2016)

1125-49-2288 **Pierre R. Marechal\*** (pr.marechal@gmail.com), Institut de Mathématiques de Toulouse, Université Paul Sabatier, 31400 Toulouse, France. *A general mollifier approach to the regularization of linear ill-posed problems.*

The use of mollifiers for the regularization of linear inverse problems finds its roots in the late 80's and early 90's. Two approaches have developed independently: the well known approximate inverses on the one hand, based on duality in Hilbert spaces, and Fourier synthesis on the other hand, which belongs to variational methods. Both approaches have in common that, prior to any technical choice, a target object is clearly defined in terms of the unknown true object: the initial ill-posed problem is replaced by that of recovering a smoothed version of the unknown object, smoothness being expressed in terms of convolution. Here, we propose a general construction for the variational approach which, in addition to extending the realm of applicability, also offers a lot of flexibility in the choice of the target object. We also provide convergence results for this approach. (Received September 20, 2016)

1125-49-2327 **Wilfrid D. Gangbo\*** ([wgangbo@math.ucla.edu](mailto:wgangbo@math.ucla.edu)), Mathematics Department, University of California, Los Angeles, Los Angeles, CA CA 90095-1, and **Bernard Dacorogna** and **Olivier Kneuss**. *Paths of minimal lengths on the set of exact differential  $k$ -forms*.

We initiate the study of optimal transportation of exact differential  $k$ -forms and introduce various distances as minimal actions. Our study involves dual maximization problems with constraints on the codifferential of  $k$ -forms. When  $k < n$ , only some directional derivatives of a vector field are controlled. This is in contrast with prior studies of optimal transportation of volume forms ( $k = n$ ), where the full gradient of a scalar function is controlled. Furthermore, our study involves paths of bounded variations on the set of  $k$ -currents. (Received September 21, 2016)

1125-49-2349 **Evan Cosgrove\*** ([ecosgrove2011@my.fit.edu](mailto:ecosgrove2011@my.fit.edu)), 1081 Hoyt Ct. NE, Palm Bay, FL 32907, **Ugur G. Abdulla** ([abdulla@fit.edu](mailto:abdulla@fit.edu)), 150 W University Blvd, Melbourne, FL 32901, and **Jonathan Goldfarb** ([jgoldfar@my.fit.edu](mailto:jgoldfar@my.fit.edu)), 150 W University Blvd, Melbourne, FL 32901. *Optimal Control of the Coefficients in Second Order Parabolic Free Boundary Problems*.

Inverse Stefan type free boundary problem for the second order parabolic PDEs with unknown coefficient is considered. Optimal control framework is employed where coefficient of the PDE and free boundary are components of the control vector. We prove the Frechet differentiability in Besov spaces and derive the formula for the Frechet gradient under the minimal regularity assumptions on the data. Necessary condition for the optimality is formulated and projective gradient method in Besov-Hilbert spaces framework is implemented. Numerical results of model examples are presented. (Received September 20, 2016)

1125-49-2374 **Isabelle Kemajou-Brown\*** ([elisabeth.brown@morgan.edu](mailto:elisabeth.brown@morgan.edu)), 1700 E Cold Spring Lane, Baltimore, MD 21084, **Olivier Menoukeu-Pamen**, Liverpool, United Kingdom, and **Zhongyang Sun**, Tianjin 300071, Tianjin, 300071, Peoples Rep of China. *A Risk-Sensitive Maximum Principle for Markov Regime-Switching Jump-Diffusion Systems and Applications*.

In this paper, we derive a general stochastic maximum principle for a risk-sensitive type optimal control problem of Markov regime-switching jump-diffusion processes. The results are obtained via a logarithmic transformation and the relationship between adjoint variables and the value function. We apply the results to study both a linear-quadratic optimal control problem and a risk-sensitive benchmarked asset management problem for Markov regime-switching models. (Received September 20, 2016)

1125-49-2503 **Mila Nikolova\*** ([nikolova@cmla.ens-cachan.fr](mailto:nikolova@cmla.ens-cachan.fr)), CMLA, CNRS, ENS Cachan, Universite Paris-Saclay, 94235 Cachan, France. *Bayesian statistics in variational inverse problems*. Preliminary report.

We address inverse problems where an unknown signal or image has to be recovered from noisy and often incomplete data. Typical solutions are defined as minimizers of variational objectives where a data term is based on the log-likelihood and prior information is incorporated in a regularization term. These variational objectives are conceived using several approaches – PDE's, regularization, statistics, along with some adjustments. Commonly, the designed variational objectives are presented in a Bayesian estimation framework.

We focus on the Bayesian setup in variational methods. Bayesian estimators minimise the Bayes' risk which includes a posterior model and a loss function. The popular 0-1 loss leads to the MAP (maximum a posteriori) estimator. MAP is directly related to variational objectives. A pitfall of the MAP interpretation of variational objectives is that knowledge on the data model and the prior is usually violated. Another loss is quadratic yielding the posterior mean (PM) estimator. The PM solution with a given non-normal prior equals the MAP solution for a different prior. Other distortions arise with discretization refinement. Current Bayesian modelling in inverse problems deserves revision. We present state of the art results and suggest further directions. (Received September 20, 2016)

1125-49-2794 **Ahmad R Almomani\*** ([almomaar@clarkson.edu](mailto:almomaar@clarkson.edu)), Clarkson Ave. Box 5815, Potsdam, NY 13699. *Constraint Handling for Wireless Energy Transfer Application with filter Particle Swarm Optimization*.

When the derivative is hard to compute or unavailable in most real-world, then we use Derivative-Free Optimization(DFO) solvers. The presence of constraints brings difficulties in searching step since the search space has to be restricted to a feasible region. We introduce new algorithm for global method that combine filter method for constraints with Particle Swarm Optimization (PSO) method. We apply the new algorithm on Wireless Energy Transfer and compare it with current solvers using performance and data profiles. (Received September 20, 2016)

1125-49-2928 **Ram N. Mohapatra\*** ([ram.mohapatra@ucf.edu](mailto:ram.mohapatra@ucf.edu)), Department of Mathematics, University of Central Florida, Orlando, FL 32816, and **Ram U. Verma** ([verma99@msn.com](mailto:verma99@msn.com)), Department of Mathematics, University of North Texas, Denton, TX. *Hybrid Classes of Duality Models for Discrete Minmax Fractional Programming Problems.*

Second order duality for Minmax problems have been studied by various researchers in a variety of situations. In this paper a new class of second-order duality models for discrete minmax fractional programming problems is introduced. Application to strong duality, weak duality and converse duality are also considered. (Received September 20, 2016)

1125-49-3007 **Rachid Ait Maalem Lahcen\*** ([rachid@ucf.edu](mailto:rachid@ucf.edu)), Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Optimal methods to withstand cyber attacks: two case studies.*

Cyber security is of paramount importance for individuals, states and nations at large. With the mushrooming of attackers, there is considerable research on determining motives behind such attacks. In this paper we have studied geo-physical crime mappings and cyber attacks on Ukrainian power grids. We shall also mention about optimal packet scan against malicious attacks on smart grids. There is considerable evidence that identification of different factors in a complex system may require Minmax technique from optimization Theory. (Received September 20, 2016)

## 51 ► Geometry

1125-51-5 **Tanvir Prince\*** ([tprince@hostos.cuny.edu](mailto:tprince@hostos.cuny.edu)), 500 Grand Concourse, B 408, New York City, NY 10451. *Is there a Simpson's line for a quadrilateral?*

Given four lines in general position (none are parallel to each other, no three are concurrent – that is we have six different point of intersection in the plane), we show that there exist an unique point in the plane whose feet of the perpendicular to these four lines lie on a straight line.

This is an extension from the case of three line where any point on the circumcircle of a triangle has the property that the feet of the perpendicular to three sides of the triangle lie on a line. (Received May 02, 2016)

1125-51-178 **Barry Minemyer\*** ([minemyer.1@osu.edu](mailto:minemyer.1@osu.edu)), Columbus, OH 43210. *Approximating continuous maps by isometries.*

The Nash-Kuiper Theorem states that the collection of  $C^1$ -isometric embeddings from a Riemannian manifold  $M^n$  into  $\mathbb{E}^N$  is  $C^0$ -dense within the collection of all smooth 1-Lipschitz embeddings provided that  $n < N$ . This result is now known to be a consequence of Gromov's more general  $h$ -principle. There have been some recent extensions of the Nash-Kuiper Theorem to Euclidean polyhedra, which in some sense provide a very specialized discretization of the  $h$ -principle. In this talk we will discuss these recent results and provide generalizations to the setting of isometric embeddings of spaces endowed with indefinite metrics into Minkowski space. The new observation is that, when dealing with Minkowski space, the assumption "1-Lipschitz" can be removed. Thus, we obtain results about isometric embeddings that are  $C^0$ -dense within the collection of *all* continuous maps. (Received August 09, 2016)

1125-51-369 **Lina Wu\*** ([lwu@bmcc.cuny.edu](mailto:lwu@bmcc.cuny.edu)), 529 West 42nd Street, Apt. 5K, New York, NY 10036. *History of Geometry's Origin and Development and Connection to Various Mathematical Fields.* Preliminary report.

Applying Geometry in daily life dated back to ancient Egyptian period. At approximately 3,000 BC, Egyptians began using Geometry to solve many difficult unanswered problems such as surveying of land, construction of buildings and astronomy related to "Earth Measurement". Ancient Geometry was used during early Egyptian, Greek, Indian and Chinese civilization. From 9th century thru 10th century, the use of Geometry was popular in Islamic Art Design. From 11th century thru 15th century, Sacred Geometry was established as a special geometry in Religious Art. From 14th century thru 17th century, Geometry was generally used in Renaissance Western Art. From 18th century thru 19th century, the creation of Non-Euclidean Geometry became the origin of Modern Geometry. The creation of Differential Geometry came from the connection between Geometry and Calculus. The origin of Algebraic Geometry was the connection between Geometry and Algebra. The growing study of Geometry has contributed to the expanding development in geometric-related mathematical fields. (Received August 30, 2016)

1125-51-497 **Yih Sung\*** ([ysung26@wisc.edu](mailto:ysung26@wisc.edu)), Department of Mathematics, Van Vleck Tower 480 Lincoln Dr., Madison, WI 53706. *Rational Points over Finite Fields on A Family of Higher Genus Curves And Hypergeometric Functions.*

In this paper we investigate the relation between the number of rational points over a finite field  $\mathbb{F}_p^n$  on a family of higher genus curves and their periods in terms of hypergeometric functions. For the case  $y^\ell = x(x-1)(x-\lambda)$  we find a closed form in terms of hypergeometric functions associated with the periods of the curve. For the general situation  $y^\ell = x^{a_1}(x-1)^{a_2}(x-\lambda)^{a_3}$  we show that the number of rational points is a linear combination of hypergeometric series, and we provide an algorithm to determine the coefficients involved. (Received September 04, 2016)

1125-51-543 **Leonardo Di Giosia, Jahanger Habib, Lea Kenigsberg, Dylanger Pittman and Weitao Zhu\*** ([wz1@williams.edu](mailto:wz1@williams.edu)). *Isoperimetric Tiling in the Hyperbolic Plane.*

We seek a least-perimeter tile of the hyperbolic plane of given area. Goodman-Strauss proves for example that a non-isosceles triangle tiles the plane if a unique linear combination of the angles of a certain type equals  $2\pi$ . We generalize his theorem and show that there is a triangular tile of area  $A$  if and only if  $0 < A < \pi$ . Furthermore, we provide some results and conjectures on general polygonal tiles. (Received September 07, 2016)

1125-51-557 **Andrew Michael Sanders\*** ([sanderandy@gmail.com](mailto:sanderandy@gmail.com)). *The geometry of locally homogeneous complex manifolds.*

Given an Anosov subgroup  $\Gamma$  of a complex, semisimple Lie group  $G$ , the work of Guichard-Wienhard and Kapovich-Leeb-Porti furnishes cocompact domains of proper discontinuity in various flag varieties associated to  $G$ . The quotients of these domains give rise to locally homogeneous complex manifolds modelled on the flag variety. In this talk, we will discuss some features of the topology and geometry of these locally homogeneous manifolds. In particular, we will compute the cohomology ring and Picard group for a number of important examples. This is joint work with David Dumas. (Received September 06, 2016)

1125-51-589 **Jacob Shulkin\*** ([jshulkin@umich.edu](mailto:jshulkin@umich.edu)), 931 Greenwood Ave, Apt 4, Ann Arbor, MI 48104. *Collineation Maps Between Tori.*

The classical Fundamental Theorem of Affine Geometry states that for  $n \geq 2$ , any bijection of  $n$ -dimensional Euclidean space that maps lines to lines (as sets) is given by an affine map. Analogous statements have since also been proven in projective and hyperbolic geometry. We consider the space-form analogue of the above problem, and solve it for tori: Any bijection of an  $n$ -dimensional torus ( $n \geq 2$ ) that maps lines to lines, is given by an affine map. This talk is based on joint work with Wouter Van Limbeek (University of Michigan). (Received September 07, 2016)

1125-51-605 **Dmitri Tymoczko\*** ([dmitri@princeton.edu](mailto:dmitri@princeton.edu)), 310 Woolworth Center, Princeton, NJ 08544. *Voice Leading and Composition.*

Musical chords can be represented as points in orbifolds, quotient spaces arising from listeners' relative indifference to octave and note order. "Voice leadings" correspond to vectors in these spaces and represent ways of moving between chords. Understanding these spaces is a central issue for composers and theorists. In particular, we have reason to ask whether there are sensible, non-trivial, and generally applicable techniques for working within the space of all possible chords—that is principled and aurally meaningful compositional strategies more general than the limited idioms of existing styles.

In my talk I describe progress on this question, detailing a large network of isomorphisms relating almost all possible chords in almost all possible scales. Using these isomorphisms it is possible to transfer musical techniques among almost arbitrary harmonic domains. I illustrate with excerpts from "The Thousand Faces of Form," an orchestral piece with live visual accompaniments reflecting the geometrical structure of its harmonies. (Received September 07, 2016)

1125-51-609 **Sergii Myroshnychenko\*** ([smyroshn@kent.edu](mailto:smyroshn@kent.edu)), Kent State University, Department of Mathematical Sciences, Summit St, Kent, OH 44242, and **Dmitry Ryabogin** ([ryabogin@math.kent.edu](mailto:ryabogin@math.kent.edu)). *On polytopes with congruent projections or sections.*

Let  $2 \leq k \leq d-1$  and let  $P$  and  $Q$  be two convex polytopes in  $\mathbb{E}^d$ . Assume that their projections,  $P|H$ ,  $Q|H$ , onto every  $k$ -dimensional subspace  $H$ , are congruent. We show that  $P$  and  $Q$  or  $P$  and  $-Q$  are translates of each other. We also prove an analogous result for sections by showing that  $P = Q$  or  $P = -Q$ , provided the polytopes contain the origin in their interior and their sections,  $P \cap H$ ,  $Q \cap H$ , by every  $k$ -dimensional subspace  $H$ , are congruent. (Received September 07, 2016)

1125-51-610 **Sergii Myroshnychenko\*** ([smyroshn@kent.edu](mailto:smyroshn@kent.edu)), Kent State University, Department of Mathematical Sciences, Summit St, Kent, OH 44242. *On a functional equation related to a pair of hedgehogs with congruent projections.*

Hedgehogs are geometrical objects that describe the Minkowski differences of arbitrary convex bodies in the Euclidean space  $\mathbb{E}^n$ . We prove that two hedgehogs in  $\mathbb{E}^n$ ,  $n \geq 3$ , coincide up to a translation and a reflection in the origin, provided that their projections onto any two-dimensional plane are directly congruent and have no direct rigid motion symmetries. Our result is a consequence of a more general analytic statement about the solutions of a functional equation in which the support functions of hedgehogs are replaced with two arbitrary twice continuously differentiable functions on the unit sphere. (Received September 07, 2016)

1125-51-732 **Samuel A Ballas\*** ([ballas@math.fsu.edu](mailto:ballas@math.fsu.edu)), **Daryl Cooper** and **Arielle Leitner**. *Classification of generalized cusps.*

Roughly speaking, a generalized cusp is a properly convex manifold with virtually nilpotent fundamental group that admits a foliation by strictly convex hyper surfaces. A motivating example is an end of a finite volume hyperbolic manifold. In this talk we will describe the geometry of these generalized cusps and describe their moduli space. This is joint work with Daryl Cooper and Arielle Leitner. (Received September 10, 2016)

1125-51-750 **Leo Di Giosia, Jahanger Habib** and **Lea Kenigsberg\***, [Lea.kenigsberg@stonybrook.edu](mailto:Lea.kenigsberg@stonybrook.edu), and **Weitao Zhu** and **Dylinger Pittman**. *Isoperimetry in Euclidean Space with Different Perimeter and Volume Densities.*

The Euclidean log convex density theorem, proved by Gregory Chambers, asserts that in Euclidean space with a log-convex density spheres about the origin are perimeter minimizing. We prove a conjecture of Alvino et al., extending this result to appropriate perimeter density  $r^k$  and volume density  $r^m$ . We seek to extend this result further to other perimeter and volume densities. (Received September 10, 2016)

1125-51-754 **Serge Tabachnikov\*** ([tabachni@math.psu.edu](mailto:tabachni@math.psu.edu)). *Skewers.*

The skewer of a pair of skew lines in space is their common perpendicular. To configuration theorems of plane projective geometry involving points and lines (such as Pappus or Desargues) there correspond configuration theorems in space: points and lines in the plane are replaced by lines in space, the incidence between a line and a point translates as the intersection of two lines at right angle, and the operations of connecting two points by a line or by intersecting two lines at a point translate as taking the skewer of two lines. These configuration theorems hold in elliptic, Euclidean, and hyperbolic geometries. This correspondence principle extends to plane configuration theorems involving polarity. For example, the theorem that the three altitudes of a triangle are concurrent corresponds to the Petersen-Morley theorem that the common normals of the opposite sides of a space right-angled hexagon have a common normal. I shall discuss two approaches to the proof, an elliptic and a hyperbolic one. I shall also discuss the skewer versions of the Sylvester problem: given a finite collection of pairwise skew lines such that the skewer of any pair intersects at least one other line at right angle, do all the line have to share a skewer? (Received September 10, 2016)

1125-51-840 **Christopher Allen Manon\*** ([cmanon@gmu.edu](mailto:cmanon@gmu.edu)), 4400 University Drive, MS: 3F2, Exploratory Hall, Fairfax, VA 22030. *Contraction of a Hamiltonian K-space.*

I will describe how to construct the contraction  $X_0$  of a Hamiltonian  $K$ -space  $X$ . In terms of symplectic and algebraic geometry, the contraction  $X_0$  is very similar to  $X$  yet it comes equipped with a Hamiltonian  $K \times T$  action for  $T \subset K$  a maximal torus. I'll also discuss how contraction emerges algebraically from the horospherical contraction operation of Popov, and its relationship to recent work of Harada and Kaveh on Newton-Okounkov bodies. This is joint work with Joachim Hilgert and Johan Martens. (Received September 12, 2016)

1125-51-1070 **Sl-ghi Choi\*** ([sl-ghi.choi@mavs.uta.edu](mailto:sl-ghi.choi@mavs.uta.edu)), 411 South Nedderman Drive, 478 Pickard Hall, Arlington, TX 76019. *Image reconstruction from limited view Radon data using GPCA.* Preliminary report.

Image reconstruction in various types of tomography requires inversion of the Radon transform and its generalizations. While there are many stable and robust algorithms for such inversions from reasonably well sampled data, most of these algorithms fail when applied to limited view data. The talk discusses a method of stable reconstruction from limited view data for functions, whose support is a union of finitely many circles. Our method is based on a modified version of GPCA (General Principle Component Analysis) and some results from algebraic geometry. (Received September 14, 2016)

1125-51-1372 **Leonardo DiGiosia, Jahangir Habib, Lea Kenigsberg and Dylanger Pittman\*** ([dsp1@williams.edu](mailto:dsp1@williams.edu)), 1436 Paresky, Williamstown, MA 01267, and **Weitao Zhu**. *Double Bubbles In Borell Space*. Preliminary report.

The Double Bubble Theorem says that the least-perimeter way to enclose and separate two prescribed volumes in  $\mathbb{R}^N$  is the standard double bubble, consisting of three spherical caps meeting at 120 degrees. We seek the optimal double bubble in Borell Space,  $\mathbb{R}^N$  with density  $e^{r^2}$ . For  $N = 1$  we show that the solution is sometimes two contiguous intervals and sometimes three contiguous intervals. In higher dimensions we think that the solution is sometimes a deformed standard double bubble (tending to a sphere plus bisecting disc for large volumes) and sometimes concentric spheres (e.g. for one volume small and the other large). (Received September 16, 2016)

1125-51-1397 **E Cabral Balreira\***, One Trinity Place, Department of Mathematics, San Antonio, TX 78201, and **Saber Elaydi** and **Rafael Luis**. *Global Dynamics and Geometry of Competitive Maps*. Preliminary report.

We introduce a new geometric notion of monotonicity that generalizes the standard ideas of planar competition maps. We develop new methods to understand the global dynamics of monotone maps and characterize basin of attraction of fixed points. (Received September 16, 2016)

1125-51-1498 **Edgar A Bering, Gabriel Conant and Jonah Gaster\*** ([gaster@bc.edu](mailto:gaster@bc.edu)). *On the complexity of finite subgraphs of the curve graph*.

We say a graph has property  $\mathcal{P}_{g,p}$  when it is an induced subgraph of the curve graph of a surface of genus  $g$  with  $p$  punctures. Two well-known graph invariants, the chromatic and clique numbers, can provide obstructions to  $\mathcal{P}_{g,p}$ . We introduce a new invariant of a graph, the *nested complexity length*, which provides a novel obstruction to  $\mathcal{P}_{g,p}$ . For the curve graph this invariant captures the topological complexity of the surface in graph-theoretic terms; indeed we show that its value is  $6g - 6 + 2p$ , i.e. twice the size of a maximal multicurve on the surface. As a consequence we show that large ‘half-graphs’ do not have  $\mathcal{P}_{g,p}$ , and we deduce quantitatively that almost all finite graphs which pass the chromatic and clique tests do not have  $\mathcal{P}_{g,p}$ . We also reinterpret our obstruction in terms of the first-order theory of the curve graph, and in terms of RAAG subgroups of the mapping class group (following Kim and Koberda). Finally, an examination of multipartite subgraphs allows us to compute the upper density of the curve graph, and to conclude that clique size, chromatic number, and nested complexity length are not sufficient to determine  $\mathcal{P}_{g,p}$ . (Received September 17, 2016)

1125-51-1536 **Giuseppe Martone\*** ([gmartone@usc.edu](mailto:gmartone@usc.edu)) and **Tengren Zhang**. *Positively ratioed representations*.

Anosov representations are specific homomorphisms from the fundamental group of a closed surface with negative Euler characteristic to a Lie group. We study certain length functions that arise in this setting. Positively ratioed representations are Anosov and they satisfy a positivity property with respect to one such length function. Examples are Hitchin and maximal representations. In this context, we relate the length of a shortest simple closed curve (which we show it exists) to the topological entropy of the length function. This is joint work with Tengren Zhang. (Received September 17, 2016)

1125-51-1958 **David A Herron\*** ([david.a.herron@uc.edu](mailto:david.a.herron@uc.edu)) and **Stephen M Buckley**. *Quasi-Hyperbolic Geodesics are Hyperbolic Quasi-Geodesics*. Preliminary report.

Each open connected planar region  $\Omega$  with two or more boundary points carries a unique maximal constant curvature metric deemed its Poincaré hyperbolic metric, and the length distance induced on  $\Omega$  gives rise to a non-Euclidean model of geometry called hyperbolic geometry in  $\Omega$ . Understanding hyperbolic geometry has important consequences in complex analysis and dynamics, quasiconformal mapping theory, Teichmüller theory, and scores of other areas.

The hyperbolic metric is notoriously difficult to compute, and hyperbolic distances as well as hyperbolic geodesics are even harder to determine. Fortunately there is a substitute, the quasi-hyperbolic metric, and in many instances—not all—quasi-hyperbolic geometry is bi-Lipschitz equivalent to hyperbolic geometry. But, how similar are these geometries?

We explain our ideas which show that in *any* hyperbolic plane domain, the hyperbolic and quasi-hyperbolic quasi-geodesics are exactly the same curves. Our techniques permit us to establish the marvelous fact that these two geometries are simultaneously Gromov hyperbolic (or not). Nonetheless, there are some important differences. (Received September 19, 2016)

1125-51-2040 **R Goldin\*** ([rgoldin@gmu.edu](mailto:rgoldin@gmu.edu)), MS 3F2, 4400 University Drive, Fairfax, VA 22039. *Positivity in Schubert Calculus, with Symplectic Analogs.*

Schubert calculus is the study of (complex) intersections of a specific set of varieties in the flag manifold; the intersection numbers and their generalizations to equivariant cohomology and K-theory are manifestly positive, defined in an appropriate sense. I will present results toward finding positive formulas for (some of) the structure constants in the corresponding rings, with generalizations to the symplectic category. Part of this work is joint with Allen Knutson, and part with Susan Tolman. (Received September 19, 2016)

1125-51-2060 **Julius L. Shaenson\***, DRL Labs, 209 S. 33rd street, Philadelphia, PA 19096. *Topological String Junctions.*

(from joint work with A. Grassi, J. Halverson, W. Taylor, F. Ruehle) The method of topological string junctions is a mathematical interpretation of something from F-theory in physics in the context of Calabi-Yau spaces used to represent possible physical theories. This talk will mention some approaches to understand higher dimensional situations that have arisen or might arise, using topological string junctions and deformations, in particular to determine Lie algebras and representations associated to various 3-folds of interest. (Received September 19, 2016)

1125-51-2305 **Elisheva Adina Gamse\*** ([adina.gamse@utoronto.ca](mailto:adina.gamse@utoronto.ca)). *Vanishing theorems in the cohomology ring of the moduli space of parabolic bundles.*

Let  $\Sigma$  be a compact connected oriented 2-manifold of genus  $g$ , and let  $p$  be a point on  $\Sigma$ . We define a space  $S_g(t)$  consisting of certain irreducible representations of the fundamental group of  $\Sigma \setminus p$ , modulo conjugation by  $SU(N)$ . This space has interpretations in algebraic geometry, gauge theory and topological quantum field theory; in particular if  $\Sigma$  has a Kähler structure then  $S_g(t)$  is the moduli space of parabolic vector bundles of rank  $N$  over  $\Sigma$ .

For  $N=2$ , Weitsman considered a tautological line bundle on  $S_g(t)$ , and proved that the  $(2g)$ th power of its first Chern class vanishes, as conjectured by Newstead. In this talk I will present his proof and then outline my extension of his work to  $SU(N)$  and to  $SO(2n+1)$ . (Received September 20, 2016)

1125-51-2385 **Ellie Dannenberg\***, MSCS Department UIC, 322 Science and Engineering Offices (MC 249), 851 S Morgan St, Chicago, IL 60607. *Circle Packings on Surfaces with Complex Projective Structures.*

The classical circle packing theorem of Koebe, Andreev, and Thurston says that given a triangulation  $\tau$  of a closed, orientable surface, there is a unique constant curvature metric on the surface so that the surface with this metric admits a circle packing with dual graph  $\tau$ . Circles are also key objects in studying complex projective surfaces. Kojima, Mizushima, and Tan give a definition of a circle packing on such a surface. Unlike in the metric case, there is a deformation space of complex projective circle packings with combinatorics given by  $\tau$ . This space is parametrized by cross ratio type coordinates. Kojima, Mizushima, and Tan describe this space in the case where  $\tau$  has 1 vertex. I expand some of their results to classes of higher vertex triangulations. (Received September 20, 2016)

1125-51-2484 **Kathryn Leonard\*** ([kleonard.ci@gmail.com](mailto:kleonard.ci@gmail.com)), 1 University Dr, Camarillo, CA 93012. *Modeling Shapes of Objects with Undergraduates.*

Mathematical modeling of shapes of 2D objects is crucial for building artificial vision systems that mimic human capability. The topic offers a wide range of interesting problems for research-level mathematicians and undergraduate students alike. We present a few research-level questions and describe some related undergraduate-level projects ranging from theoretical to experimental to applied. We also share a time-tested philosophy for identifying suitable initial projects for new undergraduate researchers. (Received September 20, 2016)

1125-51-2532 **William Soller** ([sollerw@southwestern.edu](mailto:sollerw@southwestern.edu)) and **Kristen McCrary\*** ([mccraryk@southwestern.edu](mailto:mccraryk@southwestern.edu)). *Existence, Uniqueness, and Cost-Optimizing Results of Mathematical Trusses.*

Mathematical trusses are abstract geometric objects which are applied to model or design various physical structures. Truss research typically involves questions of construction (existence and uniqueness of truss structures) as well as optimization (minimizing the cost of the truss). In this work, we present necessary and sufficient conditions for existence and uniqueness of certain restricted trusses (called grid trusses), as well as arguments for non-uniqueness in other grid trusses. We also apply calculus of variations techniques to explore the cost-reducing effect of cutting corners on four-beam trusses. This work extends known results on three-beam corner trusses. (Received September 20, 2016)

1125-51-2537 **Osman Yardimci\*** (ozy0003@tigermail.auburn.edu), 221 Parker Hall, Auburn, AL 36849. *On the Principle of Planimeters.*

We studied the mathematics of various mechanical devices, called planimeters, which before the time of computers were used to determine the area of an arbitrary two-dimensional shape. Numerous papers were written on this subject. By combining the intuitive and rigorous approaches, we explained the reason behind the linear and polar planimeters in a way accessible for students with basic calculus background. We also considered variants of the planimeters to measure other quantities of the planar regions. (Received September 20, 2016)

1125-51-2735 **Hiraku Abe\*** (hirakuabe@globe.ocn.ne.jp), **Lauren DeDieu**, **Federico Galetto** and **Megumi Harada**. *Flat families of Hessenberg varieties with an application to Newton-Okounkov bodies.* Preliminary report.

Hessenberg varieties are subvarieties of the full flag variety. In this talk, I will concentrate on Lie type A. I will talk about a flat degeneration of a regular semisimple Hessenberg variety to a regular nilpotent Hessenberg variety whose special fiber is reduced, and I will explain how we can use this flat family to compute some Newton-Okounkov bodies of the Peterson variety of dimension 2. Along the way, we will also see that any regular nilpotent Hessenberg variety is a local complete intersection; this is a generalization of a result in Erik Insko's PhD thesis. This is a joint work with Lauren DeDieu, Federico Galetto, and Megumi Harada. (Received September 20, 2016)

1125-51-2802 **Rachel A. Neville\*** (neville@math.colostate.edu), **Patrick Shipman**, **Mark Bradley**, **Francis Motta** and **Daniel Pearson**. *Topological Measure of Order on Lattice Patterns.*

Pattern forming systems can give rise to perfect or near perfect lattice patterns. For instance, bombarding a binary surface with a broad ion beam can lead to hexagonal arrays of nanodots. It is necessary to be able to measure the degree of order present in a pattern to identify not only variation in the pattern, but also defects. With D. Pearson, M. Bradley, F. Motta, and P. Shipman, we propose a topological method relying on persistent homology to measure the degree of order present in a pattern. There are several standard methods of measuring order using Fourier methods, the autocorrelation function and a nearest neighbor measure. We will compare these methods with our topological method for various lattice patterns. (Received September 20, 2016)

## 52 ► Convex and discrete geometry

1125-52-61 **Metin Alper Gur\*** (mgur@indiana.edu), 831 East 3rd Street, Bloomington, IN 47405. *Hypersurfaces with central convex cross-sections.*

The compact transverse cross-sections of a cylinder over a central ovaloid in  $\mathbb{R}^n$ ,  $n \geq 3$ , with hyperplanes are central ovaloids. A similar result holds for quadrics (level sets of quadratic polynomials in  $\mathbb{R}^n$ ,  $n \geq 3$ ). Their compact transverse cross-sections with hyperplanes are ellipsoids, which are central ovaloids.

In  $\mathbb{R}^3$ , Blaschke, Brunn, and Olovjanischnikoff found results for compact, convex surfaces that motivated B. Solomon to prove that these two kinds of examples provide the only complete, connected, smooth surfaces in  $\mathbb{R}^3$ , whose ovaloid cross-sections are central. We generalize that result to all higher dimensions, proving: *If  $M^{n-1} \subseteq \mathbb{R}^n$ ,  $n \geq 4$ , is a complete, connected, smooth hypersurface, which intersects at least one hyperplane transversally along an ovaloid, and every such ovaloid on  $M$  is central, then  $M$  is either a cylinder over a central ovaloid or a quadric.* (Received July 03, 2016)

1125-52-250 **T. Bisztriczky\*** (tbisztri@ucalgary.ca). *On triangulations of simple polygons.* Preliminary report.

We present a brief survey of such triangulations, and revisit one (equidiameter) stemming from recent joint work with A. Bezdek. (Received August 18, 2016)

1125-52-340 **Déborah Oliveros\*** (dolivero@matem.unam.mx), Área de la Investigación Científica, Circuito Exterior, Ciudad Universitaria., Coyacan, 04510 México, Mexico. *Variants on Helly's Theorems over Algebraic Subsets of  $\mathbb{R}^d$ .* Preliminary report.

We will discuss some interesting behavior of Helly type theorems over different subsets in  $\mathbb{R}^d$  in particular, some extensions of prior work in  $\mathbb{Z}^d$  or in  $\mathbb{Z}^{d-k} \times \mathbb{R}^k$ , etc. as well as consider the situation for some algebraic structures, in particular for arbitrary subgroups of  $\mathbb{R}^d$  or for the difference between a lattices and some of its sublattices. (Received August 28, 2016)

1125-52-432 **Galyna V Livshyts\*** ([glivshyts6@math.gatech.edu](mailto:glivshyts6@math.gatech.edu)), Georgia Institute of Technology, Department of Mathematics, 686, Cherry st NW, Atlanta, GA 30318. *On an extension of Minkowski's theorem for measures.*

The Minkowski theorem asserts that every centered measure on the unit sphere which is not concentrated on any great subsphere is the surface area measure of the (unique) convex body.  $L_p$ -Brunn-Minkowski theory has called for extensions of this theorem in which the surface area measure is replaced, for example, by cone volume measure of a convex body. Borozcky, Lutwak, Stancu, Saraglou, Yang, Zhang, and many others have contributed to the study of this topic.

In this talk we discuss another natural extension of Minkowski's theorem, in which the surface area measure is replaced by the surface area measure with respect to an underlying measure in  $\mathbb{R}^n$ , with certain concavity and homogeneity properties. This new theorem has several consequences. Firstly, it helps to establish uniqueness and existence of a solution of certain PDE in the class of even support functions of convex sets; this result is a weaker version of the Log-Minkowski conjecture. Secondly, we use this theorem to obtain an extension of the solution to Shephard's problem for some measures, after extending the notion of a projection appropriately. Thirdly, we prove an analogue of Aleksandrov's theorem about unique determination of a symmetric convex body with areas of its projections for certain measures. (Received September 01, 2016)

1125-52-459 **Alexey Garber\*** ([alexey.garber@utrgv.edu](mailto:alexey.garber@utrgv.edu)). *Helly numbers for crystals and cut-and-project sets.*

Helly number  $h(S)$  of a set  $S \subseteq \mathbb{R}^d$  is the smallest positive integer  $n$  such that, if any  $n$  sets from a finite family of convex sets intersect at point of  $S$ , then all sets from the same family intersect at point of  $S$ .

Helly numbers were studied for different point sets, in particular it was proven by Helly that  $h(\mathbb{R}^d) = d + 1$ , and it was proven by Doignon that  $h(\mathbb{Z}^d) = 2^d$ .

In this talk we will prove existence of Helly numbers for each periodic discrete point set (crystal) and for certain quasiperiodic point sets (cut-and-project sets). (Received September 02, 2016)

1125-52-460 **Alexey Glazyrin\*** ([alexey.glazyrin@utrgv.edu](mailto:alexey.glazyrin@utrgv.edu)), One West University Blvd, School of Mathematical & Statistical Sciences, LHSB 2.520, Brownsville, TX 78520. *Contact graphs of ball packings.*

I will talk about packings of Euclidean balls with possibly different radii. As the main result, I will show how to obtain a new upper bound for the average kissing number in the three-dimensional case. (Received September 02, 2016)

1125-52-468 **Oleg R Musin\*** ([oleg.musin@gmail.com](mailto:oleg.musin@gmail.com)), One West University blvd, Mathematics, LHSB 2.522, Brownsville, TX 78520. *Minimal spherical representation of graphs.*

Any graph  $G$  can be embedded in a Euclidean space as a two-distance set with the minimum distance  $a$  if the vertices are adjacent and distance  $b$  otherwise. The Euclidean representation number of  $G$  is the smallest dimension in which  $G$  is representable. In this talk we consider spherical and  $J$ -spherical representation numbers of  $G$ . We give exact formulas for these numbers using multiplicities of polynomials that are defined by the Cayley-Menger determinant. We show that using W. Kuperberg's theorem the representation numbers can be found explicitly for the join of graphs. (Received September 02, 2016)

1125-52-525 **Jon Lee\*** ([jonxlee@umich.edu](mailto:jonxlee@umich.edu)). *Volume-based comparison for some polytopes arising in optimization.*

We look at using volume as a measure for comparing polytopal relaxations of some nonlinear sets that naturally arise in the spatial branch-and-bound approach to global optimization. By getting closed formulae for the volumes of various parametric families of relaxations, we give guidance for algorithm developers and users. This is joint work with Emily Speakman. (Received September 05, 2016)

1125-52-537 **S Brazitikos, S Dann\*** ([susanna.dann@tuwien.ac.at](mailto:susanna.dann@tuwien.ac.at)), **A Giannopoulos** and **A Koldobsky**. *On the average volume of sections of convex bodies.*

The average section functional  $as(K)$  of a centered convex body in  $\mathbb{R}^n$  is the average volume of the central hyperplane sections of  $K$ :

$$as(K) = \int_{S^{n-1}} |K \cap \xi^\perp| d\sigma(\xi).$$

We study the question if there exists an absolute constant  $C > 0$  such that for every  $n$ , for every centered convex body  $K$  in  $\mathbb{R}^n$  and for every  $1 \leq k \leq n - 1$ ,

$$as(K) \leq C^k |K|^{\frac{k}{n}} \max_{E \in \text{Gr}_{n-k}} as(K \cap E).$$

We observe that the case  $k = 1$  is equivalent to the hyperplane conjecture. We show that this inequality holds true in full generality if one replaces  $C$  by  $CL_K$  or  $Cd_{\text{ovr}}(K, \mathcal{BP}_k^n)$ , where  $L_K$  is the isotropic constant of  $K$  and  $d_{\text{ovr}}(K, \mathcal{BP}_k^n)$  is the outer volume ratio distance from  $K$  to the class  $\mathcal{BP}_k^n$  of generalized  $k$ -intersection bodies. We also compare  $\text{as}(K)$  to the average of  $\text{as}(K \cap E)$  over all  $k$ -codimensional sections of  $K$ . We examine separately the dependence of the constants on the dimension in the case where  $K$  is in some of the classical positions as well as the natural lower dimensional analogue of the average section functional. (Received September 06, 2016)

1125-52-612      **Michael N. Bleicher\*** ([bleicher@math.wisc.edu](mailto:bleicher@math.wisc.edu)), PMB 4369 PO BOX 2428, Pensacola, FL 32513. *Some problems in Geometry and its presentation.* Preliminary report.

We will discuss the following problems:

1. Isoperimetric problems.
2. Packing sets with  $n$ -similar set.
3. The Dunkel Winkel Problem.
4. The hospital guerney problem.

Also we will discuss the importance of good problems in the trrsching of Geometry (Received September 07, 2016)

1125-52-657      **Yoav Kallus\*** ([yoav@santafe.edu](mailto:yoav@santafe.edu)), Santa Fe Institute, 1399 Hyde Park Rd, Santa Fe, NM 87501. *Pessimal packing shapes.*

Stanislaw Ulam reportedly conjectured that spheres are the worst case for the optimal packing density among convex solids. The conjecture is curious because the same doesn't happen in 2D. I show that spheres are a local minimum of the optimal packing density among convex, centrally-symmetric shapes. Similar techniques also show that higher dimensional spheres are not local minima, that regular heptagons are a local minimum in 2D, and that the sphere is a local maximum for the optimal covering density. (Received September 08, 2016)

1125-52-800      **Tudor I. Zamfirescu\*** ([tuzamfirescu@gmail.com](mailto:tuzamfirescu@gmail.com)). *Discs held in cages.*

A *cage* is the 1-skeleton of a 3-dimensional polytope. In how many ways can a disc (2-dimensional ball) be held by a regular tetrahedral cage? And if the tetrahedron is not necessarily regular? And if we have a pentahedron instead? Such questions will be answered in this talk. (Received September 12, 2016)

1125-52-829      **Matthew Alexander\*** ([malexan5@kent.edu](mailto:malexan5@kent.edu)), **Martin Henk** and **Artem Zvavitch**. *A discrete version of Koldobsky's slicing inequality.*

In this talk we will discuss an answer to a question of Alexander Koldobsky and present a discrete version of his slicing inequality. We let  $\#K$  be a number of integer lattice points contained in a set  $K$ . We show that for each  $d \in \mathbb{N}$  there exists a constant  $C(d)$ , depending on  $d$  only, such that for any origin-symmetric convex body  $K \subset \mathbb{R}^d$  containing  $d$  linearly independent lattice points

$$\#K \leq C(d) \max_{\xi \in S^{d-1}} (\#(K \cap \xi^\perp)) \text{vol}_d(K)^{\frac{1}{d}},$$

where  $\xi^\perp$  is the hyperplane orthogonal to a unit vector  $\xi$ . We show that  $C(d)$  can be chosen asymptotically of order  $O(1)^d$  for hyperplane slices. Additionally, we will discuss some special cases and generalizations for this inequality. (Received September 12, 2016)

1125-52-904      **Woden Kusner\*** ([wkusner@gmail.com](mailto:wkusner@gmail.com)). *Critical packings, rigidity, and the radius function.* Preliminary report.

There are a number of classical problems in geometric optimization that ask for the "best" configuration of points with respect to some function. We are interested in the relationships between various notions of criticality for such functions on configuration spaces, in particular the injectivity or packing radius. This is not a Morse function, but it has been observed to be Morse-like, in that the topological notion of regularity can be defined in an analogous way. Furthermore, there is a geometric interpretation from rigidity theory that characterizes configurations as critical by the existence of a strut measure. (Received September 13, 2016)

1125-52-928      **Thomas Hales\*** ([hales@pitt.edu](mailto:hales@pitt.edu)), 416 Thackeray Hall, Math Department, University of Pittsburgh, Pittsburgh, PA 15260, and **Wöden Kusner**. *Packings of Regular Pentagons in the Plane.*

We determine a densest packing of congruent regular pentagons in the plane. More specifically, we prove the pentagonal ice-ray conjecture of Henley (1986), and Kuperberg and Kuperberg (1990), which asserts that an optimal packing of congruent regular pentagons in the plane is a double lattice, formed by aligned vertical columns of upward pointing pentagons alternating with aligned vertical columns of downward pointing pentagons.

This talk will go into some of the history of the packings of regular pentagons, including a packing described by Albrecht Dürer in 1525, the pentagonal ice-ray that appears in Chinese lattice design around 1900, and developments starting with Penrose tilings and quasi-crystals. Some of the main ideas of our computer-assisted proof will be described, including the use of a meet-in-the-middle algorithm, inspired from cryptography.

The preprint is on the arXiv (same title). (Received September 13, 2016)

1125-52-1202 **Włodzimierz Kuperberg\*** ([kuperw1@auburn.edu](mailto:kuperw1@auburn.edu)), Department of Mathematics and Statistics, Auburn University, Auburn, AL 36849. *Extensive parallelograms and double-lattice packings*. Preliminary report.

A parallelogram inscribed in a given convex disk  $K$  in the plane is *extensive* if each of its sides is at least as long as one-half of the affine diameter of  $K$  parallel to the side. A packing of the plane with congruent copies of  $K$  is a *double-lattice* packing if it is the union of two lattice packings, one by translates of  $K$ , and the other by translates of  $-K$ , where the two underlying lattices are translates of each other. The speaker, jointly with Greg Kuperberg (1990) proved that each convex disk  $K$  admits a double-lattice packing of density at least  $\sqrt{3}/2 = 0.866\dots$ . For the regular pentagon and heptagon the densest double-lattice packings were found, of density  $(5 - \sqrt{5})/3 = 0.92131\dots$  and  $0.8926\dots$ , respectively. They conjectured that the densest double-lattice packing with regular pentagons is of maximum density among all packings with its congruent copies. Recently, some results were obtained by Kallus & Kusner, and Hales & Kusner, indicating that a complete proof of the conjecture may appear soon. In this talk some overlooked questions concerning double lattice packings will be discussed, including a conjecture for double-lattice packings analogous to the classical theorem of Lászlo Fejes Tóth about lattice packings. (Received September 15, 2016)

1125-52-1296 **Doyon Kim\*** ([dzk0028@auburn.edu](mailto:dzk0028@auburn.edu)), 1954 Keystone Drive, Auburn, AL 36830. *Variations of Toeplitz' Conjecture*.

In 1911, Otto Toeplitz made a conjecture asserting that every Jordan curve in the plane contains four points forming the corners of a square. There are numerous positive partial results, including the case of convex curves, but the general question is still unsolved.

In this talk we consider variations of Toeplitz' conjecture. We say that a polygon  $P$  is strongly inscribed in a Jordan curve  $C$ , if all of its vertices are on  $C$  and the interior of  $P$  is a subset of the region enclosed by  $C$ . We study the problem of characterization of those triangles  $T$ , for which every Jordan curve has a strongly inscribed triangle similar to  $T$ . We prove that if  $T$  is a triangle such that all three angles of the triangle  $T$  are greater than 45 degree then there is a Jordan curve  $C$  that does not have a strongly inscribed triangle similar to  $T$ .

Then we show that for every Jordan curve  $C$  and a triangle  $T$ , there is a triangle  $T'$  similar to  $T$ , completely inside of  $C$  with at least two vertices of  $T'$  are on  $C$ . We also prove the similar statement for quadrilateral  $D$ : For every Jordan curve  $C$  and a quadrilateral  $D$ , there is a quadrilateral  $D'$  similar to  $D$ , completely inside of  $C$  with at least two vertices on  $C$ . (Received September 16, 2016)

1125-52-1550 **David Walter Stoner\*** ([dstoner@college.harvard.edu](mailto:dstoner@college.harvard.edu)), 208 Ashwood Drive, Aiken, SC 29801, and **Zoe Wellner, Ryan Chen, Frederick Huang** and **Maxwell Poley**. *On Convex and Higher Dimensional Extensions of Conway's Thrackle Theorem*.

A *thrackle* is defined as a drawing of a simple graph into the plane such that every pair of edges intersects exactly once, either at a common vertex or a transversal intersection point. In 1952, John Conway introduced these thrackles to the literature and conjectured that all thrackles satisfy  $|E| \leq |V|$ , where  $V$  and  $E$  denote the sets of vertices and edges, respectively. The conjecture has been resolved for several specific classes of graph representations, including those for which all edges are drawn as line segments. This result motivates the study of generalizations of the linear thrackle conjecture. In particular, we present analogues of Conway's conjecture when edges are replaced by convex hulls of vertex sets, and when these cell complexes are drawn in higher dimensional Euclidean spaces. We use results from algebraic topology and extremal combinatorics to establish upper bounds on the size of  $|E|$  in these extensions. In doing so, we find several new classes of families of sets for which Conway's conjecture holds and is tight. (Received September 17, 2016)

1125-52-1569 **Rebecca F. Durst** ([rfd1@williams.edu](mailto:rfd1@williams.edu)), **Max Hlavacek\*** ([mhlavacek@g.hmc.edu](mailto:mhlavacek@g.hmc.edu)) and **Chi Huynh** ([nhuynh30@gatech.edu](mailto:nhuynh30@gatech.edu)). *Classification of All Crescent Configurations on Four and Five Points*.

Erdős once wrote, "my most striking contribution is, no doubt, my problem on the number of distinct distances." Consequent to his work on the problem, he conjectured that, for sufficiently large  $N$ , it is impossible to place  $N$  points in general position on the plane with  $N - 1$  distinct distances so there is a distance occurring exactly  $i$  times for every  $1 \leq i \leq N - 1$ . These have been termed *crescent configurations* due to the increase in distance

multiplicities. Although significant work has been done by C. Pomerance, I. Palásti and others, nothing is yet known about the existence of these configurations on  $n$  points for  $n > 8$ . We take a new approach to studying these using techniques from distance geometry and graph theory that have allowed us to provide a method for classifying all configurations on  $n$  points up to isomorphism. Furthermore, we have proven that there exist only three possible realizations on four points and have decreased the number of configurations on five points from 12,600 to 27 final realizations. We then return to Erdős' question on existence with a new effective method for turning previously intractable problems into a more solvable form. This is joint work with Steven J. Miller and Eyvindur A. Palsson. (Received September 18, 2016)

1125-52-1750 **Braxton A Carrigan\*** (carriganb1@southernct.edu) and **Bruce W Atkinson**. *On the Bounds of a Golden Triangulation Refinements.*

We are concerned with triangulating point sets with golden triangles, which are isosceles triangles where the ratio of the longer side to the shorter side is the golden ratio. We will start with some basic results concerning vertices, side lengths, and areas of the triangles and their relationships to the golden ratio and classify all convex polygons which can be triangulated with golden triangles and determine which of them can be done in a minimal sense. In an attempt to establish a refinement process for golden triangulations we define a splitting process that produces a proper refinement and preserves specific desirable properties. Furthermore we will analyze lower bound criteria for finding efficient refinement algorithms, and establish some basic upper bound techniques. (Received September 19, 2016)

1125-52-1814 **Dominique Guillot\*** (dguillot@udel.edu), Department of Mathematical Sciences, University of Delaware, Newark, DE 19716, and **Mahya Ghandehari** and **Tina Torkaman**. *Critical exponents of graphs.*

Given a positive semidefinite matrix  $A$  with positive entries and a real number  $a$ , the entrywise power of  $A$  is obtained by taking the  $a$ -th power of each entry of  $A$ . Whether or not the resulting matrix must be positive semidefinite is a non-trivial problem solved in 1977 by FitzGerald and Horn. Motivated by applications in high-dimensional statistics, we examine when powering-up matrices with a structure of zeros encoded by a graph preserves positivity. I will discuss recent progress on the problem, and connections with the geometry of cones of structured positive semidefinite matrices. (Received September 19, 2016)

1125-52-1978 **John C. Bowers** (bowersjc@jmu.edu) and **Philip L. Bowers\*** (bowers@math.fsu.edu). *The 3-dimensional incidence geometry of circle space with applications to the geometry of circle frameworks in the Riemann sphere.* Preliminary report.

Topologically, the space of circles in the Riemann sphere is a punctured real projective 3-space. More useful to the conformal geometer is the **geometry** of circle space, a natural incidence structure of points, lines, and planes that can be used to understand the geometry of circle frameworks. In this talk we will give both intrinsic and extrinsic descriptions of this incidence geometry, define circle frameworks, and consider their rigidity. We will address the existence of circle frameworks with prescribed inversive distance data decorating the edges and suggest a Menger-type embedding theorem for them. (Received September 19, 2016)

1125-52-2116 **Jonathan Dahl\***, dahlj@lafayette.edu. *Alexandrov curvature of convex hypersurfaces in Hilbert space forms.*

We show convex hypersurfaces in Hilbert space forms have corresponding lower bounds on Alexandrov curvature. This extends earlier work of Buyalo, Alexander, Kapovitch, and Petrunin for convex hypersurfaces in Riemannian manifolds of finite dimension. (Received September 19, 2016)

1125-52-2124 **Cynthia Vinzant\*** (clvinzan@ncsu.edu). *Convex algebraic geometry.*

Convex algebraic geometry is the study of convex sets defined by real polynomial inequalities, using tools from both convexity and algebraic geometry. Important examples include spectrahedra, hyperbolicity cones, convex hulls of varieties, and cones of nonnegative polynomials. I will define these objects and discuss their relation to problems in semidefinite programming and other areas in optimization. (Received September 19, 2016)

1125-52-2245 **Karoly Bezdek\*** (karoly.bezdek@gmail.com), 2500 University Drive N.W., Calgary, Alberta T2N1N4, Canada, and **Zsolt Langi**. *Aspects of soft ball packings.*

In the talk we survey recent selected results on the density, kissing numbers, and contact numbers for soft ball packings in Euclidean (resp., normed) spaces. This is a joint work with Zs. Langi (Univ. of Tech., Budapest, Hungary). (Received September 20, 2016)

1125-52-2370 **John C. Bowers\*** ([bowersjc@jmu.edu](mailto:bowersjc@jmu.edu)), ISAT-CS Building MSC 4103, 701 Carrier Drive, Harrisonburg, VA 22807, **Philip L. Bowers** ([bowers@math.fsu.edu](mailto:bowers@math.fsu.edu)), 208 Love Building, 1017 Academic Way, Tallahassee, FL 32306-4510, and **Kevin Pratt**, Computer Science & Engineering Department, Storrs, CT 06269-4155. *Cauchy Rigidity of Convex c-Polyhedra.*

A c-polyhedron is a generalization of circle packings on the sphere to circle patterns with specified inversive distances between adjacent circles where the underlying 1-skeleton need not be a triangulation. In this talk we prove that any two convex c-polyhedra with inversive congruent faces are inversive congruent. The proof follows the pattern of Cauchy's proof of his celebrated rigidity theorem for convex Euclidean polyhedra. The trick in applying Cauchy's argument in this setting is in constructing hyperbolic polygons around each vertex in a c-polyhedron on which a variant of Cauchy's arm lemma can be applied. (Received September 20, 2016)

1125-52-2445 **Ciprian S. Borcea** ([borcea@rider.edu](mailto:borcea@rider.edu)), Rider University, Lawrenceville, NJ 08648, and **Ileana Streinu\*** ([istreinu@smith.edu](mailto:istreinu@smith.edu)), Smith College, Northampton, MA 01063. *Periodic auxetics.*

In materials science, auxetic behavior refers to the rather counter-intuitive property of a material becoming laterally wider when stretched and thinner when compressed.

We have recently proposed a purely geometric criterion, valid in arbitrary dimensions, for characterizing auxetic one-parameter deformations for periodic frameworks, which relies on the evolution of the periodicity lattice. A deformation path will be auxetic when the Gram matrix for a basis of periods gives a curve with all tangents in the positive semidefinite cone, analogous to a causal line in special relativity.

A special situation of auxetic behavior arises from expansive mechanisms, defined by the stronger property that the distance between any pair of vertices increases or stays the same. For two-dimensional periodic frameworks, expansiveness can be explained and explored in terms of periodic pointed pseudo-triangulations. An essential ingredient in the proof is our recent generalization, from finite to periodic frameworks, of a 150 year old theorem of James Clerk Maxwell relating stressed planar frameworks and their liftings to 3D polyhedral surfaces. (Received September 20, 2016)

1125-52-2447 **Andrzej Szymanski\*** ([andrzej.szymanski@sru.edu](mailto:andrzej.szymanski@sru.edu)), Department of Mathematics and Statistics, Slippery Rock University of PA, Slippery Rock, PA 16057. *On L-structures and other related convexity structures.* Preliminary report.

We analyze some relationships between K-simplicial structures, L-structures, and L\*-spaces. A question due to H. Ben-El-Mechaiekh is answered by showing that K-simplicial structures coincide with L-structures provided they are T. We construct new examples distinguishing L\*-structures from L-structures and we also characterize absolute retracts in terms of L-structures. (Received September 20, 2016)

1125-52-2814 **Michelle R. Cordier\*** ([michelle.r.cordier@gmail.com](mailto:michelle.r.cordier@gmail.com)), 316 Washington Ave, Wheeling, WV 26003. *Using Rotations, If You Can Hide Behind It, Can You Hide Inside It?*

Let  $K, L$  be convex sets in  $\mathbb{R}^3$ . If every projection of  $K$  can be rotated to be contained in the corresponding projection of  $L$ , does that imply that  $K$  is contained in  $L$ ? (Received September 20, 2016)

1125-52-2952 **T. H. Wears\*** ([wearsth@longwood.edu](mailto:wearsth@longwood.edu)). *Lorentzian Ricci Solitons on Solvable Lie groups.* Preliminary report.

We present the preliminary results of an investigation of Lorentzian Ricci soliton metrics on solvable Lie groups. Focusing on solvable Lie groups of dimensions four, five and six, we use the automorphisms of the corresponding Lie algebra to reduce the number of free parameters in an arbitrary left invariant Lorentzian metric by establishing canonical forms for the metrics on the particular Lie group in question. We then analyze the resulting canonical forms to classify the left invariant metrics that are algebraic Ricci solitons. We present the results for several solvable Lie groups showing that the structure of Lorentzian Ricci solitons is quite rich and that a solvable Lie group can support inequivalent and isometrically distinct Lorentzian soliton metrics. (Received September 20, 2016)

## 53 ► Differential geometry

- 1125-53-2      **Nemat Abazari\*** ([nematabazari@gmail.com](mailto:nematabazari@gmail.com)), University of Mohaghegh Ardabili, Department of Mathematics, Faculty of Mathematical Sciences, 56199-1136 Ardabil, Iran, and **Ilgin Sager** ([ilginsager81@gmail.com](mailto:ilginsager81@gmail.com)), Missouri University of Science and Technology, Department of Mathematics and Statistics, Rolla, MO 65409-0020. *On the Stationary Acceleration of the Frenet Curves in 3-Dimensional Lie Groups.*
- In this paper, the stationary acceleration of the spherical general helix in a 3-dimensional Lie group is studied by using a bi-invariant metric. The relationship between the Frenet elements of the stationary acceleration curve in 4-dimensional Euclidean space and the intrinsic Frenet elements of the Lie group is outlined. As a consequence, the corresponding curvature and torsion of these curves are computed. In Minkowski space, for the curves on a timelike surface, to have a stationary acceleration, a necessary and sufficient condition is refined. (Received April 14, 2016)
- 1125-53-101      **Stefan Müller\*** ([smueller@georgiasouthern.edu](mailto:smueller@georgiasouthern.edu)), Department of Mathematical Sciences, Georgia Southern University, 65 Georgia Ave. Room 3008 P.O. Box 8093, Statesboro, GA 30460.  *$C^0$ -characterization of symplectic embeddings via Lagrangian embeddings.*
- We prove that an embedding of a (small) ball into a symplectic manifold is symplectic if and only if it preserves the shape invariant. The latter is, in brief, the set of all cohomology classes that can be represented by the pull-back of a fixed primitive of the symplectic form by a Lagrangian embedding of a fixed manifold and of a given homotopy type. The proof is based on displacement information about (non)-Lagrangian submanifolds that comes from J-holomorphic curve methods. The definition of shape preserving does not involve derivatives and is preserved by uniform convergence (on compact subsets). In particular, we derive a new proof of the well-known  $C^0$ -rigidity of symplectic embeddings (and diffeomorphisms). An advantage of our techniques is that they avoid the cumbersome distinction between symplectic and anti-symplectic, and also work well in the contact setting (which will be discussed only if time permits). We moreover demonstrate that the shape is often a natural language in symplectic topology. The talk is based on the preprint arXiv:1607.03135. (Received July 27, 2016)
- 1125-53-175      **Bahar Acu\*** ([bacu@usc.edu](mailto:bacu@usc.edu)), 3620 S Vermont Ave KAP 104, Los Angeles, CA 90089-2532. *On Fillability of Higher Dimensional Contact Manifolds and Their Symplectic Mapping Class Group Relations.*
- In this talk, we will examine filling properties of higher dimensional contact manifolds and show that the symplectization of a  $(2n+1)$ -dimensional contact manifold given by an iterated planar open book decomposition, which will be defined in the talk, can be foliated by a special class of planar curves. We will also examine symplectomorphisms of higher dimensional symplectic manifolds yielding a generalization of the classical chain relation on surfaces. The second part of the talk is joint work with Russell Avdek. (Received September 20, 2016)
- 1125-53-176      **Bahar Acu\*** ([bacu@usc.edu](mailto:bacu@usc.edu)), 3620 S Vermont Ave KAP 104, Los Angeles, CA 90089-2532. *On Fillability of Higher Dimensional Contact Manifolds Supporting Iterated Planar Open Books.*
- The Weinstein conjecture asserts that certain vector fields carry closed orbits. It was proven for all closed 3-dimensional manifolds by Taubes, but it is still open in higher dimensions. In this talk, we show that a  $(2n+1)$ -dimensional contact manifold supporting an iterated planar open book decomposition, which will be defined in the talk, satisfies the Weinstein conjecture. (Received September 20, 2016)
- 1125-53-239      **Ryad Ghanam\*** ([raghanam@vcu.edu](mailto:raghanam@vcu.edu)), PO Box 8095, Doha, Qatar, 8095, Qatar, and **Gerard Thompson** ([gerard.thompson@utoledo.edu](mailto:gerard.thompson@utoledo.edu)), Toledo, OH 43606. *Non-solvable subalgebras of  $g L(4, R)$ .* Preliminary report.
- All the simple, then semi-simple, subalgebras of  $gl(4, R)$  are found. Each such semi-simple subalgebra acts by commutator on  $gl(4, R)$ . In each case the invariant subspaces are found and the results used to determine all possible subalgebras of  $gl(4, R)$  that are not solvable. (Received August 17, 2016)
- 1125-53-398      **Lisandra Hernandez-Vazquez\*** ([lisandra@math.stonybrook.edu](mailto:lisandra@math.stonybrook.edu)) and **Gueo Grantcharov**. *Poisson Structures on Twistor Spaces of Hyperkähler and HKT Manifolds.*
- HKT structures (an abbreviation for hyperkähler with torsion) were first introduced in String Theory as the structures induced on the target manifolds of  $(4,0)$ - supersymmetric sigma models with Wess-Zumino term. From a mathematical viewpoint, compact HKT manifolds share many properties with Kähler manifolds. We

characterize HKT structures in terms of a nondegenerate complex Poisson bivector on a hypercomplex manifold and extend the characterization to the twistor space. After considering the flat case in some detail, we show that the twistor space of a hyperkähler manifold admits a holomorphic Poisson structure. We briefly mention the relation to quaternionic and hypercomplex deformations on tori and K3 surfaces. (Received August 31, 2016)

1125-53-419 **Igor Belegradek\*** (ib@math.gatech.edu). *Spaces of nonnegatively curved surfaces.*

The talk will be a survey of my recent work on topological properties of the space of complete metrics of nonnegative curvature on simply-connected surfaces, i.e., the plane and the 2-sphere. (Received September 01, 2016)

1125-53-501 **Ian M Adelstein\***, ian.adelstein@trincoll.edu, and **Mary R Sandoval**. *The  $G$ -invariant spectrum and non-orbifold singularities.*

We consider the  $G$ -invariant spectrum of the Laplacian on an orbit space  $M/G$  where  $M$  is a compact Riemannian manifold and  $G$  acts by isometries. We generalize the Sunada-Pesce-Sutton technique to the  $G$ -invariant setting to produce pairs of isospectral non-isometric orbit spaces. One of these spaces is isometric to an orbifold with constant sectional curvature whereas the other admits non-orbifold singularities and therefore has unbounded sectional curvature. We therefore show that constant sectional curvature and the presence of non-orbifold singularities are inaudible properties of the  $G$ -invariant spectrum. (Received September 04, 2016)

1125-53-534 **Michael R Benfield, Helge Kristian Jensen and Irina A Kogan\*** (iakogan@ncsu.edu). *Jacobians with prescribed eigenvectors.*

We consider the problem of constructing all possible maps from an open subset  $\Omega \subset \mathbb{R}^n$  to  $\mathbb{R}^n$ , such that the set of eigenvector fields of the Jacobian matrix of each of these maps contains a given set of  $m \leq n$  independent vector fields on  $\Omega$ . Our initial motivation for considering this problem comes from the geometric study of hyperbolic conservation laws. This problem is, however, of independent geometric interest and, in turn, leads to an interesting systems of overdetermined systems of PDEs, which can be studied via classical integrability theorems and their appropriate generalizations. (Received September 05, 2016)

1125-53-558 **Andrew Michael Sanders\*** (sanderandy@gmail.com). *Riemannian metrics on character varieties of closed surface groups.*

We will present a general method for constructing Riemannian metrics on the  $G$ -character variety of a closed, orientable surface group where  $G$  is a semisimple Lie group. The Riemannian metrics we construct are compatible with the Atiyah-Bott-Goldman symplectic structure and generalize the construction of the Weil-Petersson metric on Teichmüller space and the Hitchin  $L^2$ -metrics on the moduli space of  $G$ -Higgs bundles. (Received September 06, 2016)

1125-53-583 **Leonardo S. Digiosia\*** (digiosia@lclark.edu), 3002 North Mountain Road, Boise, ID. 83702, Boise, ID 83702, and **Lea Kenigsberg, Dylanger Pittman, Jay Habib and Weitao Zhu**. *The Log Convex Density Conjecture in Hyperbolic Space.*

The isoperimetric problem with a density or weighting seeks to enclose prescribed weighted area with minimum weighted perimeter. According to Chambers' recent proof of the Log Convex Density Conjecture, for many densities on  $R^n$  the answer is a sphere about the origin. We generalize his results from  $R^n$  to the  $H^n$  with related but different volume and perimeter densities. (Received September 07, 2016)

1125-53-762 **Yi Lin\*** (yilin@georgiasouthern.edu), 65 Georgia Ave. Room 2306, Statesboro, 30458. *Convexity property of Hamiltonian transversely symplectic manifolds.*

In this talk, we introduce the notion of a Hamiltonian action on a transversely symplectic foliation. This provides a framework to study the Hamiltonian actions on many interesting singular (possibly non-Hausdorff) symplectic spaces, such as symplectic orbifolds, symplectic quasi-folds (by E. Prato), and the leaf spaces of characteristic Reeb foliations in both contact and co-symplectic geometries. We explain that under reasonable conditions, the components of a moment map introduced by us are still Morse-Bott functions with even indexes. This in particular leads to a foliated version of the Atiyah-Guillemin-Sternberg-Kirwan convexity theorem. This talk is based on a recent joint work with R. Sjamaar. (Received September 11, 2016)

1125-53-844 **Chen He\*** (he.chen@husky.neu.edu). *GKM graphs for odd dimensional manifolds with torus actions.*

Let torus  $T$  act on a manifold  $M$ , if the equivariant cohomology  $H_T^*(M)$  is a free module of  $H_T^*(pt)$ , then according to the Chang-Skjelbred Lemma,  $H_T^*(M)$  can be determined by the 1-skeleton  $M_1$  consisting of fixed points and 1-dimensional orbits. Goresky, Kottwitz and MacPherson considered the case where  $M$  is an algebraic manifold

and  $M_1$  is 2-dimensional, and introduced a graphic description of equivariant cohomology. In this paper, we follow those ideas to consider the case where  $M$  is an odd-dimensional (possibly non-orientable) manifold and  $M_1$  is 3-dimensional, and give a similar graphic description of equivariant cohomology. (Received September 12, 2016)

1125-53-1051 **Ziva Myer\*** (zmyer@brynmawr.edu). *The Generating Family Cohomology Ring for Legendrian Submanifolds*. Preliminary report.

A contact structure on a smooth manifold is a geometric structure given by a maximally non-integrable hyperplane field. Legendrian submanifolds are the submanifolds of the largest possible dimension that are everywhere tangent to the contact structure. There are many examples of Legendrian submanifolds that are isotopic as smooth submanifolds, but are not equivalent via an isotopy through Legendrian submanifolds. To show this, Legendrian invariants have been constructed through a variety of techniques. I will discuss how I am extending one such invariant, Generating Family Cohomology, by constructing a product structure. The construction uses moduli spaces of Morse flow trees – spaces of intersecting gradient trajectories of functions whose critical points encode Reeb chords of the Legendrian submanifold. This product lays the foundation for an A-infinity algebra that shows, in particular, that Generating Family Cohomology has a ring structure. (Received September 14, 2016)

1125-53-1053 **Ziva Myer\*** (zmyer@brynmawr.edu). *Product Structures for Generating Family Cohomology of Legendrian Submanifolds*. Preliminary report.

In contact geometry, invariants of Legendrian submanifolds in 1-jet spaces have been obtained through a variety of techniques. I will discuss how I am enriching one invariant, Generating Family Cohomology, by constructing a product structure on the cohomology groups. The construction uses moduli spaces of Morse flow trees – spaces of intersecting gradient trajectories of functions whose critical points encode Reeb chords of the Legendrian. This product structure lays the foundation for an A-infinity algebra structure for the Legendrian that shows, in particular, that this product gives Generating Family Cohomology a ring structure. (Received September 14, 2016)

1125-53-1530 **Andrew Zimmer\*** (aazimmer@uchicago.edu). *Rigidity of divisible domains in flag manifolds*.

Many symmetric spaces of non-compact type can be embedded as a domain in some compact homogeneous space. For instance, real hyperbolic space embeds as a domain in real projective space and the projective automorphisms of the domain coincide with the isometry group (this is the Klein model). This embedding is quite flexible in that there exists perturbations which are non-symmetric but still have large projective automorphism group (for instance co-compact). Perhaps more surprisingly, in real projective space there also exist non-symmetric domains whose projective automorphism group is discrete, co-compact, and not quasi-isometric to any symmetric space. In this talk I will discuss some recent rigidity results for domains in other compact homogeneous spaces (complex/quaternionic projective space and certain Grassmannians). Some of this is joint work with Wouter van Limbeek. (Received September 17, 2016)

1125-53-1577 **Yiannis Loizides\*** (yiannis.loizides@mail.utoronto.ca) and **Eckhard Meinrenken**. *Norm-square localization for Hamiltonian  $LG$ -spaces*. Preliminary report.

Let  $G$  be a compact, connected, simply connected Lie group, and let  $LG$  denote the loop group. There is a one-one correspondence between proper Hamiltonian  $LG$ -spaces and compact *quasi*-Hamiltonian  $G$ -spaces. One of the authors ([M]) has proposed a definition for the quantization of a quasi-Hamiltonian  $G$ -space as an element in the twisted K-homology of  $G$  (the latter is related to the ring of positive energy representations of  $LG$  via the Freed-Hopkins-Teleman theorem). We prove a ‘norm-square localization’ formula for the quantization of a quasi-Hamiltonian  $G$ -space, with terms indexed by the components of the critical set of the norm-square of the moment map of the corresponding Hamiltonian  $LG$ -space. An important application is to give a new proof of the quantization-commutes-with-reduction theorem for quasi-Hamiltonian spaces. (Received September 18, 2016)

1125-53-1622 **Teresa Arias-Marco, Emily Dryden, Carolyn Gordon, Asma Hassannezhad, Allie Ray\*** (allie.ray@trincoll.edu) and **Elizabeth Stanhope**. *Spectral Geometry of the Steklov Problem on Orbifolds*.

An orbifold is a space locally modeled on  $\mathbb{R}^n/\Gamma$ , where  $\Gamma$  is a discrete group acting properly discontinuously on  $\mathbb{R}^n$ . We will begin by discussing some classical results about geometric properties of manifolds determined by the Laplace and Steklov spectrum. After introducing the concept of an orbifold, we will then look at how these

results transfer (or fail to transfer) to the orbifold setting. One result is that the Steklov spectrum determines the number of singular points on the boundary of a compact 2-dimensional orbifold but does not detect the presence of interior singular points. (Received September 18, 2016)

1125-53-1929 **Brian Klatt\*** ([brn.klitt@gmail.com](mailto:brn.klitt@gmail.com)), 1517 RAVENA ST, Bethlehem, PA 18015. *Compact Gradient Shrinking Ricci Solitons, the Hitchin-Thorpe Inequality, and Spin Structures.*

If an Einstein metric exists on a compact topological 4-manifold, there is a topological implication which was discovered by Hitchin and Thorpe and involves the Euler number and signature:  $2\chi - 3|\tau| \geq 0$ . It has been conjectured by H.D. Cao that this obstruction still holds when the Einstein condition is relaxed to Gradient Ricci Soliton,  $Rc(g) + \nabla^2 f = \lambda g$ , where  $f$  is a function called the potential. I will briefly discuss this problem and some partial progress. Namely, if one assumes that the underlying manifold has a spin structure, I will show that the Hitchin-Thorpe inequality is true for gradient Ricci solitons. The only known examples of compact gradient shrinking Ricci solitons with a non-constant potential function in dimension 4 are, however, not spin manifolds. Is it possible that a compact gradient shrinking Ricci soliton on a spin manifold necessarily has a constant potential function (i.e. the soliton metric is an Einstein metric)? I will discuss this question and directions for future study. (Received September 19, 2016)

1125-53-2559 **Ibrahim Unal\*** ([iunal@metu.edu.tr](mailto:iunal@metu.edu.tr)), Middle East Technical University, Department of Mathematics, Cankaya, 06800 Ankara, Turkey. *Gauss Maps of Embeddings into Calibrated Manifolds and the h-Principle.*

Introduction of potential theory on calibrated manifolds by Harvey and Lawson has brought many techniques and tools to understand the geometry of these special manifolds. One of these is the notion of  $\phi$ -free submanifolds. These submanifolds are analogues of totally real submanifolds in complex geometry and play a similar role to construct  $\phi$ -convex domains in a calibrated manifold with calibration  $\phi$ . By studying the Gauss maps of embeddings into calibrated manifolds with special holonomy, we get important results about the topology of  $\phi$ -free submanifolds, especially in  $G_2$ -geometry. Moreover, we prove that the h-principle holds for  $\phi$ -free embeddings for coassociative calibration in  $\mathbb{R}^7$ , for Cayley calibration in  $\mathbb{R}^8$  and for quaternionic calibration in  $\mathbb{H}^n$ .

In this talk, after a quick introduction to calibrated manifolds, I will talk about the recent results about the geometry of  $\phi$ -free submanifolds of  $G_2$  and  $Spin(7)$ -manifolds. (Received September 20, 2016)

1125-53-2565 **Jesse W. Hicks\*** ([j.hicks@aggiemail.usu.edu](mailto:j.hicks@aggiemail.usu.edu)), 3581 E Silvercreek Dr, Washington, UT 84780. *Classification of Spacetimes with Symmetry.* Preliminary report.

Spacetimes with symmetry play a critical role in Einstein's Theory of General Relativity. Missing from the literature is a correct, usable, and computer accessible classification of such spacetimes. This research fills this gap; specifically, we

- i) give a new and different approach to the classification of spacetimes with symmetry using modern methods and tools such as the Schmidt method and computer algebra systems, resulting in ninety-two spacetimes;
- ii) create digital databases of the classification for easy access and use for researchers;
- iii) create software to classify any spacetime metric with symmetry against the new database;
- iv) compare results of our classification with those of Petrov and find that Petrov missed six cases and incorrectly normalized a significant number of metrics;
- v) classify spacetimes with symmetry in the book *Exact Solutions to Einstein's Field Equations Second Edition* by Stephani, Kramer, Macallum, Hoenselaers, and Herlt and in Komrakov's paper *Einstein-Maxwell equation on four-dimensional homogeneous spaces* using the new software.

(Received September 20, 2016)

1125-53-2878 **Jonathan Simone\*** ([js3fv@virginia.edu](mailto:js3fv@virginia.edu)). *Symplectically replacing plumbings with Euler characteristic 2 4-manifolds.*

The (generalized) rational blowdown technique is a procedure in which a linear plumbing of  $D^2$ -bundles over  $S^2$  is excised from a smooth 4-manifold and replaced by a rational homology ball. This was introduced by Fintushel and Stern, generalized by Park, and shown to be a symplectic operation by Symington. Based on work by Lisca, it is known precisely which linear plumbings can be symplectically rationally blown down. In this talk, we will produce a complete list of linear plumbings that can be "symplectically replaced" by 4-manifolds of Euler characteristic 2. We call such plumbings "2-replaceable." We will then construct "2-replaceable trees" and finally use symplectic cut and paste to produce an exotic rational surface. (Received September 20, 2016)

1125-53-2941 **Sonja Hohloch, Silvia Sabatini, Daniele Sepe and Margaret Symington\***  
(symington\_mf@mercuer.edu). *Beyond toric blow-ups.*

Blowing up and down is an important tool in the study of symplectic manifolds. In dimension four, equivariant blow-ups of symplectic four-manifolds equipped with a  $T^2$ -action or an  $S^1$ -action are well understood. In this talk I will describe a blow-up that respects an  $S^1 \times \mathbf{R}$ -action but no  $T^2$ -action. This blow-up can be “big” in the sense that given certain toric manifolds of dimension four, the exceptional sphere introduced may have greater area than can be achieved by a toric blow-up. I will discuss both topological and symplectic aspects of this blow-up, and explain how it can be implemented on certain completely integrable Hamiltonian systems with two degrees of freedom. (Received September 20, 2016)

1125-53-2984 **Geoffrey Scott\*** (gscott@math.toronto.edu). *Dirac geometry of folded symplectic and b-symplectic structures.*

The simplest examples of singularities that appear in presymplectic geometry and Poisson geometry are described by folded symplectic forms and b-symplectic forms, respectively. In this talk, I will show how certain techniques and celebrated results from folded and b-symplectic geometry generalize to the context of Dirac geometry, which is a common generalization of both presymplectic and Poisson geometry. (Received September 20, 2016)

1125-53-3063 **François Ziegler\*** (fziegler@georgiasouthern.edu), Department of Mathematical Sciences, Georgia Southern University, Statesboro, GA 30460-8093. *Symplectic and Contact Imprimitivity.*

A famous theorem of Mackey characterizes those unitary  $G$ -modules  $V$  that are induced from a closed subgroup  $H \subset G$  by the presence of a *system of imprimitivity* based on  $G/H$ : that is, a  $G$ -invariant, commutative  $C^*$ -subalgebra of  $\text{End}(V)$  whose spectrum is, as a  $G$ -space, homogeneous and isomorphic to  $G/H$ . In this work, we similarly characterize those hamiltonian  $G$ -spaces  $X$  that are induced from  $H$  (in the sense of Kazhdan-Kostant-Sternberg, 1978) by the presence of a (symplectic) *system of imprimitivity* based on  $G/H$ : that is, a  $G$ -invariant, Poisson commutative subalgebra  $\mathfrak{f}$  of  $C^\infty(X)$ , consisting of functions whose hamiltonian flow is complete, and such that the image of the moment map  $X \rightarrow \mathfrak{f}^*$  is homogeneous and isomorphic to  $G/H$ . Likewise, we characterize induced Kostant-Souriau bundles over hamiltonian  $G$ -spaces by the presence of a (contact) system of imprimitivity. This result is a key ingredient in the Mackey ‘normal subgroup analysis’ of hamiltonian and Kostant-Souriau  $G$ -spaces. (Received September 20, 2016)

## 54 ► General topology

1125-54-234 **Song Yu\*** (song.yu@pomona.edu), 170 E 6th St, Mailbox 1839, Claremont, CA 91711. *Symmetries of Graphs in Homology Spheres.*

In this paper we will explore symmetries of spatial graphs in the family of homology spheres. We will generalize previous findings in the 3-sphere and see how concepts of rigidity, chirality and intrinsic linking interact with automorphisms of spatial graphs. Our results include a rigid symmetry theorem for 3-connected graphs, a classification of realizable automorphisms of complete graphs and examples of unrealizable automorphisms of Petersen graphs. (Received August 17, 2016)

1125-54-556 **Lynne Yengulalp\*** (lyengulalp1@udayton.edu), 300 College Park, Dayton, OH 45469. *Strategies in the Banach-Mazur game, pi-Noetherian type, and cellularity.* Preliminary report.

For regular spaces,  $X$ , we study the relationship (if any) between the cellularity of  $X$  and the pi-Noetherian type of  $X$ . This study is part of the goal of determining on which regular spaces the nonempty player can reduce a winning coding strategy in the Banach-Mazur game to a winning 2-tactic. (Received September 06, 2016)

1125-54-915 **Sayan Mukherjee\*** (sayan@stat.duke.edu), 112 Old Chemistry Building, PO 90251, Durham, NC 27708, and **Tingran Gao, Katherine Turner, Doug Boyer, Washington Mio and Jacek Brodzki.** *Modeling shapes and surfaces.*

I will introduce a geometric/topological transform that allows for the modeling of shapes and surfaces without requiring landmarks. We discuss applications of this to morphological analysis of primates as well as the analysis of melanoma. I will also outline how the transform can measure distances between shapes as well as place probability models on shapes and surfaces. I will also discuss approaches using conformal geometry to model surfaces. (Received September 13, 2016)

1125-54-1333 **David B Damiano** and **Emily T Winn\*** (etwinn17@g.holycross.edu). *Topological Modeling of Force Networks in Granular Media*. Preliminary report.

Granular materials, or collections of solid macroscopic particles in contact with each other, play an important role in chemistry, pharmaceutical sciences, and agriculture. Despite their importance, little is understood about the force networks formed from inter-particle contacts in these materials because of the difficulty of modeling large networks. The challenge lies in finding a model that is specific enough to characterize the local topology of a cell while still being robust enough to capture information of the entire network. A recently developed method that may answer this dilemma is the swatch and cloth, which uses adjacency graphs and statistics to model random cell networks. In this talk, we discuss how the swatch and cloth model can be applied to these force networks to facilitate comparisons of materials and their bulk properties. We also address applying swatch and cloth to communities, or partitioned sub regions, of a force network to examine different parts of one network. (Received September 16, 2016)

1125-54-1490 **Michael Willis\***, 141 Cabell Drive, Kerchof Hall, P.O. Box 400137, Charlottesville, VA 22904-4137. *A Colored Khovanov Homotopy Type*.

Let  $L$  be a link in  $S^3$  with Khovanov homology  $Kh(L)$ . The recently defined Khovanov homotopy type  $\mathcal{X}(L)$  is a spectrum satisfying  $H^*(\mathcal{X}(L)) \cong Kh(L)$ . In this talk, I will describe how stabilization of the homotopy type of infinite torus braids allows for a definition of a colored Khovanov homotopy type  $\mathcal{X}_n(L)$  satisfying  $H^*(\mathcal{X}_n(L)) \cong Kh_n(L)$ , the colored Khovanov homology of  $L$ . Time permitting, I will also discuss more general infinite braids. (Received September 17, 2016)

1125-54-1624 **Malachi Alexander**, **Selina Foster\*** (selina.foster@gmail.com), **Gianni Krakoff** and **Jennifer McLoud-Mann**. *Hexagonal Mosaic Knots*.

In their 2008 paper, *Quantum Knots and Mosaics*, Lomonaco and Kauffman introduce mosaic knots on square tilings of the plane. This paper generalizes the tiles, terms, and theorems from square tilings to hexagonal tilings. Additionally, we develop further constructions with properties unique to hexagonal mosaic knots, including minimal hextile number and algebraic saturation diagrams. (Received September 18, 2016)

1125-54-1820 **Włodzimierz J. Charatonik**, Department of Mathematics and Statistics, 400 W. 12th St., Rolla, MO 65409, and **Şahika Şahan\***, Department of Mathematics and Statistics, 400 W. 12th St., Rolla, MO 65409. *Zero-Dimensional Spaces Homeomorphic to Their Cartesian Squares*.

We show that there exist uncountably many zero-dimensional compact metric spaces homeomorphic to their Cartesian squares as well as their  $n$ -fold symmetric products. (Received September 19, 2016)

1125-54-2152 **C Caruvana\*** (christopher.caruvana@unt.edu) and **R Kallman**. *Descriptive Set Theory and Borel Probabilities*.

We will study certain sets of Borel probabilities on a Polish space and introduce a  $\sigma$ -ideal of sets which extends the  $\sigma$ -ideal of meager sets which, in many models of ZFC, is a strict extension. We will also show that the  $\sigma$ -ideal introduced strictly extends the  $\sigma$ -ideal of universally null sets and discuss some applications of Martin's Axiom. (Received September 19, 2016)

1125-54-2550 **Sarah J. Tymochko\*** (sjtymo17@g.holycross.edu) and **David B. Damiano**. *A Topological Analysis of Retinal Vasculature*. Preliminary report.

Vasculature in the retina can be used to screen for diseases such as diabetic retinopathy and glaucoma. Healthy retinal blood vessels are straight or slightly curved but diseased vessels have increased tortuosity and curvature. We propose a new method of detecting tortuosity and distinguishing between healthy and diseased vasculature using techniques from topological data analysis including zero dimension persistence homology. This method is applied to a dataset of fundus images of retinas that are healthy or diseased with diabetic retinopathy or glaucoma. (Received September 20, 2016)

1125-54-2753 **Candice Renee Price\*** (cprice@sandiego.edu). *A Discussion on the Tangle Model: An Application of Topology*.

The tangle model was developed in the 1980's by professors DeWitt Sumner and Claus Ernst. This model uses the mathematics of tangles to model protein-DNA binding. An  $n$ -string tangle is a pair  $(B, t)$  where  $B$  is a 3-dimensional ball and  $t$  is a collection of  $n$  non-intersecting curves properly embedded in  $B$ .  $N$ -string tangles are formed by placing  $2n$  points on the boundary of  $B$ , and attaching  $n$  non-intersecting curves inside  $B$ . Tangles, like knots and links, are studied through their diagrams. In the tangle model for DNA site-specific recombination, one is required to solve simultaneous equations for unknown tangles which are summands of observed DNA knots

and links. This discussion will give a review of the tangle model including definitions (Received September 20, 2016)

## 55 ► Algebraic topology

1125-55-37 **Mohammad A. Obiedat\*** ([mohammad.obiedat@gallaudet.edu](mailto:mohammad.obiedat@gallaudet.edu)), 800 Florida Avenue NE, Washington, DC 20002. *A Note on the Construction of Complex and Quaternionic Vector Fields on Spheres.*

A relationship between real, complex, and quaternionic vector fields on spheres is given by using a relationship between the corresponding standard inner products. The number of linearly independent complex vector fields on the standard  $(4n - 1)$ -sphere is shown to be twice the number of linearly independent quaternionic vector fields plus  $d$ , where  $d = 1$  or  $3$ . (Received June 13, 2016)

1125-55-60 **Moo K Chung\*** ([mkchung@wisc.edu](mailto:mkchung@wisc.edu)), 2015 Canterbury Road, Madison, WI 53711. *Constructing large-scale brain networks with billions of connections via persistent homology.* Preliminary report.

L1-penalty based sparse systems are usually parameterized by a tuning parameter that determines the sparsity of the system. How to choose the right tuning parameter is a fundamental and difficult problem in learning the sparse system. In this talk, by treating the tuning parameter as another dimension, persistent homological structures over the parameter space can be constructed. The method is applied in building large-scale network obtained from functional magnetic images of the human brain. By taking every voxel in images as network nodes, we can build the brain network with billions of connections. The constructed large-scale network is used to address the scientific question of if our thought pattern is heritable. This is a joint work with Paul Rathouz of University of Wisconsin-Madison, Benjamin Lahey of University of Chicago and David Zald of Vanderbilt University. The talk is based on arXiv:1509.04771. (Received July 01, 2016)

1125-55-75 **Noureen Khan\*** ([noureen.khan@unt.edu](mailto:noureen.khan@unt.edu)), 7400 University Hills Blvd, Dallas, TX 75241. *On Classification of Virtual Rational Tangles.*

We study virtual generalization of rational tangles that form basis for equivalence classes of virtual knots and links. New invariants, virtual tangle number and the coloring method are introduced for the classification of virtual rational tangles. We use contemporary and combinatorial arguments to show that any two virtual rational tangles are equivalent if and only if their tangle numbers are equal. (Received July 11, 2016)

1125-55-189 **Yu Pan\*** ([yp37@math.duke.edu](mailto:yp37@math.duke.edu)). *Exact Lagrangian fillings of Legendrian  $(2, n)$  torus knots.*

For a Legendrian  $(2, n)$  torus knot or link with maximal Thurston-Bennequin number, Ekholm, Honda, and Kálmán constructed  $C_n$  exact Lagrangian fillings, where  $C_n$  is the  $n$ -th Catalan number. We show that these exact Lagrangian fillings are pairwise non-isotopic through exact Lagrangian isotopy. To do that, we compute the augmentations induced by the exact Lagrangian fillings  $L$  to  $\mathbb{Z}_2[H_1(L)]$  and distinguish the resulting augmentations. (Received September 20, 2016)

1125-55-210 **Mikael Vejdemo-Johansson\*** ([mvj@math.csi.cuny.edu](mailto:mvj@math.csi.cuny.edu)), 2800 Victory Boulevard, 1S-215, Staten Island, NY 10314. *Persistent homology and algebraic foundations: a survey.*

Persistent homology is a cornerstone of contemporary topological data analysis. Since the field got started, several algebraic developments have had concrete algorithmic and conceptual repercussions: opening up new analysis methods, and new computational methods emerging from the algebraic foundations.

Many ongoing directions of research today introduce sheaves as a next step in extending these algebraic foundations, and this session includes several speakers working in these directions. For this talk, we will look at the existing history, with an overview of existing algebraic foundations and their effects. (Received August 12, 2016)

1125-55-325 **Omer Bobrowski** and **Matthew Kahle\***, [mkahle@math.osu.edu](mailto:mkahle@math.osu.edu), and **Primoz Skraba**, Ljubljana, Slovenia. *Maximally persistent cycles in random geometric complexes.*

The topology of random simplicial complexes has been the subject of intensive study over the past ten years or so. A number of papers have identified phase transitions for homology to appear or disappear, computed estimates for Betti numbers, and so on. One of the justifications for this emerging field is as a null hypothesis for topological data analysis.

With this application to topological inference in mind, we study the persistent homology of random geometric complexes. Essentially we ask: given a set of random points in  $d$ -dimensional space, how large is the largest  $k$ -dimensional hole? We measure the size of the hole topologically, as death radius / birth radius, for either the Vietoris-Rips filtration or the Cech filtration. We obtain upper and lower bounds for every  $k$  and  $d$ , agreeing up to a constant factor. (Received August 26, 2016)

1125-55-437 **Daniel Ramras\*** ([dramras@iupui.edu](mailto:dramras@iupui.edu)), 402 N. Blackford, LD 270, Indianapolis, IN 46202. *Spaces of flat connections.*

I'll discuss some results regarding spaces of flat connections on vector bundles over surfaces, and over the 3-dimensional Heisenberg manifold. The aim in this work is to understand the homotopy types of these spaces after stabilizing with respect to rank, by combing geometric methods with ideas from stable homotopy theory. (Received September 01, 2016)

1125-55-514 **Neil Fullarton** and **Andrew Putman\*** ([andyp@nd.edu](mailto:andyp@nd.edu)). *The high-dimensional cohomology of congruence subgroups of the mapping class group.*

I will show that the level  $l$  congruence subgroup of the mapping class group has a vast amount of unstable rational cohomology in its cohomological dimension. (Received September 04, 2016)

1125-55-532 **Kathryn Bryant\*** ([kthrynbrant@gmail.com](mailto:kthrynbrant@gmail.com)), 14 E Cache la Poudre St, Colorado Springs, CO 80903. *Converting  $d$ -invariants into lattice points: A visualization technique to aid in knot slicing.*

This talk will focus on a particular technique used to prove/disprove knot sliceness and it will serve as an invitation to other researchers to help push the technique further. In the 2011 paper "The slice-ribbon conjecture for 3-stranded pretzel knots," Greene and Jabuka make use of Donaldson's diagonalization theorem and a theorem about  $d$ -invariants (due to Ozsváth and Szabó) from Heegaard-Floer theory in order to classify the slice knots within the family of 3-stranded pretzel knots. They give the necessary condition for a slice knot  $K$  that the number of vanishing  $d$ -invariants  $d(Y, \mathfrak{s})$ , where  $Y$  is the 2-fold branched cover of  $K$  and  $\mathfrak{s}$  is a  $\text{spin}^c$  structure on  $Y$ , must exceed or equal  $\sqrt{|\det K|}$ , where  $\det K$  is the knot determinant of  $K$ . The speaker used this criterion to study sliceness of 5-stranded pretzel knots and, after making a necessary refinement, translated the  $d$ -invariant criterion into one about lattice points. The goal of this talk is to explain this lattice point criterion and look for new ways to study it. (Received September 06, 2016)

1125-55-958 **Thomas Wanner\*** ([twanner@gmu.edu](mailto:twanner@gmu.edu)), Department of Mathematical Sciences, George Mason University, Fairfax, VA 22030. *Topological Microstructure Analysis Using Persistence Landscapes.*

Phase separation mechanisms can produce a variety of intricate microstructures, which often are difficult to characterize in a quantitative way. In recent years, a number of novel topological metrics for microstructures have been proposed, which measure essential connectivity information and are based on techniques from algebraic topology. Such metrics are inherently computable using computational homology and provide a significant data reduction, passing from complicated patterns to discrete information. It is therefore natural to wonder what type of information is actually retained by the topology. In this talk, we show that averaged persistence landscapes can be used to recover central system information in the Cahn-Hilliard theory of phase separation. More precisely, we show that topological information of evolving microstructures alone suffices to accurately detect both concentration information and the actual decomposition stage of a data snapshot. Considering that persistent homology only measures discrete connectivity information, regardless of the size of the topological features, these results indicate that the system parameters in a phase separation process affect the topology considerably more than anticipated. (Joint work with Pawel Dlotko.) (Received September 13, 2016)

1125-55-1112 **Mark C Hughes\*** ([hughes@mathematics.byu.edu](mailto:hughes@mathematics.byu.edu)). *A neural network approach to computing knot invariants.*

In recent years neural networks have received a great deal of attention due to their remarkable ability to detect subtle and very complex patterns in large data sets. They have become an important machine learning tool and have been used extensively in many fields, including computer vision, fraud detection, artificial intelligence, and financial modeling.

Knots are topological objects which have been studied for much longer, typically using topological, geometric, and algebraic tools. Numerous knot invariants can be defined which reflect interesting properties of these knots.

In this talk I will outline an approach to using neural networks to model invariants of knots. Depending on the invariant of interest, these approaches can yield surprisingly accurate models. I will show how the predictions

made by these models can be used in certain cases to compute values of invariants such as the slice genus and Ozsváth-Szabó  $\tau$ -invariant for previously unknown knots. (Received September 14, 2016)

1125-55-1147 **Justin M Curry\*** ([curry.justin@gmail.com](mailto:curry.justin@gmail.com)). *The Role of (Co)Sheaves in TDA*. Preliminary report.

Many constructions in topological data analysis can be viewed as sheaves or cosheaves: Persistence, Mapper, Vineyards, the Persistent Homology Transform, and so on. With certain niceness assumptions, these (co)sheaves are constructible and can be viewed as representations of the entrance path category, which is associated to a particular stratification of a topological space  $X$ . However, the ability to pass back to the open sets of  $X$  allows us to compare different stratifications and quiver representations. This provides a unifying framework for combining the representation theory of quivers with sampling theory, the techniques of interleaving (which provides a distance between sheaves), and density results. (Received September 15, 2016)

1125-55-1151 **Magnus Bakke Botnan, Justin Curry and Elizabeth Munch\*** ([emunch@albany.edu](mailto:emunch@albany.edu)). *The interleaving distance for posets*.

The interleaving distance has been shown to be an incredibly powerful tool in Topological Data Analysis (TDA), as various incarnations provide distances between persistence diagrams, Reeb graphs, and merge trees. In this work, we have extended the notion of interleavings to encompass cosheaves arising from a functor defined on a poset. One result of this machinery is that it provides a definition for the interleaving distance for the mapper graph. In addition, we can approximate the Reeb graph interleaving distance by the interleaving distance for its mapper which opens the door to computational possibilities. (Received September 15, 2016)

1125-55-1154 **Matthew L Wright\*** ([wright5@stolaf.edu](mailto:wright5@stolaf.edu)) and **Michael Lesnick**. *Multidimensional Persistence: A Practical Approach*.

Multidimensional persistent homology is highly relevant in the analysis of noisy data, as it offers the ability to filter by two or more parameters simultaneously. However, practical software for using multidimensional persistence in applications is still in its infancy. In this talk, I will highlight recent computational advances that facilitate the use of two-dimensional persistence to extract features from noisy data. This work is part of the development of the RIVET software for computing and visualizing two-dimensional persistence modules, in collaboration with Mike Lesnick. (Received September 15, 2016)

1125-55-1181 **Cliff Joslyn, Kathleen Nowak, Brenda Praggastis, Emilie Purvine\*** ([emilie.purvine@pnnl.gov](mailto:emilie.purvine@pnnl.gov)) and **Michael Robinson**. *Computations for Local and Pseudo Sections in Real-world Sheaves*. Preliminary report.

Sheaves of vector spaces on abstract simplicial complexes (ASCs) can represent the consistency between multiple data feeds: vector spaces record the possible values returned by a source, and restriction maps encode the consistency relationships among interacting sources as coded in an ASC. Global sections of a sheaf can be computed via sheaf cohomology, indicating readings from the sources which are consistent by equality according to the restriction maps and the structure of the ASC. But actual measurement of real-valued data fail equality conditions due to noise and resolution issues. The GPS on a cell phone and a fitness watch may both be on the same person, and return slightly different coordinates, but we may want to consider them consistent up to some tolerance. We address these issues through two closely related new concepts. Local sections cover the ASC with sets of data feeds that are maximally consistent, revealing the quality of a sheaf assignment. Pseudo and approximate sections can establish a consistency structure to represent degree of tolerance or uncertainty. The degree of approximation tolerance establishes a filtration of maximal local sections, from all sensors possibly being distinct at zero tolerance to all sensors being equivalent at maximal tolerance. (Received September 15, 2016)

1125-55-1225 **Gregory Henselman\*** ([grh@seas.upenn.edu](mailto:grh@seas.upenn.edu)). *Matroids, Morse Theory, and Fast Persistent Homology Computations*. Preliminary report.

The object of this talk is to introduce a set of axioms on a family of flats in a matroid, and the combinatorial theory of finite linear chain complexes that results. This theory is interesting in its own right, with structural analogs to homology, persistence, duality, and, for based complexes, discrete Morse Theory. There is a natural correspondence between linear-algebraic chain complexes and certain of those introduced; this correspondence renders several structures, notably Morse constructions and 1-parameter persistence modules, open to combinatorial methods of analysis that are both elementary and simple. Applications include greatly simplified proofs of results which have historically absorbed much effort in the community of applied topologists, and new insight into structures which remain poorly understood, such as multidimensional persistence. Time permitting, we

will discuss future directions in combinatorial sheaf cohomology and applications in algorithm design. No prior knowledge of matroid theory will be assumed. (Received September 15, 2016)

1125-55-1241 **Amit Patel\*** ([akpatel@colostate.edu](mailto:akpatel@colostate.edu)), Department of Mathematics, 1874 Campus Delivery, Fort Collins, CO 80523-1874. *Classification of Constructible Cosheaves.*

A covering space of a manifold  $M$  is equivalent to a functor from the fundamental groupoid of  $M$  to the category of sets. This is the classic classification theorem for covering spaces. In general, a locally constant cosheaf valued in a category  $C$  is equivalent to a functor from the fundamental groupoid of  $M$  to  $C$ . Suppose  $S$  is a topological stratification of  $M$ . An  $S$ -constructible cosheaf valued in a category  $C$  is equivalent to a functor from the entrance path category of  $(M, S)$  to  $C$ . The entrance path category, introduced by MacPherson, acts as the fundamental groupoid for stratified spaces. I will discuss this classification theorem for constructible cosheaves. (Received September 15, 2016)

1125-55-1265 **Francesco Vaccarino\*** ([francesco.vaccarino@gmail.com](mailto:francesco.vaccarino@gmail.com)). *Multidimensional persistence vs diagram and Hochschild cohomology.* Preliminary report.

Motivated by the attempt to build on the top of reasonable data (e.g. weighted networks) so-called abstract BF-theories (a kind of topological quantum field theories), we will extend the usual approach of multidimensional persistent (co-)homology by considering diagrams of incidence algebras of finite posets. By using results of Gerstenhaber et al. we will show what is the link between diagram cohomology, Hochschild cohomology, and multidimensional persistence. thus connecting seemingly uncorrelated research areas as topological data analysis and topological quantum field theory. (Received September 15, 2016)

1125-55-1266 **Peter Bubenik\*** ([peter.bubenik@ufl.edu](mailto:peter.bubenik@ufl.edu)). *An Introduction to Topological Data Analysis.*

I will give an introduction to how topology can be used to summarize the geometry of data in a way allows further analysis using statistics and machine learning. Here are the key ingredients. Data is encoded geometrically. Tools from topology are used to convert this to algebra. A fundamental algebraic result and accompanying matrix algorithm allows us to compute an elegant summary. We then map this to a Hilbert space. I will show how these ideas apply to a biological application. (Received September 15, 2016)

1125-55-1272 **Luke Wolcott\*** ([luke.wolcott@gmail.com](mailto:luke.wolcott@gmail.com)). *Interactive software for topological data analysis.*

I will demo new software for playing with 2D data sets and persistence barcodes, meant to help develop intuition for what a barcode can tell us. One can save and load data sets, easily add and remove points, and watch how the barcode changes in realtime. The software builds on the JavaPlex demo written for Processing by Mikael Vejdemo-Johansson. I will discuss possible future development for interacting with 3D live data. (Received September 15, 2016)

1125-55-1287 **Peter Bubenik\*** ([peter.bubenik@ufl.edu](mailto:peter.bubenik@ufl.edu)). *Discovering Geometry using Topological Data Analysis.* Preliminary report.

In one approach to topological data analysis, persistent homology is used to recover the homology of an object from which we have sampled finitely many points. In this analysis, the long-lived features describe this homology and the short-lived features are considered to be noise. I will advocate an opposing view: the short-lived features reveal the underlying geometry and can be crucial to analyzing the data. (Received September 15, 2016)

1125-55-1540 **Vin de Silva, Elizabeth Munch and Anastasios Stefanou\*** ([astefanou@albany.edu](mailto:astefanou@albany.edu)). *Theory of interleavings on categories with lax  $[0, \infty)$ -action and the hom tree functor.*

The interleaving distance is a powerful tool in TDA which has been shown to provide a metric for such topological signatures as persistence diagrams and Reeb graphs. In this talk we generalize the idea of interleavings to a broader class of objects, namely categories with lax  $[0, \infty)$ -action. This allows us to show that many commonly used distances, such as the  $L_\infty$  and Hausdorff metrics, are in fact special cases of interleaving distances. In addition, there is a natural way to define morphisms between these categories that generalizes the stability results of TDA to a broad class of objects by showing that the morphisms are 1-Lipschitz. As an application of this result, we will give an example of such a morphism, known as the hom-tree functor, which provides a new bound on the Reeb graph interleaving distance. (Received September 17, 2016)

1125-55-1597 **Michael Lesnick\*** (mlesnick@princeton.edu) and **Andrew Blumberg**. *Universality of the Homotopy Interleaving Distance.*

As a step towards establishing homotopy-theoretic foundations for the theory of persistent homology, we introduce and study *homotopy interleavings* between filtered topological spaces. These are homotopy-invariant analogues of *interleavings*, objects commonly used in topological data analysis to articulate stability and inference theorems. Whereas ordinary interleavings can be interpreted as pairs of “approximate isomorphisms” between filtered spaces, homotopy interleavings can be viewed as pairs of “approximate weak equivalences.”

Our main results are that homotopy interleavings induce an extended pseudometric  $d_{HI}$  on filtered spaces, and that this is the universal pseudometric satisfying natural stability and homotopy invariance axioms. We also show that  $d_{HI}$  (or more generally, any pseudometric satisfying these two axioms and an additional “homology bounding” axiom) can be used to formulate lifts of several fundamental topological data analysis theorems from the algebraic (homological) level to the level of filtered spaces. (Received September 18, 2016)

1125-55-1614 **Shaun Harker** (sharker@math.rutgers.edu), Department of Mathematics, Hill Center for the Mathematical Sciences, 110 Frelinghuysen Rd., Piscataway, NJ 08854-8019, **Miroslav Kramar** (kramar.miroslav.e1@tohoku.ac.jp), Advanced Institute for Material Research, 2-1-1 Katahira, Aoba-ku, Sendai, 980-8577, Japan AIMR Main Building, 4B, **Rachel Levanger\*** (rachel@math.rutgers.edu), Department of Mathematics, Hill Center for the Mathematical Sciences, 110 Frelinghuysen Rd., Piscataway, NJ 08854-8019, and **Konstantin Mischaikow** (mischaik@math.rutgers.edu), Department of Mathematics, Hill Center for the Mathematical Sciences, 110 Frelinghuysen Rd., Piscataway, NJ 08854-8019. *A comparison framework for interleaved persistence modules.*

Pursuant to the theme of this special session, understanding of the role of noise in computational topological measurements, we give a recent result in the theory of persistent homology that can be used to rigorously track noise introduced in the computation of persistence diagrams. We illustrate the use of this analytical framework by looking closely at common approximation techniques such as sub-sampling and discretization, and through other operations in, e.g. image processing, such as image smoothing. In each case, we contrast this result with the typical formulation of uniform errors achieved in terms of the Bottleneck distance, which can be seen as a type of sup-norm on the space of persistence diagrams. (Received September 18, 2016)

1125-55-1639 **Lew Ludwig\*** (ludwigl@denison.edu), 100 West College Street, Granville, OH 43023. *Knot mosaics - something for everyone.*

Since their inception in 2008 by Lomonaco and Kauffman, knot mosaics have provided a wealth of areas to study - mosaic number, composition, and tabulation to name a few. In this presentation we consider a wide array of questions about knot mosaics that require very little perquisite knowledge, ideal for students or faculty members with little experience in knot theory. We will consider questions that cover a range of interests: knot theory, combinatorics, graph theory, and even computer science. This presentation is intended for a general audience. (Received September 18, 2016)

1125-55-1725 **Amelia Tebbe\*** (tebbe2@illinois.edu). *Blowing up Atomic Functors.*

A functor from finite sets to chain complexes is called atomic if it is completely determined by its value on a particular set. In this talk, we present a new resolution for these atomic functors, which allows us to easily compute their Goodwillie polynomial approximations. By a rank filtration, any functor from finite sets to chain complexes is built from atomic functors. Computing the linear approximation of an atomic functor is a classic result involving partition complexes. Robinson constructed a bicomplex, which can be used to compute the linear approximation of any functor. We hope to use our new resolution to similarly construct bicomplexes that allow us to compute polynomial approximations for any functor from finite sets to chain complexes. (Received September 20, 2016)

1125-55-1936 **Jody Trout\*** (jody.trout@dartmouth.edu), 6188 Kemeny Hall, Hanover, NH 03755. *On the isomorphism between topological and geometric K-homology.* Preliminary report.

Let  $X$  be a topological space. Using ideas of Paul Baum, we will describe an isomorphism between the Baum-Douglas geometric  $K$ -homology of  $X$  (using  $\text{spin}^c$ -manifolds) and the topological  $K$ -homology of  $X$  (via the Bott spectrum) at the level of cycles, improving a result of Martin Jakob. (Received September 19, 2016)

1125-55-1941 **Keith Penrod\*** ([keith.penrod@morehouse.edu](mailto:keith.penrod@morehouse.edu)), 830 Westview Dr, Atlanta, GA 30314. *Higher Big Homotopy Groups.*

J Cannon and G Conner defined the “big fundamental group.” We extend that notion to higher dimensions, to define “big homotopy groups” for all dimensions, and consider the long exact homotopy sequence of a pair in this setting. (Received September 20, 2016)

1125-55-1943 **Julianna Tymoczko\*** ([jtymoczko@smith.edu](mailto:jtymoczko@smith.edu)), Department of Mathematics and Statistics, Smith College, Northampton, MA 01063. *Equivariant cohomology of certain affine Springer fibers.*

The (finite) Springer fiber is a subvariety of the flag manifold whose geometry encodes information about the representations of the symmetric group. We describe an infinite analogue: the affine Springer fiber sitting inside the affine Grassmannian. We will explicitly describe generating sets for certain affine Springer fibers, using the combinatorial methods of Goresky-Kottwitz-MacPherson. Part of this work is joint with Claudia Yun of Smith College. (Received September 19, 2016)

1125-55-2149 **Sarah Yeakel\*** ([syeakel@math.umd.edu](mailto:syeakel@math.umd.edu)). *Derivatives in Goodwillie Calculus.*

Goodwillie’s calculus of functors is a method of approximating a homotopy functor with easier “polynomial” functors, building a Taylor tower where the homogeneous pieces are classified by certain spectra, called the derivatives of the functor. It is known that the derivatives of the identity functor of spaces form an operad and composable functors have a chain rule. We will talk about some extra structure that the derivatives possess and extensions to other settings. (Received September 19, 2016)

1125-55-2277 **Leona Sparaco\*** ([lsparaco@math.fsu.edu](mailto:lsparaco@math.fsu.edu)). *Symmetries of Link Complements and Their Character Varieties.* Preliminary report.

The  $SL_2(\mathbb{C})$  character variety of a hyperbolic 3-manifold  $M$  is essentially the set of all representations of the fundamental group of  $M$  to  $SL_2(\mathbb{C})$  up to conjugation. This complex algebraic set encodes many properties of the manifold  $M$ . This talk explores the impact of a symmetry of the manifold on its character variety, focusing on some examples like the Borromean rings. (Received September 20, 2016)

1125-55-2307 **Victoria R Gerardi\*** ([vrgerardi@email.wm.edu](mailto:vrgerardi@email.wm.edu)), **Sarah Day** and **Yu-Min Chung**. *Computational Topology in Texture Classification.*

Computational topology has proven to be very effective in extracting information in areas such as biology and GIS to collect meaningful data that otherwise may be noisy or chaotic. This talk is focused on computational topology as a tool to classify 2-dimensional digital images of textures. Texture classification is important because being able to distinguish between the different images of materials is something that has many real world applications in fields such as medical imaging, industrial tests, etc. Texture classification is very difficult due to the amount of noise that often exists in digital images. Using the software Perseus, the persistent homology groups of distinct textures such as pasta, canvas, foam, etc. were calculated and then analyzed. The data was then tested as a classification method for textures, and was found to be a relatively functioning classification method between the different textures. (Received September 20, 2016)

1125-55-2329 **Ann Sizemore, Chad Giusti\*** ([cgiusti@seas.upenn.edu](mailto:cgiusti@seas.upenn.edu)), **Richard F Betzel** and **Danielle S Bassett**. *Topological structures in human brain networks.*

Encoding brain regions and their connections as a network of nodes and edges captures many of the possible paths along which information can be transmitted as humans process and perform complex behaviors. Here, we investigate both densely connected groups of nodes that could perform local computations as well as larger patterns of interactions that would allow for parallel processing. We detect cliques in the average structural connectomes of 8 healthy adults scanned in triplicate and discover the presence of more high-dimensional cliques than expected in null networks constructed via wiring minimization. This provides architecture through which brain network can perform rapid, local processing. Complementary to their study of locally dense structures, we locate persistent cycles and study their minimal representatives, which represent structure around which information may flow in either diverging or converging patterns. These cycles exist consistently across subjects, differ from those observed in null model networks, and – importantly – link regions of early and late evolutionary origin in long loops, underscoring their unique role in controlling brain function. (Received September 20, 2016)

1125-55-2457 **Hein van der Holst\*** ([hvanderholst@gsu.edu](mailto:hvanderholst@gsu.edu)), 30 Pryor St SW, Atlanta, GA 30303, and **Robin Thomas** and **Sergey Norin**. *Decomposing 2-cycles.*

For a graph  $G = (V, E)$ , a 2-cycle  $A = [a_{e,f}]$  is an  $E \times E$  matrix such that  $a_{e,f} = 0$  if  $e$  and  $f$  have a common vertex and each row and each column of  $A$  is a circulation on  $G$ . Examples of 2-cycles are 2-cycles coming from

pairs of disjoint cycles of  $G$ . Also on each subgraph of  $G$  that is a subdivision of  $K_5$  or  $K_{3,3}$ , there is a 2-cycle. It has been a conjecture that each 2-cycle can be written as a sum of these types of 2-cycles. For symmetric matrices, the presenter proved this in his work on a polynomial-time algorithm for finding a linkless embedding of a graph. For general matrices, this has recently been disproved by Barnett.

In this talk, we give a finite list of types of 2-cycles such that each 2-cycle is a sum of 2-cycles from this list. This solves a problem which has been open for over 40 years. We also show that for Kuratowski-connected graphs, it suffices to have 2-cycles coming from pairs of disjoint cycles of  $G$  and 2-cycles on subgraphs of  $G$  that are subdivisions of  $K_5$  or  $K_{3,3}$ . (Received September 20, 2016)

1125-55-2517 **Leanne Elizabeth Merrill\*** (leannem@uoregon.edu), Department of Mathematics, Fenton Hall, University of Oregon, Eugene, OR 97403. *Algebraic  $v_n$  self maps of spectra at the prime 2.*

A central question of algebraic topology is to understand homotopy classes of maps between finite cell complexes. The Nilpotence Theorem of Hopkins-Devinatz-Smith together with the Periodicity Theorem of Hopkins-Smith describes non-nilpotent self maps of finite spectra. The Morava K-theories  $K(n)_*$  are extraordinary cohomology theories which detect whether a finite spectrum  $X$  supports a  $v_n$  self map. Such maps are known to exist for each finite spectrum  $X$  for an appropriate  $n$  but few explicit examples are known. Working at the prime 2, we use a technique of Palmieri-Sadofsky to produce algebraic analogs of  $v_n$  maps that are easier to detect and compute. We reproduce the existence proof of Adams's  $v_1^4$  map on the Mod 2 Moore spectrum, and work towards a  $v_2^2$  map for a small value of  $i$ . (Received September 20, 2016)

1125-55-2595 **Joshua Lee Mike\*** (mike@math.utk.edu) and **Vasileios Maroulas.** *Combinatorial Hodge Theory for Equitable Kidney Paired Donation.* Preliminary report.

The problem of Kidney Paired Donation (KPD) has traditionally been approached within an integer programming framework. Here we adopt computational topology methods to find kidney exchange cycles. Employing Hodge theory, we decompose the edge flow describing the KPD pool into three parts. The curl portion of the flow represents local cycles and is trivial here. The gradient portion creates a scoring that we use to measure inequity in the kidney exchange. This scoring measures typical cases of disparity within a KPD pool, specifically that under demanded pairs and highly sensitized patients have lower scores than typical patient-donor pairs. The last portion of the decomposition is used to guide our search for kidney exchange cycles by capturing the 1-cohomology of the kidney exchange graph and investigating the tendency of the donations to occur in cycles. Further results demonstrate that PD pair score and the chance to obtain a kidney are positively correlated when using top trading cycles and chains; in contrast, we show that our method eliminates disparity in a KPD pool, i.e. the chance to obtain a kidney through our method is independent of score. (Received September 20, 2016)

1125-55-2703 **Philip C Puente\*** (philippuente@my.unt.edu), 1155 Union Circle #311430, Denton, TX 76203. *The braid group of crystallographic complex reflection groups.*

Crystallographic complex reflection groups have infinite order and stabilize a lattice in complex space. For the special case of affine Weyl groups, Viet Dung showed that the fundamental group of the space of regular orbits has a presentation mimicking the Coxeter presentation of the reflection group—one need only remove the condition that the generators have finite order. Malle conjectured that the same holds for all crystallographic complex reflection groups with their Coxeter-like presentations. We discuss some recent results on this conjecture obtained by adapting Viet Dung's method of semi-cell complexes to the setting of complex reflection groups. A complication in this new setting arises from the fact that the subgroup stabilizing a point may not be generated by reflections, unlike in the case of finite reflection groups. (Received September 20, 2016)

1125-55-2759 **Brittany Terese Fasy\*** (soc-info@montana.edu), MSU GSoC, Bozeman, MT 59717. *Topological Descriptors.* Preliminary report.

Over the past 20 years, topological data analysis (TDA) has developed as a field of mathematical research. Theoretical contributions have been made from researchers in mathematics, theoretical computer science, statistics, and machine learning. Other disciplines have started using techniques developed in TDA in order to have a new lens through which to view their data. In this talk, we will discuss some of the different topological descriptors that can be used to describe point clouds and images. We will also explain how these descriptors can be used for hypothesis testing, using an application in astronomy to motivate our discussion. (Received September 20, 2016)

1125-55-2764 **Ben Reid\*** ([bwr@uoregon.edu](mailto:bwr@uoregon.edu)). *Constructing a finite spectrum with a  $v_2$  self map at  $p = 3$ .* Certain families of periodic elements in the stable homotopy groups of spheres arise from non-nilpotent self maps of spectra with special homotopic properties. The Nilpotence and Periodicity theorems describe these  $v_n$  self maps and how to detect them using the Morava K theories. It is known that for each finite  $n$ , we can find a finite spectrum that has a  $v_n$  self map, but few concrete examples exist. Working at the prime 3, we use a technique of Palmieri and Sadofsky to construct algebraic analogs of the  $v_n$  self maps that are easier to compute, known as  $u_i$  self maps. In particular, we prove a theorem about the relation between  $u_i$  self maps and Margolis homology, and use it to produce a finite spectrum with a  $v_2$  self map. (Received September 20, 2016)

1125-55-2901 **Jisu Kim\*** ([jisuk1@andrew.cmu.edu](mailto:jisuk1@andrew.cmu.edu)), Department of Statistics, Carnegie Mellon University, 5000 Forbes Ave, Baker Hall 132, Pittsburgh, PA. *Statistical Inference on Topological Data Analysis*. Preliminary report.

Persistent homology robustly extracts topological features from sampled manifolds and level sets of sampled functions. To estimate persistent homology and measure its randomness, a confidence set can be computed for the sampled persistent homology. Such confidence set is constructed by combining bootstrap from statistics and stability theorem from computational topology. From the confidence set, topological signal and topological noise can be separated from the persistent homology. (Received September 20, 2016)

1125-55-3042 **Peter J. Bonventre\*** ([pjb9ub@virginia.edu](mailto:pjb9ub@virginia.edu)) and **Luis Pereira**. *Equivariant dendroidal sets and the homotopy theory of equivariant operads*.

In this talk, we will explore new frameworks for studying equivariant operads. The main tool is our category of genuine equivariant trees, which synthesizes the dendroidal category of trees  $\Omega$  with the orbit category of a finite group  $O_G$ . Using this, we define equivariant dendroidal sets, as well as a combinatorial candidate for  $G$ - $\infty$ -operads via a horn lifting condition. Moreover, we begin the process of showing that these concepts model the same homotopy theory as colored simplicial  $G$ -operads by building model structures and Quillen functors comparing them. (Received September 20, 2016)

1125-55-3101 **Juan S. Villeta-Garcia\*** ([villeta2@illinois.edu](mailto:villeta2@illinois.edu)). *Stabilizing Spectral Functors of Exact Categories*. Preliminary report.

Algebraic K-Theory is often thought of as “the” universal additive invariant of rings (or more generally, exact categories). Often, however, functors on exact categories don’t satisfy additivity. We will describe a procedure (due to McCarthy) that constructs a functor’s universal additive approximation, and apply it to different local coefficient systems, recovering known invariants of rings (K-Theory, THH, etc.). We will talk about what happens when we push these constructions to the world of spectra, and tie in work of Lindenstrauss and McCarthy on the Taylor tower of Algebraic K-Theory. (Received September 21, 2016)

## 57 ► *Manifolds and cell complexes*

1125-57-145 **Tyrone Ghaswala** and **Rebecca Winarski\*** ([winarskr@uwm.edu](mailto:winarskr@uwm.edu)). *Lifting Homeomorphisms and Cyclic Branched Covers of the Sphere*.

Birman and Hilden ask: given finite branched cover  $X$  over  $S^2$ , does every homeomorphism of  $S^2$  lift to a homeomorphism of  $X$ ? For covers of degree 2, the answer is yes, but the answer is sometimes yes and sometimes no for higher degree covers. In joint work with Ghaswala, we completely answer the question for cyclic branched covers. When the answer is yes, there is an embedding of the mapping class group of  $S^2$  into a finite quotient of the mapping class group of  $X$ . (Received August 03, 2016)

1125-57-429 **Eriko Hironaka\*** ([ehironaka@gmail.com](mailto:ehironaka@gmail.com)) and **Sarah Koch**. *A disconnected deformation space of rational maps*. Preliminary report.

This talk concerns the deformation space  $\text{Def}(f)$  of rational maps of pairs  $f : (\mathbb{P}^1, A) \rightarrow (\mathbb{P}^1, B)$  where  $A \subset B$ ,  $A$  and  $B$  are finite, and  $B$  contains the critical points of  $f$ . The space  $\text{Def}(f)$  was first defined by A. Epstein as a generalization of the space of combinatorially equivalent post-critically finite mapping classes studied by W. Thurston. In the post-critically finite case, Thurston showed that  $\text{Def}(f)$  is connected. In this talk I present joint work with Sarah Koch, showing that in general  $\text{Def}(f)$  may be disconnected by giving an explicit example where  $\text{Def}(f)$  has infinitely many connected components. (Received September 01, 2016)

1125-57-439 **Christine Ruey Shan Lee\*** (cleee@math.utexas.edu). *Stability and asymptotics of almost-adequate links.*

In relating quantum knot invariants such as the colored Jones polynomial to the topology of the knot complement, one often considers the stability and asymptotics of the polynomial. Adequate links form a rich class of links satisfying a diagrammatic condition, for which this approach has been particularly successful. For example, the Slope Conjecture is true for an adequate knot, and the corresponding boundary slope comes from a state surface which is easy to visualize. It is also known that the stable coefficients of the polynomial for an adequate knot give volume bounds and other topological information of the knot complement. In this talk, I will discuss a class of links diagrammatically similar to adequate links, which has analogous stability behaviors and asymptotics in the colored Jones polynomial. It is expected that various geometric/topological results known for adequate links will generalize to this class. (Received September 01, 2016)

1125-57-445 **Arunima Ray\*** (aruray@brandeis.edu), 415 South St MS 050, Waltham, MA 02453, and **Daniel Ruberman.** *4-dimensional analogues of Dehn's lemma.*

We investigate certain 4-dimensional analogues of the classical 3-dimensional Dehn's lemma, giving examples where such analogues do or do not hold, in the smooth and topological categories. In particular, we show that an essential 2-sphere  $S$  in the boundary of a simply connected 4-manifold  $W$  such that  $S$  is null-homotopic in  $W$  need not extend to an embedding of a ball in  $W$ . However, if  $W$  has abelian fundamental group (e.g. if  $W$  is simply connected) with boundary a homology sphere, then  $S$  bounds a topologically embedded ball in  $W$ . Moreover, we give examples where such an  $S$  does not bound any smoothly embedded ball in  $W$ . In a similar vein, we construct incompressible tori  $T \subseteq \partial W$  where  $W$  is a contractible 4-manifold such that  $T$  extends to a map of a solid torus in  $W$ , but not to any embedding of a solid torus in  $W$ . Moreover, we construct an incompressible torus  $T$  in the boundary of a contractible 4-manifold  $W$  such that  $T$  extends to a topological embedding of a solid torus in  $W$  but no smooth embedding. As an application of our results about tori, we address a question posed by Gompf about extending certain families of diffeomorphisms of 3-manifolds which he has recently used to construct infinite corks. (Received September 02, 2016)

1125-57-448 **Micah Chrisman\*** (mchrisma@monmouth.edu) and **Robert Todd.** *Virtual Knots and the Multi-variable Alexander Polynomial of Boundary Links in the 3-Sphere.* Preliminary report.

A two component oriented link  $L = K_1 \sqcup K_2$  in  $\mathbb{S}^3$  is said to be a boundary link if there are Seifert surfaces  $\Sigma_1, \Sigma_2$  for  $K_1, K_2$ , respectively, such that  $\Sigma_1 \cap \Sigma_2 = \emptyset$ . Given a boundary link, when can the disjoint surfaces  $\Sigma_1$  and  $\Sigma_2$  be chosen so that  $\Sigma_1$  is also a minimal genus Seifert surface for  $K_1$ ? We discuss an obstruction arising from virtual knot theory when  $K_1$  is a fibered knot. The obstruction relates the multi-variable Alexander polynomial of a boundary link (in the sense of Gutiérrez) to the Alexander polynomial of an almost classical knot (Boden-Gaudreau-Harper-Nicas-White). Examples utilizing the obstruction are given. (Received September 02, 2016)

1125-57-463 **Samuel J Taylor\*** (s.taylor@yale.edu). *Convex cocompactness and stability in mapping class groups.*

Convex cocompact subgroups of mapping class groups were introduced by Farb and Mosher and have important connections to the geometry of Teichmüller space, the curve complex, and surface group extensions. In this talk, I will discuss a characterization of such subgroups that involves only the geometry of the mapping class group. This characterization involves a strong notion of quasiconvexity, which we call stability, and captures the intuition that convex cocompact subgroups are "highly hyperbolic" subgroups of mapping class groups. I will also discuss stable subgroups of other finitely generated groups of geometric interest. (Received September 02, 2016)

1125-57-509 **Caitlin Levenson\*** (leverson@math.gatech.edu). *Invariants of Legendrian Links.*

Given a plane field  $dz - xdy$  in  $\mathbb{R}^3$ . A Legendrian knot is a knot which at every point is tangent to the plane at that point. One can similarly define a Legendrian knot in any contact 3-manifold (manifold with a plane field satisfying some conditions). In this talk, we will explore Legendrian knots in  $\mathbb{R}^3$ ,  $J^1(S^1)$ , and  $\#^k(S^1 \times S^2)$  as well as a few Legendrian knot invariants. We will also look at the relationships between a few of these knot invariants. No knowledge of Legendrian knots will be assumed though some knowledge of basic knot theory would be useful. (Received September 04, 2016)

1125-57-521 **Erica Flapan\*** (eflapan@pomona.edu), 610 N. College Ave., Department of Mathematics, Pomona College, Claremont, CA 91711. *Topological Complexity in Protein Structures*. Preliminary report.

For DNA molecules, topological complexity occurs exclusively as the result of knotting or linking of the polynucleotide backbone. By contrast, while a few knots and links have been found within the polypeptide backbones of some protein structures, non-planarity can also result from the connectivity between a polypeptide chain and inter- and intra-chain linking via cofactors and disulfide bonds. In this talk, we survey the known types of knots, links, and non-planar graphs in protein structures with and without including such bonds and cofactors. Then we present a model which could explain why certain non-planar configurations are more likely to occur than others. (Received September 05, 2016)

1125-57-606 **Colin Adams\*** (cadams@williams.edu), Department of Mathematics, Bronfman Science Center, Williams College, Williamstown, MA 01267. *Open accessible problems in knot theory*.

Knot theory provides fertile ground for undergraduate research. There are pretty pictures, students can work on explicit examples immediately, and there is deep and beautiful mathematics embedded in the subject. In this talk, we will see how one can generalize the concept of crossing number of knots to obtain a bucketful of open questions. Moreover, we can define petal number (least number of petals in a flower-like diagram that represents the knot) and come up with many more fascinating questions. (Received September 07, 2016)

1125-57-636 **Daniele Alessandrini** (daniele.alessandrini@gmail.com), **Sara Maloni\*** (sm4cw@virginia.edu) and **Anna Wienhard** (wienhard@mathi.uni-heidelberg.de). *The geometry of quasi-Hitchin symplectic Anosov representations*.

After revising the background theory of symplectic Anosov representations and their domains of discontinuity, we will focus on our joint work in progress with Daniele Alessandrini and Anna Wienhard. In particular, we will describe partial results about the homeomorphism type of the quotient of the domain of discontinuity for quasi-Hitchin representations in  $\mathrm{Sp}(4, \mathbb{C})$  acting on the Lagrangian space  $\mathrm{Lag}(\mathbb{C}^4)$  (Received September 08, 2016)

1125-57-731 **Charles Frohman\*** (charles-frohman@uiowa.edu), Department of Mathematics, The University of Iowa, Iowa City, IA 52242, and **Joanna Kania-Bartoszyńska** and **Thang Le**. *Representations of the Kauffman bracket skein algebra at roots of unity*.

Let  $F$  be a finite type surface,  $\zeta$  a primitive  $n$ th root of unity. The Kauffman bracket skein algebra  $K_\zeta(F)$  is a noncommutative algebra built from equivalence classes of framed links in  $F \times [0, 1]$  modulo the Kauffman bracket skein relations with the variable set to be  $\zeta$ . The product comes from stacking. If  $n \equiv 2 \pmod{4}$ , then the center of  $K_\zeta(F)$  is a finite extension of the coordinate ring of the  $SL_2\mathbb{C}$ -character variety of the fundamental group of  $F$ . If  $n$  is odd, the center of  $K_\zeta(F)$  is a finite extension of the coordinate ring of that part of the  $PSL_2\mathbb{C}$ -character variety of the fundamental group of  $F$  coming from representations that lift to  $SL_2\mathbb{C}$ .

We prove that there is a nonempty Zariski open subset  $V_c$  of the maximal spectrum of the center of  $K_\zeta(F)$  that parameterizes a family of irreducible representations of  $K_\zeta(F)$  all having the same dimension. If  $m = \frac{n}{\gcd(n, 4)}$ , if  $n \not\equiv 0 \pmod{4}$ , and  $F$  has at least one puncture the dimension of these representations is  $m \frac{-3e(F) - p}{2}$  where  $e(F)$  is the Euler characteristic of  $F$  and  $p$  is the number of punctures. (Received September 10, 2016)

1125-57-784 **Adam Lowrance\*** (adlowrance@vassar.edu). *Computing the Turaev genus of a link*.

The Turaev surface of a link diagram is an oriented Heegaard surface constructed using the all-A and all-B Kauffman states of the link diagram. The Turaev genus of a link is the minimum genus of a Turaev surface for any diagram of the link. In this talk, we discuss some recent computations of the Turaev genus of links with few crossings and of several infinite families of torus knots. (Received September 11, 2016)

1125-57-934 **E Flapan** and **E Davie Lawrence\*** (edlawrence@usfca.edu). *Topological symmetry groups of Möbius ladders*.

Chemists have been trying for decades to synthesize molecules with topologically interesting structures. This served as motivation for the study of symmetries of graphs embedded in  $S^3$ . Furthermore, the questions arising from chemists have led to answers that are topologically fascinating in their own right. We will define the topological symmetry group of a graph embedded in  $S^3$ , and discuss recent work on exactly what groups are realizable as topological symmetry groups for a certain class of graphs known as Möbius ladders. (Received September 13, 2016)

1125-57-1003 **Oliver J Thistlethwaite\*** ([oliver@math.ucr.edu](mailto:oliver@math.ucr.edu)), 1144 Park Hill Cir, Knoxville, TN 37909. *Seiberg-Witten Invariants, Alexander Polynomials, and Fibred Classes.*

Since their introduction in 1994, the Seiberg-Witten invariants have become one of the main tools used in 4-manifold theory. In this talk, we will show these invariants can be used to identify sufficient conditions for a 3-manifold to fibre over a circle. (Received September 13, 2016)

1125-57-1126 **Stephen Lo\***, Department of Mathematics, The University of Auckland, Private Bag 92019, Auckland, Auckland 1142, New Zealand. *The symmetric genus of metabelian groups.* Preliminary report.

The *symmetric genus* of a finite group  $G$ , denoted by  $\sigma(G)$ , is defined as the smallest non-negative integer  $g$  such that there exists a compact Riemann surface of genus  $g$  on which the group  $G$  has a faithful action as a group of automorphisms, some of which may reverse the surface's orientation. A natural question to ask is whether, for each non-negative integer  $g$ , there exists a finite group  $G$  such that  $\sigma(G) = g$ . It is known that the answer is "yes" for all  $g \not\equiv 8$  or  $14 \pmod{18}$ , and there is evidence that some of the remaining gaps in the spectrum (range of values) of the symmetric genus can be filled by considering finite metabelian groups. Our research aims to determine the symmetric genus of various families of finite metabelian groups, starting with metacyclic groups. In this talk, we will describe the method of determining the symmetric genus of finite groups using the Riemann-Hurwitz equation, which allows us to treat the problem as a purely algebraic one. We will also present results for some families of metabelian groups that we have considered, including metacyclic split extensions of the form  $C_m : C_p$  where  $p$  is a prime. (Received September 15, 2016)

1125-57-1186 **Jeff Cheeger\*** ([cheeger@cims.nyu.edu](mailto:cheeger@cims.nyu.edu)), 251 Mercer St., New York, NY 10012. *L-classes: Is there a computable local combinatorial formula?* Preliminary report.

The issue of obtaining a computable local combinatorial formula for the  $L$ -classes of a triangulated manifold has been of interest since the 1960's, or perhaps even earlier. For instance, one (unsuccessful) attempt in the late 1960's to derive such a formula gave rise to Chern-Simons invariants. In the early 1980's, the author gave a local formula for the  $L$ -classes of a triangulated manifold equipped with a piecewise flat metric. The formula is in terms of  $\eta$ -invariants of links. It gives rise to a local combinatorial formula when one chooses the metric for which all of the simplices are regular. While the formula does not involve any arbitrary choices, it has the disadvantage that in general,  $\eta$ -invariants are difficult to compute. We will indicate a current project aimed at gaining some further understanding of the relevant  $\eta$ -invariants. (Received September 15, 2016)

1125-57-1356 **Dan Margalit\*** ([margalit@math.gatech.edu](mailto:margalit@math.gatech.edu)), **Balazs Strenner** and **S. Öykü Yurttaş** ([oykuyurttas@googlemail.com](mailto:oykuyurttas@googlemail.com)). *Fast Nielsen-Thurston Classification.*

We consider two algorithmic questions concerning the mapping class group. The first is the Nielsen-Thurston classification problem: given a mapping class, determine if it is periodic, reducible, or pseudo-Anosov (and if it is periodic find the order, if it is reducible find the reducing curves, and if it is pseudo-Anosov find the stable and unstable laminations and the stretch factor). The second problem is the conjugacy problem (or even more: the conjugacy search problem). We show that both problems have polynomial time solutions. (Received September 16, 2016)

1125-57-1459 **Radmila Sazdanovic\*** ([rsazdan@ncsu.edu](mailto:rsazdan@ncsu.edu)), Department of Mathematics NCSU, Box 8205, Raleigh, NC 27695, and **Andrew Cooper** and **Vin de Silva**. *Invariants of Simplicial Complexes from Configuration Spaces.* Preliminary report.

The configuration space of  $n$  distinct points in a manifold  $X$  is a well-studied object with lots of applications. Eastwood and Huggett define graph configuration spaces  $M(G, X)$  by allowing vertices connected by an edge in  $G$  to occupy the same point in  $X$ . Our work generalizes this construction from graphs to finite simplicial complexes to obtain the space  $M(S, X)$ . In this talk we will discuss properties of homology of  $M(S, X)$  and the polynomial invariant of simplicial complexes arising as its Euler characteristic. (Received September 16, 2016)

1125-57-1471 **Jenya Sapir\*** ([jsapir2@illinois.edu](mailto:jsapir2@illinois.edu)), 1409 W Green St, Urbana, IL 61801. *Geodesics on surfaces.*

We will give an overview of various problems about geodesics on a hyperbolic surface  $S$ . We will first discuss various bounds on the number of closed geodesics on  $S$  given upper bounds on length and self-intersection number. We will then discuss some application of these counting results. For example, they can be used to show that complete geodesics with "too few" self-intersections per unit length lie in a very small part of  $S$ . This extends a result of Birman and Series for complete geodesics with finitely many self-intersections. There will be lots of pictures. (Received September 17, 2016)

1125-57-1491 **Ben Blum-Smith\*** ([ben@cims.nyu.edu](mailto:ben@cims.nyu.edu)), Courant Institute of Mathematical Sciences, 251 Mercer St., Room 607, New York, NY 10012. *When is a sphere quotient a sphere? And an application to invariant theory.*

Let  $G$  be a finite group of orthogonal transformations of  $\mathbb{R}^n$ . The unit sphere carries the structure of both a topological and piecewise-linear manifold, so that  $G$  acts on it by piecewise-linear homeomorphisms. For what  $G$  is the quotient by this action a topological sphere? A piecewise-linear sphere? We report on recent work of Christian Lange which fully answers these questions, and apply the results to the invariant theory of finite groups, as follows. The invariant ring of a finite group acting linearly on a polynomial ring over a field is usually Cohen-Macaulay, meaning that it is a finite free module over a subring generated by algebraically independent elements. However, it can fail to be Cohen-Macaulay if the group order is divisible by the characteristic of the field, and it has become an important problem in invariant theory to determine when this failure occurs. We use a connection between commutative algebra and the topology of cell complexes developed in the 70's and 80's by Hochster, Stanley, Reisner, Garsia, and Stanton to show that Lange's results can be used to give a complete solution to this problem in the case that  $G$  is a permutation group. (Received September 17, 2016)

1125-57-1557 **Anar Akhmedov and Nur Saglam\*** ([sagla004@umn.edu](mailto:sagla004@umn.edu)). *Constructions of new Lefschetz fibrations using cyclic group actions.* Preliminary report.

Using the cyclic group actions, we construct the families of Lefschetz fibrations over  $S^2$ . We also obtain more families of Lefschetz fibrations gluing these examples. If time permits, we will also talk about some applications of these Lefschetz fibrations. This is a joint work with Anar Akhmedov (Received September 18, 2016)

1125-57-1627 **Martin Bridgeman** ([bridgem@bc.edu](mailto:bridgem@bc.edu)), **Richard Canary\*** ([canary@umich.edu](mailto:canary@umich.edu)) and **François Labourie** ([francois.labourie@math.unice.fr](mailto:francois.labourie@math.unice.fr)). *Simple length spectrum rigidity.*

It is well known that a discrete, faithful representation of a closed surface group into  $\mathrm{PSL}(2, \mathbb{R})$  is determined, up to conjugacy in  $\mathrm{PGL}(2, \mathbb{R})$ , by the translation lengths of finitely many elements represented by simple closed curves. We will discuss generalizations of this result to the setting of quasifuchsian representations into  $\mathrm{PSL}(2, \mathbb{C})$  and Hitchin representations into  $\mathrm{PSL}(n, \mathbb{R})$ . We will also discuss applications of our simple length rigidity results to the study of the pressure metric. (This is joint work with Martin Bridgeman and François Labourie) (Received September 18, 2016)

1125-57-1862 **Thomas W. Tucker\*** ([ttucker@colgate.edu](mailto:ttucker@colgate.edu)), 406 Williston Rd, Po Box 163, Sagamore Beach, MA 02562, and **Undine Leopold**. *Orientation-Reversing Euclidean Symmetry of Closed Surfaces Immersed in 3-Space.* Preliminary report.

Given a finite group  $G$  of isometries of euclidean 3-space  $E^3$  and a closed surface  $S$ , an immersion  $f : S \rightarrow E^3$  is in  $G$ -general position if  $f(S)$  is invariant under  $G$ , points of  $S$  have disk neighborhoods whose images are in general position, and no singular points of  $f(S)$  lie on an axis of rotation of  $G$ . For such an immersion, there is an induced action by  $G$  on  $S$  whose generalized Riemann-Hurwitz equation (GRH), that is, orbifold  $S/G$  with branching information, satisfies certain natural restrictions. We classify which restricted GRH are realized by a  $G$ -general position immersion of  $S$ . The authors had previously done this when  $G$  is orientation-preserving. Nearly always, if the usual Riemann-Hurwitz equation for the orientation-preserving subgroup  $G^\circ$  can be realized, so can the GRH for  $G$ . (Received September 19, 2016)

1125-57-1874 **Aaron Fenyes\*** ([afenyes@math.toronto.edu](mailto:afenyes@math.toronto.edu)), Dept. of Mathematics, University of Toronto, Bahen Centre, 40 St. George St., Room 6290, Toronto, Ontario M5S 2E4, Canada. *Spectral networks craft hour.* Preliminary report.

Spectral networks are tools for cutting up local systems on surfaces and reassembling them into more convenient ones. I'll show you how they work by doing the most hands-on thing I can do with them: cutting up hyperbolic surfaces and reassembling them into flat ones. Some classic pieces of geometry, including Apollonian packings, Strebel differentials, shear coordinates, and Fenchel-Nielsen coordinates, emerge naturally from this construction. (Received September 19, 2016)

1125-57-1974 **M Kate Kearney\*** ([kearney@gonzaga.edu](mailto:kearney@gonzaga.edu)). *The Stable Concordance Genus of Knots.*

The concordance genus of a knot is the least three-genus of a knot concordant to the knot. The concordance genus is bounded below by the four-genus (or slice genus), and bounded above by the three-genus. This makes the concordance genus a valuable tool to describe the difference between these invariants. In simple cases the concordance genus is not difficult to calculate, since there are a variety of algebraic tools that give bounds for the concordance genus. Unfortunately, as the crossing number increases, it becomes increasingly difficult to find concordances. In this talk we will review some basics of knot theory and concordance and lead to a discussion of the concordance genus as a tool for studying concordance of knots. The stable concordance genus, which we

will also discuss in this talk, describes the behavior of the concordance genus of a given knot under connect sum. We will briefly define the invariant, give some examples of calculations, and discuss applications to the study of concordance. (Received September 19, 2016)

1125-57-1994 **Emily K. Vecchia\*** ([vecc2404@stthomas.edu](mailto:vecc2404@stthomas.edu)). *To what extent does the knotting of open subarcs of a closed knot configuration predict the knot type of the configuration?*  
Preliminary report.

In traditional knot theory, researchers study entanglement predominately in closed curves. In nature, however, there are many materials which are entangled but form open curves, such as DNA and proteins. To understand the knotting in these physical systems, a rigorous mathematical definition of knotting for open curves is needed. Several such definitions have been proposed, each with their own advantages and disadvantages. In this talk, we analyze to what extent the different definitions of open knotting “converge” to traditional knot theory. In particular, given an open subarc of a closed polygonal knot, we analyze to what extent the classification of the knotting in the subarc coincides with the knot type of the closed knot as a function of length for these different open knotting definitions. (Received September 19, 2016)

1125-57-1995 **Oliver J O’Keefe\*** ([okee0055@stthomas.edu](mailto:okee0055@stthomas.edu)). *Knotting Transitions in Tight Knots*.  
Preliminary report.

Glueballs are subatomic particles which are hypothesized to take the shape of tightened knotted and linked uniform-radius tubes. Under this hypothesis, one of the ways that glueballs could decay is via quantum tunneling events. In such an event, the tube fully passes through itself at a random self-contact along the tube. The goal of this research is to determine all of the possible knot type changes which can occur from quantum tunneling events in model glueballs. (Received September 19, 2016)

1125-57-2070 **Kin Hei A. Mak\*** ([km6hb@virginia.edu](mailto:km6hb@virginia.edu)). *An alternative notion of symplectic 4-manifolds of general type*. Preliminary report.

The Seiberg-Witten invariants of minimal complex surfaces of general type have been understood for a long time to be as simple as possible while remaining nontrivial: such surfaces have a single Seiberg-Witten basic class, up to sign. A natural generalization of the notion of general type to the symplectic case is the condition that a symplectic 4-manifold have symplectic Kodaira dimension 2. However, the set of basic classes for such a symplectic manifold may be large; in particular this class of symplectic 4-manifolds does not reflect the behavior of general type surfaces as far as Seiberg-Witten theory. Here we describe a geometric-topological condition on a symplectic 4-manifold that guarantees it has just one Ozsváth-Szabó basic class up to sign. As a consequence, we provide a calculation of the Ozsváth-Szabó 4-manifold invariants for a large class of general type surfaces, independent of the conjectured equivalence between Ozsváth-Szabó and Seiberg-Witten invariants of 4-manifolds. (Received September 19, 2016)

1125-57-2090 **Thang Le\*** ([1etu@math.gatech.edu](mailto:1etu@math.gatech.edu)) and **Jon Paprocki**. *Skein algebra and quantum Teichmuller space of surfaces*.

We show how to decompose the Kauffman bracket skein algebra of a surface into elementary blocks corresponding to the triangles in an ideal triangulation of the surface. This gives an easy proof of the existence of the quantum trace map of Bonahon and Wong relating the skein algebra and the quantum Teichmuller space. (Received September 19, 2016)

1125-57-2200 **Katherine Walsh\*** ([k3walsh@math.arizona.edu](mailto:k3walsh@math.arizona.edu)). *Patterns and Stability in the Colored Jones Polynomial*.

The colored Jones polynomial assigns to each knot a sequence of Laurent polynomials. This talk will explain some of the patterns and stability in the coefficients of these polynomials. We will look at what knot information is encoded in various polynomials that appear in the higher order stable sequences. We will also explore how this stability relates to other stabilities in families of knots related to each other by adding twists. (Received September 19, 2016)

1125-57-2239 **Eric G Samperton\*** ([egsamp@math.ucdavis.edu](mailto:egsamp@math.ucdavis.edu)). *Computational complexity and 3-manifolds and zombies*.

We consider the computational complexity of counting homomorphisms from 3-manifold groups to fixed finite groups  $G$ . Let  $G$  either be non-abelian simple or  $S_m$ , where  $m \geq 5$ . Then counting homomorphisms from fundamental groups of 3-manifolds to  $G$  is  $\#P$ -complete. It follows that determining when the fundamental group of a 3-manifold admits a nontrivial homomorphism to  $G$  is NP-complete. In particular, for fixed  $m \geq 5$ , it is NP-complete to decide when a 3-manifold admits a connected  $m$ -sheeted cover.

These results follow from an analysis of the action of the pointed mapping class group  $\text{Mod}_*(\Sigma_g)$  on the set of homomorphisms  $X_g := \{\pi_1(\Sigma_g) \rightarrow G\}$ . We build on ideas of Dunfield-Thurston that were originally used in the context of random 3-manifolds. In particular, we show that when  $g$  is large enough, there exists a subgroup of  $\text{Mod}_*(\Sigma_{2g})$  that acts on  $X_g^2$  in a manner that allows us to produce gadgets encoding reversible logic gates. Our construction can be considered as a classical analogue of topological quantum computing. This is joint work with Greg Kuperberg. (Received September 20, 2016)

1125-57-2336 **Moshe Cohen\*** ([mcohen@vassar.edu](mailto:mcohen@vassar.edu)) and **Elia Saini**. *Extending a result of Nazir and Yoshinaga to distinguish more interesting hyperplane arrangements*. Preliminary report.

The moduli space of an arrangement is the space of all combinatorially equivalent arrangements; we are motivated by the topology of this space. When a line is added in a relatively trivial way to an arrangement whose moduli space has just one component, the resulting line arrangement also has a moduli space of just one component by a result of Nazir and Yoshinaga appearing in a paper on arrangements of eight lines. The first author used this to study arrangements of ten lines with more interesting moduli spaces.

We extend this result to hyperplane arrangements of higher dimension. (Received September 20, 2016)

1125-57-3070 **William Ballinger, Ching-Yun Hsu, Wyatt Mackey, Yi Ni, Tynan Ochse and Faramarz Vafaee\*** ([vafaee@caltech.edu](mailto:vafaee@caltech.edu)), 1200 E California Blvd, Pasadena, CA 91125. *The prism manifold realization problem*.

The spherical manifold realization problem asks which spherical three-manifolds arise by surgery on knots in  $S^3$ . In recent years, the realization problem for C,T,O,I-type spherical manifolds has been solved, leaving the D-type spherical manifolds (aka prism manifolds) as the only remaining case. Every prism manifold can be parametrized as  $P(p, q)$ , for a pair of relatively prime integers  $p > 1$  and  $q$ . We determine a complete list of prism manifolds  $P(p, q)$  that can be realized by positive integral surgery on knots in  $S^3$  when  $q < 0$ . The general methodology undertaken to obtain the classification is similar to that of Greene for lens spaces. The arguments rely on tools from Floer homology and lattice theory, and are primarily combinatorial in nature. (Received September 20, 2016)

## 58 ► *Global analysis, analysis on manifolds*

1125-58-1001 **Victor Guillemin, Eva Miranda and Jonathan Weitsman\*** ([j.weitsman@neu.edu](mailto:j.weitsman@neu.edu)). *On geometric quantization of b-symplectic manifolds*.

We study a notion of pre-quantization for  $b$ -symplectic manifolds. We use it to construct a formal geometric quantization of  $b$ -symplectic manifolds equipped with Hamiltonian torus actions with nonzero modular weight. We show that these quantizations are finite dimensional  $T$ -modules. (Received September 13, 2016)

1125-58-1317 **Paul Frank Baum\*** ([pxb6@psu.edu](mailto:pxb6@psu.edu)), Mathematics Department, McAllister Building, Penn State University, University Park, PA 16802. *Beyond Ellipticity*.

This talk applies K-homology to solve the index problem for a class of hypoelliptic (but not elliptic) differential operators on contact manifolds. K-homology is the dual theory to K-theory. An explicit formula will be given for the K-cycle (i.e., the element in geometric K-homology) determined by certain Fredholm differential operators in the Heisenberg calculus. The index theorem of this talk indicates how the analytic versus geometric K-homology setting provides an effective framework for extending formulas of Atiyah–Singer type to non-elliptic Fredholm operators. Given an index problem, the K-homology framework provides a guide and hint as to what the solution of that index problem might be. The above is joint work with Erik van Erp. (Received September 16, 2016)

1125-58-1619 **Lisa Jeffrey\*** ([jeffrey@math.toronto.edu](mailto:jeffrey@math.toronto.edu)), Mathematics Dept., Univ. of Toronto, Toronto, ON M4T3B4, Canada. *Real loci in symplectic manifolds*.

Let  $M$  be a symplectic manifold and let  $\sigma$  be an antisymplectic involution on  $M$ . The real locus is the fixed point set of the involution. It is a Lagrangian submanifold. Suppose also  $M$  is equipped with the Hamiltonian action of a torus  $T$ . It is possible to define a compatibility between  $T$  and  $M$ . This set of ideas was introduced in a 1983 paper by Hans Duistermaat.

In this talk I will describe some developments in this field since Duistermaat's foundational paper. My contributions in this area are joint work with Liviu Mare, and (in a separate project) with Nan-Kuo Ho, Khoa Dang Nguyen and Eugene Xia. (Received September 18, 2016)

1125-58-1751 **Caleb Mayer\*** (c1mayer@haverford.edu), 184 Forest Street, Oberlin, OH 44074, and **Robert Dougherty-Bliss** and **Heidi Whiteside**. *Global Dynamics of a Cancer Stem Cell Treatment Model*. Preliminary report.

We provide global stability arguments for a cancer treatment model with chemotherapy and radiotherapy that accounts for the cancer stem cell hypothesis. Employing the method of localization of compact invariant sets, we resolve the global dynamics of the no-treatment, constant radiation, and combination chemotherapy and radiotherapy cases. In our analysis of the combination treatment model, we show that the presence of a chemotherapy agent lowers the required radiation strength for a globally asymptotically stable cure state. (Received September 19, 2016)

1125-58-2188 **Rebecca Blaire Hoehne\*** (rhoehne01@saintmarys.edu), **Zachary Abernathy**, **Kristen Abernathy**, **Claire Burgess** and **Kelsey Brown**. *Global Dynamics of a Colorectal Cancer Treatment Model with Cancer Stem Cells*. Preliminary report.

We present and analyze a mathematical model of the treatment of colorectal cancer using a system of nonlinear ordinary differential equations. The model describes the effectiveness of immunotherapy and chemotherapy for treatment of tumor cells and cancer stem cells (CSCs). The effects of CD8<sup>+</sup>T cells, natural killer cells, and interleukin proteins on tumor cells and CSCs under the influence of treatment are also illustrated. Using the method of localization of compact invariant sets, we present conditions on treatment parameters to guarantee a globally attracting tumor clearance state. Numerical simulations and sensitivity analyses of the model are examined using biologically sound parameters to assess the validity of the model. (Received September 19, 2016)

1125-58-2826 **Phillip Andreae\*** (pvandreae@meredith.edu). *Analytic torsion, the eta invariant, and closed differential forms on spaces of metrics*.

We introduce an interpretation of certain invariants constructed from Laplace spectral data on a compact Riemannian manifold as regularized integrals of closed differential forms on the space of Riemannian metrics, or more generally on a space of metrics on a vector bundle. We apply this idea to both the Ray-Singer analytic torsion and the eta invariant, explaining their dependence on the metric used to define them with a Stokes' theorem argument. We also introduce analytic multi-torsion, a generalization of analytic torsion, in the context of certain manifolds with local product structure; we prove that it is metric independent in a suitable sense. (Received September 20, 2016)

## 60 ► Probability theory and stochastic processes

1125-60-169 **Jin Sixian\*** (sixian.jin@cgu.edu), 710 N. College Ave., Claremont, CA 91711, and **Schellhorn Henry** and **Vives Josep**. *Dyson type formula for pure jump Lévy processes and applications*. Preliminary report.

In this talk, we obtain a Dyson type formula for pure jump Lévy processes, that is, we represent the conditional expectation of a functional of pure jump Lévy processes as a convergent series in terms of the Malliavin derivatives evaluated along a "frozen path". When the target random variable depends on a discrete trajectory of Lévy processes, this series can be simplified to a backward Taylor expansion. These series representations turn out to be useful for different applications. In particular in Quantitative Finance, we present the application on the option pricing problem of Lévy quadratic model. (Received August 07, 2016)

1125-60-227 **Radu Balan\*** (rvbalan@math.umd.edu), Department of Mathematics, College Park, MD 20742. *Statistics of the Stability Bounds in the Phase Retrieval Problem*. Preliminary report.

In this talk we present estimates of the tail-bound for the distribution of the Lipschitz constants in the phase retrieval problem. Specifically it is known that if the frame  $\{f_1, \dots, f_m\}$  for  $C^n$  is phase retrievable then there are constants  $a_0$  and  $b_0$  so that for every  $x, y \in C^n$ :

$$a_0 \|xx^* - yy^*\|_1^2 \leq \sum_{k=1}^m \left| \langle x, f_k \rangle \right|^2 - \left| \langle y, f_k \rangle \right|^2 \leq b_0 \|xx^* - yy^*\|_1^2.$$

Assume  $f_1, \dots, f_m$  are independent realizations with entries from  $CN(0, 1)$ . In this talk we establish estimates for the probability  $P(a_0 > a)$ . (Received August 15, 2016)

1125-60-281 **Jinho Baik\*** ([baik@umich.edu](mailto:baik@umich.edu)). *Totally asymmetric simple exclusion process on a ring.*  
 We discuss the effect of the system size to the particle fluctuations of interacting particle systems in the KPZ universality class. In particular we consider the TASEP (totally asymmetric simple exclusion process) on a ring in the large ring, large time limit with constant average particle density for two types of initial conditions. This system is equivalently to a periodic TASEP and also a directed last passage site percolation with periodic weights. The hydrodynamic limit of the periodic TASEP is same as the one for the usual TASEP for all large time, but the particle fluctuations agree only up to certain time scale depending on the system size. We discuss especially what happens at the critical time scale. This is a joint work with Zhipeng Liu. (Received August 22, 2016)

1125-60-391 **Randall Herrell, Renming Song, Dongsheng Wu\*** ([dongsheng.wu@uah.edu](mailto:dongsheng.wu@uah.edu)) and **Yimin Xiao.** *Sample Paths of the Solution to the Fractional Stochastic Heat Equation Driven by a Fractional-Colored Noise.* Preliminary report.  
 Let  $\{u(t; x); t \in [0; T], x \in \mathbf{R}^d\}$  be the solution to the linear fractional stochastic heat equation driven by a fractional noise in time with correlated spatial structure. We first prove the existence and uniqueness of the solution process  $u$ , then study various path properties of  $u$  with respect to the time and to the space variable, respectively. In particular, we derive exact uniform moduli of continuity and Chung- type laws of iterated logarithm. (Received August 31, 2016)

1125-60-397 **Christian Houdre\*** ([houdre@math.gatech.edu](mailto:houdre@math.gatech.edu)). *Asymptotics in Sequence Comparison.*  
 Both for random words and random permutations, I will present a panoramic view of recent results on the asymptotic law of the, centered and normalized, length of their longest common (and increasing) subsequences. (Received August 31, 2016)

1125-60-569 **Chen Xu\*** ([cxu60@math.gatech.edu](mailto:cxu60@math.gatech.edu)) and **Christian Houdre** ([houdre@math.gatech.edu](mailto:houdre@math.gatech.edu)). *Concentration of Geodesics in Directed Bernoulli Percolation.* Preliminary report.  
 We study the deviation of the geodesics from the main diagonal in directed last passage site percolation with i.i.d. Bernoulli weights on the vertices of a  $n \times n$  grid. At first, when  $n$  is large enough, all the geodesics are shown to be concentrated in a cylinder, centered on the main diagonal and of width of order  $n^{(2\kappa+2)/(2\kappa+3)}\sqrt{\ln n}$ , where  $1 \leq \kappa < \infty$  is the curvature of the shape function at  $(1, 1)$ . Next, via an iterative scheme, the width order is improved to  $n^{(2\kappa+2)/(2\kappa+3)+\epsilon}$ , for any  $\epsilon > 0$ . Finally, we will show the generality of this methodology by applying to other related models such as directed first passage site percolation and the longest common subsequence problem. This talk is based on a joint work with C. Houdré. (Received September 06, 2016)

1125-60-740 **Ming Liao\*** ([liaoamin@auburn.edu](mailto:liaoamin@auburn.edu)), Department of Mathematics and Statistics, Auburn University, AL 36849. *Invariant Markov processes under Lie group actions.*  
 The invariance of probability distributions under various transformations plays an important role in the probability theory, such as translation invariant Markov processes in Euclidean spaces, which are called Levy processes and are characterized by independent and stationary increments. In this talk, we will present a more general theory of invariant Markov processes under Lie group actions. (Received September 10, 2016)

1125-60-752 **Guillaume Barraquand\*** ([barraquand@math.columbia.edu](mailto:barraquand@math.columbia.edu)), Department of Mathematics, Columbia University, 2990 Broadway, New York, NY 10027. *The facilitated exclusion process and random growth in a half space.*  
 The *facilitated exclusion process* is an interacting particle system on the integer line which is a variant of TASEP. Each particle hops to an empty site on the right at rate 1, but the move is allowed only when the site on the left is occupied. This process was introduced in physics to model the motion of particles in glasses: particles move towards lower density region (exclusion rule), but they move only when stimulated by other neighbouring particles (facilitation rule).

For this process, we will show several limit theorems for the fluctuations of particle positions towards different laws from random matrix theory: GUE, GOE and GSE Tracy-Widom distribution. We will also explain how this process is related to random growth processes in a half space and discuss universal aspects of the effect of the boundary for such systems.

Joint work with Jinho Baik, Ivan Corwin and Toufic Suidan. (Received September 10, 2016)

1125-60-755 **Daniel Dobbs\*** ([dobbsd@trine.edu](mailto:dobbsd@trine.edu)), One University Ave, Angola, IN 46703. *Small deviations for time-changed Brownian motions and applications to second-order chaos.*

We prove strong small deviations results for Brownian motion under independent time-changes satisfying their own asymptotic criteria. We then apply these results to certain stochastic integrals which are elements of second-order homogeneous chaos. This is joint work with Tai Melcher. (Received September 10, 2016)

1125-60-767 **Scott Spencer\*** ([spencer@math.gatech.edu](mailto:spencer@math.gatech.edu)). *A Learning Theory Approach to Compressive Sensing.*

We will discuss the utility of *Vapnik-Chervonenkis dimension* in demonstrating that Gaussian measurement matrices have the *Restricted Isometry Property* (of order  $s$ ) when the number of measurements is on the order of  $s \log(n/s)$ . We also investigate the sign-linear embeddings of 1-bit compressed sensing. One can give short arguments concerning a Restricted Isometry Property of such maps using the VC-dimension of sparse hemispheres. This approach has a natural extension to the presence of additive white noise prior to quantization. Noisy one-bit mappings are shown to satisfy an RIP when the metric on the sphere is given by the noise. (Received September 11, 2016)

1125-60-852 **Farzad Sabzikar\*** ([sabzikar@iastate.edu](mailto:sabzikar@iastate.edu)), 2103 Snedecor Hall, Department of Statistics, Iowa State University, Ames, IA 50011, and **Donatas Surgailis** ([donatas.surgailis@mii.vu.lt](mailto:donatas.surgailis@mii.vu.lt)), Institute of Mathematics and Informatics, Akademijos 4, LT-08663 Vilnius, Lithuania, Lithuania. *Invariance principle for tempered fractionally integrated time series.*

We establish functional limit theorems for a recently proposed class of time series models. Tempered fractionally integrated time series generalize the well-known fractionally integrated model by applying a tempered fractional difference operator in place of the usual fractional difference operator. The new times series are useful to model turbulence, where they extend the classical Kolmogorov 5/3 model to include low frequencies. (Received September 20, 2016)

1125-60-918 **G erard Ben Arous, Manuel Cabezas and Alexander Fribergh\*** ([fribergh@dms.umontreal.ca](mailto:fribergh@dms.umontreal.ca)), Pavillon Andr e-Aisenstadt, 2920, chemin de la Tour, Montr al, QC H3T 1J4, Canada. *The ant in a labyrinth.*

One of the most famous open problem in random walks in random environments is to understand the behavior of a simple random walk on a critical percolation cluster, a model known as the ant in the labyrinth. I will present new results on the scaling limit for the simple random walk on the critical branching random walk in high dimension. In the light of lace expansion, we believe that the limiting behavior of this model should be universal for simple random walks on critical structures in high dimensions. (Received September 13, 2016)

1125-60-923 **Lisa B Hartung\***, 251 Mercer Street, New York, NY 10012. *Diffusion processes on branching Brownian motion.*

Branching Brownian motion (BBM) is a classical process in probability, describing a population of particles performing independent Brownian motion and branching according to a Galton Watson process. In this talk we show how one can construct a class of one-dimensional diffusion processes on the particles of BBM that are symmetric w.r.t. the limits of random martingale measures. These measures are associated with the extended extremal process of branching Brownian motion and are supported on a Cantor-like set (joint work with S. Andres and A. Bovier). (Received September 13, 2016)

1125-60-935 **Reza Gheissari\*** ([reza@cims.nyu.edu](mailto:reza@cims.nyu.edu)) and **Aukosh Jagannath**. *Relaxation to equilibrium in spherical spin glasses.*

Spherical spin glasses are an archetypal model of quenched disorder and glassy behavior in statistical physics: for a spherical spin glass of system size  $N$ , the energy landscape is known to have exponentially many local minima as  $N \rightarrow \infty$ . This induces a rich and complicated geometry in the corresponding Gibbs measure at low temperature, where the mass concentrates in energy bands around the local minima [Subag 16]. We consider the natural reversible dynamics with respect to the Gibbs measure (the Langevin dynamics) and in particular, the time it takes for the dynamics to relax to equilibrium. We prove that whereas at high temperatures, the time to relax to equilibrium is  $O(1)$ , at low temperatures the model is in a *glassy phase* marked by timescales to equilibrium that are exponentially long in the system size; this transition is deeply linked to the complexity of the geometry of the Gibbs measure in the static low temperature regime. (Received September 13, 2016)

1125-60-936 **Louis-Pierre Arguin\*** ([louis-pierre.arguin@baruch.cuny.edu](mailto:louis-pierre.arguin@baruch.cuny.edu)). *The local extrema of the Riemann zeta function on the critical line.*

A recent conjecture of Fyodorov, Hiary & Keating states that the maxima of the modulus of the Riemann zeta function on an interval of the critical line behave similarly to the maxima of a simple spin glass system. In this talk, we will discuss recent progress on this conjecture. We will highlight the connections between the number theory problem and the probabilistic models including the branching random walk and the characteristic polynomial of random matrices. This is joint work with D. Belius (Zurich), P. Bourgade (NYU) and A. Harper (Cambridge). (Received September 13, 2016)

1125-60-938 **Subhabrata Sen\*** ([ssen90@stanford.edu](mailto:ssen90@stanford.edu)), 390 Serra Mall, Sequoia Hall Rm 229, Stanford, CA 94305. *Optimization on Sparse Random Graphs and its Applications.*

Many optimization problems on sparse random (hyper)graphs are currently intractable to analysis by purely combinatorial techniques. Statistical Physicists predict that the analysis often “simplifies” considerably in the “large degree” limit. In this talk, we will see how to make this idea rigorous by analyzing certain graph cut problems and random constraint satisfaction problems. Our techniques lead to connections between combinatorial problems on sparse random graphs and the study of spin glasses and random matrices. Parts of this talk will be based on joint work with Amir Dembo and Andrea Montanari. (Received September 13, 2016)

1125-60-941 **D Wanduku\*** ([dwanduku@georgiasouthern.edu](mailto:dwanduku@georgiasouthern.edu)), Department of Mathematical Sciences, Georgia Southern University, 65 Georgia Ave. Room 3042, P.O. Box 8093, Statesboro, GA 30460. *A Generalized Stochastic SEIRS Epidemic Dynamic Model for Vector-borne Diseases with three Distributed Delays and Nonlinear Incidence.*

A general stochastic SEIRS triple delay epidemic dynamic model for vector-borne diseases with nonlinear incidence rate is presented. Two of the distributed delays account for the varying incubation period of the infectious agent in the vector and host, and the third distributed delay accounts for the varying immunity period to the disease. Furthermore, the disease dynamics is influenced by random environmental perturbations in the disease transmission and natural death processes. The basic reproduction number—both in the presence and absence of noise—are computed. In addition, the stochastic asymptotic properties of the system— asymptotic stability of the equilibria and asymptotic behavior of the system in the neighborhood of equilibria—are presented. Moreover the stability results are exhibited in several real life scenarios and the significance of the results are presented. Numerical simulation results are presented. (Received September 13, 2016)

1125-60-994 **Antonio Auffinger** ([tuca@northwestern.edu](mailto:tuca@northwestern.edu)), 2033 Sheridan Road, Department of Mathematics, Northwestern University, Evanston, IL 60208, and **Si Tang\*** ([sitang@galton.uchicago.edu](mailto:sitang@galton.uchicago.edu)), 5747 S. Ellis Avenue, Department of Statistics, University of Chicago, Chicago, IL 60637. *On the time constant of high dimensional first passage percolation.*

The talk focuses on the behavior of the time constant  $\mu(e_1)$  in first passage percolation on  $\mathbb{Z}^d$  in high dimension. We prove that if the passage times have finite mean, then

$$\lim_{d \rightarrow \infty} \frac{\mu(e_1)d}{\log d} = \frac{1}{2a},$$

where  $a \in [0, \infty]$  is a constant that depends only on the behavior of the distribution of the passage times at 0. For the same class of distributions, we also prove that the limit shape is not an Euclidean ball, nor a  $d$ -dimensional cube or diamond, provided that  $d$  is large enough. (Received September 13, 2016)

1125-60-1045 **G. S. Ladde\*** ([gladde@usf.edu](mailto:gladde@usf.edu)), Department of Mathematics and Statistics, University of South Florida, 4202 East Fowler Avenue, CMC 342, Tampa, FL 33620-5700. *Fundamental Properties of Solutions of Systems of Nonlinear Stochastic Differential Equations.* Preliminary report.

The main objective of this work is to introduce an innovative method for global existence and uniqueness of solution processes for a class of nonlinear and nonstationary stochastic systems of differential equations. We note that the classical sufficient conditions for the global existence and uniqueness of solution processes are too restrictive, and the Khasminskii-type approach is either not feasible or not easily applicable. In this work, an innovative approach is developed. The idea is seeking a one-to-one nonlinear transformation that reduces a given system of stochastic differential equations that possesses the either classical or Khasminskii-type conditions. Applications are given to illustrate the role and scope of developed results. (Received September 14, 2016)

1125-60-1057 **Mohammad Foondun** and **Ngartelbaye Guerngar\*** (nzg0017@auburn.edu), 221 Parker Hall, Auburn University, Auburn, AL 36849, and **Erkan Nane**. *Large time behavior for the solution of the fractional stochastic heat equation in bounded domains.*

We consider the following fractional stochastic partial differential equation on  $D$  an open bounded subset of  $\mathbb{R}^d$  for  $d \geq 1$

$$\partial_t u_t(x) = -\frac{1}{2}(-\Delta)^{\frac{\alpha}{2}} u_t(x) + \xi \sigma(u_t(x)) \dot{W}(t, x) \quad \text{for } \alpha \in (0, 2]$$

where the fractional Laplacian is the infinitesimal generator of a symmetric  $\alpha$ -stable process in  $\mathbb{R}^d$ ,  $\xi$  is a parameter in  $\mathbb{R}$ ,  $\sigma$  is a Lipschitz continuous function and  $\dot{W}(t, x)$  is a Gaussian noise white in time and white or coloured in space.

We show that under Dirichlet conditions, in the long run, the  $p^{\text{th}}$ -moment of the solution grows exponentially fast for large values of  $\xi$ . However when  $\xi$  is very small we observe eventually an exponential decay of the  $p^{\text{th}}$ -moment of this same solution. Foondun and Nualart (On the behaviour of stochastic heat equations on bounded domains. *ALEA Lat. Am. J. Probab. Math. Stat.* 12 (2015), no. 2, 551–571.) established the large time behavior for  $\alpha = 2$ . We extend their results to the case  $\alpha \in (0, 2)$ . (Received September 14, 2016)

1125-60-1122 **Sunday A Asogwa\*** (saa0020@auburn.edu), 221 Parker hall, Department of Mathematics & Statistics, Auburn University, Auburn, AL 36849, and **Erkan Nane**. *Intermittency fronts for space-time fractional stochastic partial differential equations in  $(d + 1)$  dimensions.* Preliminary report.

In this talk, we study the intermittency fronts of the following space-time fractional stochastic heat type equation

$$\partial_t^\beta u_t(x) = -\nu(-\Delta)^{\alpha/2} u_t(x) + I_t^{1-\beta}[\sigma(u) \dot{W}(t, x)]$$

in  $(d + 1)$  dimensions, where  $\nu > 0$ ,  $\beta \in (0, 1)$ ,  $\alpha \in (0, 2]$ ,  $d < \min\{2, \beta^{-1}\}\alpha$ ,  $\partial_t^\beta$  is the Caputo fractional derivative,  $(-\Delta)^{\alpha/2}$  is the generator of an isotropic stable process,  $\dot{W}(t, x)$  is space-time white noise, and  $\sigma : \mathbb{R} \rightarrow \mathbb{R}$  is Lipschitz continuous. The fact that these fronts grow linearly with time is quite surprising here since the operator studied here is fractional in time. Precisely, for the choices of  $\alpha = 2$  and  $d \in \{1, 2, 3\}$ , we prove intermittency fronts for higher moments; which essentially measure how fast the peaks spread in space. (Received September 15, 2016)

1125-60-1125 **Po-Keng Cheng\***, Math Tower B-148, Stony Brook University, Stony Brook, NY 11794, and **Frank J. Fabozzi** and **Stoyan Stoyanov**. *Transitions among States behind Interactive Agent Model.*

In this paper, we introduce a simple heterogeneous agent mechanism, where the distribution of returns generated from the mechanism match stylized facts in financial markets. We introduce one more key factor, the length of evaluations on performances between strategies, which also has a significant influence on price fluctuations. To investigate the transitions among states, we introduce a Markov transition matrix, Perron-Frobenius transition matrix, and Inertia. Our results show the stickiness of states switching from one to another, and the longer length of performance evaluations would generate more complex dynamic price fluctuations. We link our simple heterogeneous agent mechanism with Markov trajectory entropy and provide a total score and probability density functions of representations under two states as applications for the mechanism. (Received September 15, 2016)

1125-60-1153 **Shannon Starr\*** (s1starr@uab.edu) and **Meg Walters**. *Phase Uniqueness for the Mallows Model.*

The Mallows model on permutations on  $\{1, \dots, n\}$  is a probability wherein  $P(\pi)$  is proportional to  $q$ -raised-to-the-power  $I(\pi)$ , where  $q > 0$  is the parameter of the model, and  $I(\pi)$  is the number of pairs  $i < j$  such that  $\pi(i) > \pi(j)$ . This model was considered by Bhatnagar and Peled, and culminating with Basu and Bhatnagar a regenerative process was discovered in the background, especially useful for describing the limit where  $0 < q < 1$  stays fixed and  $n$  goes to infinity so  $\{1, \dots, n\} \rightarrow \{1, 2, \dots\}$ . In another scaling regime,  $q = q_n = \exp(-\beta/n)$ , Sumit Mukherjee proved a LDP and identified the explicit rate function for the limiting permuton. This may be viewed as a Gibbsian field for a mean field system. We prove that for the Gibbsian system, there is a unique equilibrium state, aka "phase uniqueness." Our proof uses the cavity step, which in this case is just the dynamic programming principle, which is related to the regenerative structure of Bhatnagar, et al. (Received September 18, 2016)

1125-60-1179 **Jan Rosinski\*** ([rosinski@math.utk.edu](mailto:rosinski@math.utk.edu)), University of Tennessee, Department of Mathematics, 227 Ayres Hall, Knoxville, TN 37934. *Some isomorphism identities for Poissonian infinitely divisible processes.*

We propose isomorphism type identities for nonlinear functionals of Poissonian infinitely divisible (ID) processes. Such identities can be viewed as an analogy of the Cameron-Martin formula for Poissonian ID processes but with random translations, or perturbation identities. The applicability of such identities relies on a precise understanding of Levy measures of stochastic processes and their representations. We will illustrate this approach on examples. (Received September 15, 2016)

1125-60-1209 **Yihren Wu\*** ([yihren.wu@hofstra.edu](mailto:yihren.wu@hofstra.edu)). *The Information Content of Volatility Demand.* Preliminary report.

This study derives a vega-weighted demand for volatility by modifying the approach of Ni, Pan, and Poteshman (2008). We show that the volatility demand can be estimated through a simple approximation that uses only the aggregate trading volumes of put and call options, for instance S&P 500 options. Instead of predicting the directional changes of volatility, we focus on determining whether the volatility demand has information content on the volatility of volatility. Our empirical results show that estimated volatility demand has significant non-directional volatility information, which could be realized directly by trades on VVIX. The changes in volatility demand have incremental effects on the changes in the future volatility of volatility, after controlling for other factors. Since our volatility demand is derived based on public information, the economic source of this predictability could be traced by both information advantage and market inefficiency, which possesses more practical implications than Ni, Pan, and Poteshman (2008), whose analyses depend on non-public information. (Received September 15, 2016)

1125-60-1321 **Samuel N Stechmann** and **Scott A Hottovy\***, 1246 Ramblewood Dr, Annapolis, MD 21409. *Cloud regimes as phase transitions.*

Clouds are repeatedly identified as a leading source of uncertainty in future climate predictions. Of particular importance are stratocumulus clouds, which can appear as either (i) closed cells that reflect solar radiation back to space or (ii) open cells that allow solar radiation to reach the Earth's surface. Here we show that these clouds regimes – open versus closed cells – fit the paradigm of a phase transition. In addition, this paradigm characterizes pockets of open cells (POCs) as the interface between the open- and closed-cell regimes, and it identifies shallow cumulus clouds as a regime of higher variability. This behavior can be understood using an idealized model for the dynamics of atmospheric water as a stochastic diffusion process. With this new conceptual viewpoint, ideas from statistical mechanics could potentially be used for understanding uncertainties related to clouds in the climate system and climate predictions. (Received September 16, 2016)

1125-60-1338 **John P. Travis\*** ([travis@mc.edu](mailto:travis@mc.edu)), Box 4025, Mathematics, Mississippi College, Clinton, MS 39058. *An OER textbook for Calculus-based Probability and Statistics using Mathbook XML.* Preliminary report.

In this presentation, I will demonstrate a new OER textbook for an introductory calculus-based probability and statistics course. This online textbook utilizes Mathbook XML and allows for multiple output formats. These formats include an online option that incorporates active Sage and WeBWorK elements. A collection of related WeBWorK online homework problems will also be shown. Results from piloting the text in a course during Fall 2016 will be presented. (Received September 16, 2016)

1125-60-1431 **Laura Villafuerte Altuzar\*** ([1va5@hotmail.com](mailto:1va5@hotmail.com)), 909 S Whitney Way Apt 3, Madison, WI 53711, and **Juan Carlos Cortes**, Instituto de Matematica Multidisciplinar, 46022 Valencia, Valencia, Spain. *Mean square power series solutions and its applications in solving linear second order random differential equations.*

In this talk we construct, by means of random power series, a solution of second order linear differential equations containing uncertainty through its coefficients and initial conditions. By assuming appropriate hypotheses on the data, we prove that the constructed random power series solution is mean square convergent.

Several examples that illustrate the theoretical results, are included. (Received September 16, 2016)

1125-60-1504 **Mark L Huber\*** ([autotomic@gmail.com](mailto:autotomic@gmail.com)), 392 Cardinal LN, Upland, CA 91786. *Fast biased user-specified relative error estimates.*

The output of random computer simulations is often analyzed using estimates derived from classical statistics. However, the flexibility to generate a random amount of data gives the ability to create a new type of estimate where the relative error of the estimate does not depend on the quantity being estimated, but is instead specified by the user ahead of time. While estimates for means of continuous random variables have long been known,

only recently have these been extended to estimating the mean of discrete random variables such as Bernoulli and Poisson. The first versions of these estimates were biased: in this talk I will show how biased versions of these estimates can shrink the relative error of the estimate more quickly. In the case of Poisson data, fewer samples are required to estimate the mean (on average) within a specified relative error with specified failure rate than are needed by the Central Limit Theorem approximation. (Received September 17, 2016)

1125-60-1510 **Michael Damron, Wai-Kit Lam\*** (lamw@indiana.edu) and **Xuan Wang**. *Asymptotics for 2D critical first-passage percolation.*

We consider the first-passage percolation on  $\mathbb{Z}^2$  with i.i.d. weights, whose distribution function satisfies  $F(0) = p_c = 1/2$ . Denote  $T(0, \partial B(n))$  as the passage time from the origin to the boundary of the box  $[-n, n]^2$ . We characterize the limit behavior of  $T(0, \partial B(n))$  by the conditions on the distribution function  $F$ . We also give exact conditions under which  $T(0, \partial B(n))$  will have uniformly bounded mean or variance. These results answer several questions of Kesten and Zhang from the '90s. We also prove a CLT under a minimal moment assumption when both the mean and variance go to infinity as  $n \rightarrow \infty$ . The main tool involves a new relation between first-passage percolation and invasion percolation: up to a constant factor, the passage time in critical first-passage percolation has the same first-order behavior as the passage time of an optimal path constrained to lie in an embedded invasion cluster. This is joint work with Michael Damron and Xuan Wang. (Received September 17, 2016)

1125-60-1511 **Michael Damron, Jack Hanson** and **Wai-Kit Lam\*** (lamw@indiana.edu). *The size of the boundary in the Eden model.*

The Eden model, a special case of first-passage percolation, is a stochastic growth model in which an infection that initially occupies the origin of  $\mathbb{Z}^d$  spreads to neighboring sites at rate 1. Infected sites are colonized permanently; that is, an infected site never heals. It is known that at time  $t$ , the infection occupies a set  $B(t)$  of vertices with volume of order  $t^d$ , and the rescaled set  $B(t)/t$  converges to a convex, compact limiting shape. In joint work with M. Damron and J. Hanson, we partially answer a question of K. Burdzy, concerning the order of the size of the boundary of  $B(t)$ . We show that, in various senses, the boundary is relatively smooth, being typically of order  $t^{d-1}$ . This is in contrast to the fractal behavior of interfaces characteristic of percolation models. (Received September 17, 2016)

1125-60-1609 **Asma Azizi\*** (aazizibo@tulane.edu), 1740 baronne street, ap#450, new orleans, LA 70113, **James Mac Hyman** (mhyman@tulane.edu), department of Math, tulane university, new orleans, LA 70118, and **Jeremy Dewar** (jdewar@tulane.edu), department of Math, tulane university, new orleans, LA 70118. *A Bipartite Network Generation Using Joint Degree Distribution*. Preliminary report.

In this work, we propose a new algorithm to construct bipartite network using joint degree (degree-degree) distribution. Bipartite networks are a representation of many different real-world interactions, such as actor-movie networks, authoring networks, occurrence networks, and sexual partnership networks. Despite the huge application of bipartite network, a few work has been done on construction of them which mostly use average degree or degree distribution. We propose a new algorithm for construction of bipartite networks, and demonstrate the ability of this algorithm to preserves some properties of networks like degree distribution of nodes and degree distribution of neighbors, i.e degree and joint degree distributions. We implement our algorithm on some real world networks, including romance network. We demonstrate that in cases that average degree of nodes is low, particularly for romance network or any other sexual partnership networks, our generated network predict with surprising accuracy the behavior of the real world, while in other cases there is a measurable discrepancy between random and real ones, indicating the essence of considering additional social structure that is not captured by the random network. (Received September 18, 2016)

1125-60-1774 **Ivan Z Corwin\*** (ivan.corwin@gmail.com). *A drunk walk in a drunk world.*

In a simple symmetric random walk on  $\mathbb{Z}$ , a particle jumps left or right with 50% chance independently at each time and space location. What if the jump probabilities are taken to be random themselves (e.g. uniformly distributed between 0% and 100%). In this talk we will describe the effect of this random environment on a random walk, in particular focusing on a new connection to the Kardar-Parisi-Zhang universality class and to the theory of quantum integrable systems. No prior knowledge or background will be expected. (Received September 19, 2016)

1125-60-1852 **Adina Oprisan\***, oprisana@canisius.edu. *An invariance principle for additive functionals of Semi-Markov processes.*

In this paper we study a class of additive functionals of semi-Markov processes  $W(t) = \int_0^t f(X_s) ds$ , where  $\{X_t, t \geq 0\}$  is an ergodic semi-Markov process with invariant distribution  $\pi$  and  $f \in L^2(\pi)$ . Using a martingale decomposition approach we proved that the sequence  $W_t^n = \frac{1}{\sigma\sqrt{nm}} \int_0^{nt} f(X_s) ds$  converges in distribution to the Wiener process  $W$  on  $\mathcal{D}([0, \infty), E)$ , with  $m = E_\pi(X_1)$ . The martingale decomposition is also crucial in proving that the aforementioned convergence in distribution admits an almost sure version - a functional almost sure central limit theorem. Applications to some autoregressive processes will be also discussed. (Received September 19, 2016)

1125-60-1887 **Jerzy Szulga\*** (szulgje@auburn.edu), Department of Mathematics and Statistics, Auburn University, Auburn, AL 36849. *Operators on discrete random chaos modeling quantum phenomena.*

Discrete random chaos, a.k.a. Walsh series (discrete harmonic analysis) or **toy Fock space** (quantum physics), is just an algebra spanned by Rademacher random variables, becoming a specifically structured Hilbert space. The plethora of operators, such as Parthasarathy's conservation, creation, and annihilation operators, discretized by Paul-Andre Meyer (or other such as discrete Malliavin derivative or Ornstein-Uhlenbeck operators), can be expressed algebraically by compositions of two simple symmetries. The compositions form a **signed group**, where all elements either commute or anticommute, and are square roots of either 1 or -1 (defining **signature**). Any (at most countable) signed group can be embedded isomorphically into the aforementioned symmetry group, which also possesses a patterned matrix representation. The spanning symmetries are simple contractions but not Hilbert-Schmidt operators. Yet they can be endowed with a Hilbert space structure via so called **rigged Hilbert space**. The arising patterns are classified with respect to commutativity/anticommutativity properties as well as in regard to signatures. (Received September 19, 2016)

1125-60-1953 **Ram Sharan Adhikari\*** (radhikari@rsu.edu), Rogers state University, Claremore, OK 74017. *A weak modified Euler-Maruyama method based on trapezoidal rule for a class of stochastic differential equations and mean square stability results.*

In this work we introduce a new class of weak numerical scheme for solving systems of Itô stochastic differential equations (SDEs). We call it Modified Euler-Maruyama scheme based on trapezoidal rule. Weak order of convergence one is established under the suitable conditions. We also discuss about the numerical performance of our method with some examples. This work also aims to determine the mean-square stability region of the weak modified Euler method for linear stochastic differential equations with multiplicative noises. (Received September 19, 2016)

1125-60-1955 **Riddhipratim Basu and Shirshendu Ganguly\*** (sganguly@berkeley.edu), 307 Evans Hall, UC Berkeley, Berkeley, CA 94720. *SO(N) Lattice Gauge Theory, under strong coupling.*

Lattice Gauge theories have been studied in the physics literature as discrete approximations to quantum Yang-Mills theory for a long time. Primary statistics of interest in these models are expectations of the so called "Wilson loop variables". In his recent seminal work Chatterjee (2015) rigorously established a gauge-string duality, by solving, what is known as the "Makeenko-Migdal" equations in the physics literature. This allows one to write the Wilson loop expectations in Lattice Gauge theories in a certain limit, as a sum of certain weights over string trajectories.

In this talk we will discuss some recent results towards a better geometric understanding of the string trajectories using correspondence to combinatorial objects such as decorated trees and non-crossing partitions. Using connections with Free Probability theory, the planar case can be explicitly described. We will also mention some structural results in higher dimensional lattice gauge theory which falls outside the exactly solvable regime. All the necessary background will be reviewed. Several open questions will be presented as well.

The talk will be based on joint work with Riddhipratim Basu (Stanford). (Received September 19, 2016)

1125-60-2089 **Peter Otto** (potto@willamette.edu), **Benjamin Savoie** (bsavoie@umflint.edu), **Ana Wright** (awright@willamette.edu) and **Renjun Zhu\*** (renjun.zhu@berkeley.edu). *Mixing Times for the Generalized Rook's Walk.* Preliminary report.

Our work focuses on using path coupling, a powerful probabilistic tool, to find bounds on the mixing times of a class of Markov chains. The mixing time of a Markov chain measures the rate of convergence to its stationary distribution. This mixing time is of interest for sampling and simulations of random processes. The Markov chains we are investigating are restrictions on the random rook's walk on a  $d$  dimensional chessboard, which can

also be considered random walks on the Cartesian powers of certain groups of circulant graphs. We prove bounds on the mixing times of these Markov chains, extending and generalizing previous results for the unrestricted case of the rook's walk. (Received September 19, 2016)

1125-60-2145 **Douglas Rizzolo\*** ([drizzolo@udel.edu](mailto:drizzolo@udel.edu)). *Connections between random matrices and pattern-avoiding permutations.*

We will discuss recent results on the structure of random pattern-avoiding permutations. The focus will be a connection between random  $k \cdots 321$ -avoiding permutations and the eigenvalues of a random matrix diffusion. In particular, we will give an invariance principle relating the RSK algorithm applied to a uniformly random  $k \cdots 321$ -avoiding permutation to the eigenvalues of a Hermitian Brownian bridge conditioned to have trace 0. (Received September 19, 2016)

1125-60-2208 **Nils Detering\*** ([detering@pstat.ucsb.edu](mailto:detering@pstat.ucsb.edu)), Statistics and Applied Probability, University of California, Santa Barbara, CA 93106-3110, and **Thilo Meyer-Brandis** and **Konstantinos Panagiotou**. *Bootstrap Percolation in Directed Inhomogeneous Random Graphs.*

Bootstrap percolation is a process that is used to model the spread of an infection on a given graph. In the model considered here each vertex is equipped with an individual threshold. As soon as the number of infected neighbors exceeds that threshold, the vertex gets infected as well and remains so forever. In this paper we perform a thorough analysis of bootstrap percolation on a novel model of directed and inhomogeneous random graphs, where the distribution of the edges is specified by assigning two distinct weights to each vertex, describing the tendency of it to receive edges from or to send edges to other vertices. Under the assumption that the limiting degree distribution of the graph is integrable we determine the typical fraction of infected vertices. Our model allows us to study settings that were outside the reach of current methods, in particular the prominent case in which the degree distribution has an unbounded variance. Among other results, we quantify the notion of "systemic risk", that is, to what extent local adverse shocks can propagate to large parts of the graph through a cascade, and discover novel features that make graphs prone/resilient to initially small infections. (Received September 19, 2016)

1125-60-2237 **Hyojung Lee\*** ([s1drs@unist.ac.kr](mailto:s1drs@unist.ac.kr)), 50 UNIST-gil, Eonyang-eup, Ulju-gun, Ulsan, 114dong 808ho, Ulsan, 44919, and **Chang Hyeong Lee** ([chlee@unist.ac.kr](mailto:chlee@unist.ac.kr)) and **Sunmi Lee** ([mathever@gmail.com](mailto:mathever@gmail.com)). *Stochastic methods for epidemic models: An application to the 2009 H1N1 Influenza outbreak in Korea.*

In this paper, we applied stochastic methods to influenza transmission models, the 2009 H1N1 influenza transmission dynamics in Korea. Gillespie's stochastic simulation algorithm (SSA) is one of well-known exact simulation methods. But since the epidemic model includes large population size and fast infection process, the SSA has the low efficiency caused by the Monte-Carlo procedure. However, the moment closure method (MCM) shows reduced computational time and efficient costs with the accuracy. We also present simulation results that there is a considerable discrepancy between the results of stochastic and deterministic models especially when a number of initial infectious individuals is small. (Received September 20, 2016)

1125-60-2259 **Carl P Dettmann\*** ([carl.dettmann@bris.ac.uk](mailto:carl.dettmann@bris.ac.uk)). *Random geometric graphs on nonuniform and fractal measures.* Preliminary report.

A random geometric graph (RGG) is a set of points defined by a Poisson point process (PPP) together with links between points whose mutual distance is less than a threshold (or more generally, occur with a distance dependent probability). In RGG with a finite expected number of points, there are two main transitions, percolation, the appearance of a large connected component, and connectivity, when all points are connected in a multi-hop fashion. In many cases it is known that connectivity occurs simultaneously with the connection of the last isolated point, and that isolated points are asymptotically Poisson distributed. One assumption underlying these results is that the density of points does not vanish within or towards the boundary of the support of the PPP. Here, we consider PPP defined using a variety of nonuniform and fractal measures that break this assumption, and explore the consequences for the above properties. These ideas are relevant to mobile ad-hoc networks, in which the points are mobile devices, links are pairwise connections, and the distribution is often in practice far from uniform. (Received September 20, 2016)

1125-60-2261 **Carl P Dettmann\*** ([carl.dettmann@bris.ac.uk](mailto:carl.dettmann@bris.ac.uk)), **Orestis Georgiou** and **Georgie Knight**. *Spectral statistics of random geometric graphs.*

We study the spectrum of random geometric graphs using random matrix theory. We look at short range correlations in the level spacings via the nearest neighbour and next nearest neighbour spacing distribution

and long range correlations via the spectral rigidity  $\Delta_3$  statistic. These correlations in the level spacings give information about localisation of eigenvectors, level of community structure and the level of randomness within the networks. We find that the spectral statistics of random geometric graphs fits the universality of random matrix theory. In particular, the short range correlations are very close to those found in the Gaussian orthogonal ensemble of random matrix theory. For long range correlations we find deviations from Gaussian orthogonal ensemble statistics towards Poisson. We compare with previous results for Erdos-Renyi, Barabasi-Albert and Watts-Strogatz random graphs where similar random matrix theory universality has been found. (Received September 20, 2016)

1125-60-2309 **Joe J Klobusicky\*** (klobuj@rpi.edu), Department of Mathematical Science, Rensselaer Polytechnic Institute, Troy, NY 12180, and **Govind Menon**. *A hydrodynamic limit theorem for a minimal model of grain boundary evolution.*

We prove exponential concentration estimates and a strong law of large numbers for a particle system that is the simplest representative of a general class of models for 2D grain boundary coarsening. The proof relies on a concentration inequality for an urn model studied by Pittel, and Maurey's concentration inequality for Lipschitz functions on the permutation group. (Received September 20, 2016)

1125-60-2351 **Jiarui Chu\*** (jichu@davidson.edu) and **Laurie Heyer**. *Random Unit Bar and Unit Square Visibility*. Preliminary report.

In visibility problems, two unit bars are visible to each other if an unobstructed vertical line of sight can be drawn between them. Similarly, two unit squares in a two-dimensional space are mutually visible if there is an unobstructed vertical or horizontal sightline between them. The study of visibility problems and visibility graphs are motivated by Very-Large-Scale-Integration (VLSI) layout design problems, and have applications in robot navigation, hidden-surface removal, and computer-aided software-engineering (CASE) tools. Although there is a rich body of research on visibility, no result has been published on the probabilistic aspects of visibility problems. We assume the locations of the bars and squares to be uniformly distributed, and study three problems. We derive the probability density function and expected value for the number of bars required to cover the top bar. Then we generalize the result to random squares in a two-dimensional space. Finally, we consider unit bar visibility as a model for random graphs by studying the probabilities that a random unit bar visibility graph has certain properties, such as being cyclic or connected. (Received September 20, 2016)

1125-60-2446 **Hoi Huu Nguyen\***, 231 W 17TH AVE, Columbus, OH 43210. *The circular law for eigenvalues of random stochastic matrices.*

Let  $X$  be a matrix sampled uniformly from the set of stochastic / doubly stochastic matrices of size  $n$ . We show that the empirical spectral distribution of the eigenvalues of  $n^{1/2}(X - EX)$ , as  $n$  grows to infinity, converges to the circular law almost surely. (Received September 20, 2016)

1125-60-2449 **Eunghyun Lee\*** (eunghyun.lee@nu.edu.kz), Astana, Kazakhstan, and **Dong Wang**, Singapore, Singapore. *Probability distribution for a particle's position in the inhomogeneous totally asymmetric zero range process.*

In this talk we consider the totally asymmetric zero range process on the integer lattice in which sites are equipped with exponentially distributed random clocks with different rates. We provide the formula for the distribution of the  $m$ -th rightmost particle's position given an arbitrary initial state of finitely many particles. In particular, we find the formula represented by a Fredholm determinant of a certain trace-class operator for a special initial state that all particles are at a single site. Finally, we give a result for asymptotics. (Received September 20, 2016)

1125-60-2473 **Wei-Kuo Chen\*** (wkchen@umn.edu), 537 Vincent Hall, 206 Church St. SE, Minneapolis, MN 55455, and **Antonio Auffinger**, **Madeline Handschy** and **Gilad Lerman**. *The energy landscape of the Sherrington-Kirkpatrick model.*

The Sherrington-Kirkpatrick (SK) model is a mean-field spin glass introduced by theoretical physicists in order to explain the strange behavior of certain alloy, such as CuMn. Despite of its seemingly simple formulation, it was conjectured to possess a number of fruitful properties. This talk will be focused on the energy landscape of the SK model. First, we will present a formula for the maximal energy in Parisi's formulation. Second, we will give a description of the energy landscape by showing that near any given energy level between zero and maximal energy, there exist exponentially many equidistant spin configurations. Based on joint works with Auffinger, Handschy, and Lerman. (Received September 20, 2016)

1125-60-2609 **Abbas M Alhakim\*** (aa145@aub.edu.lb) and **S Molchanov**. *Random walks on infinitely generated dense groups*. Preliminary report.

We consider symmetric random walks on dense groups of the real line, generated by a countable set of rationally independent real numbers which is equipped with an arbitrary probability distribution  $\{p_k\}$ . We obtain a precise result for the probability of return to zero in terms of a modified Bessel function. We then derive asymptotic expansions for this return probability. In particular it is shown that the leading asymptotics is sometimes determined by the tail of  $\{p_k\}$  while in some other cases it is determined by the lower part and yet in some other cases both the tail and the lower part contribute equally. To this end, various classes of distributions with light, moderately heavy and heavy tails are investigated. (Received September 20, 2016)

1125-60-2610 **Katelynn D Kochalski\*** (kdk7rn@virginia.edu), 141 Cabell Dr, Kerchof Hall, PO Box 400137, Charlottesville, VA 22904. *Fluid Limit for a Batched Processor Sharing Queue*. Preliminary report.

We consider a sequence of single server queues operating under a service policy that incorporates batches into processor sharing. Assuming the limiting system is critical, we find a fluid limit for the measure-valued processes that describe each system in the sequence. (Received September 20, 2016)

1125-60-2630 **John E Haga\*** (hagaj@wit.edu), 550 Huntington Avenue, Department of Applied Mathematics, Wentworth Institute of Technology, Boston, MA 02115. *Levy Processes in a Step 3 Nilpotent Lie Group*.

The infinitesimal generators of Lévy processes in Euclidean space are pseudodifferential operators with symbols given by the Levy-Khintchine formula. This classical analysis relies heavily on Fourier analysis which, in the case when the state space is a Lie group, becomes much more subtle. Still the notion of pseudo-differential operators can be extended to connected, simply connected nilpotent Lie groups by employing the Weyl functional calculus. With respect to this definition, the generators of Levy processes in the simplest step 3 nilpotent Lie group  $G$  are pseudodifferential operators which admit  $C_c(G)$  as its core. (Received September 20, 2016)

1125-60-2709 **Gerandy Brito\*** (gerandy@uw.edu), Department of Mathematics, University of Washington. Box 354350, Seattle, WA 98195, and **Christopher Hoffman**, Department of Mathematics, University of Washington. Box 354350, Seattle, WA 98195. *On a family of measures with finitely many infinite geodesics in planar first passage percolation*.

The study of infinite geodesics in first passage percolation have received a lot of attention since the work of Hoffman [05] and [08], Damron and Hanson [14] and more recently Ahlberg and Hoffman [16]. In this talk we discuss further advances in this topic presenting a family of measures for which it is possible to completely characterize the set of infinite geodesics. Based on joint work with Christopher Hoffman. (Received September 20, 2016)

1125-60-2951 **Kenneth Stephen Berenhaut\*** (berenhks@wfu.edu), Department of Mathematics and Statistics, Wake Forest University, Winston-Salem, NC 27106. *Some problems related to networks, random walks and choice*. Preliminary report.

In this talk, we consider a variety of problems related to networks. Topics considered include employment of random walks in addressing questions regarding clustering of nodes, communities and modularity. We will also touch on questions of choice with connections to graphs. (Received September 20, 2016)

1125-60-3072 **Flavia C Sancier-Barbosa\*** (fsancier@antiochcollege.edu). *Simulating Option Pricing Models with Hereditary Structure*.

We discuss the derivation and simulation of continuous option pricing models that have stochastic volatility with hereditary structure. That is, the stock dynamics follows a nonlinear stochastic functional differential equation. In particular, two models are discussed: one in which the drift and volatility terms have a continuous dependence on past stock prices, and one in which the continuous past dependence may include present stock prices. The models are an extension of the well-known Black-Scholes framework. The option pricing formulas are the result of an equivalent (local) martingale measure and therefore are written as conditional expectations that can be simulated via Monte Carlo methods. (Received September 20, 2016)

1125-60-3107 **Ivan Corwin\***, Columbia University, Mathematics Department, 2990 Broadway, New York, NY 10027. *KPZ fluctuations in exactly solvable models, I*.

Some random growth models admit concise and exact formulas describing expectations of various observables of interest. These models and their solvability spring from certain algebraic structures such as representation theory and quantum integrable systems. By studying these examples, we are able to gain predictions for the

universal behaviors of a much wider class of random growth models – the so-called Kardar-Parisi-Zhang (KPZ) universality class.

We will touch on some of the models discussed earlier in the short course and on some new ones, such as directed last-passage percolation, positive temperature directed polymers, the (totally) asymmetric simple exclusion process, the KPZ stochastic partial differential equation, and some others. We sketch a proof of the asymptotic fluctuation scaling and statistics for one of these models and indicate how this generalizes to the broader class. (Received September 21, 2016)

1125-60-3108 **Timo Seppäläinen\***, Mathematics Department, Van Vleck Hall, 480 Lincoln Dr., Madison, WI 53706. *Stationary versions and fluctuation exponents for the exactly solvable stochastic corner growth model.*

This talk describes the derivation of the limiting shape and the fluctuation exponents for a stochastic growth model on the planar integer lattice known as the corner growth model. When the random weights assigned to lattice points that determine the speed of growth have exponential distributions, this model is exactly solvable. A particular feature of its solvability is that it possesses a tractable stationary version that is suitably invariant under lattice translations. Utilizing the stationary version enables one to perform explicit calculations and in particular to determine fluctuation exponents.

The planar corner growth model is a member of the so-called Kardar-Parisi-Zhang (KPZ) universality class. All members of the class are expected to share the same fluctuation exponents, but this remains a conjecture. This lecture is part of the AMS Short Course on Random Growth Models. (Received September 21, 2016)

1125-60-3130 **Firas Rassoul-Agha\***, Mathematics Department, 100 S 1400 E, Salt Lake City, UT 84109. *Busemann functions, geodesics, and the competition interface for directed percolation.*

We introduce the last-passage percolation model (LLP) and relate it to several much-studied stochastic models such as the corner growth model, queues in series, and the totally asymmetric simple exclusion process. By letting two infections from different seeds compete for space, the growth model turns into one of competition. We show how certain Busemann function limits can be proved with the help of results from queuing theory. These functions carry information about the large scale behavior of the system. They provide equations for the limiting shape and can be used to prove existence, uniqueness, and coalescence of geodesics, under mild regularity assumptions on the shape. Busemann functions are also useful to study the interface between the two growing infections. Planar directed LPP has the advantage over undirected first passage percolation in that it has an exactly solvable case that provides a window to the deeper properties of the entire class of models. This is the case where vertex weights are exponentially or geometrically distributed. The distribution of the Busemann functions becomes then fairly explicit. This leads to a number of precise results, such as closed-form expressions for the limit shape function and for the limiting angle of the competition interface. (Received September 21, 2016)

1125-60-3157 **Philippe Sosoe\***, Center for Math. Sciences and Applications, 20 Garden St., Cambridge, MA 02318. *Concentration in First-Passage Percolation*

The order of the fluctuations of the passage time from the origin to a distant vertex  $x$  in first-passage percolation (FPP) has been the subject of intense investigation. It is believed that the variance grows as  $\|x\|_1^{2\chi}$ , for  $\chi > 0$ . In two dimensions, the exponent  $\chi$  is expected to be typical of the KPZ universality class:  $\chi = 1/3$ . This has been confirmed rigorously for some special models of last passage percolation, but remains mysterious in general.

In these lectures, I will discuss bounds on the order of fluctuations in FPP in  $\mathbb{Z}^d$ , for general  $d$ , focusing on two results. First, exponential concentration on a linear scale for the passage times, as proved by Kesten. Secondly, I will present an estimate of order  $\|x\|_1 / \log \|x\|_1$  for the variance. Such a bound was first derived by Benamini, Kalai and Schramm (BKS) for Bernoulli edge weights. The BKS result was generalized to other weight distributions, first by Benaim-Rossignol, and then Damron, Hanson, and myself. Although far from optimal, this sublinear bound remains the best known to date. I will explain how these results follow from supplementing concentration results with simple observations about the global effect of varying the passage time across single edges. (Received September 28, 2016)

1125-60-3158 **Michael Damron\***, School of Mathematics, Skiles Building, 686 Cherry St., Atlanta, GA 30332. *Introduction to random growth models (2 lectures)*

Random growth models come from physics and biology and can describe the motion of interfaces or the spread of infections. Mathematically, they can show interesting non-traditional limiting behavior. Two examples are first-passage percolation (FPP) and last-passage percolation (LPP). An infection is set on a  $d$ -dimensional lattice and spreads across edges using nonnegative random passage times  $t_e$  on edges in FPP, and on vertices in LPP.

In FPP, infections take paths of least time, but in LPP, they take directed paths of maximal time. An infection from  $x$  infects  $y$  at time  $T(x, y)$  and at time  $t$ , an infection from 0 occupies a region  $B(t)$  of  $\mathbb{Z}^d$ .

First, we will cover some basic probability, then focus on convergence of the rescaled region  $B(t)/t$  to a limiting shape  $\mathcal{B}$ . Little is known about  $\mathcal{B}$  except in exactly solvable cases, but we will discuss conjectured properties (differentiability and curvature of  $\partial\mathcal{B}$ ) and some proved properties, along with recent variational formulas. Then we will move to rate of convergence to  $\mathcal{B}$ , which is studied through the geometry of geodesics and scaling exponents. (Received September 30, 2016)

1125-60-3159 **Jack Hanson\***, Department of Mathematics, Convent Ave. at 138th St., New York, NY 10031. *Infinite geodesics, asymptotic directions, and Busemann functions*

In first-passage percolation (FPP), a finite geodesic between vertices  $x$  and  $y$  is an optimizing path for the random metric  $T(\cdot, \cdot)$ . An infinite geodesic is an infinite path whose finite subpaths are finite geodesics. There are many natural questions about the structure of infinite geodesics: the number of distinct infinite geodesics, whether they are asymptotically directed, and whether a doubly infinite geodesic can exist.

Much can be said about these questions under an unproven curvature assumption on the model's limiting shape – e.g., that every infinite geodesic has an asymptotic direction. Busemann functions were brought to FPP as a tool for proving similar statements under minimal assumptions. For instance, they have been used to prove that there exist more than two disjoint infinite geodesics without any unverified assumptions.

Busemann functions have the form  $B(x, y) = \lim_k [T(x, z_k) - T(y, z_k)]$  for  $(z_k)_k$  the vertices on an infinite geodesic.  $B(x, y)$  encodes the relative favorability of the points  $x$  and  $y$  for infecting  $z_k$  for  $k$  large, and so the asymptotics of  $B$  control the direction of geodesics to  $(z_k)$ . We will discuss the limiting behavior of Busemann functions and its relationship to properties of infinite FPP geodesics. (Received September 30, 2016)

1125-60-3179 **Ivan Corwin\*** ([ivan.corwin@gmail.com](mailto:ivan.corwin@gmail.com)), Columbia University, Mathematics Department, 2990 Broadway, New York, New York 10027. *KPZ fluctuations in exactly solvable models, II.*

Some random growth models admit concise and exact formulas describing expectations of various observables of interest. These models and their solvability spring from certain algebraic structures such as representation theory and quantum integrable systems. By studying these examples, we are able to gain predictions for the universal behaviors of a much wider class of random growth models the so-called Kardar-Parisi-Zhang (KPZ) universality class. We will touch on some of the models discussed earlier in the short course and on some new ones, such as directed last-passage percolation, positive temperature directed polymers, the (totally) asymmetric simple exclusion process, the KPZ stochastic partial differential equation, and some others. We sketch a proof of the asymptotic fluctuation scaling and statistics for one of these models and indicate how this generalizes to the broader class. (Received November 01, 2016)

## 62 ► Statistics

1125-62-259 **Stephanie Ann Allen\*** ([stephanie.a.allen95@gmail.com](mailto:stephanie.a.allen95@gmail.com)), 591 Pierce Creek Road, Binghamton, NY 13903, and **Ye Ye, David Madras** and **Greg Anthony Zanotti**. *Change-point Detection Methods for Body-Worn Video.*

Body-worn video (BWV) cameras are increasingly utilized by police departments to provide a record of police-public interactions. However, large-scale BWV deployment produces terabytes of data per week, necessitating the development of effective computational methods to identify salient changes in video. In work carried out at the 2016 RIPS program at IPAM, UCLA, we present a novel two-stage framework for video change-point detection. First, we employ state-of-the-art machine learning methods including convolutional neural networks and support vector machines for scene classification. We then develop and compare change-point detection algorithms utilizing hidden Markov models, time series analysis, and maximum likelihood estimation to identify noteworthy changes. We test our framework on detection of vehicle exits and entrances in a BWV dataset provided by the Los Angeles Police Department and achieve over 90% recall and nearly 70% precision — demonstrating robustness to rapid scene changes, extreme luminance differences, and frequent camera occlusions. (Received August 19, 2016)

1125-62-299 **Joshua M. Whitlinger\*** ([whitlingerjm@vcu.edu](mailto:whitlingerjm@vcu.edu)), 4118 Grace Harris Hall, 1015 Floyd Avenue, Richmond, VA 23220, and **Edward L. Boone** and **Ryad Ghanam**. *Estimating the Fraction in Fractional Differential Equations using a Bayesian Approach.*

The extraction of natural gas from the earth has been shown to be governed by differential equations concerning flow through a porous material. Recently, models such as fractional differential equations have been developed to

model this phenomenon. One key issue with these models is estimating the fraction of the differential equation. Traditional methods such as maximum likelihood, least squares and even method of moments are not available to estimate this parameter as traditional calculus methods do not apply. In this work we develop a Bayesian approach to estimate the fraction of the differential equation and use this approach to adequately quantify the uncertainties associated with the error and predictions. A simulation study is presented as well to assess the utility of the modeling approach. (Received August 24, 2016)

1125-62-443 **Genki Kusano\*** ([genksn@gmail.com](mailto:genksn@gmail.com)), 6-3, Aoba, Aramaki, Aoba-ku, Sendai, Miyagi 980-8578, Japan, and **Kenji Fukumizu** and **Yasuaki Hiraoka**. *Kernel method for persistence diagrams*.

Topological data analysis (TDA) is an emerging mathematical concept for characterizing shapes in complex data. In TDA, persistence diagrams are widely recognized as a useful descriptor of data, and can distinguish robust and noisy topological properties. In this talk, we will propose a kernel method on persistence diagrams to develop a statistical framework in TDA. The proposed kernel satisfies the stability property and provides explicit control on the effect of persistence by a weight function. Furthermore, the method allows a fast approximation technique. The method is applied into practical data and the results show the advantage of our method compared to other relevant methods on persistence diagrams. (Received September 02, 2016)

1125-62-477 **Emanuele Ventura\***, [emanuele.ventura@aalto.fi](mailto:emanuele.ventura@aalto.fi). *Finite phylogenetic complexity and combinatorics of tables*.

In this talk, we will discuss the phylogenetic complexity of an abelian group. This is an invariant introduced in a seminal work by Sturmfels and Sullivant, in the context of group-based models. We will show that the phylogenetic complexity is finite for every abelian group. This is a joint work with Mateusz Michałek. (Received September 03, 2016)

1125-62-582 **Sarah Robinson\***, C308B Life Sci F. Davison, Athens, GA 30602, and **David Burton**. *Functional Data Analysis of Copy Number Alterations in Bladder Cancer Tumors*.

Genomic structural changes known as copy number alterations (CNAs) have a role in tumor progression. CNAs are changes in the chromosome where regions are either amplified or deleted. It is thought that bladder cancer subgroups have varying CN profiles that are similar within groups but differ across groups. We analyze array comparative genomic hybridization (aCGH) data from 93 bladder cancer patients whose profiles are in muscle invasive and non-muscle invasive subgroups. We are treating these CN profiles as functions across the entire chromosome, and using functional data analysis tools for inference. We use Bayesian, wavelet-based, functional response regression to characterize the CN profiles of muscle invasive and non-muscle invasive patients. We develop simulated aCGH profiles in order to test these methods. We find that our wavelet bases method using a fixed effects model for functional regression confirms the results of prior research about CN amplifications at 1q23.3, 1q21.2, and 11q13.2. Further, we find on chromosome 11 of our dataset that RRM1, RIN1, FG19, and ANO1, genes which are known to be associated with bladder cancer, are altered due to the effects of muscle invasiveness within a tumor. (Received September 06, 2016)

1125-62-764 **Jason Morton\*** ([morton@math.psu.edu](mailto:morton@math.psu.edu)). *Slow inconsistent statistics*. Preliminary report. Traditionally categorical data analysis (e.g. generalized linear models) works with simple, flat datasets akin to a single table in a database with no missing data. What if we have a distributed database with many partial local tables, and data is streaming in too fast for the local tables ever to be globally consistent? I'll discuss how some hints from physics (e.g. quantum nonlocality) could help define sensible algebraic models in this scenario. (Received September 11, 2016)

1125-62-777 **Donald Richards\***, Department of Statistics, Penn State University, University Park, PA 16802. *Distance Correlation: A New Tool for Detecting Association and Measuring Correlation Between Data Sets*.

The difficulties of detecting association and establishing causation have fascinated mankind since time immemorial. Democritus, the Greek philosopher, noted the importance of proving causality when he wrote, "I would rather discover one cause than gain the kingdom of Persia."

To address the difficulties of establishing causation, statisticians have developed many inferential techniques. Perhaps the most well-known method stems from Karl Pearson's correlation coefficient, which was introduced in the late 19th century based on ideas of Francis Galton.

I will introduce in this lecture the recently-devised distance correlation coefficient and describe its advantages over Pearson's and other measures of correlation. We will apply the distance correlation coefficient to data from

large astrophysical databases, where it is desired to classify galaxies. The lecture also will analyze data arising from a study of the association between state-by-state homicide rates and the stringency of state gun laws.

The lecture will review a remarkable singular integral arising in the theory of distance correlation coefficients. We will show extensions of this integral to truncated Maclaurin expansions of the cosine function and in the theory of spherical functions on symmetric cones. (Received September 11, 2016)

1125-62-1027 **Seth Sullivant\*** (smsull12@ncsu.edu). *Introduction to Algebraic Statistics.*

I will provide an introduction and overview of some of the main research areas and results in algebraic statistics. (Received September 14, 2016)

1125-62-1044 **Emmanuel A Appiah\*** (nuelap@live.com), Department of Mathematics and Statistics, University of South Florida, 4202 East Fowler Avenue, CMC 342, Tampa, FL 33620-5700, and **G. S. Ladde** (gladde@usf.edu), Department of Mathematics and Statistics, University of South Florida, 4202 East Fowler Avenue, CMC 342, Tampa, FL 33620-5700. *Nonlinear Hybrid Dynamic Modeling for Time-to-Event Processes.* Preliminary report.

The log-logistic distribution provides a useful alternative to the Weibull distribution for the parametric modeling of survival data where the hazard rate is non-monotonic. In this work, we present an innovative alternative non-linear hybrid model of survival data. The presented approach is motivated by state and parameter estimation of time-to-event processes in biological, chemical, engineering, epidemiological, medical, military, multiple-markets and social dynamic processes under the influence of discrete-time intervention processes. The approach tested in the context of survival data set. (Received September 14, 2016)

1125-62-1214 **Morgan G Escalera\*** (escalerm@rose-hulman.edu). *Sovereign Adaptive Risk Modeling.*

In the wake of the 2008 financial crisis, the FSB (Financial Stability Board) and the BCBS (Basel Committee on Banking Supervision) created a list of Globally Systematically Important Banks with the intention of determining which financial institutions were important enough to the global market that their failure would result in total systemic collapse. The purpose of this research paper is to use econometrics and statistical analysis to create a mathematical model that generalizes the BCBS's five criteria that define the financial institutions and apply them to governmental bodies. These five criteria are size, interconnectedness, cross-jurisdictional activities, complexity, and substitutability. The model is created by utilizing a series of weighted directed graphs to analyze the interconnectedness of central banks in the system as well as creating a "market valuation" of each country based on their one-year bond yields and a complete regard of their holdings. The original application of the model was for the troubled economy of Greece in the Eurozone. Time permitting, we plan to discuss the cases of other countries such as Argentina or Saudi Arabia. (Received September 15, 2016)

1125-62-1304 **Susan Holmes\*** (susan@stat.stanford.edu), Statistics Dept, Sequoia Hall, 390 Serra Mall, Stanford, CA 94305. *Statistical proof and the problem of irreproducibility.*

Data currently generated in the fields of ecology, medicine, climate science and neuroscience often contain tens of thousands of measured variables. Statistical analyses can result in publications whose results are irreproducible.

The field of modern statistics has had to revisit the standard hypothesis testing paradigm. A first step has consisted in the correction for multiplicity in the number of possible variables selected as significant using multiple hypotheses correction and FDR control (Benjamini, Hochberg, 1995). However this does not solve the problem of post-selection inference where the same data is used to choose models and evaluate them. Recent work by groups at the Wharton school (Brown et al.) and at Stanford (Taylor et al.) address these issues.

It remains that the complexity of software and flexibility of choices in tuning parameters can bias the output towards inflation of significant results; neuroscientists recently revisited the problem in the context of fMRI studies.

Formal correction methods cannot accommodate the flexibility available to today's statisticians. I will also present ways that open source tools (Huber, 2015) and publication of code and data can enhance reproducibility. (Received September 16, 2016)

1125-62-1343 **Elizabeth S. Allman** (e.allman@alaska.edu), **James H. Degnan** (jamdeg@unm.edu) and **John A. Rhodes\*** (j.rhodes@alaska.edu). *Identifiability of species trees from gene tree clades and splits.*

Due to population genetic effects, trees describing the evolutionary histories of individual genes may differ from the tree describing the history of the species from which they are drawn. While the multispecies coalescent models this process, model-based inference of a species tree from gene trees remains computationally formidable. An attractive option is to summarize a collection of gene trees through the clades or splits they display, and base inference on only the clade or split frequencies. But for this to work, it is essential that the species tree be

identifiable from these summary statistics. Viewing the spaces of clade and split probabilities as algebraic varieties, we establish this fact. Of particular interest is that while the notion of split is unrooted, split probabilities generally retain enough information to identify not only the species tree topology, but also its root. (Received September 16, 2016)

1125-62-1596 **Matthew Pietrosanu\*** ([pietrosa@ualberta.ca](mailto:pietrosa@ualberta.ca)), Edmonton, Alberta, Canada, and **Giseon Heo** ([gheo@ualberta.ca](mailto:gheo@ualberta.ca)). *Pseudo-Multidimensional Persistent Homology for Landmark-Free Analysis of Point-Cloud Datasets for Medical Imaging and the Determination of Manual Segmentation Reliability.*

Shape comparison, particularly of “soft” objects such as skin-covered or internal structures of the human body, is a difficult problem relevant to computer vision, rehabilitation medicine, dentistry/orthodontics, and beyond, and is commonly employed in the screening, diagnosis, and management of physical deformities. Current methods, however, are often qualitative, rely on visual inspection, or require the manual selection of landmark points, and are thus subjectively influenced by the choice of reviewer.

Seeking to improve upon current methods, we apply persistent homology, a technique in computational topology useful for recovering the underlying topology of a point-cloud dataset. In particular, we extend the standard one-variable distance filtration by incorporating point-cloud curvature estimates to distinguish between otherwise topologically-equivalent manifolds. We present this new technique for *pseudo-multidimensional persistence* as robust against landmark selection and the variability inherent in manual segmentation. Furthermore, we demonstrate an application of this technique to the comparison of maxillary sinuses between multiple patients and to the determination of the reliability of repeated manual segmentations performed by a particular reviewer. (Received September 18, 2016)

1125-62-1717 **Nikita Alexeev** and **Vishesh Karwa\***, [vkawra@seas.harvard.edu](mailto:vkawra@seas.harvard.edu), and **Debdeep Pati**, **Sonja Petrović**, **Mateja Raič**, **Liam Solus**, **Dane Wilburne**, **Robert Williams** and **Bowei Yan**. *Exact goodness-of-fit tests for Stochastic Block Models.*

Stochastic Block models (SBM) with unknown block structure widely used to detect communities in real world networks. Testing the goodness of fit of such models is a challenging task due to the fact that the parameters of an SBM are usually estimated from a single observed network. Usual asymptotic tests are not valid. We develop a finite sample goodness-of-fit tests for three different variants of Stochastic Block models with unknown blocks. The finite sample test is based on the posterior predictive distribution of the SBM with unknown blocks. A key building block for sampling from this distribution is sampler from fibers of models with known block assignments. Sampling from these fibers is carried out using Markov bases. As intermediate results, we describe the Markov bases and the marginal polytope of Stochastic Block models with known block assignments. (Received September 19, 2016)

1125-62-1739 **Carlos Amendola\*** ([amendola@math.tu-berlin.de](mailto:amendola@math.tu-berlin.de)). *Maximum Likelihood Estimate Homotopy Tracking for Twisted Toric Models.* Preliminary report.

The study of maximum likelihood estimates (MLEs) for toric models has been one of the successes of Algebraic Statistics. In the setting of a ‘twisted’ toric model, which corresponds to a scaling of the original parametrization map, we prove that it is possible to track a homotopy path whose endpoints correspond to the different MLEs of two twisted models that may have distinct ML degrees. We illustrate with examples such as the Veronese model, and present how this idea could be applied to help compute MLEs for toric models efficiently.

This talk is based on a Mathematics Research Communities: Algebraic Statistics project, developed by the ‘Likelihood Geometry’ group. (Received September 19, 2016)

1125-62-1765 **Kaie Kubjas\*** ([kaie.kubjas@gmail.com](mailto:kaie.kubjas@gmail.com)) and **Dimitra Kosta**. *Maximum likelihood geometry for group-based phylogenetic models.* Preliminary report.

Based on Matsen’s work on inequalities for group-based phylogenetic models in Fourier coordinates, we study polynomial equations and inequalities that cut out group-based models in original coordinates. Moreover, we investigate the maximum likelihood (ML) degree of group-based models in small instances. Besides the ML degree of the interior of the model, we also look at the ML degrees of boundary components. This talk is based on joint work with Dimitra Kosta. (Received September 19, 2016)

1125-62-1825 **Miguel del Alamo\*** ([miguel.delalamo@stud.uni-goettingen.de](mailto:miguel.delalamo@stud.uni-goettingen.de)), Institute for Mathematical Stochastics, Georg-August-University Goettingen, Goldschmidtstrasse 7, 37077 Goettingen, Germany, and **Axel Munk** and **Housen Li**. *Variational Multiscale Estimators for Nonparametric Regression and Statistical Inverse Problems.*

In the context of nonparametric regression and statistical inverse problems, we introduce and analyze a variational approach called MultIscale Nemirovski-Dantzig (MIND) estimator. This estimator can be viewed as a multiscale extension of the Dantzig selector that relies on early ideas of Nemirovski. More precisely, the MIND minimizes a homogeneous Sobolev norm subject to the constraint that the multiresolution norm of the residuals be bounded by a universal threshold. We will show that the MIND attains almost minimax optimal convergence rates with respect to the  $L^q$ -risk,  $1 \leq q \leq \infty$ , both almost surely and in expectation. Furthermore, it partially adapts to the smoothness of the function to be estimated over a certain range of Sobolev classes.

In this talk we will review this approach and illustrate its performance in several Monte Carlo simulations and real data examples from cell microscopy. (Received September 19, 2016)

1125-62-1915 **Megan Owen\*** ([megan.owen@lehman.cuny.edu](mailto:megan.owen@lehman.cuny.edu)). *Means and a Central Limit Theorem in tree space.*

The space of metric phylogenetic trees introduced by Billera, Holmes, and Vogtmann (2001) is a polyhedral cone complex. It is also non-positively curved or CAT(0), so there is a unique shortest path (geodesic) between any two trees and a well-defined notion of a mean tree for a given set of trees. I will discuss some properties of the mean tree and give a partial Central Limit Theorem for when the mean is in a top-dimensional cone or on a co-dimension one boundary of one. The later result is joint work with Dennis Barden and Huiling Le. (Received September 19, 2016)

1125-62-1920 **Habib Najm\*** ([hnnajm@sandia.gov](mailto:hnnajm@sandia.gov)), **Khachik Sargsyan**, **Xun Huan**, **Joseph Oefelein**, **Guilhem Lacaze** and **Zachary Vane**. *Uncertainty Quantification with Model Error.*

In the context of Bayesian model calibration, statistical model error representations have been employed, whose parameters have been inferred jointly with other model parameters. The calibration of this model error representation provides for a suitably fitted correction on model predictions to bridge the gap with calibration observables. Extending these methods to physical methods has led to the development of strategies for embedding such model error representations within the model, to ensure satisfaction of various constraints.

This talk will provide an overview of our developments in this regard. We will discuss the basics of the construction, including a number of variants relying on different simplifications, and will outline its utility in different situations including those with or without data noise. We will also present a demonstration in a chemical model, and one in large eddy simulation of turbulent flow. (Received September 19, 2016)

1125-62-1991 **Elizabeth Gross\*** ([elizabeth.gross@sjsu.edu](mailto:elizabeth.gross@sjsu.edu)) and **Seth Sullivant**. *The maximum likelihood threshold of a graph.*

The maximum likelihood threshold of a graph is the smallest number of data points that guarantees that maximum likelihood estimates exist almost surely in the Gaussian graphical model associated to the graph. In this talk, we will show that this graph parameter is connected to the theory of combinatorial rigidity. In particular, if the edge set of a graph  $G$  is an independent set in the  $(n-1)$ -dimensional generic rigidity matroid, then the maximum likelihood threshold of  $G$  is less than or equal to  $n$ . This connection allows us to make several new statements regarding the existence of the maximum likelihood estimator in situations of limited data. (Received September 19, 2016)

1125-62-2054 **John P Higgins\*** ([higgins4@vt.edu](mailto:higgins4@vt.edu)). *De-identification through re-identification: using high-dimensional distance to mitigate statistical disclosure.*

When the Census Bureau releases public-use data files, they must be sure to not disclose any individual's information for ethical reasons. To protect public-use data, the Center for Disclosure Avoidance Research (CDAR) uses various methods such as adding noise and row swapping, but there are no currently comprehensive techniques to validate the efficacy of such methods. This research aims to re-identify subjects from the American Housing Survey (AHS) to those in an external dataset using high-dimensional Euclidean distance with an additive penalty to account for missing values and close but not exact matches. Ultimately, the prevalence of missing values in this study leads to an interesting conclusion that the results of re-identification studies are directly limited by the quality of the both the internal and the external data used. (Received September 19, 2016)

1125-62-2127 **Alexandra Dwyer\*** (amd3550@gmail.com) and **Rachel Strow** (rs249049@muhlenberg.edu). *How do characteristics of police departments impact crime?* Preliminary report.

Police/civilian interaction has become a highly discussed and controversial issue in the United States. After considering what things impact police/civilian interactions, we became particularly interested in the characteristics and policies of police forces, and how they may impact crime. To answer this question we gathered data on crime rates from the Uniform Crime Reporting database, demographic data from the US Census Bureau, and data on police force personnel and policies from the Law Enforcement Management and Administration Survey. The explanatory variables we looked at include demographics of police forces, number of officers per 10,000 residents, the use of merit pay, residential requirements, and a community policing plan. We used statistical analysis to determine which, if any, of these predictor variables has an effect on crime. (Received September 19, 2016)

1125-62-2269 **Vindya Kumari Pathirana\*** (vindya.pathirana@uconn.edu), Department of Mathematics, University of Connecticut, and **Kandethody M Ramachandran** (ram@usf.edu), Department of Mathematics and Statistics, University of South Florida. *Mahalanobis Based k-Nearest Neighbor Forecasting versus Time Series Forecasting Methods*. Preliminary report.

Foreign exchange (FX) rate forecasting is a challenging area of study. Various linear and nonlinear methods have been used to forecast FX rates. As the currency data are nonlinear and highly correlated, forecasting through nonlinear dynamical systems is becoming more relevant. The k-nearest neighbor (k-NN) algorithm is one of the most commonly used nonlinear pattern recognition and forecasting methods that outperforms the available linear forecasting methods for the high frequency foreign exchange data. As the k neighbors are selected according to a distance function, the choice of distance plays a key role in the k-NN procedure. The time series forecasting method, Auto Regressive Integrated Moving Average process (ARIMA) is considered as one of the superior forecasting method for time series data. In this work, we compare the performances of Mahalanobis distance based k-NN forecasting procedure with the traditional ARIMA based forecasting method. In addition, the forecasts were transformed into a technical trading strategy to create buy and sell signals. The two methods were evaluated for their forecasting accuracy and trading performances. (Received September 20, 2016)

1125-62-2289 **Morgan E Engle\*** (englem@southwestern.edu) and **Penny Phan** (phan@southwestern.edu). *Green Math: Models of Greenhouse Gasses*.

We will present mathematical models that analyze data on CO2 emissions for different countries across time. Some factors we consider are fossil fuels, livestock, land use and agriculture. We will illustrate how parameter values, dependent on agricultural practices, account for methane emissions in diverse socio-economic regions of the world. (Received September 20, 2016)

1125-62-2290 **Marechal and Vanhems\*** (a.vanhems@tbs-education.fr), Toulouse Business School, 20 bd Lascrosses BP 7010, 31068 Toulouse, France. *Solving inverse problems in econometrics using a mollification approach*.

The overall purpose of this research is to provide new tools for the analysis of structural econometric models. Our focus will be on nonparametric procedures that permit estimation of causal effects while avoiding the strong assumptions required by parametric procedures or deconvolution models. In the first case, our research is motivated by the analysis of structural causal models with endogeneity. Endogeneity may be due to omitted variables, measurement errors, or simultaneity. Nonparametric regression with endogeneity gives rise to an ill-posed inverse problem. In the second case, our objective is to reconstruct the density function of a random variable from the observation of noisy data, which also leads to an ill-posed inverse problem. Our contribution is to solve both problems using a mollification approach. Mollification has been investigated for linear ill-posed equations, specifically for deconvolution in signal and image processing, by Lannes et al (1987), and has been recently extended by Bonnefond and Marechal (2009) and Marechal (2016) to more general situations. We investigate the use of this technique in the econometric context. (Received September 20, 2016)

1125-62-2323 **Steve Chung, Aaron Pinkerton, Hunter Rehm and Michelle Yu\*** (myu18@g.holycross.edu). *A Bayesian Approach to Predicting the Outcome of Endovenous Laser Ablation*.

Varicose veins affect more than 40 million people in the United States. Endovenous Laser Ablation (EVLA) is a common and highly effective treatment method for varicose veins. The success of EVLA may depend on several key variables: laser power, time, length of vein, laser energy, and linear endovenous energy density (LEED), as well as combinations of these variables. In this study, we consider the Bayesian and parametric logistic regression models to predict the treatment outcome from these key variables. We use the Monte Carlo cross validation to

assess the models. Our finding indicates that the Bayesian logistic regression is a better predictive model than the parametric logistic regression. (Received September 20, 2016)

1125-62-2386 **Mosisa G. Aga\*** ([maga@aum.edu](mailto:maga@aum.edu)), 7061 Senators Dr, Montgomery, AL 36117.  
*Bootstrapping Time Series Models.*

The bootstrap is a general approach to statistical inference based on building a sampling distribution for a statistic by re-sampling from the data at hand. In this talk we will discuss the application of bootstrap to dependent data structures in general and to certain time series models in particular. (Received September 20, 2016)

1125-62-2399 **Andrea Arnold\*** ([anarnold@ncsu.edu](mailto:anarnold@ncsu.edu)) and **Alun L. Lloyd.** *Nonlinear filtering for periodic time-varying parameter estimation in an epidemic model.*

Many systems arising in biological applications are subject to periodic forcing, such as seasonal forcing in epidemiological systems. In these types of applications, the forcing parameter is not only time-varying in nature but also has a periodic structure. We present an approach to estimate periodic time-varying parameters using nonlinear filtering. In particular, we employ the ensemble Kalman filter (EnKF) to estimate the periodic seasonal contact rate in an epidemic model for the spread of measles.

In the literature, most parameter estimation methodology is aimed at parameters treated as constant or as drifting over time with no imposed structure. The proposed technique imposes a periodic structure on the contact rate by treating it as a piecewise function consisting of parameters that change, e.g., each month. The augmented EnKF implementation also allows for estimation of model initial conditions as well as the reporting probability, which is vital for predicting under-represented data such as the incidence data considered here. In particular, results are demonstrated on real time-series data of measles case reports from various regions, including monthly reported cases in Baltimore, MD, (1928-1963) and weekly reported cases in England and Wales (1948-1967). (Received September 20, 2016)

1125-62-2459 **Monica Christine Jackson\*** ([monica@monicajackson.net](mailto:monica@monicajackson.net)), **Kimberly Sellers, Calandra Moore, Jada Johnson, Jessica Lyle, Israel Almodovar, Emma Veach and Ismael Xique.** *Race matters: analyzing the relationship between colorectal cancer mortality rates and various factors within respective racial groups.*

Colorectal cancer (CRC) is the third leading cause of mortality due to cancer (with over 50,000 deaths annually), representing 9% of all cancer deaths in the U.S. In particular, the African-American CRC mortality rate is among the highest reported for any race/ethnic group. Meanwhile, the CRC mortality rate for Hispanics is 15–19% lower than that for non-Hispanic Caucasians. While factors such as obesity, age, and socioeconomic status are known to associate with CRC mortality, do these and other potential factors correlate with CRC death in the same way across races? This research linked data obtained from several sources to examine geographic and racial/ethnic differences, and develop a spatial regression model that adjusted for several factors that may attribute to health disparities among ethnic/racial groups. This analysis showed that sunlight, obesity, and socio-economic status were significant predictors of CRC mortality. The study is significant because it not only verifies known factors associated with the risk of CRC death but, more importantly, demonstrates how these factors vary within different racial groups. Accordingly, education on reducing risk factors for CRC should be directed at specific racial groups above and beyond creating a generalized education plan. (Received September 20, 2016)

1125-62-2502 **Eva Gjekmarkaj\*** ([egjekmarkaj@smith.edu](mailto:egjekmarkaj@smith.edu)), 46 Whittier Street, Florence, MA 01062, and **Jessica Lynn Perry** ([jperry@smith.edu](mailto:jperry@smith.edu)), 10 Aldrich Street, Northampton, MA 01060. *An Examination of the Relationship between Financial Characteristics and Portfolio Performance.* Preliminary report.

Our goal is to predict the performance of a given financial portfolio on the basis of equity characteristics. Possible ways to study this question include multiple regression and principal components analysis. We focus this study on principal components as a method for summarizing and reducing the number of predictor variables. We use the identified principal components as predictor variables in a regression analysis on portfolio performance. (Received September 20, 2016)

1125-62-2632 **Francis Edward Su\*** ([su@math.hmc.edu](mailto:su@math.hmc.edu)), Department of Mathematics, Harvey Mudd College, 301 Platt Blvd, Claremont, CA 91106. *Data Mining the MathFeed News App.* Preliminary report.

The MathFeed News App is an iPhone app I've written that gathers together news stories and blogs about mathematics from across the web. What makes a story a 'math' story? How should one decide whether a story

is 'popular' enough to go into the app? I will discuss the many interesting math and data mining questions that present themselves, that would, if implemented, have immediate practical application. (Received September 20, 2016)

1125-62-2637 **Riley S Abel\*** (rileyabel@gmail.com) and **Kimihiro Noguchi** (rileyabel@gmail.com). *Nonparametric Multiple Comparisons Using Log Odds of the Relative Effects*. Preliminary report.

Nonparametric multiple comparisons can be very useful, but must be used carefully to avoid nontransitivity paradoxes. Using the example of nontransitive dice, we show how to resolve the issue of these paradoxes in nonparametric multiple comparisons by creating a reference distribution. Then, we suggest a new testing procedure using the log odds transformation and a Box-type finite-sample approximation. We conclude by analyzing the dataset from a neuropsychological study, highlighting the effectiveness of the log odds transformation as a convenient effect size. (Received September 20, 2016)

1125-62-2866 **Daniel T Shanahan\*** (dshanahan@lsu.edu), 103 School of Music, Louisiana State University, Baton Rouge, LA 70803. *A Diachronic Approach to Musical Prototypicality with Dynamic Time Warping*. Preliminary report.

As musical styles change, ideas that were once common begin to wane into obscurity as new ideas emerge. Music theorists have employed many paradigms to discuss this, including "arcs of prototypicality", in which musical ideas begin to coalesce as they are used more often, and splinter over time into new ideas. This study employs a newly-curated dataset of more than fifty thousand melodic themes to examine the possible arcs of prototypicality that occur in the 17th and 18th centuries. The corpus was derived from the RISM incipit database, and has been curated to include composer birth and death dates, as well as nationality. Specifically, it employs the time series alignment algorithm known as dynamic time warping (DTW), in which distances of feature vectors are calculated and aligned independently of the time series. While other distance metrics might be useful in approaching this problem (e.g. Levenshtein), the ability of DTW to perform distance measurements while aligning the features with time series make it particularly suitable for aligning and comparing musical ideas over time. This paper will discuss the benefits of DTW, and will provide a comparison of the results of multiple distance metrics, and their ability to compare melodic relationships in large corpora. (Received September 20, 2016)

1125-62-2932 **Alen Alexanderian\*** (alexanderian@ncsu.edu), alexanderian@ncsu.edu, and **Arvind K Saibaba**, asaibab@ncsu.edu. *Scalable algorithms for D-optimal design of experiments for large-scale Bayesian linear inverse problems*.

We address the problem of D-optimal experimental design for infinite-dimensional Bayesian linear inverse problems governed by PDEs. The goal is optimal placement of sensors where observational data is collected, so as to maximize the expected information gain. That is, we rely on a Bayesian D-optimal criterion, given by the expected Kullback-Leibler divergence from prior to posterior. In the infinite-dimensional Hilbert space setting, assuming Gaussian prior and noise models, the analytic expression for the resulting optimal experimental design (OED) objective function is given by the log-determinant of a perturbation of the identity by a prior-preconditioned data misfit Hessian operator. We introduce efficient randomized linear algebra methods to render the computation of OED objective function and its gradient tractable in large-scale applications. Numerical results illustrating our framework will be provided in the context of D-optimal sensor placement for reconstruction of initial concentration field in an advection-diffusion problem. (Received September 20, 2016)

1125-62-3032 **Deidra Andrea Coleman\*** (dcoleman@philander.edu), **Brian J Reich** and **Donald E. K. Martin**. *A Bayesian False Discovery Approach to Syndromic Surveillance*.

We give a procedure to detect outbreaks using epidemiological data while controlling the Bayesian False Discovery Rate (BFDR). The procedure entails choosing an appropriate Bayesian model that captures the spatial dependency inherent in epidemiological data and considers all days of interest, selecting a test statistic based on a chosen measure that provides the magnitude of the maximum spatial cluster for each day, and identifying a cutoff value that controls the BFDR for rejecting the collective null hypothesis of no outbreak over a collection of days for a specified region. We use our procedure to analyze botulism-like syndrome data collected by the North Carolina Disease Event Tracking and Epidemiologic Collection Tool (NC DETECT). (Received September 20, 2016)

1125-62-3066 **Kelly Bodwin\*** (kbodwin@email.unc.edu), **Kai Zhang** and **Andrew Nobel**. *Coherent Itemset Mining*. Preliminary report.

It is often of interest to find associations between variables based on observed binary data. This problem has been previously studied under the heading of "frequent itemset mining" or "association rule mining". However,

these classical approaches break down in the presence of non-identically distributed samples, which we argue is a common structure in real datasets of interest. In this paper, we propose a new model that accounts for differences in sample behavior while maintaining a common underlying variable dependence structure. We then introduce an algorithm that makes use of this model to identify groups of associated variables, which we refer to as coherent itemsets. The Coherent Itemset Mining (CIM) algorithm relies on an iterative update procedure that adaptively selects variable sets based on statistical testing principles. It is designed to run efficiently for high dimensional data. The CIM algorithm is tested on a variety of simulations as well as real datasets in genetics and text analysis. (Received September 20, 2016)

1125-62-3119 **Daniele Bigoni, Youssef Marzouk\*** ([ymarz@mit.edu](mailto:ymarz@mit.edu)) and **Alessio Spantini**. *Measure transport, variational inference, and low-dimensional maps.*

We will discuss how transport maps, i.e., deterministic couplings between probability measures, can enable useful new approaches to Bayesian computation. A first use involves a combination of optimal transport and Metropolis correction; here, we use continuous transportation to transform typical MCMC proposals into adapted non-Gaussian proposals. Second, we discuss a variational approach to Bayesian inference that constructs a deterministic transport map from a reference distribution to the posterior, without resorting to MCMC. Independent and unweighted samples can then be obtained by pushing forward reference samples through the map. Making either approach efficient in high dimensions, however, requires identifying and exploiting low-dimensional structure. We present new results relating sparsity of transport maps to the conditional independence structure of the target distribution, and discuss how this structure can be revealed through the analysis of certain average derivative functionals. The resulting inference algorithms involve either the direct identification of sparse maps or the composition of low-dimensional maps. We demonstrate our approaches on Bayesian inference problems arising in spatial statistics and in partial differential equations. (Received September 21, 2016)

1125-62-3147 **Michael P. Johnson\*** ([michael.johnson@umb.edu](mailto:michael.johnson@umb.edu)), Department of Public Policy & Public Affairs, 100 Morrissey Blvd., McCormack Hall Room 2017C, Boston, MA 02125-3393. *Community Data Analytics: Localized Data Analysis and Decision Modeling in the Era of ‘Big Data’ and ‘Smart Cities’*. Preliminary report.

Community-based organizations use data for program design, services provision and strategic planning. However, CBOs often have limited ability to identify, access and apply these data. Thus, CBOs may make decisions on the basis of inadequate data, or limited understanding of the local environment, or limited ability to generate mission-aligned solutions. Community data analytics (CDA) uses local know-how and clearly-articulated values in order to transform data into action. CDA is rooted in principles of operations research and management science for public benefit. These principles include: active participation by local stakeholders to identify problems of interest; a critical perspective on issues of problem and solution design; appropriate technology, and local control of solution implementation. This presentation features two examples of current research in community data analytics: planning and decision support for land use in declining regions and blighted communities, and community-derived measures of local economic development success. These cases yield insights into ways that urban communities can assert control over local development at a time when the smart cities and big data movements represent as-yet unfulfilled potential for local development and revitalization. (Received September 21, 2016)

## 65 ► Numerical analysis

1125-65-29 **Kailash C. Patidar\*** ([kpatidar@uwc.ac.za](mailto:kpatidar@uwc.ac.za)), Department of Math. and Appl. Mathematics, University of the Western Cape, Private Bag X17, Bellville, 7535, South Africa. *NSFDMs as FOFDMs for robust simulation of parameter sensitive partial differential equations.*

In this talk, we will discuss some classical approaches for designing exponentially fitted methods to solve parameter sensitive ordinary differential equations, in particular, those which are singularly perturbed, and their limitations when one needs to extend them to solve systems of such equations as well as singularly perturbed partial differential equations. Noting the importance of the uniform convergence of the fitted methods, for which they are very popular in this research community, we will then discuss how certain issues are effectively resolved in our recent work and what are the problems that may be tackled further using the approaches that are based on NSFDM philosophy. (Received May 27, 2016)

1125-65-143 **Mei Yang\*** ([mei.yang@mavs.uta.edu](mailto:mei.yang@mavs.uta.edu)), 300 College Street. Apt.B, Arlington, TX 76010, and **Ren-cang Li**. *Heavy Ball Minimal Residual Method for Least-squares Problems.*

The heavy ball minimal residual (HBMR) method is presented for solving overdetermined least-squares problem  $\min_x \|Ax - b\|_2$ , where  $A$  is a sparse matrix. HBMR method seeks optimal approximate solutions of the least-squares problem by minimizing the residual norm  $\|A^T \text{right}\|_2$  over both the Krylov subspace obtained by the restarted Golub-Kahan bidiagonalization process and the information of the Krylov subspaces in the previous cycles. Numerical experiments are reported to show the advantages of the HBMR method. (Received September 17, 2016)

1125-65-182 **Deanna Needell\*** ([dneedell@cmc.edu](mailto:dneedell@cmc.edu)), 850 Columbia Ave, Claremont, CA 91711, and **Rachel Ward**. *Batched Stochastic Gradient Descent with Weighted Sampling.*

We analyze a batched variant of Stochastic Gradient Descent (SGD) with weighted sampling distribution for smooth and non-smooth objective functions. We show that when the batches can be distributed computationally that a significant speedup in the convergence rate is possible. We propose several computationally efficient schemes to approximate the optimal weights, and compute the proposed sampling distribution for the least-squares and hinge loss problems. We show both analytically and experimentally that substantial gains can be obtained using this hybrid approach. (Received August 09, 2016)

1125-65-260 **Zhu Xinyun\*** ([zhu\\_x@utpb.edu](mailto:zhu_x@utpb.edu)), Department of Mathematics, University of Texas of the Permian Basin, Odessa, TX 79762, and **Hongtao Fan** and **Bing Zheng**. *A relaxed positive semi-definite and skew-Hermitian splitting preconditioner for non-Hermitian generalized saddle point problems.* Preliminary report.

For non-Hermitian saddle point linear systems, Pan, Ng and Bai presented a positive semi-definite and skew-Hermitian splitting (PSS) preconditioner (Pan et al. Appl. Math. Comput. 172, 762–771 2006), to accelerate the convergence rate of the Krylov subspace iteration methods like the GMRES method. In this paper, a relaxed positive semi-definite and skew-Hermitian (RPSS) splitting preconditioner based on the PSS preconditioner for the non-Hermitian generalized saddle point problems is considered. The distribution of eigenvalues and the form of the eigenvectors of the preconditioned matrix are analyzed. Moreover, an upper bound on the degree of the minimal polynomial is also studied. Finally, numerical experiments of a model Navier-Stokes equation are presented to illustrate the efficiency of the RPSS preconditioner compared to the PSS preconditioner, the block diagonal preconditioner (BD), and the block triangular preconditioner (BT) in terms of the number of iteration and computational time. (Received August 19, 2016)

1125-65-287 **Albert Fannjiang\*** ([fannjiang@math.ucdavis.edu](mailto:fannjiang@math.ucdavis.edu)), **Pengwen Chen** and **Gi-Ren Liu**. *Phase Retrieval and Ptychography with Coded Diffraction Patterns.*

We discuss the convergence properties of various alternating projections algorithms for phase retrieval with coded diffraction patterns. We propose a novel initialization method that produces high-quality initial guesses and we present an error bound for the initialization method. (Received August 23, 2016)

1125-65-313 **Fidele F Ngwane\*** ([ngwanef@mailbox.sc.edu](mailto:ngwanef@mailbox.sc.edu)), 807 Hampton street, walterboro, SC 29488, and **Samuel N Jator** ([jators@apsu.edu](mailto:jators@apsu.edu)), Department of mathematics, Austin Peay State University, Clarksville, TN 37044. *Integrating oscillatory Hamiltonian systems via a block algorithm with an automatic error estimate based on a trigonometrically-fitted second derivative extended backward differentiation formula.*

We derived a trigonometrically-fitted continuous second derivative extended backward differentiation formula whose coefficients are functions of the frequency and the step size. The continuous form is used to construct a trigonometrically-fitted block second derivative extended backward differentiation formula (TBSDEBDF) for numerically integrating oscillatory Hamiltonian systems in a block-by-block approach and the Hamiltonian function is shown to conserve energy. The convergence and stability properties of the TBSDEBDF are discussed and numerical examples are presented to illustrate the accuracy of the method. In the case where the analytic solution is not available, we use error estimates to show the accuracy of the method. (Received August 25, 2016)

1125-65-329 **Pete Casazza** and **Xuemei Chen\*** ([xchen@math.usfca.edu](mailto:xchen@math.usfca.edu)). *Frame Scalings: A Condition Number Approach.*

Scaling frame vectors is a simple and noninvasive way to construct tight frames. However, not all frames can be modified to tight frames in this fashion, so in this case we explore the problem of finding the best conditioned frame by scaling, which is crucial for applications like signal processing. We conclude that this problem is equivalent to solving a convex optimization problem involving the operator norm, which is unconventional since this problem

was only studied in the perspective of Frobenious norm before. We also further study the Frobenious norm case in relation to the condition number of the frame operator, and the convexity of optimal scalings. (Received August 27, 2016)

1125-65-352 **Francisco Javier Sayas\*** (fjsayas@udel.edu), Department of Mathematical Sciences, University of Delaware, Newark, DE 19716. *Exotic transmission problems for wave equations and how they really are semidiscrete integral equations.*

Several problems related to scattering of acoustic, elastic, or electromagnetic waves can be reformulated using retarded potentials and their associated boundary integral operators. I will focus on simple transmission problems for the wave equation and show how the Galerkin semidiscretizations in space of some time-domain boundary integral equations are equivalent to stable dynamical systems subject to interface excitation. I will then start from the end, presenting more general dynamical systems and show how we can pull back to stable systems of time-domain integral equations describing scattering by obstacles with piecewise constant material properties. (This talk condenses work developed with Antonio Laliena from the University of Zaragoza, Tianyu Qiu from Rice University, and Alexander Rieder from the Technical University of Vienna.) (Received August 29, 2016)

1125-65-376 **Jeb B. Collins\*** (jbcoll112@gmail.com) and **Jonathan Hauenstein.** *Improving solutions to fully nonlinear second order elliptic PDEs with numerical algebraic geometry endgame techniques.* Preliminary report.

Fully nonlinear second order elliptic partial differential equations are very difficult to solve as the nonlinearity in the second order term causes issues with defining a weak form for the problem. The vanishing viscosity method was developed to approximate these equation with a fourth order perturbation term. The solution is then approximated by reducing the effect of this perturbation term. In this work, homotopy continuation techniques used in numerical algebraic geometry are utilized to vanish the perturbation term effectively for such differential equations. (Received August 30, 2016)

1125-65-393 **Leo Rebholz\*** (rebholz@clemson.edu) and **Mengying Xiao** (mengyix@clemson.edu). *Improved accuracy in algebraic splitting methods for Navier-Stokes equations.*

This paper studies a new alteration of Yosida algebraic splitting methods for the Navier-Stokes equations. By applying the usual or pressure-corrected Yosida splitting techniques to discretizations written in terms of velocity and pressure updates ( $u^{n+1} - u^n, p^{n+1} - p^n$ ), we show that the accuracy is increased by one full order in  $\Delta t$  without any additional cost in the respective methods. Proofs of the convergence results are given both in linear algebraic and finite element frameworks. Several numerical tests are given which reveal the (sometimes dramatic) improvement in accuracy offered by the proposed fix. (Received August 31, 2016)

1125-65-407 **Ron Buckmire\*** (rbuckmir@nsf.gov), 4201 Wilson Boulevard, Division of Undergraduate Education, Directorate for Education and Human Resources, Arlington, VA 22230. *A NSFD Scheme For Discretizing Radial Derivatives: History, Application and Significance.*

Standard finite difference schemes often have difficulties when used to obtain solutions to singular boundary value problems, especially when one is interested in knowing the solution of such problems near the origin. The history, application and significance of the development of a nonstandard finite difference scheme which can be used to numerically approximate derivatives in polar and spherical coordinates more accurately than standard schemes will be discussed. (Received August 31, 2016)

1125-65-480 **Siwei Duo\*** (sddy9@mst.edu) and **Yanzhi Zhang.** *Numerical methods for non-local diffusion equation.*

In recent years, the non-local diffusion equation arises much attention which has been widely used in various areas. The fractional Laplacian is a non-local operator which serves as a prototype operator for modeling non-local diffusion processes and can be used to describe new phenomena that are absent from the standard diffusions. However, the nonlocality introduces considerable challenges in both mathematical analysis and numerical simulations. In this talk, we present two numerical methods for solving the non-local diffusion equation with fractional Laplacian. The accuracy and convergence of these methods are analyzed. Also, some numerical examples are presented to compare the accuracy of these methods with other available methods in the literature. (Received September 03, 2016)

1125-65-592 **Yingwei Wang\*** ([wyshtj@gmail.com](mailto:wyshtj@gmail.com)), Department of Mathematics, Purdue University, West Lafayette, IN 47907. *Two-level spectral methods for nonlinear differential equations with multiple solutions*. Preliminary report.

In recent years, there has been a growing interest in the study of nonlinear differential equations with multiple solutions. In this talk, I will present a two-level algorithm based on spectral methods and regeneration homotopy to solve the second order elliptic equation with nonlinearity of polynomial type. Both theoretical and numerical results verify that the proposed method enjoys the following merits: (i) it guarantees multiple solutions; (ii) the computational cost is relatively small; (iii) it is of high-order accuracy. (Received September 07, 2016)

1125-65-838 **Robert Plato\*** ([plato@mathematik.uni-siegen.de](mailto:plato@mathematik.uni-siegen.de)), Department of Mathematics, University of Siegen, Walter-Flex-Str. 3, Siegen, 57068. *Some new results on Lavrentiev regularization for linear accretive ill-posed problems*.

In this talk we consider Lavrentiev regularization for solving linear accretive ill-posed problems in Hilbert spaces. We present converse and saturation results both for exact and noisy data, and quasi-optimality results for parameter choices are obtained as a by-product. New results on adjoint source representations are also given.

## References

1. R. Plato, Converse results, saturation and quasi-optimality for Lavrentiev regularization of accretive problems, 2016, [arXiv.org:1607.04879](https://arxiv.org/abs/1607.04879).
2. R. Plato, P. Mathé, and B. Hofmann, Optimal rates for Lavrentiev regularization with adjoint source conditions, 2016. TU Chemnitz, Fakultät für Mathematik, Preprint 2016-3.

(Received September 12, 2016)

1125-65-850 **Hristo V Kojouharov\*** ([hristo@uta.edu](mailto:hristo@uta.edu)), Department of Mathematics, The University of Texas at Arlington, Arlington, TX 76019-0408, **Daniel T Wood**, Vaccine and Infectious Disease Division, Fred Hutchinson Cancer Research Center, Seattle, WA 98109-1024, and **Dobromir T Dimitrov**, Vaccine and Infectious Disease Division, Fred Hutchinson Cancer Research Center, Seattle, WA 98109-1024. *A Class of Nonstandard Finite Difference Methods for Autonomous Dynamical Systems*.

A class of dynamically consistent numerical methods are analyzed for general productive-destructive systems (PDS). Based on this approach, a nonstandard finite difference method for solving autonomous dynamical systems is constructed. It is designed so that it preserves the positivity of solutions and the local behavior of the dynamical system near equilibria. The proposed numerical method is computationally efficient and easy to implement. Applications to several specific biological systems are also presented. (Received September 12, 2016)

1125-65-855 **Wenyu Lei\*** ([wenyu@math.tamu.edu](mailto:wenyu@math.tamu.edu)), Texas A&M University, 3368 TAMU, College Station, TX 77843-3368, **Joseph E. Pasciak** ([pasciak@math.tamu.edu](mailto:pasciak@math.tamu.edu)), Texas A&M University, 3368 TAMU, College Station, TX 77843-3386, and **Andrea Bonito** ([bonito@math.tamu.edu](mailto:bonito@math.tamu.edu)), Texas A&M University, 3386 TAMU, College Station, TX 77843-3386. *Numerical Approximation of a Variational Problem on a Bounded Domain involving the Fractional Laplacian*.

We present a non-conforming method for the variational problem on a bounded domain involving the fractional Laplacian. We first derive an alternative integral representation of the bilinear form corresponding to the variational problem involving solutions of elliptic problems defined on  $\mathbb{R}^d$ . The numerical approximation of the action of the corresponding stiffness matrix consists of three stages: (i) Apply a SINC quadrature scheme to approximate the integral representation by a finite sum where each term involves the solution of an elliptic partial differential equation defined on  $\mathbb{R}^d$ ; (ii) Reduce each term to a truncated problem on a bounded domain; (iii) Use the finite element method to approximate the solution of the truncated problem. The consistency error analysis is discussed together with the numerical implementation of the entire algorithm. The results of computations illustrate the error behavior in terms of the mesh size restricted to the bounded domain, the domain truncation parameter and the quadrature spacing parameter. (Received September 12, 2016)

1125-65-943 **Xu Zhang\***, 410 Allen Hal, 175 President's Circle, Mississippi State, MS 39762. *Superconvergence of Immersed Finite Element Methods*.

Immersed finite element method (IFEM) is a class of finite element methods (FEM) that can solve interface problems with unfitted meshes. Superconvergence is a phenomenon that the order of convergence at certain points is higher than the maximum order of convergence of numerical solutions. In this talk, we introduce some superconvergence properties of IFEM for one dimensional interface problems. The key step in our analysis is the construction of generalized orthogonal polynomials with discontinuous weight function. We will show that IFE

functions perfectly fit into the framework of generalized orthogonal polynomials. Finally, we will demonstrate that IFE solutions inherit all desired superconvergence properties from standard FEM. This is a joint work with Waixiang Cao and Zhimin Zhang. (Received September 13, 2016)

1125-65-948 **Melody Alsaker\*** (alsaker@gonzaga.edu). *Electrical Impedance Tomography Imaging of Experimental Data Using a D-bar Method with an Optimized Prior.*

Electrical Impedance Tomography (EIT) is a medical imaging technology wherein the internal electrical properties of a body are reconstructed, via a mathematical inversion process, using only surface measurements. There are many potential applications of EIT, including human thoracic imaging. EIT has advantages in its low cost, portability, and absence of ionizing radiation. However, EIT involves solving a nonlinear and extremely ill-posed inverse problem, so images suffer from poor spatial resolution. Recent advancements in direct, nonlinear D-bar reconstruction methods have produced an algorithm in which enhanced spatial resolution is achieved by embedding prior information, in the form of organ boundaries and conductivity estimates, directly into the algorithm. This method has been previously shown to be effective on reconstructions of simulated thoracic data. We present here a new method for constructing the prior, in which conductivity estimates and regularization parameters are automatically selected so as to optimize a nonlinear Fourier transform crucial to the method, thus minimizing guesswork and increasing efficiency. The method is demonstrated to be effective in enhancing spatial resolution of reconstructions of experimentally collected data from agar thoracic phantoms. (Received September 14, 2016)

1125-65-1161 **Shuonan Wu\*** (sxw58@psu.edu) and **Jinchao Xu**. *Multiphase Allen-Cahn and Cahn-Hilliard models and their discretizations with the effect of pairwise surface tensions.*

In this talk, the mathematical properties and numerical discretizations of multiphase models that simulate the phase separation of an N-component mixture are studied. For the general choice of phase variables, the unisolvent property of the coefficient matrix involved in the N-phase models based on the pairwise surface tensions is established. Moreover, the symmetric positive-definite property of the coefficient matrix on an  $(N-1)$ -dimensional hyperplane can be proved equivalent to the some physical condition for pairwise surface tensions. The N-phase Allen-Cahn and N-phase Cahn-Hilliard equations can then be derived from the free-energy functional. An interesting property is that the resulting dynamics of concentrations are independent of phase variables chosen. Finite element discretizations for N-phase models can be obtained as a natural extension of the existing discretizations for the two-phase model. The energy dispersion of the numerical solutions can be proved and numerically observed under some restrictions pertaining to time step size. Numerical experiments including the evolution of triple junctions and the spinodal decomposition in a quaternary mixture are described in order to investigate the effect of pairwise surface tensions. (Received September 15, 2016)

1125-65-1191 **Yingwei Wang\*** (wywshtj@gmail.com), Department of Mathematics, Purdue University, West Lafayette, IN 47907. *Müntz-Galerkin methods and applications to mixed Dirichlet-Neumann boundary value problems.*

In general, solutions to the Laplacian equation enjoy relatively high smoothness. However, they can exhibit singular behaviors at domain corners or points where boundary conditions change type. In this talk I will focus on the mixed Dirichlet-Neumann boundary conditions for Laplacian equation, and discuss how singularities in this case adversely affect the accuracy and convergence of standard numerical methods. Then, starting from the celebrated Weierstrass theorem on polynomial approximation, I will describe the approximation theory related to the so called Müntz polynomials, which can be viewed as a generalization of usual polynomials. Additionally, I will illustrate the idea of Müntz-Galerkin methods, and show that how they can overcome the difficulties to achieving high order accuracy for the problems with singularities. (Received September 15, 2016)

1125-65-1192 **Jin Wang\***, Department of Mathematics, University of Tennessee at Chattanooga. *Computing fluid-structure interaction.*

The interactions between fluid flows and immersed solid structures are nonlinear multi-physics phenomena that have applications to a wide range of scientific and engineering disciplines. Mathematically, such problems are described by systems of nonlinear PDEs linking the dynamics of the fluid and structure. In this talk, I will give a brief overview of representative numerical techniques currently available for computing fluid-structure interaction problems, with a focus on methods of the immersed boundary type. I will also highlight some recently developed methods which are capable of handling problems involving sophisticated structures described by detailed constitutive laws. (Received September 15, 2016)

1125-65-1275 **Yuan Liu\*** (y1686@msstate.edu), **Yingda Cheng** and **Chi-Wang Shu**. *A simple bound-preserving sweeping technique for conservative numerical approximations.*

In this talk, we introduce a simple bound-preserving sweeping procedure for conservative numerical approximations. Conservative schemes are of importance in many applications, yet for high order methods, the numerical solutions do not necessarily satisfy maximum principle. This paper constructs a simple sweeping algorithm to enforce the bound of the solutions. It has a very general framework acting as a postprocessing step accommodating many point-based or cell average-based discretizations. The method is proven to preserve the bounds of the numerical solution while conserving a prescribed quantity designated as a weighted average of values from all points. The technique is demonstrated to work well with a spectral method, high order finite difference and finite volume methods for scalar conservation laws and incompressible flows. Extensive numerical tests in 1D and 2D are provided to verify the accuracy of the sweeping procedure. (Received September 15, 2016)

1125-65-1279 **Yinlin Dong\*** (yinlin@uta.edu), University of Texas at Arlington, Department of Mathematics, Arlington, TX 76019, and **Zhengjie Wang**. *Construction Methodology of Weighted Upwind Compact Scheme.*

High order compact scheme has good resolution, correspondingly, good dispersion properties. On the other hand, high order compact scheme has been hampered by dissipation's seesaw game between stencils and combined scheme. In this paper, a methodology to compromise dissipation and dispersion in constructing Weighted Upwind Compact Scheme has been derived. Also, several numerical test cases have been shown. (Received September 15, 2016)

1125-65-1307 **Yingjie Liu\***, 686 Cherry Street, Skiles Building, Atlanta, GA 30332. *Central Schemes on Overlapping Cells for Solving MHD Equations on Triangular Meshes.*

This talk is based on a recently submitted paper joint with Zhiliang Xu (U. of Notre Dame) We develop new central DG-type methods on overlapping cells for solving nonlinear MHD equations on triangular meshes. The methods are fully conservative for the magnetic field. New features are introduced to reduce the complexity. These methods also take advantage of the nice feature of central schemes to avoid dealing with Riemann problems at discontinuities of the electromagnetic field. (Received September 16, 2016)

1125-65-1322 **Constantin Bacuta\*** (bacuta@udel.edu), 501 Ewing Hall, Newark, DE, and **Klajdi Qirko** (kqirko@udel.edu), 501 Ewing Hall, Newark, DE. *A saddle point least squares method for mixed variational formulations.*

We present a Saddle Point Least Squares method for discretizing first and second order boundary value problems written as primal mixed variational formulations. For the mixed formulation we assume a stability LBB condition and a data compatibility condition at the continuous level. For the proposed discretization method a discrete inf – sup condition is automatically satisfied by natural choices of test spaces (first) and corresponding trial spaces (second). The discretization and the iterative approach does not require nodal bases for the trial space, and a multilevel preconditioner acting on the discrete test space can be adopted to speed up the approximation process. The stopping criterion is based on matching the order of the the iteration error with the the order of the expected discretization error. Applications of the method include discretizations of second order PDEs with oscillatory or rough coefficients and first order systems of PDEs, such as *div – curl* systems and time-harmonic Maxwell equations. (Received September 16, 2016)

1125-65-1389 **Xiaobing Feng\*** (xfeng@math.utk.edu), Department of Mathematics, The University of Tennessee, Knoxville, TN 37996, **Stefan Schnake** (schnake@math.utk.edu), Department of Mathematics, The University of Tennessee, Knoxville, TN 37996, and **Michael Neilan** (neilan@pitt.edu), Department of Mathematics, University of Pittsburgh, Pittsburgh, PA 15260. *Finite element and discontinuous Galerkin methods for linear elliptic PDEs in non-divergence form.*

In this talk I shall present some newly developed finite element (FE) and discontinuous Galerkin (DG) methods for approximating strong solutions of a class of linear elliptic PDEs in non-divergence form whose leading coefficients are only continuous. Such PDEs are building blocks of fully nonlinear Hamilton-Jacobi-Bellman equations arising from stochastic optimal control and financial mathematics. The proposed numerical methods can use either  $C^0$  or  $L^2$  finite element spaces, they are very simple to implement and can be done using standard FE or DG codes. On the other hand, the convergence analysis of the methods is quite involved and very technical, it requires to establish a FE and a DG discrete Calderon-Zygmund theory, which will be the focus of this talk. Numerical experiments will be presented to demonstrate the effectiveness of the proposed FE and DG methods. This talk is based on two recent joint works with Michael Neilan of University of Pittsburgh and Stefan Schnake of the University of Tennessee. (Received September 16, 2016)

- 1125-65-1403 **Jangwoon Lee\***, 1301 College Avenue, Fredericksburg, VA 22401, and **Lee and Hwang**. *An Optimization Based Domain Decomposition Method for Stochastic Optimal Control Problems.*

A nonoverlapping domain decomposition method for optimal control problems for partial differential equations with random inputs is presented. The domain decomposition is effected through a an auxiliary optimization problem. This results in an multi-objective optimization problem involving the given functional and the auxiliary functional. The existence of an optimal solution to the multi-objective optimization problem is proved as are convergence estimates as the parameters used to regularize the problem (penalty parameters) and to combine the two objective functionals tend to zero. An optimality system for the optimal solution is derived and used to define a gradient method. Convergence results are obtained for the gradient method and the results of some numerical experiments are obtained. (Received September 16, 2016)

- 1125-65-1429 **Azmy S. Ackleh** ([ackleh@louisiana.edu](mailto:ackleh@louisiana.edu)), 104 E. University Circle, Lafayette, LA 70506, **Baoling Ma** ([baoling.ma@millersville.edu](mailto:baoling.ma@millersville.edu)), P.O. Box 1002, millersville, PA 17551, and **Tingting Tang\*** ([tang@louisiana.edu](mailto:tang@louisiana.edu)), 104 E. University Circle, Lafayette, LA 70503. *A high resolution finite difference method for a model of structured Susceptible-Infected populations coupled with the environment.*

We develop a general model describing a structured Susceptible-Infected (SI) population coupled with the environment. This model applies to problems arising in ecology, epidemiology and cell biology. The model consists of a system of quasilinear hyperbolic partial differential equations coupled with a system of non-linear ordinary differential equations that represents the environment. We develop a second order high resolution finite difference scheme to approximate the solution of the model. Convergence of this scheme to a weak solution with bounded total variation is proved. Numerical simulations are provided to demonstrate the high resolution property of the scheme and an application to a multi-host wildlife disease model is explored. (Received September 16, 2016)

- 1125-65-1455 **Wenbin Chen, Daozhi Han and Xiaoming Wang\*** ([wxm@math.fsu.edu](mailto:wxm@math.fsu.edu)), Department of Mathematics, Florida State University, Tallahassee, FL 32306. *Uniquely solvable and energy stable decoupled schemes for Cahn-Hilliard-Stokes-Darcy system.*

We present two novel decoupled numerical schemes for solving the Cahn-Hilliard-Stokes-Darcy (CHSD) model for two-phase flows in karstic geometry. In the first numerical scheme, we explore a fractional step method (operator splitting) to decouple the phase-field (Cahn-Hilliard equation) from the velocity field (Stokes-Darcy uid equations). To further decouple the Stokes-Darcy system, we introduce a first order pressure stabilization term in the Darcy solver in the second numerical scheme so that the Stokes system is decoupled from the Darcy system and hence the CHSD system can be solved in a fully decoupled manner. We show that both decoupled numerical schemes are uniquely solvable and energy stable. Numerical results are presented to demonstrate the accuracy and efficiency of our schemes. (Received September 16, 2016)

- 1125-65-1460 **Sean Ryan Breckling\*** ([sean.breckling@unlv.edu](mailto:sean.breckling@unlv.edu)), 4505 S. Maryland Pkwy., Las Vegas, NV 89154. *A Numerical study of the Navier Stokes- $\alpha$  Deconvolution Model with Pointwise Mass Conservation.*

This paper presents efficient, universally stable finite element schemes for the Navier Stokes- $\alpha$  (NS $\alpha$ ) deconvolution model in both two and three dimensions. The accuracy of these schemes is enhanced by van Cittert approximate deconvolution, as well as through the choice of pointwise divergence-free discrete spaces. Finite element analysis is provided, which includes results for stability and well-posedness, as well as optimal convergence results for both velocity and pressure. Finally, several numerical experiments are presented which demonstrate the performance of NS $\alpha$  against direct numerical simulations of the Navier-Stokes equations, as well as the advantages pointwise divergence-free finite elements. (Received September 16, 2016)

- 1125-65-1479 **Jonathan D Hauenstein\***, 152C Hurley Hall, Notre Dame, IN 46556, and **Owen Coss, Hoon Hong and Daniel K Molzahn**. *Computing the real equilibrium points of the standard Kuramoto model.* Preliminary report.

The standard Kuramoto model (all-to-all uniform coupling) is used to describe synchronization behavior of a large set of oscillators. The equilibrium points of this model can be computed by solving a system of polynomial equations using algebraic geometry. We develop an approach to compute only the real equilibrium points by reducing down to solving a univariate polynomial and compare it with other computational algebraic geometric approaches. Analyzing this univariate approach allows us to prove that, asymptotically, the maximum number of real equilibrium points grows at the same rate as the number of complex equilibrium points. Although the maximum number of real equilibrium points is still an open question for four or more oscillators, we conjecture

an upper bound for any number of oscillators which generalizes the known cases and is obtained with explicitly provided natural frequencies. (Received September 17, 2016)

1125-65-1600 **James G Nagy\*** ([nagy@mathcs.emory.edu](mailto:nagy@mathcs.emory.edu)), Atlanta, GA 30322. *Kronecker product approximations for image reconstruction problems.*

Many image reconstruction problems can be formulated as large scale linear systems. In many applications it is important to compute solutions of these linear very efficiently. This is often done by exploiting structure of the matrices. In this talk we describe how Kronecker arise naturally in many imaging applications, and how their properties can be exploited in both direct factorization methods and in iterative algorithms. (Received September 18, 2016)

1125-65-1604 **Xiaojing Ye\*** ([xye@gsu.edu](mailto:xye@gsu.edu)), Atlanta, GA. *Decentralized consensus optimization on networks with delayed and stochastic gradients.*

In decentralized computing, nodes in a network privately hold parts of the objective function and need to collaboratively solve for the consensual optimal solution of the total objective, while they can only communicate with their immediate neighbors during updates. In real-world networks, it is often difficult and sometimes impossible to synchronize these nodes, and as a result they have to use stale (and stochastic) gradient information which may steer their iterates away from the optimal solution.

In this talk, we first present an overview of decentralized consensus optimization by addressing the main differences from traditional optimization and its new computational challenges, and then focus on a decentralized consensus algorithm by taking the delays of gradients into consideration. We show that, as long as the random delays are bounded in expectation and a proper diminishing step size policy is employed, the iterates generated by this algorithm still converge to a consensual optimal solution. Convergence rates of both objective and consensus are derived. Numerical results on some synthetic optimization problems and on real seismic tomography will also be presented. (Received September 18, 2016)

1125-65-1606 **Carmeliza Navasca\*** ([cnavasca@gmail.com](mailto:cnavasca@gmail.com)), Birmingham, AL. *The regularization of the alternating least squares for low rank tensor approximation.*

In this talk, we will discuss the role of the alternating least squares (ALS) and its variants in low rank tensor decomposition. Moreover, we will describe some recent results on the convergence based on the Kurdyka-Lojasiewicz inequality and the acceleration of the ALS algorithm. Finally, we will show some numerical experiments which verify the local convergence rate and compare various algorithms. (Received September 18, 2016)

1125-65-1610 **Daniel Szyld\*** ([szyld@temple.edu](mailto:szyld@temple.edu)), Philadelphia, PA, **Steven Shank** ([sshank@mit.edu](mailto:sshank@mit.edu)), Cambridge, MA, and **Valeria Simoncini**, Bologna, Italy. *Classical iterative methods for the solution of Generalized Lyapunov Equations.*

There has been a flurry of activity in recent years in the area of solution of matrix equations. In particular, a good understanding has been reached on how to approach the solution of large scale Lyapunov equations. An effective way to solve Lyapunov equations is to use Galerkin projection with appropriate extended or rational Krylov subspaces. Computations are performed usually with low rank storage.

In this talk we focus on generalized Lyapunov equations, which have additional terms in the matrix equation. Several authors have proposed to use approximations to conjugate gradients or to BiCGStab, appropriately preconditioned, where the basis vectors (matrices) and iterates are “truncated” throughout the algorithm to keep all these elements represented by low-rank matrices.

In the present work, we propose a return to classical iterative methods, and consider instead stationary iterations, where the classical theory of splittings applies, and we present a new theorem on the convergence when the linear system at each step is solved inexactly.

Numerical experiments show the competitiveness of the proposed approach. (Received September 18, 2016)

1125-65-1649 **Mikheil Tutberidze\*** ([mikheil.tutberidze@iliauni.edu.ge](mailto:mikheil.tutberidze@iliauni.edu.ge)), 3/5 Kakutsa Cholokashvili ave., 0162 Tbilisi, Rep of Georgia, and **Akaki Gvalia** ([akaki.gvalia@gmail.com](mailto:akaki.gvalia@gmail.com)), 59a David Aghmashenebeli str., 2000 Zestafoni, Rep of Georgia. *On the Numerical Integration of Initial-Boundary Value Problem to One Nonlinear Parabolic Equation.*

In the present work the difference scheme for initial-boundary value problem to following nonlinear parabolic equation

$$\frac{\partial U}{\partial t} = \frac{\partial}{\partial x} \left( k \left( x, t, U, \frac{\partial U}{\partial x} \right) \frac{\partial U}{\partial x} \right) + f(x, t, U)$$

is considered. For the mentioned difference scheme the convergence of its solution to the solution of the source problem is proved when certain conditions hold. For the same difference scheme the comparison theorems and the existence and uniqueness of its solution is proved for the same conditions. The iteration process for finding difference scheme solution is constructed and its convergence is proved. (Received September 18, 2016)

1125-65-1653 **Zhu Wang** ([wangzhu@math.sc.edu](mailto:wangzhu@math.sc.edu)), Department of Mathematics, University of South Carolina, Columbia, SC 29208, and **Lili Ju\*** ([ju@math.sc.edu](mailto:ju@math.sc.edu)), Department of Mathematics, University of South Carolina, Columbia, SC 29208. *Exponential Time Differencing Gauge Method for Incompressible Viscous Flows.*

To design efficient and accurate time integration schemes for numerically simulating incompressible viscous flows, such as those governed by Stokes or Navier-Stokes equations, the discretization and coupling of velocity and pressure need to be treated carefully for stability and consistency. The gauge formulation introduces a gauge variable and an auxiliary field for the fluids equations, and the resulting system contains a coupled momentum equation and a kinematic equation with certain consistent boundary conditions. In this talk, we present an exponential time differencing multistep method for solving the gauge system with high-order temporal accuracy. In particular, the momentum equation is completely decoupled from the kinematic equation in the discrete level at each time step and is solved by explicit exponential time stepping schemes. We rigorously prove that the first order exponential time differencing scheme is unconditionally stable for the Stokes flow. A compact representation of the method for problems on rectangular domains is also proposed, which makes FFT-based fast solvers available for the resulting fully discrete problems. Various numerical experiments are carried out to demonstrate the accuracy and stability of the method. (Received September 18, 2016)

1125-65-1690 **Rundong Du\*** ([rdu@gatech.edu](mailto:rdu@gatech.edu)), Atlanta, GA, and **Haesun Park** ([hpark@cc.gatech.edu](mailto:hpark@cc.gatech.edu)), Atlanta, GA. *Hybrid Clustering of Data with Contents and Links based on Nonnegative Matrix Factorization.*

Nonnegative matrix factorization (NMF) has been successfully applied to text and graph clustering. In reality, some data sets have both text contents and graph structure, such as papers with citations and online articles with hyperlinks. By jointly optimizing the objectives of NMF for text and graph clustering, we achieve better clustering results than clustering merely based on the text or on the graph. We propose an effective algorithm for the joint objective based on the block coordinate descend (BCD) method. Substantial experimental results show improvements of clustering quality and the potential application to citation recommendations of papers and patents. (Received September 18, 2016)

1125-65-1693 **Jennifer Erway\*** ([erwayjb@wfu.edu](mailto:erwayjb@wfu.edu)), Winston-Salem, NC. *Compact Representations of Quasi-Newton Matrices.*

The compact representation of a limited-memory quasi-Newton matrix has applications in both numerical linear algebra and optimization. In this talk, we review this representation of BFGS matrices, and then present the generalization of this—allowing one to compute the compact representation for any member of the Broyden convex class. Additionally, we discuss several specific applications in linear algebra as well as an application in optimization. (Received September 18, 2016)

1125-65-1845 **Dan Bates\*** ([bates@math.colostate.edu](mailto:bates@math.colostate.edu)). *Tuning of tolerances in polynomial homotopy continuation.* Preliminary report.

Homotopy continuation refers to the use of a homotopy function to compute the solutions of a system of equations, typically with numerical methods. In this talk, we consider the particular case of polynomial homotopy continuation for solving polynomial systems of equations, solved using predictor-corrector methods. Such methods include a wide range of choices, both discrete (e.g., the order of the predictor method) and continuous (e.g., various tolerances). There is an art to choosing and adapting these settings to solve particular problems, sometimes making these methods difficult for users unaccustomed to making such choices. In this talk, we report on recent progress in automating some of these choices, in the context of the redevelopment of the Bertini software package. (Received September 19, 2016)

1125-65-1879 **Susanne C. Brenner**, Department of Mathematics, Center for Computation & Technology, Louisiana State University, Baton Rouge, LA 70803, **Christopher B. Davis\***, Department of Mathematics, Tennessee Technological University, Cookeville, TN 38505, and **Li-yeng Sung**, Department of Mathematics, Center for Computation & Technology, Louisiana State University, Baton Rouge, LA 70803. *A domain decomposition method for a partition of unity method applied to fourth order problems.* Preliminary report.

In this talk, we investigate the use of a type of additive Schwarz domain decomposition method for a partition of unity method applied to fourth order problems. The numerical algorithm will be presented and analyzed and numerical examples will be given to demonstrate the effectiveness of the method. (Received September 19, 2016)

1125-65-1909 **Sara Pollock\*** ([sara.pollock@wright.edu](mailto:sara.pollock@wright.edu)), Dept. of Mathematics and Statistics, Wright State University - 120MM, 3640 Colonel Glenn Highway, Dayton, OH 45435-0001. *Pseudo-time adaptive regularization for quasilinear elliptic PDE.*

The approximation of solutions to nonlinear elliptic problems, particularly those of nonmonotone type, presents unique challenges not encountered in the related problems of linear and nonlinear monotone type. In particular, internal layers in the diffusion coefficient may need to be uncovered during the solution process, which can be highly unstable. Without introduced stabilization, direct linearization methods may produce sequences of divergent iterates which do not approximate the PDE solution. I will discuss adaptive regularization strategies to solve the discrete nonlinear equations induced by finite element discretizations of nonmonotone quasilinear PDE, and discuss the ramifications of adaptive regularization control for running efficient simulations starting from a coarse mesh. (Received September 19, 2016)

1125-65-1922 **Bo Dong\***, 285 Old Westport Road, North Dartmouth, MA 02747. *A new optimally convergent HDG method for Korteweg-de Vries type equations.* Preliminary report.

We develop and analyze a new hybridizable discontinuous Galerkin (HDG) method for solving third-order Korteweg-de Vries type equations. The method is defined by a discrete version of the characterizations of the exact solution in terms of local problems and transmission conditions, and the exact solution  $u$  and its derivatives  $q := u_x$  and  $p := u_{xx}$  are approximated by piecewise-polynomials of degree  $k \geq 0$ . We carry out stability analysis in the general case and error analysis in the linear case, and we find conditions on the stabilization parameters in the numerical traces under which the method is stable and has optimal convergence rates for  $u, q$  and  $p$  in  $L^2$ -norm. Numerical results in both linear and nonlinear cases are displayed, which validate the optimal convergence rates in our error analysis. (Received September 19, 2016)

1125-65-1947 **Joseph A Eichholz\*** ([eichholz@rose-hulman.edu](mailto:eichholz@rose-hulman.edu)), 5500 Wabash Ave, Terre Haute, IN 47803. *A new multi-grid method for the obstacle problem.* Preliminary report.

The obstacle problem is a standard example of a variational inequality of the first kind. Variational inequalities naturally arise in a wide range of applications, for example, elastoplasticity, contact mechanics, heat control problems, and options pricing problems in finance. Due to the inequality in the formulation of the obstacle problem, the standard quadratic finite element method only achieves  $\mathcal{O}(N^{-3/4+\epsilon})$  convergence, where  $N$  is the number of degrees of freedom and  $\epsilon > 0$  arbitrary. We introduce a new multi-grid algorithm for solving the obstacle problem in which we solve the obstacle problem, capture the free boundary, refine the mesh, and solve the problem on a new mesh. We extend previous work to demonstrate that the method has nearly optimal convergence order  $\mathcal{O}(N^{-1+\epsilon-p})$ , where  $p$  is the degree of the basis. Numerical evidence is provided indicating superior performance to standard finite element methods. (Received September 19, 2016)

1125-65-1985 **Vladimir Druskin\*** ([druskin1@slb.com](mailto:druskin1@slb.com)), **Alexander Mamonov**, **Andrew Thaler** and **Mikhail Zaslavsky**. *Solution of inverse hyperbolic problems via data-driven discrete-time reduced order models.*

We compute reduced order model (ROM) from discrete time-domain data for hyperbolic wave equations and obtain ROM's block-tridiagonal realization via data-driven QR transform. Equivalent reformulation of this realization as a Galerkin projection of the underlying PDE allows us direct imaging of a PDE coefficient, suppressing nonlinear artifacts such as multiple reflections. Justification of our approach is based on an intriguing connection of the ROMs with the Krein-Marchenko-Gelfand-Levititan method. We show applications to seismic exploration and medical ultrasound imaging. (Received September 19, 2016)

1125-65-1987 **Nourridine Siewe\***, nourridine@nimbios.org, and **Abdul-Aziz Yakubu, Abhay R Satoskar and Avner Friedman**. *Granuloma Formation in Leishmaniasis: A Mathematical Model*.

Leishmaniasis is a disease caused by the Leishmania parasites. The two common forms of leishmaniasis are cutaneous leishmaniasis (CL) and visceral leishmaniasis (VL). VL is the more severe of the two and, if untreated, may become fatal. The hallmark of VL is the formation of granuloma in the liver or the spleen. In this paper, we develop a mathematical model of the evolution of granuloma in the liver. The model is represented by a system of partial differential equations and it includes migration of cells from the adaptive immune system into the granuloma; the rate of the influx is determined by the strength of the immune response of the infected individual. It is shown that parasite load decreases as the strength of the immune system increases. Furthermore, the efficacy of a commonly used drug, which increases T cells proliferation, increases in a person with stronger immune response. The model also provides an explanation why, in contrast to humans, mice recover naturally from VL in the liver. (Received September 19, 2016)

1125-65-2006 **Vladimir Druskin\*** (druskin1@slb.com), **Alexander Mamonov** and **Mikhail Zaslavsky**. *Multi-scale reduced order models with S-fraction sparsification for large wave propagation problems*.

We solve large wave propagation problems by splitting computational domains into coarse cells and computing reduced order models (ROMs) of the cell Dirichlet-to-Neumann maps via rational Krylov subspace projection. The novelty of our formulation is the matrix Stieltjes continued fraction (S-fraction) representation of the obtained ROM allowing its block-tridiagonal realization and thus computationally efficient special discretization of the total problem as a sparse network (of graph-Laplacian type) with matrix-valued coefficients, in particular suitable for modern high performance computing platforms. We show numerical result for a 3D multi-phase anisotropic elastic problem. (Received September 19, 2016)

1125-65-2023 **Dmitriy Zhigunov\*** (dzhigunov@email.wm.edu), College of William & Mary, Jones Hall, Room 100, 200 Ukrop Way, Williamsburg, VA 23185, and **Yu-Min Chung** (ychung@wm.edu), College of William & Mary, Jones Hall, Room 100, 200 Ukrop Way, Williamsburg, VA 23185. *(Un)Stable Manifold Computation via Forward-Backward Iteration*.

The stable and unstable manifolds of an autonomous differential equation can be shown to satisfy a boundary value problem on an infinite interval. To approximate the manifold, we truncate the interval to a finite one, and discretize the differential equation through finite difference methods. This leads to a “forward-backward” iterated scheme. The subsequent errors due to truncation and discretization are discussed through numerical experiments and analytically. (Received September 20, 2016)

1125-65-2061 **Laurent Demanet\***, 77 Massachusetts Ave, 2-247, Cambridge, MA 02139. *Phase tracking for model reduction of high-frequency waves*.

Interpretation of seismic data in terms of coherent atomic events can be formulated as a hard, nonconvex optimization problem. The same is true of the interpretation of the solution of a Helmholtz equation in terms of elementary phases from geometrical optics. I will present a method that empirically finds the global minimum of this functional in the case of simple synthetic seismic records, even when events cross. The idea is to slowly grow a trust region for phases and amplitudes in a way that reminds of continuation, or tracking. I will also discuss applications to low-frequency extrapolation and full waveform inversion. Joint work with Yunyue Elita Li. (Received September 19, 2016)

1125-65-2123 **Wujun Zhang\*** (wujun@math.rutgers.edu), Hill Center, 110 Frelinghuysen Rd., Piscataway, NJ 08854, and **Shawn Walker**. *Finite element methods for nematic liquid crystals with variable degree of orientation*.

We consider the simplest one-constant model, put forward by J. Ericksen, for nematic liquid crystals with variable degree of orientation. The equilibrium state is described by a director field  $\mathbf{n}$  and its degree of orientation  $s$ , where the pair  $(s, \mathbf{n})$  minimizes a sum of Frank-like energies and a double well potential. In particular, the Euler-Lagrange equations for the minimizer contain a degenerate elliptic equation for  $\mathbf{n}$ , which allows for line and plane defects to have finite energy.

We present a structure preserving discretization of the liquid crystal energy with piecewise linear finite elements that can handle the degenerate elliptic part without regularization, and show that it is consistent and stable. We prove  $\Gamma$ -convergence of discrete global minimizers to continuous ones as the mesh size goes to zero. We develop a quasi-gradient flow scheme for computing discrete equilibrium solutions and prove it has a strictly monotone energy decreasing property. We present simulations in two and three dimensions to illustrate the

method's ability to handle non-trivial defects. Our results include electric and colloidal effects. This work is joint with R.H. Nochetto and S. Walker. (Received September 19, 2016)

1125-65-2183 **Adel Faridani\*** ([faridani@oregonstate.edu](mailto:faridani@oregonstate.edu)). *Numerical implementation of  $\pi$ -line reconstruction formulas in tomography.*

$\pi$ -line inversion formulas give rise to a versatile class of reconstruction algorithms in computed tomography that can be adapted to a variety of data acquisition geometries. These formulas share a certain view dependent derivative of the data function which has to be implemented accurately in each case. This is especially important when the source curve is not circular and standard discretization schemes may perform poorly. This talk will present and analyze a general method to derive accurate discretizations for the view dependent derivative. (Received September 19, 2016)

1125-65-2229 **Bryce D Wilkins\*** ([bryce.wilkins@usma.edu](mailto:bryce.wilkins@usma.edu)), United States Military Academy, Department of Mathematical Sciences, PO Box 4440, West Point, NY 10996, and **Randy Boucher** ([randy.boucher@usma.edu](mailto:randy.boucher@usma.edu)), United States Military Academy, Department of Mathematical Sciences, PO Box 4440, West Point, NY 10996. *An Unsteady Two-Dimensional Complex Variable Boundary Element Method for Modeling Heat Transport Problems.*

Solving potential problems, such as those that occur in the analysis of steady-state heat transfer, electrostatics, ideal fluid flow, and groundwater flow, is important in several fields of engineering, science, and applied mathematics. One technique for numerically approximating the solution to these potential problems is the Complex Variable Boundary Element Method (CVBEM). Typically, applications of the CVBEM have been limited to steady-state solutions of the Laplace equation. In this work, by adding a time component, the CVBEM is extended to modeling applications of the two-dimensional transient Laplace equation.

The problem considered is a rectangular two-dimensional domain. The underpinning of the modeling approach is to resolve the global problem into two subproblems; a transient and a steady-state subproblem. The transient component of the problem is modeled by a generalized Fourier series expansion. The steady-state component of the problem is solved by application of the CVBEM. The global solution is the sum of the solutions from the two modeling components.

The mechanics and solution success of the new technique of coupling a generalized Fourier series and a CVBEM approximation in solving the two-dimensional transient Laplace equation are discussed. (Received September 20, 2016)

1125-65-2263 **Martin Gutting\*** ([gutting@mathematik.uni-siegen.de](mailto:gutting@mathematik.uni-siegen.de)), University of Siegen, Geomathematics Group, ENC, Walter-Flex-Strasse 3, 57072 Siegen, Germany. *Parameter Choices for Fast Harmonic Spline Approximation.*

The approximation by harmonic trial functions allows the construction of the solution of boundary value problems in geoscience, e.g., in terms of harmonic splines. Due to their localizing properties regional modeling or the improvement of a global model in a part of the Earth's surface is possible with harmonic splines. Fast multipole methods have been developed for certain cases of the occurring kernels to obtain a fast matrix-vector multiplication. This reduces the numerical effort of the matrix-vector multiplication from quadratic to linear in reference to the number of points for a prescribed accuracy of the kernel approximation while inducing a small error due to the approximation of the kernel. The application of the fast multipole method to spline approximation which also allows the treatment of noisy data requires the choice of a smoothing parameter. We investigate different methods to (ideally automatically) choose this parameter with and without prior knowledge of the noise level. Thereby, the performance of these methods is considered for different types of noise in a large simulation study with regard to applications in gravitational field modeling and boundary value problems where the boundary is the known surface of the Earth itself. (Received September 20, 2016)

1125-65-2390 **Umberto Villa\*** ([uvilla@ices.utexas.edu](mailto:uvilla@ices.utexas.edu)), The University of Texas at Austin, Inst. Computational Engineering and Sciences, 201 E. 24th Street, Stop C0200, Austin, TX 78712, and **Peng Chen** and **Omar Ghattas**. *Bayesian Inverse Problems Governed by Stochastic PDE Models.*

Inverse problems arise when we seek to determine unknown parameters from observational data and mathematical models that relate the parameters to the data. These problems arise across all areas of science, engineering, medicine, and beyond.

Inverse problems are often ill-posed; that is, multiple values of the parameters may be consistent with the data to within the noise. Bayesian inference provides a systematic framework for quantifying the resulting uncertainty in the parameters. However, this formulation presents significant challenges. First, the parameter to be inferred

is often a spatially correlated field, resulting in a high dimensional parameter space after discretization. Second, the forward model is often computationally expensive to solve, particularly when it takes the form of PDEs. Finally, for many applications of practical interests, the model is often inadequate, that is it only partially captures the complex dynamics of the physical system leading to a discrepancy between its outputs and reality.

In this talk, we focus on how to model inadequacy by introducing a stochastic term in the governing equations, and how to efficiently solve the resulting Bayesian inverse problem. An application to Bayesian calibration of turbulence closure models is presented. (Received September 20, 2016)

1125-65-2392 **Xiaobing Feng** and **Stefan Schnake\*** ([schnake@math.utk.edu](mailto:schnake@math.utk.edu)). *An enhanced finite element method for a class of variational problems exhibiting the Lavrentiev gap phenomenon.*

This talk concerns itself with numerical approximations to the minimizers of functionals that exhibit the Lavrentiev gap phenomenon - a defect from the singularities of said minimizers. This phenomenon arises in physical problems, especially ones from material science such as nonlinear elasticity and microstructure theory. Standard finite element discretizations applied to functionals with this phenomenon fail to approximate both the true minimizer and the minimum value of the integral. We introduce a simple and novel cutoff technique that allows the finite element method to converge to the true minimizer. Several numerical tests will be shown towards the end of the talk. (Received September 20, 2016)

1125-65-2397 **Joshua Lee Padgett\*** ([josh\\_padgett@baylor.edu](mailto:josh_padgett@baylor.edu)), 1721 S 9th St, Apt #206, Waco, TX 76706. *An approach to the numerical solution of multidimensional stochastic Kawarada equations via adaptive operator splitting.*

This talk concerns the numerical solution of multidimensional nonlinear Kawarada equations. The stochastically influenced degenerate reaction-diffusion equations exhibit strong singularities and play an important role in numerous industrial applications. Moving mesh strategies and operator splitting are utilized throughout the approach to yield favorable adaptive grids in both space and time. Highly efficient and effective nonuniform difference schemes are developed. It is shown that the numerical solution acquired not only approximates the theoretical solution satisfactorily, but also preserves the required positivity, monotonicity and stability of the solution when proper constraints are satisfied. The latter is particularly crucial to quenching-combustion simulations. Numerical experiments are given to illustrate and demonstrate our conclusions. (Received September 20, 2016)

1125-65-2488 **Aritra Dutta\*** ([d.aritra2010@knights.ucf.edu](mailto:d.aritra2010@knights.ucf.edu)), University of Central Florida, Mathematics Department, 4000 Central Florida Blvd, Orlando, FL 32816, and **Xin Li**. *A Problem of Weighted Low-Rank Approximation of Matrices and its Applications in Machine Learning.*

We study the numerical solutions to a special weighted low-rank approximation problem. We propose two different analogs of the problem and establish their numerical solutions with convergence analysis. We also demonstrate the performance of our algorithms on real data and report the improvements in performance when weight is learned from the data over other state of the art methods. (Received September 20, 2016)

1125-65-2491 **Sarah Ryan Black\*** ([sblack@icasa.nmt.edu](mailto:sblack@icasa.nmt.edu)) and **Emily Kim Miller** ([emiller@icasa.nmt.edu](mailto:emiller@icasa.nmt.edu)). *Automated Computation for Symbolic and Graphical Representations of Separatrices for N-Dimensional Dynamical Systems.*

In dynamical systems, the separatrix is formally defined as the boundary of two or more regions exhibiting different behaviors. Finding an exact analytic expression for the separatrix of a system is occasionally very difficult and often impossible; numeric approximations are usually the only option to express these important boundaries. Separatrices allow us to gain insight into dynamical systems when other methods are insufficient or impractical. In the literature, there are several well-known approaches to finding numeric approximations of separatrices; these approaches focus on systems with basins of attraction and saddle nodes. In this paper, the authors propose an algorithm for finding separatrices that is not dependent on the dynamical system containing basins of attraction or saddle nodes. In fact, the algorithm will find a separatrix between any two regions that exhibit divergent behavior. Examples presented will focus on using the algorithm to construct numeric approximations of separatrices in 2-dimensional and 3-dimensional nonlinear autonomous dynamical systems. However, the algorithm itself is dimensionless and can be extended to higher dimensional systems. (Received September 20, 2016)

1125-65-2510 **A. Bass Bagayogo\*** ([abagayogo@ustboniface.ca](mailto:abagayogo@ustboniface.ca)), 200 avenue de la cathedrale, Winnipeg, Manitoba R2H0H7, Canada. *Numerical Simulation of Hydraulic Fracking and Induced Earthquakes.*

The hydraulic fracking process uses high-pressure injections of fluid to break apart rock and release trapped oil and natural gas to enhance the production of hydrocarbons. Both fracking and wastewater injections can increase the fluid pressure in the natural pores and fractures in rock or change the state of stress on existing faults which produce sudden release of energy causes the seismic waves that triggered the ground to shake. Hydraulic fracturing involves not only underground injections composed mostly of water, but also a mixture of chemical additives. These chemicals range from toxic biocides and surfactants, to corrosion inhibitors and many are also used by others industries. In this talk I will provide examples of natural hydraulic fracking and situations in which fracking are used in industrial problems, I will describe the governing equations in 1D-2D as well as 3D models of hydraulic fracking, which involve a coupled system nonlinear partial-integro-differential equations as well as a free boundary condition. I will discuss the challenges for efficient and robust numerical modeling of the 2D-3D hydraulic fracking problem. The computational efficiency of these techniques is demonstrated with Symbolic-Numeric results. (Received September 20, 2016)

1125-65-2636 **Weiwei Zhang\*** ([weiweizhang@kings.edu](mailto:weiweizhang@kings.edu)), King's College, Wilkes-Barre, PA 18711. *Mathematical modeling of fungal growth.* Preliminary report.

Mathematical modeling of the fungal growth has been studied since 1980s. The research would be applied in toxic control and environmental control, biological fuel, food industry and others. Some successful models to predict the fungal growth needed to solve the mixed hyperbolic-parabolic PDE systems. Different numerical methods were applied to understand the properties, such as the external and internal substrate concentrations, the biomass density, the hyphal tip density and others. Modifications to the existing numerical schemes and hence the numerical results will be presented in this talk. There are also discussions of the construction of the PDE system. (Received September 20, 2016)

1125-65-2650 **Christopher Moro\*** ([chrismoro13@gmail.com](mailto:chrismoro13@gmail.com)), **Padmanabhan Seshaiyer** and **Carmen Caiseda.** *Predicting methane concentration in the atmosphere through mathematical modeling, computation and simulation.*

The mathematics of sustainability is a current research area that generates global interest. Climate change and greenhouse gases have been at the center of scientific debate, showing a common interest of protecting our environment. Methane is identified as an important greenhouse gas with a warming radiative forcing that is 25 times greater than carbon dioxide. It is also an important gas present in the emissions from the US oil and natural gas industries. Therefore methane concentration levels are recently being discussed between these industries and the EPA to regulate methane emissions in the atmosphere. In this talk we present the mathematical modeling of methane concentration in air. We use a benchmark problem to study the convergence rate of the numerical schemes. The numerical solution of the advection-diffusion equation by the use of the Finite Difference theta-method is obtained and implemented for both: constant and linear diffusion coefficient. Finally a parameter estimation code is implemented in order to simulate best the experimental data to be obtained. (Received September 20, 2016)

1125-65-2664 **Emily E Schaal\*** ([eeschaal@email.wm.edu](mailto:eeschaal@email.wm.edu)) and **Yu-Min Chung** ([ychung@wm.edu](mailto:ychung@wm.edu)). *Center Manifolds via Lyapunov-Perron.*

The Lyapunov-Perron (L-P) operator is a theoretical method used to show the existence of the invariant manifold. In 2005, M.S. Jolly and R. Rosa presented an algorithm for solving systems with center manifolds based on discretizing the L-P operator. However, this discretization can be difficult and expensive to implement. First, we provide detailed proofs for the construction of the center manifold by the L-P operator under the Jolly-Rosa framework. Second, we present an algorithm based on a boundary value formulation of the operator. Importantly, the algorithm is simple and can be adopted using any generic scheme, such as the Runge-Kutta methods. We implement the algorithm, test it with several examples, and discuss applications. (Received September 20, 2016)

1125-65-2713 **Fred J Hickernell\*** ([hickernell@iit.edu](mailto:hickernell@iit.edu)), Department of Applied Mathematics, Illinois Institute of Technology, RE 208, 10 W. 32nd St., Chicago, IL 60616. *The Trio Identity for Cubature Error.*

Xiaoli Meng introduced the trio identity for expectations of random variables during an invited talk of the 2016 Joint Mathematics Meetings. Here we extend his ideas to multidimensional integration problems, which may also be written as expectations of random variables:  $\mu = \mathbb{E}[f(X)] = \int f(x) \nu(dx)$ . Here  $X$  is a random variable

following probability measure  $\nu$ . Such integrals are approximated by weighted sums,  $\hat{\mu} = \sum_{i=1}^n w_i f(X_i)$ . We show how the error,  $\mu - \hat{\mu}$ , can be represented as the product of the confounding, the discrepancy, and the variation. This trio identity holds for deterministic, randomized, and Bayesian settings. (Received September 20, 2016)

1125-65-2751 **Robert Lipton, Paul Sinz\*** (psinz1@lsu.edu) and **Michael Stuebner**. *Maximum Energy Concentration Inside Composite Structures*.

A systematic method for identifying the worst case load amongst all boundary loads of a fixed energy is introduced. Here the worst case load is defined as the load which delivers the largest fraction of input energy into a prescribed subdomain of interest. This leads to an eigenvalue problem, for which the largest eigenvalue is the maximum fraction of energy which can concentrate in the subdomain. The associated eigenfunctions are the worst case solutions. The properties of these eigenfunctions motivates a particular Generalized Finite Element Method (GFEM) called the Multiscale Spectral GFEM (MS-GFEM), developed by Babuška and Lipton (2011). (Received September 20, 2016)

1125-65-2796 **Tulsi Upadhyay\*** (tulsi.upadhyay@usm.edu), Tulsi Upadhyay, 202 S 30th Avenue, Apt 204, Hattiesburg, MS 39401. *Invariant Densities of Frobenius-Perron Operator Related to Random Maps*.

Let  $\tau = \{\tau_1, \tau_2, \dots, \tau_r; p_1, p_2, \dots, p_r\}$  be a random map. The nonsingular transformations,  $\tau_1, \tau_2, \dots, \tau_r$ , are defined from  $[0, 1]$  to itself, and  $p_1, p_2, \dots, p_r$  are probabilities such that at each iteration the possibility of selecting a map  $\tau_i$  is  $p_i$ ,  $1 \leq i \leq r$ . We use piecewise linear polynomials in maximum entropy principle to approximate invariant densities of the Frobenius-Perron operator related to some random maps. The  $L^1$  errors between the exact and approximated invariant densities are also depicted.

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**Keywords:** Frobenius-Perron operator; invariant density;  $L^1$  error; nonsingular transformation; random map. (Received September 20, 2016)

1125-65-2935 **Fei Xue\*** (fxue@clemson.edu), Department of Mathematical Sciences, Martin Hall O-203, Clemson University, Clemson, SC 29634, and **Minghao Wu** (mwrostan@syr.edu), Department of Mathematics, 215 Carnegie Building, Syracuse University, Syracuse, NY 13244. *Fast and robust computation of right-most eigenvalues of large matrices*. Preliminary report.

We propose a matrix exponential transformation for computing the right-most eigenvalues of a large matrix  $A$  or a matrix pencil  $(A, B)$ . Robust numerical algorithms for computing these eigenvalues are needed for linear stability analysis of dynamic systems and hence have important applications. Since exponential maps the right-most eigenvalues of  $A$  to dominant eigenvalues of  $\exp(A)$  that are easily captured by eigenvalue algorithms, the most important technique for the success of this approach is fast and robust computation of the exponential matrix vector product  $\exp(hA)v$ . We discuss polynomial and rational approximations to this matrix vector product using Leja interpolation points. Numerical results demonstrate the effectiveness of the proposed algorithms. (Received September 20, 2016)

1125-65-3155 **Roger Ghanem\*** (ghanem@usc.edu), University of Southern California. *Recent advances in probabilistic modeling for multiscale and multiphysics problems*.

Complex systems whose driving mechanisms cannot be easily learned from observations of their behavior are ubiquitous. Two important issues must generally be addressed when using probabilistic methods to tackle these problems. First, the specific probabilistic models adopted will themselves bias inferences in often unclear fashion. This refers both to prior models and to knowledge updating mechanisms. Second, a probabilistic approach, by necessity, embeds a given problem in a statistical ensemble of problems, typically increasing the analytical or computational requisite for solving the problem. This talk will describe recent advances in adapting the polynomial chaos methodologies for handling coupled interacting systems, with probabilistic constraints and parameters. This approach addresses both challenges alluded to above while, at the same, presenting a consistent mathematical formalism for analysis. The approach is based on polynomial chaos decompositions that provide a characterization of random variables (or random processes) as functional forms in terms of other, underlying, fluctuations that can be thought of as causing the randomness of interest. This hierarchical decomposition is consistent with multi-physics, multi-scale and multi-disciplinary paradigms. (Received September 23, 2016)

## 68 ► Computer science

1125-68-77 **Henry Segerman\*** ([segerman@math.okstate.edu](mailto:segerman@math.okstate.edu)). *Navigating the three-sphere via quaternions.*

Hypernom is a virtual reality, phone or tablet game, in which the player navigates the three-sphere by changing the orientation of the device they are playing on. The orientation of the device is interpreted as a unit quaternion, which acts on the three-sphere by quaternion multiplication. This navigation scheme is surprisingly comprehensible for players. I will discuss the qualitative behaviour of this control scheme, and the mathematical explanation for this behaviour. (Received July 11, 2016)

1125-68-396 **Diego Cifuentes\*** ([diegcif@mit.edu](mailto:diegcif@mit.edu)), 77 Massachussets Avenue, Office 32-D760, Cambridge, MA 02139, and **Pablo A Parrilo**. *Chordal networks of polynomial ideals.*

We introduce a novel representation of structured polynomial ideals, which we refer to as *chordal networks*. The sparsity structure of a polynomial system is often described by a graph that captures the interactions among the variables. Chordal networks provide a computationally convenient decomposition of a polynomial ideal into simpler (triangular) polynomial sets, while preserving its underlying graphical structure. We show that many interesting families of polynomial ideals admit compact chordal network representations (of size linear in the number of variables), even though the number of components could be exponentially large. Chordal networks can be computed for arbitrary polynomial systems using a refinement of the chordal elimination algorithm from (Cifuentes2014). Furthermore, they can be effectively used to obtain several properties of the variety, such as its dimension, cardinality and equidimensional components, as well as an efficient probabilistic test for radical ideal membership. We apply our methods to examples from algebraic statistics and vector addition systems; for these instances, algorithms based on chordal networks outperform existing techniques by orders of magnitude. (Received August 31, 2016)

1125-68-692 **Gauri Joshi\*** ([gjoshi@alum.mit.edu](mailto:gjoshi@alum.mit.edu)), 1101 Kitchawan Road, Yorktown Heights, NY 10598. *Using Coding to Reduce Delay in Content Access.*

Codes are primarily designed to provide reliability against channel noise, or disk failures and errors. In this talk I will discuss how redundancy in coded content can also be exploited to speed-up content access. One example is content download from an  $(n,k)$  coded distributed storage system. Requesting all  $n$  coded chunks and waiting for any  $k$  to be downloaded can significantly reduce access latency. Another example is streaming communication, which places delay constraints on the packets of the content by requiring them to be delivered fast and in order. Coding can help ensure smooth playback of the stream with minimum interruptions. I will conclude by presenting some open questions about code design for such delay-sensitive applications. (Received September 09, 2016)

1125-68-975 **Venkatesan Guruswami** and **Mary Wootters\*** ([marykw@stanford.edu](mailto:marykw@stanford.edu)). *Repairing Reed-Solomon Codes.*

Reed-Solomon (RS) codes are often used in distributed storage. However, recently it's been observed that the traditional recovery algorithms for RS codes are substantially sub-optimal in this setting, and the quickly-developing field of regenerating codes has provided some much better solutions.

In this work, we show that, in fact, RS codes are much better for distributed storage than you might think! Our main result is that, in some parameter regimes, RS codes themselves are optimal regenerating codes, among MDS codes with linear repair schemes. Moreover, we give a characterization of MDS codes with good linear repair schemes which holds in any parameter regime, and which can be used to give non-trivial repair schemes for RS codes in other settings. (Received September 13, 2016)

1125-68-997 **Bangyan Wen\*** ([ztong@otterbein.edu](mailto:ztong@otterbein.edu)), 110 Marlene Dr., Westerville, OH 43081, and **Yi Lin**. *The answer to the P/NP problem is  $P \neq NP$  — proof via logical analysis.*

Assertion 1: The answer to the P/NP problem is  $P \neq NP$ . The problems having no polynomial time DTM solutions are denoted -P. The first proof: Defining NP is to research -P, and NTM is not used as only DTM, NTM must be used for some problems in -P, thus ,  $P \neq NP$ .

The second proof: Assume  $NP = P$ , then we have that the NTM NP algorithms are DTM P algorithms. However, the NP algorithms require parallel multi-valued and random guess which can finish in a glance, which does not exist in real life. Thus to avoid the confusions, we have  $P \neq NP$ .

Assertion 2: According to the current understanding with self-contradictory, neither  $NP = P$  nor  $P \neq NP$  is provable. 1) To show  $NP = P$ , we need to show for all problems  $X$ ,  $(X \in NP) \wedge (X \in P)$  . It is known now that, thus the existence of P requires the evidence of the real existence of P1 ( $P=P1$ ). However, all NP problems

depending on NP algorithms do not have evidence of P1, which means the proof cannot be finished. 2) To show  $NP \neq P$ , we need to show there exists problem  $x: (x \in NP) \wedge \neg(x \in P)$ . The evidence of the real nonexistence of P1 will be rejected, because it is known now that the nonexistence of P (denote  $\neg P$ ) is not equivalent to the nonexistence of  $\neg P : (\neg P \neq P1)$ , thus the proof cannot be done. (Received September 13, 2016)

1125-68-1087 **Erdal Imamoglu\*** (eimamogl@math.fsu.edu) and **Mark van Hoeij** (hoeij@math.fsu.edu). *Computing Hypergeometric Solutions of Second Order Linear Differential Equations using Quotients of Formal Solutions and Integral Bases.*

We present an algorithm for computing hypergeometric solutions of a second order linear differential operator  $L$  with rational function coefficients. Our algorithm searches for solutions of the form

$$\exp\left(\int r dx\right) \cdot (r_0 \cdot {}_2F_1(a_1, a_2; b_1; f) + r_1 \cdot {}_2F_1'(a_1, a_2; b_1; f))$$

where  $r, r_1, r_2, f \in \overline{\mathbb{Q}(x)}$ , and  $a_1, a_2, b_1 \in \mathbb{Q}$ . Our algorithm has two components. The first tries to simplify  $L$  using normalized integral bases. The goal is to reduce  $r_1$  to 0. The second component tries to find  $a_1, a_2, b_1, f$  using quotients of formal solutions, modular reduction, Hensel lifting, and rational reconstruction. (Received September 15, 2016)

1125-68-1242 **Noga Ron-Zewi\*** (nogazewi@gmail.com). *Locally-testable and locally-decodable codes.*

Locally-testable and locally-decodable codes are special families of error-correcting codes that admit highly efficient algorithms that detect and correct errors in transmission in sublinear time, probing only a small number of entries of the corrupted codeword. These codes have arisen from a variety of motivations within the theory of computation, and the study of these codes had a significant effect on coding theory as well.

In this talk I will survey the motivations for the study of locally-testable and locally-decodable codes within the theory of computation and some of the state-of-the-art results. I will also highlight some of the most interesting challenges that remain. (Received September 15, 2016)

1125-68-1556 **Kim Laine\*** (kim.laine@microsoft.com). *Machine Learning on Encrypted Data.*

Modern machine learning techniques can be used to build incredibly accurate predictive models for various tasks in health, finance, and other industries. Conversely, given such a model, anyone can use it to extract valuable information from their private data.

In this talk we focus on new results in using machine learning models (inference) on private encrypted data. Time permitting, we will also comment on the significantly more difficult problem of training models in a privacy-preserving way. (Received September 18, 2016)

1125-68-2096 **Charles C Earl\*** (charles.earl@gmail.com). *AI and Communities of Color: What Questions Should We Be Asking?* Preliminary report.

Recent advances in deep learning have enabled impressive advances in computer vision and natural language understanding. This is evidenced by the conversational bots (e.g. Siri) that we use daily and the self-driving cars that are slowly making it onto our roads.

However we have increasingly seen these technologies applied in ways that are problematic for communities of color – predictive policing as one example. In this talk I will attempt to unpack implications of state of the art AI and then look at analytic avenues for holding these systems accountable. (Received September 19, 2016)

1125-68-2462 **Parikshit Gopalan, Guangda Hu, Swastik Kopparty, Shubhangi Saraf, Carol Wang and Sergey Yekhanin\***, yekhanin@microsoft.com. *Maximally recoverable codes.*

Data storage applications require erasure-correcting codes with prescribed sets of dependencies between data symbols and parity symbols (topology). A code as above is Maximally Recoverable (MR), if it corrects all erasure patterns that are information theoretically correctable given the dependency constraints. Applications furthermore need codes over small finite fields in order to facilitate encoding and decoding operations. In this talk we survey the state of the art in maximally recoverable codes and present the first super-polynomial lower bound for the field size of MR codes (in any topology). (Joint work with Parikshit Gopalan, Guangda Hu, Swastik Kopparty, Shubhangi Saraf, and Carol Wang). (Received September 20, 2016)

1125-68-2677 **Kolten C Pearson** (koltenpearson@gmail.com), Kolten Pearson, Orem, UT 84097, and **Megan Searles\*** (megan.d.searles@gmail.com), 999 E 450 N #21, Provo, UT 84606. *Using Data Analytics to Teach Computers the Difference between Rembrandt and Monet.* Preliminary report.

Data analytics is a new field that uses ideas from mathematics, computer science, and statistics to take large sources of data to derive new results. One technique in data analytics is artificial neural networks. These are

inspired by biological neural networks and allow computers to learn based on observational data. In our research we applied this to art classification. Specifically, we fed a large art database through an artificial neural network to have the computer learn how to classify other art images. Now, we are analyzing how different network architectures compare when faced with the task at hand. Our research looks into the discrepancies between different networks in order to find the optimal way to teach our computers to classify art. (Received September 20, 2016)

1125-68-2969 **Robert S Owor\*** ([robert.owor@asurams.edu](mailto:robert.owor@asurams.edu)), Department of Mathematics and CS, Albany, GA 31705, and **Zephyrinus C Okonkwo**. *A Block Chain Mathematical Algorithm for Secure Data Authentication, Storage, Transmission, Reception and Distribution*.

The goal of this paper is to present our initial findings on the relationship between Block Chain technology and elliptical curve cryptography on the one hand and its application to cyber security access control and authentication protocols on the other. We examine elliptical curve cryptography and the principles of Block Chain technology and how they can be applied to the improvement of secure data authentication, storage, reception, transmission and distribution using block chain mathematical algorithms. We next critically evaluate the current state and limitations of data encryption and decryption, authentication and access control protocols. We then introduce and discuss a modified Generalized Block Chain Mathematical algorithm and the potential application of this algorithm to addressing key problem areas in secure data authentication, storage, transmission, reception, and distribution. (Received September 20, 2016)

## 70 ► *Mechanics of particles and systems*

1125-70-1381 **Tim Krumwiede\*** ([krumwiede@math.utk.edu](mailto:krumwiede@math.utk.edu)), 2704 Sood Rd Apt 9, Knoxville, TN 37921, and **Tim P Schulze**. *Surface Energy in Bond-Counting Models*. Preliminary report.

Continuum models in computational material science require the choice of a surface energy function, based on properties of the material of interest. I will show how to use atomistic stochastic bond-counting models and crystal geometry to inform this choice. We will examine some of the difficulties that arise in the comparison between these models due to differing types of truncation. New crystal geometry methods are required when considering materials with non-Bravais lattice structure, resulting in a multi-valued surface energy. I will present these methods in the context of the two-dimensional material graphene in a way that correctly predicts its equilibrium shape. (Received September 16, 2016)

1125-70-2066 **Xiaolin Wang\*** ([wxiaolin@umich.edu](mailto:wxiaolin@umich.edu)), 530 Church St., Department of mathematics, Ann Arbor, MI 48109, and **Silas Alben** ([alben@umich.edu](mailto:alben@umich.edu)), 530 Church St, Department of mathematics, Ann Arbor, MI 48109. *Dynamics and locomotion of flexible foils in a frictional medium*.

Flexible beams with Coulomb friction force can be applied to model the locomotion of footless animals like snakes. The lack of limbs distinguishes their kinematics from other common modes of locomotion including flying and swimming, and exhibits unique dynamic behaviors. In this work, we study a nonlinear beam under frictional forces, driven by periodic base excitations. We consider the cases where the base is fixed and free to translate. The key control parameters are the excitation amplitude, beam rigidity, and frictional coefficients. We study dynamical phenomena including resonances and multiply-periodic states for the fixed base, and the speed and efficiency of locomotion for the freely translating base. Numerical simulations and linearized models are used to explain the dynamic behaviors. (Received September 19, 2016)

## 74 ► *Mechanics of deformable solids*

1125-74-1067 **Yue Chen\*** ([ychen5@aum.edu](mailto:ychen5@aum.edu)), Auburn University at Montgomery, Department of Mathematics and Computer Scienc, 310S Goodwyn Hall, Montgomery, AL 36064, and **Chi-Sing Man** and **Kazumi Tanuma**. *Inverse problems related to monitoring depth profile of residual stress via Rayleigh-wave dispersion*.

In this talk, the inverse problem is to investigate the possibility of using Rayleigh waves to monitor the retention of the protective prestress during the lifetime of a structural component. The solution of the inverse problem is based on the direct problem to determine dispersion curves for Rayleigh waves propagating in various directions when the material parameters, texture coefficients, and initial stresses are given. We can infer the depth profiles of the residual stresses which are good approximations to the real ones. (Received September 14, 2016)

1125-74-2251 **Robert Lipton** and **Eyad Said\***, Department Of Mathematics, Baton Rouge, LA. *Damage Modeling using Dissipation Potentials for State Based Peridynamic Fracture Evolution.*

We formulate a non-local cohesive model for calculating the deformation inside a cracking body. The physical properties in this model include elastic and softening behavior and are assigned at each point. Three-point interaction is also included in this model by means of the hydrostatic stress. We work within the small deformation setting and use the peridynamic formulation. The strains are calculated as difference quotients and the constitutive relation is non-local cohesive law relating the strain to the force that is motivated by Lennard-Jones model. At each instant of the evolution we identify a process zone where strains lie above a threshold value. Perturbation analysis shows that jump discontinuities within the process zone can become unstable and grow as enough strains in the peridynamic neighborhood exceeds the threshold. We derive an explicit inequality that shows that the size of the process zone is controlled by the ratio given by the length scale of non-local interaction divided by the characteristic dimension of the sample. We conclude by calibrating the model to the physical properties of the body. (Received September 20, 2016)

## 76 ► Fluid mechanics

1125-76-146 **Eleftherios Gkioulekas\*** ([eleftherios.gkioulekas@utrgv.edu](mailto:eleftherios.gkioulekas@utrgv.edu)), University of Texas Rio Grande Valley, School of Mathematical and Statistical Scienc, 1201 West University Drive, Edinburg, TX 78539-2999. *Multilocality and fusion rules on the generalized structure functions in two-dimensional and three-dimensional Navier-Stokes turbulence.*

Using the fusion rules hypothesis for three-dimensional and two-dimensional Navier-Stokes turbulence, we generalize a previous non-perturbative locality proof to multiple applications of the nonlinear interactions operator on generalized structure functions of velocity differences. We shall call this generalization of non-perturbative locality to multiple applications of the nonlinear interactions operator *multilocality*. The resulting cross-terms pose a new challenge requiring a new argument and the introduction of a new fusion rule that takes advantage of rotational symmetry. Our main result is that the fusion rules hypothesis implies both locality and multilocality in both the IR and UV limits for the downscale energy cascade of three-dimensional Navier-Stokes turbulence and the downscale enstrophy cascade and inverse energy cascade of two-dimensional Navier-Stokes turbulence. We stress that these claims relate to non-perturbative locality of generalized structure functions on all orders, and not the term by term perturbative locality of diagrammatic theories or closure models that involve only two-point correlation and response functions. (Received August 30, 2016)

1125-76-366 **Luoding Zhu\*** ([luozhu@iupui.edu](mailto:luozhu@iupui.edu)), 402 N Blackfort Street, Indianapolis, IN 46202. *An elastic sheet interacting with a 3D non-Newtonian fluid flow.* Preliminary report.

Motivated by fluid-structure-interaction (FSI) phenomena in life sciences (e.g., erythrocytes moving in flowing blood), we consider a simple FSI model problem — interaction of an elastic sheet (fixed at the midline) with a non-Newtonian fluid in three dimensions. The non-Newtonian flow is modelled by the power law. The fluid flow is modelled by the lattice Boltzmann equations. The deformable structure and the FSI is handled by the immersed boundary (IB) method. Drag of the sheet is computed, drag scaling is studied, and effects of fluid property, Reynolds number, and sheet bending rigidity on drag and its scaling are investigated. (Received August 29, 2016)

1125-76-598 **Ling Xu\***, 530 Church Street, 2074 East Hall, Ann Arbor, MI 48109, and **Robert Krasny**, 530 Church Street, 2074 East Hall, Ann Arbor, MI 48109. *An efficient particle vortex method for vorticity dynamics in free space.* Preliminary report.

We present an efficient particle vortex method for directly simulating the fluid flow motion in free space. The method employs techniques to accelerate the computation (adaptive mesh refinement and MPI parallelism) and maintain the accuracy and stability of the solutions (vorticity remeshing, high order interpolation). For the case of inviscid flow, a comparison to the inviscid elliptic dipole computation is made to validate the method. For the viscous case, some preliminary results will be presented. (Received September 07, 2016)

1125-76-739 **Joshua Ballew\*** ([jballew@andrew.cmu.edu](mailto:jballew@andrew.cmu.edu)), Carnegie Mellon University, Department of Mathematical Sciences, 5000 Forbes Ave, Pittsburgh, PA 15213, and **Konstantina Trivisa** ([trivisa@math.umd.edu](mailto:trivisa@math.umd.edu)), University of Maryland, College Park, Department of Mathematics, College Park, MD 20742. *Well-Posedness for Systems of Fluid-Particle Interaction.*

The Navier-Stokes-Smoluchowski and Euler-Smoluchowski systems model the interaction of particles interacting with a viscous or inviscid compressible fluid, respectively. In this talk, a weak-strong uniqueness result for the Navier-Stokes-Smoluchowski system in three spatial dimensions is presented. This result depends upon an admissibility criterion of a relative entropy inequality comparing the energies of a weak solution and a potential smooth solution. For a simplified version of the Euler-Smoluchowski model in three spatial dimensions, the existence of multiple weak solutions is shown using the techniques of convex integration. A weak-strong uniqueness result when the model also has an energy inequality is also presented. (Received September 10, 2016)

1125-76-1188 **William H Mitchell\*** ([whmitchell@math.wisc.edu](mailto:whmitchell@math.wisc.edu)) and **Saverio Spagnolie.** *Viscous erosion and generalized traction integral equations.*

Motivated by problems in sedimentation and erosion, we derive a new boundary integral formulation for computing surface tractions in several Stokes flow problems, in particular for problems with a non-trivial background flow and/or a no-slip plane wall. The associated integral operators enjoy the conditioning advantages of second kind integral equations while avoiding the traditional obstacles of hypersingularity and rank deficiency. We use this formulation to study the erosion of immersed particles according to a model relating the local ablation rate to the local shear stress. In several flow configurations we find the emergence of distinct limiting body shapes involving sharp corners and ridges. These numerical results compare favorably with analytical predictions from a reduced-order model. (Received September 15, 2016)

1125-76-1503 **Rachael Thormann\***, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623. *Mathematical Model of the Motion of a Contact Lens.*

We are developing a model of the motion of a contact lens under the influence of the shear stress produced by a pressure gradient in the tearfilm. After a soft contact lens is placed on the eye, blinking deforms the lens causing it to take the shape of the eye and shift off the center of the eye. Our model will capture the mechanism of re-centering the contact lens on the eye. This is the next step towards a full model of the solid and fluid mechanics of a contact lens in the tear film. To do so, we describe the solid mechanics of the contact lens building off the suction pressure work of Maki, Ross, and Holz (Existence Theory for the Radially-Symmetric Contact Lens Equation, D. S. Ross, K. L. Maki, E. Holz, SIAM J. Appl. Math, 76(3), 827-844, (2016)). (Received September 17, 2016)

1125-76-1889 **Luan T Hoang\***, Department of Mathematics and Statistics, Texas Tech University, Box 41042, Lubbock, TX 79409, and **Vincent R. Martinez**, Tulane University. *Asymptotic expansion for solutions of Navier-Stokes equations with a non-potential body force.* Preliminary report.

We study the long-time behavior of solutions to the three-dimensional incompressible Navier-Stokes equations with periodic boundary conditions. The body forces are non-potential and decay exponentially in time. We establish an asymptotic expansion of Foias-Saut-type for weak solutions in Gevrey classes. This extends the previous results for the case of potential forces. The proof uses the Gevrey-norm technique which makes it short and significantly simplifies the conditions on the forces. (Received September 19, 2016)

1125-76-2132 **Bagisa Mukherjee\*** ([bgm102@psu.edu](mailto:bgm102@psu.edu)), 120 Ridge View Drive, Dunmore, PA 18512. *Determination of Director angle for flow aligning Nematic Liquid Crystals under Couette geometry.*

We consider steady state flow of nematic liquid crystals in a Couette geometry driven by the relative rotation of the two concentric cylinders. We use the standard Ericksen-Leslie continuum model. The director, a unit vector, represents the average molecular orientation. We assume strong anchoring boundary conditions and find an explicit expression for the director orientation in the bulk of the flow for low Ericksen number. (Received September 19, 2016)

1125-76-2206 **Longhua Zhao\*** (1xz315@case.edu), **Angela Chuang**, **Iordanka N. Panayotova**, **Kimberly S. Sheldon**, **Lydia Bourouiba** and **Laura A. Miller**. *Flying Spiders: Simulating and modeling the dynamics of ballooning.*

Certain species of spiders can use a type of aerial dispersal called “ballooning” to move from one location to another. In order to balloon, such a spider releases a silk dragline from its spinnerets and when the movement of air relative to the dragline generates enough force, the spider takes off. Yet, the detailed physics driving this process remains little understood. We developed and numerically solved a two-dimensional fully-coupled fluid-structure interaction mathematical model to identify the crucial physical phenomena driving spider ballooning. We used the immersed boundary method to solve the multi-scale motion of flexible dragline through air. We explored three critical stages for ballooning: take-off, flight, and settling. Our numerical simulations allowed us to quantify how the dynamics of ballooning is significantly influenced by the spider mass and the length of the dragline, and other key properties such as dragline bending modulus. We will discuss our results and their implications for ecological events of long-distance population dispersal. (Received September 19, 2016)

1125-76-2278 **Weiwei Chen\*** (weiwei.chen@glasgow.ac.uk), Room 509, Mathematics Building, University Garden., Glasgow, G12 8QW, United Kingdom, **Hao Gao**, Mathematics Building, University Garden., G12 8QW, United Kingdom, **Xiaoyu Luo**, Mathematics Building, University Garden., G12 8QW, United Kingdom, and **Nicholas A Hill**, Mathematics Building, University Garden., G12 8QW, United Kingdom. *Wave intensity analysis of left ventricle-systemic arteries interaction.* Preliminary report.

Wave intensity analyses (WIA) are widely used to study pressure and flow propagation in the cardiovascular system, which is a complex and nonlinear, with many vessels and bifurcations from which waves are reflected. The method separates the forward and backward components of waves in order to understand and characterize the pulsatile behaviour, and the work done by the heart. In this study, arterial wave propagation is simulated numerically in a computational model [1] that couples three-dimensional finite-strain structure-based left ventricle (LV) with a one-dimensional mathematical model of the systemic arteries (SA). The aim is to use WIA to predict peak pressures and the load on the LV arising from pathologies of the cardiovascular system and to identify biomarkers for different diseases.

[1] Chen, W.W., Gao, H., Luo, X.Y., and Hill, N.A. (2016) Study of cardiovascular function using a coupled left ventricle and systemic circulation model. *Journal of Biomechanics* (Received September 20, 2016)

1125-76-2427 **Joshua Chasen\***, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623, **Roland Sanford**, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623, **Kara Maki**, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623, and **David Ross**, School of Mathematical Sciences, Rochester Institute of Technology, Rochester, NY 14623. *The Deformation of an Eye Caused by a Contact Lens.*

In this work, we aim to better understand how the design of contact lenses can be optimized for patient comfort and ocular fit. Specifically, we study how the eye tissue deforms when a contact lens is inserted on the eye. To do so, we implement a high-order accurate scheme to solve a system of partial of differential equations modeling the elastic stresses induced in an eye by a contact lens on its surface. The suction pressure distribution produced by a radially symmetric contact lens is characterized by a system of ordinary differential equations depending on the shape of the surface of the eye, the shape of the undeformed contact lens, and the material properties of the contact lens. We explore how different contact lens shapes, contact lens material properties, and eye tissue properties affect the deformation of the eye tissue. (Received September 20, 2016)

1125-76-2530 **Laurel A Ohm\*** (ohmxx039@umn.edu). *Slender body theory: an error analysis.* Preliminary report.

Slender body theory is a commonly-used approximation in modeling the dynamics of thin fibers immersed in a viscous fluid in three dimensions. The approximation treats a thin fiber as 1D distribution of point forces, coupled with a fiber integrity condition. Despite the ubiquity of numerical methods and results based on slender body theory, very little analytical work has been done to quantify the error introduced by approximating a three-dimensional fiber as a one-dimensional force distribution, and to verify convergence of the slender body solution to the true solution as the body radius goes to zero. Here, we present a PDE reformulation of the question slender body theory aims to solve and derive a convergence result for the slender body approximation to the true solution. (Received September 20, 2016)

1125-76-2615 **Paul DeVries\***, 31-10 Thomson Ave, room E223N, Long Island City, NY 11101, and **Michael Wiley**. *Calculating Lift and Drag Force for Airfoils*. Preliminary report.

In the study of aerodynamics, an airfoil can be described as any curved structure to designed to optimize the lifting force and drag force acting upon it. Drag is the force of wind resistance acting upon an object and the lifting force is a force generated by the airfoil that counteracts the object's weight. Based on the airfoil's geometry, it is possible to calculate these two forces using calculus. Early work on this field has been mainly centered on airplane wings, but it has since been applied to wind turbine design which utilizes both drag and lift to its advantage. (Received September 20, 2016)

1125-76-2691 **John C Chrispell\*** ([john.chrispell@iup.edu](mailto:john.chrispell@iup.edu)), 210 South 10th Street, Indiana, PA 15705. *Numerical Simulation of a Fluid Ratchet using an Immersed Boundary Framework*.

Numerical simulations of a fluid ratchet for pumping both Newtonian and Viscoelastic fluids will be examined using an immersed boundary framework. Preliminary results regarding the overall flow generated by varying the shape of the ratcheting structure will be presented. (Received September 20, 2016)

1125-76-2696 **Sourav Dutta\*** ([sdutta@math.tamu.edu](mailto:sdutta@math.tamu.edu)), Texas A&M University, Department of Mathematics, 3368 TAMU, College Station, TX 77843-3368, and **Prabir Daripa**. *A numerical study of immiscible two-phase multicomponent flows in highly heterogeneous porous media*.

We will present a fast and efficient approach for solving a coupled system of elliptic and parabolic equations arising in the context of multiphase flows in porous media. Such flows are found in many different physical applications which include enhanced oil recovery processes, subsurface flows and bio-fluid flows. A new global pressure function for incompressible, multicomponent, immiscible two-phase flows will be introduced. The system of equations using the global pressure model is not as strongly coupled as the models which use the phase pressures as simulation variables. This system is numerically solved using a modern, hybrid method based on a combination of a discontinuous non-traditional finite element formulation and a time-implicit finite difference scheme based on the modified method of characteristics. Results of a theoretical convergence study and numerical comparisons with an exact solution and also with existing literature will be presented. We will conclude with a discussion of the effect of various chemical components and of the heterogeneity of the domain properties on the spontaneous formation of finger patterns and other complex flow characteristics. (Received September 20, 2016)

1125-76-2708 **Manuchehr Aminian** ([aminian@live.unc.edu](mailto:aminian@live.unc.edu)), **Francesca Bernardi\*** ([bernardi@live.unc.edu](mailto:bernardi@live.unc.edu)), **Roberto Camassa** ([camassa@amath.unc.edu](mailto:camassa@amath.unc.edu)), **Daniel M. Harris** ([dmh@email.unc.edu](mailto:dmh@email.unc.edu)) and **Richard M. McLaughlin** ([rmm@email.unc.edu](mailto:rmm@email.unc.edu)). *Tailoring Tails in Taylor Dispersion: How Boundaries Shape Chemical Deliveries in Microfluidics*. Preliminary report.

We present the results of a combined computational, theoretical and experimental study of the dispersion of a passive scalar in laminar shear flow through rectangular and elliptical channels. We show through Monte Carlo simulation, asymptotic analysis and experiments that the cross-sectional aspect ratio sets the sign of the average skewness at long times (relative to the Taylor diffusion timescale) which describes the longitudinal asymmetry of the tracer distribution. Universally, thin channels (aspect ratio  $\ll 1$ ) result in negative average skewness, whereas thick channels (aspect ratio  $\sim 1$ ) result in positive average skewness. Our analysis also allows us to define a "golden" aspect ratio which separates thin from thick channels, the value of which is remarkably similar for both the rectangle and the ellipse. Further, by examining the median of the cross-sectionally averaged distribution, we establish that negative skewness correlates with solutes arriving with sharp fronts followed by a tapering tail. The experimental results are in strong agreement with our theoretical and numerical predictions. Future directions and potential microfluidic applications will be discussed. (Received September 20, 2016)

1125-76-2732 **Aleksey S Telyakovskiy\*** ([alekseyt@unr.edu](mailto:alekseyt@unr.edu)), Department of Mathematics and Statistics, University of Nevada, Reno, NV 89557. *Approximate solution to porous media flows*. Preliminary report.

Nonlinear equations of the diffusion type describe unconfined flows in groundwater flow modeling. In case of the horizontal infiltration Dupuit assumption that the equipotential lines are vertical is often used. It makes the velocity of flow horizontal. Depending on the specific application the form of the diffusivity, that depends on the water head, can differ. For certain classes of initial and boundary conditions similarity variables can be introduced and the initial-boundary value problem for nonlinear PDE can be reduced to a boundary value problem for an ODE. In this work we construct approximate analytical solutions that respect some properties of the original problem. (Received September 20, 2016)

1125-76-2888 **Manal Badgaish\*** ([mbadgai2@masonlive.gmu.edu](mailto:mbadgai2@masonlive.gmu.edu)), Department of Mathematical Sciences, George Mason University, Fairfax, VA 22030, **Jeng-Eng Lin** ([jelin@gmu.edu](mailto:jelin@gmu.edu)), Department of Mathematical Sciences, George Mason University, Fairfax, VA 22030, and **Padmanabhan Seshaiyer** ([pseshaiy@gmu.edu](mailto:pseshaiy@gmu.edu)), Department of Mathematical Sciences, George Mason University, Fairfax, VA 22030. *Non-linear dynamics and analysis of Intracranial Saccular Aneurysms with growth and remodeling.*

In this work a new mathematical model for the interaction of blood flow with the arterial wall surrounded by cerebral spinal fluid is developed with applications to intracranial saccular aneurysms. The blood pressure acting on the inner arterial wall is modeled via a Fourier Series, the arterial wall is modeled as a spring-mass system incorporating growth and remodeling and the surrounding cerebral spinal fluid is modeled via a simplified Navier-Stokes equation. The resulting non-linear coupled fluid structure interaction problem is analyzed and a perturbation technique is employed to derive the first-order approximation solution to the system. An analytical solution is also derived for the linearized version of the problem using Laplace transforms. The solutions are validated against related work from the literature and the results suggest the biological significance of the inclusion of the growth and remodeling effects on the rupture of intracranial aneurysms. (Received September 20, 2016)

1125-76-2889 **Amy Buchmann\***, [abuchman@tulane.edu](mailto:abuchman@tulane.edu), and **Ricardo Cortez** and **Lisa Fauci**. *A Computational Model of Ciliary Beating*. Preliminary report.

Cilia, flexible hairlike appendages located on the surface of a cell, play an important role in many biological processes including the transport of mucus in the lungs and the locomotion of ciliated microswimmers. Cilia self-organize forming a metachronal wave that propels the surrounding fluid. To study this coordinated movement and motivated by the oscillator model in, we model each cilium as an elastic, actuated body whose beat pattern is driven by a geometric switch where the beat angle switches between two ‘traps’, driving the motion of the power and recovery strokes. The cilia are coupled to a viscous fluid using a numerical method based upon a centerline distribution of regularized Stokeslets. We first characterize the beat cycle and flow produced by a single cilium and then investigate the synchronization states between two cilia. (Received September 20, 2016)

1125-76-2916 **Suncica Canic\*** ([canic@math.uh.edu](mailto:canic@math.uh.edu)). *Interaction between blood flow and multi-layered structure of arterial walls.*

Fluid-structure interaction problems with composite structures arise in many applications. One example is the interaction between blood flow and arterial walls. Arterial walls are composed of several layers, each with different mechanical characteristics and thickness. In this talk we will summarize the main difficulties in studying this class of problems, and present a computational scheme for the calculation of FSI solutions. Our results reveal a new physical regularizing mechanism in FSI problems with multi-layered structures: inertia of the thin fluid-structure interface with mass regularizes evolution of FSI solutions. Implications of our theoretical results on modeling the human cardiovascular system will be discussed. This is a joint work with Boris Muha (University of Zagreb, Croatia), Martina Bukac (U of Notre Dame, US) and Roland Glowinski (UH). Numerical results with vascular stents were obtained with S. Deparis and D. Forti (EPFL, Switzerland). Collaboration with medical doctors Dr. S. Little (Methodist Hospital Houston) and Dr. Z. Krajcer (Texas Heart Institute) is also acknowledged. (Received September 20, 2016)

1125-76-2917 **Nguyenho Ho\*** ([nguyenho.ho@uc.edu](mailto:nguyenho.ho@uc.edu)), **Karin Leiderman** and **Sarah Olson**. *A 3-dimensional model of sperm motility in a Brinkman fluid.*

We investigate 3-dimensional motility of sperm in fluids with a sparse network of stationary obstacles or fibers. The Brinkman equation is used to model the average fluid flow where a resistance term is inversely proportional to the fluid permeability and represents the effect due to the presence of the fibers. The sperm flagellum is idealized as a Kirchhoff rod that can exhibit lateral or spiral waves. To solve for the local fluid velocity and angular velocity, we use the method of regularized Brinkmanlets and extend it to the case for a Kirchhoff rod that is discretized as point forces and torques. The numerical method is validated by comparing to asymptotic swimming speeds derived for a cylinder of infinite length that is propagating lateral or spiral waves in a Brinkman fluid. Similar to the asymptotics, we observe that in the case of small amplitude, swimming speed is enhanced as the permeability in the fluid is decreased. For larger amplitudes, the simulations show a non monotonic change in swimming speed as the fluid permeability is increased. This is due to the emergent amplitude and wavelengths; as the permeability is decreased, the emergent amplitude of the swimmer has a tendency to decrease due to the extra resistance of the increased stationary fibers. (Received September 20, 2016)

## 78 ► *Optics, electromagnetic theory*

1125-78-1706     **Alexey Sukhinin\*** (asukhini@uvm.edu). *Coupled microring cavities under PT-symmetry*. Preliminary report.

A new family of lasers based on parity-time(PT) symmetry has been introduced a few years ago. It is believed that such lasers have potential for greater tunability and single mode generation with high purity. The principle of PT-symmetry transpire in the dynamical interplay between optical gain and loss between laser cavities. In this talk I discuss the mathematical model and present preliminary analysis of various configurations of PT-symmetric coupled microring cavities. (Received September 18, 2016)

1125-78-2545     **Sanjeewa S K Karunarithna\*** (sanjeewa.karunarithna@ttu.edu), Texas Tech University, Department of Mathematics and Statistics, Broadway and Boston, Lubbock, TX 79409, and **Ram Iyer**, Texas Tech University, Department of Mathematics and Statistics, Broadway and Boston, Lubbock, TX 79409. *Computation of wavefront aberration in rigid gas permeable corneal lenses*.

A contact lens motion is an important issue for rigid gas permeable corneal lenses as they are smaller and move with each blink and saccade. A measurement by a wavefront aberrometer if made with a trial lens could result in different measurements depending on the position of the lens. It is important to compute the wavefront had the lens been in a different position. Using markers on the lens, we calculate a three dimensional (3D) rotation matrix that corresponds to the apparent two dimensional rotation and decentration. Such an approach has heretofore not been considered in the literature. We propose a new method to determine the wavefront for a different lens position. Our method uses the exact shape of the lens in order to compute the wavefront after a three dimensional rotation, and may be used for lenses with very high power and for relatively large rotations and decentrations. We extend the results to estimate the actual wavefront, when the lens centered on the cornea. Numerical results illustrate our method. (Received September 20, 2016)

1125-78-2646     **Sarah G Rody\*** (sgr33@drexel.edu). *Prescribed Curvature and Optics*.

I will discuss the general problem of trying to find a three dimensional surface perpendicular to a vector field  $\mathbf{W}$  that has unit length. I will use the Euler-Lagrange equations for the cost functional

$$\mathfrak{F}(\sigma) = \iint_{\sigma} |\mathbf{W} - \hat{n}|^2 dS,$$

where  $\sigma$  is a surface with unit normals  $\hat{n}$ , to give a PDE for the surface. This PDE can be solved numerically, but is highly dependent on the boundary value conditions. Finally, I will show application of this method to the passenger-side mirror problem. (Received September 20, 2016)

## 81 ► *Quantum theory*

1125-81-197     **Stefan Huber, Robert Koenig and Anna Vershynina\*** (annavershynina@gmail.com), BCAM (Basque Center for Applied Mathematics), Alameda de Mazarredo 14, 48009 Bilbao, Bizkaia, Spain. *Quantum analogues of geometric information theoretic inequalities*.

Geometric inequalities, such as the entropy power inequality or the isoperimetric inequality, relate geometric quantities, such as volumes and surface areas. Classically, these inequalities have useful applications for obtaining bounds on channel capacities, and deriving Log-Sobolev inequalities. In my talk I provide quantum analogues of certain well-known inequalities from classical Information Theory, with the most notable being the isoperimetric inequality for entropies. The latter inequality is useful for the study of convergence of certain semigroups to fixed points. In the talk I demonstrate how to apply the isoperimetric inequality for entropies to show exponentially fast convergence of quantum Ornstein-Uhlenbeck (qOU) semigroup to a fixed point of the process. The inequality representing the fast convergence can be viewed as a quantum analogue of a classical Log-Sobolev inequality. As a separate result, necessary for the fast convergence of qOU semigroup, I argue that gaussian thermal states minimize output entropy for the attenuator semigroup among all states with a given mean-photon number. (Received August 15, 2016)

1125-81-591 **Alexander Turbiner\*** ([turbiner@nucleares.unam.mx](mailto:turbiner@nucleares.unam.mx)), Instituto de Ciencias Nucleares, UNAM, AP 70-543, 04510 Mexico-City, DF, Mexico. *The Heun operator as a Hamiltonian*. Preliminary report.

It is shown that the celebrated Heun operator  $H_e = -(a_0x^3 + a_1x^2 + a_2x)\frac{d^2}{dx^2} + (b_0x^2 + b_1x + b_2)\frac{d}{dx} + c_0x$  is the Hamiltonian of the  $sl(2, R)$ -quantum Euler-Arnold top of spin  $\nu$  in a constant magnetic field. For  $a_0 \neq 0$  it is canonically-equivalent to  $BC_1(A_1)$ - Calogero-Moser-Sutherland quantum models, if  $a_0 = 0$ , ten known one-dimensional quasi-exactly-solvable (QES) problems are reproduced, and if, in addition,  $b_0 = c_0 = 0$ , then four well-known one-dimensional quantal exactly-solvable problems are reproduced. If spin  $\nu$  of the top takes (half)-integer value the Hamiltonian possesses a finite-dimensional invariant subspace and  $(2\nu + 1)$  polynomial eigenfunctions occur. Discrete systems on uniform and exponential lattices are introduced which are canonically-equivalent to one described by the Heun operator. (Received September 07, 2016)

1125-81-623 **Kenny De Commer\***, Pleinlaan 2, 1050 Brussel, Belgium. *Torsion-freeness of free quantum groups*.

In order to formulate the Baum-Connes conjecture in the setting of discrete quantum groups, R. Meyer introduced a notion of torsion-freeness for them. In this talk, we show that the duals of the free unitary quantum groups of Wang and Van Daele are torsion-free in the sense of Meyer, answering a question of C. Voigt. This is joint work with Y. Arano. (Received September 08, 2016)

1125-81-651 **Iris Cong\***, [iriscong23@ucla.edu](mailto:iriscong23@ucla.edu), and **Meng Cheng** and **Zhengan Wang**. *Topological Quantum Computation with Gapped Boundaries*.

We study fault-tolerant quantum computation with gapped boundaries. We first present the algebraic/categorical structure of gapped boundaries and boundary defects. These will be used to describe topologically protected operations and obtain quantum gates. Finally, we show how gapped boundaries of the abelian theory  $\mathcal{D}(\mathbb{Z}_3)$  can be used to perform universal quantum computation. (Received September 08, 2016)

1125-81-798 **Ian Marquette\*** ([i.marquette@uq.edu.au](mailto:i.marquette@uq.edu.au)), St Lucia, QLD 4072, Australia, and **Fazlul Hoque** and **Yao-Zhong Zhang**. *N-dimensional superintegrable systems and higher rank quadratic algebras*.

Classical and quantum superintegrable systems have a long history and they possess more integrals of motion than degrees of freedom. They have many attractive properties, wide applications in modern physics and connection to many domains in pure and applied mathematics. We overview two families of superintegrable Kepler-Coulomb systems with non-central terms and superintegrable Hamiltonians with double singular oscillators of type  $(n, N - n)$  in  $N$ -dimensional Euclidean space. We present their quadratic and polynomial algebras involving Casimir operators of  $so(N+1)$  Lie algebras that exhibit very interesting decompositions  $Q(3) \oplus so(N-1)$ ,  $Q(3) \oplus so(n) \oplus so(N-n)$  and the cubic Casimir operators. The realization of these algebras in terms of deformed oscillator enables the determination of a finite dimensional unitary representation. We present algebraic derivations of the degenerate energy spectra of these systems and relate them with the physical spectra obtained from the separation of variables. We also discuss cases related to these models involving Abelian and non Abelian monopoles interaction. (Received September 12, 2016)

1125-81-907 **Juan Cuadra**, **Pavel Etingof** and **Chelsea Walton\*** ([notlaw@temple.edu](mailto:notlaw@temple.edu)), 1805 N. Broad Street, Philadelphia, PA 19122. *No Quantum Symmetry from an algebraic point of view*.

I will discuss recent work the lack of finite dimensional Hopf actions on (quantizations of) commutative domains. This includes results on Hopf actions on Weyl algebras, universal enveloping algebras of finite dimensional Lie algebras, spherical symplectic reflection algebras, quantum polynomial algebras, twisted homogeneous coordinate rings of abelian varieties, and Sklyanin algebras. (Received September 13, 2016)

1125-81-926 **Vincent X Genest** ([vxgenest@mit.edu](mailto:vxgenest@mit.edu)) and **Luc Vinet\*** ([vinet@crm.umontreal.ca](mailto:vinet@crm.umontreal.ca)), Centre de Recherches Mathématiques, Université de Montréal, Montreal, QC H3C 3J7, Canada. *Supersymmetry and Superintegrability of the  $B_n$  Calogero Model*.

The rational  $B_n$  Calogero model for distinguishable particles is shown to be supersymmetric. The superintegrability is demonstrated by exhibiting the constants of motion that commute with the supercharges. Focusing for simplicity on the case  $n = 3$ , the symmetry algebra is found to be a generalization of the Bannai-Ito algebra that involves additional symmetries constructed in terms of the  $B_n$  Weyl group generators. (Received September 13, 2016)

1125-81-966 **Marcel Bischoff\*** ([marcel.bischoff@vanderbilt.edu](mailto:marcel.bischoff@vanderbilt.edu)), Vanderbilt University, Department of Mathematics, 1326 Stevenson Center, Nashville, TN 37240. *Generalized fixed points of conformal nets*.

We define actions of finite hypergroups by unital completely positive maps on factors. Such actions arise naturally from finite index subfactors with commutative 2-box space. Using this we can define a proper action on local conformal net of factors and show that the fixed point gives a finite index subnet. Conversely, every finite index subnet arises from a unique proper action of a hypergroup. Using Longo-Rehren subfactors and  $\alpha$ -induction we get a classification of possible actions for completely rational nets. In this case, the hypergroup is necessarily formed by double cosets  $V \setminus F / V$  where  $F$  is a categorifiable fusion ring and  $V$  the fusion ring of representations of the net. (Received September 13, 2016)

1125-81-973 **Tianyuan Xu\*** ([tianyuan@uoregon.edu](mailto:tianyuan@uoregon.edu)), Department of Mathematics, University of Oregon, Eugene, OR 97403. *Fusion rules for some tensor categories attached to Kazhdan-Lusztig cells in Coxeter groups*. Preliminary report.

We recall Lusztig's construction of tensor categories attached to Kazhdan-Lusztig cells in Coxeter groups, and present some results on the Grothendieck rings of these categories. These categories were first constructed for finite and affine Weyl groups using perverse sheaves on flag manifolds, and the construction was later generalized to all Coxeter groups using the language of Soergel bimodules, but the Grothendieck rings are known to be isomorphic to Lusztig's asymptotic Hecke algebras, allowing us to study these rings using more elementary methods. We give explicit descriptions of the structure of these Grothendieck rings for certain cases. In particular, we discuss cases where the concerned tensor category is fusion and provide examples where the Grothendieck rings coincide with free fusion semi-rings arising from partition quantum groups. (Received September 13, 2016)

1125-81-1199 **De Bie, Genest\*** ([vxgenest@mit.edu](mailto:vxgenest@mit.edu)) and **Vinet**. *The  $Z_n^2$  Dirac-Dunkl operator and a higher rank Bannai-Ito algebra*.

In this talk, I will discuss the  $n$ -dimensional Dirac-Dunkl operator associated with the reflection group  $Z_n^2$ . I will exhibit the symmetries of this operator, and describe the invariance algebra they generate. The symmetry algebra will be identified as a rank- $n$  generalization of the Bannai-Ito algebra. Moreover, I will explain how a basis for the kernel of this operator can be constructed using a generalization of the Cauchy-Kovalevskia extension in Clifford analysis, and how these basis functions form a basis for irreducible representations of Bannai-Ito algebra. Finally, I will conjecture on the role played by the multivariate Bannai-Ito polynomials in this framework. (Received September 15, 2016)

1125-81-1424 **Ram Band, Gregory Berkolaiko and Tracy Weyand\*** ([tracy\\_veyand@baylor.edu](mailto:tracy_veyand@baylor.edu)). *Critical Points in the Spectrum of Infinite Periodic Graphs*.

We consider infinite periodic graphs  $\Gamma$  that are formed by translating infinitely many copies of a fundamental domain  $G$ . We are interested in the spectrum of the Schrödinger operator acting on the graph  $\Gamma$ . According to Floquet-Bloch theory, we can find this spectrum by calculating the spectrum of the magnetic Schrödinger operator acting on a fundamental domain  $G$  and then taking the union over the Brillouin zone, which is the set of all possible magnetic fluxes.

Therefore, we can consider each eigenvalue as a function of magnetic flux, and we are interested in where critical points occur. While most critical points occur on the boundary of the Brillouin zone, counterexamples have shown that this is not always true. We will show that if the fundamental domain is a tree and the eigenvalue is generic, then the critical points will occur on the boundary. (Received September 16, 2016)

1125-81-1546 **Zhengan Wang and Modjtaba Shokrian Zini\*** ([shokrian@math.ucsb.edu](mailto:shokrian@math.ucsb.edu)). *VOAs as thermodynamical limit of Anyonic chains*. Preliminary report.

I will first define the anyonic  $\mathfrak{su}(2)_k$  antiferro-magnetic chains and review the evidences showing that in the scaling limit, we obtain a Minimal model with central charge  $1 - \frac{6}{(k+1)(k+2)}$ . Next, I will restrict to the case  $k = 2$  (Ising model), get into the details of the convergence of the open (and periodic) chains to the chiral (and full) Minimal model Vertex Operator Algebras (VOA) with central charge  $\frac{1}{2}$ , and review what is known rigorously in the mathematics literature. I will then try to give a precise meaning to the notion of scaling limit from a computational point of view. This, and other necessary concepts that will be defined, can be used to ask and hopefully give an answer to the following question rigorously: "Can Minimal models be simulated efficiently on a Quantum computer?". The answer is affirmative for the case of  $k = 2$ . I will also try to show the difficulties of answering this question for the general level  $k > 2$ . This is a preliminary work on the efficient simulation of CFTs on Quantum Computers. (Received September 17, 2016)

1125-81-1583 **John C Schotland\*** (schotland@umich.edu). *An Inverse Problem in Quantum Optics.*

We consider the scattering of entangled two-photon states in deterministic and random media. We investigate the influence of the entanglement of the incident field on the entanglement of the scattered field. A related inverse problem is described. (Received September 18, 2016)

1125-81-1647 **Maria Clara Nucci\*** (mariaclara.nucci@unipg.it), Department of Mathematics and Informatics, University of Perugia, 06123 Perugia, Italy. *Quantizing with Noether symmetries; linearity and superintegrability; unusual Lagrangian systems: an overview.*

We will illustrate our recent work on the following research themes: the role of Noether symmetries in quantizing classically Hamiltonian systems; the linearization of maximally superintegrable systems; the transformation of odd-order systems (e.g., in biology) into Lagrangian systems. (Received September 18, 2016)

1125-81-1689 **James E Tener\*** (jtener@math.ucsb.edu). *On classification of vertex operator algebras by their representation categories.* Preliminary report.

In this talk, I will present work in progress with Meng Cheng and Zhenghan Wang on the subject of classification of vertex operator algebras whose representation theory is given by a fixed modular tensor category. Perhaps the best studied problem in this area is the classification of “holomorphic” vertex operator algebras, i.e. those VOAs whose representation theory is simply  $\text{Vec}$ , with small central charge. However in this talk we will focus on a complementary problem, that of classifying, modulo holomorphic VOAs, those VOAs whose representation theory is given by a fixed non-trivial modular tensor category. Using techniques of Terry Gannon, we will explore the landscape of this classification problem for some small modular tensor categories. (Received September 18, 2016)

1125-81-3078 **Ian Swenson Marshall\*** (iswenmar@gmail.com). *On Brauer-Picard groups of fusion  $p$ -categories.*

A fusion  $p$ -category is a fusion category whose Frobenius-Perron is a power of prime number  $p$ . For odd primes  $p$ , all such categories are group theoretical. For  $p=2$ , such categories are equivalent to a  $\mathbb{Z}/2\mathbb{Z}$ -extension of a group theoretical category. We examine the computation of outer autoequivalences as a method for computing Brauer-Picard groups of these categories. This talk is based on joint work with Dmitri Nikshych. <https://arxiv.org/abs/1603.04318> (Received September 21, 2016)

## 82 ► *Statistical mechanics, structure of matter*

1125-82-594 **Karl E. Liechty\*** (klichty@depaul.edu) and **P. M. Bleher**. *Orthogonal polynomials and the six-vertex model.*

The six-vertex model is a two-dimensional exactly solvable model in statistical physics. When equipped with certain boundary conditions, the partition function of this model is expressed exactly in terms of orthogonal polynomials on the real line, and the thermodynamic limit of the partition function is given by asymptotic analysis of the orthogonal polynomials. I’ll talk about a decade (plus) old project of computing the asymptotics of the partition function for domain wall boundary conditions, and also more recent work extending these results to other related boundary conditions. (Received September 07, 2016)

1125-82-1868 **Tong Yang\*** (matyang@cityu.edu.hk), Y6517 Academic 1, City University of Hong Kong, Hong Kong, Peoples Rep of China. *Measure Valued Solutions to the Boltzmann Equation.*

In this talk, we will present some recent results on the well-posedness, regularity and large time behavior of the measure valued solutions. The results are from several joint works with Yong-Kum Cho, Yoshinori Morimoto, Shuaijun Wang and Huijiang Zhao. (Received September 19, 2016)

1125-82-1908 **S Joo\*** (sjoo@odu.edu), **A Contreras**, **C Garcia-Azpeitia** and **C Garcia-Cervera**. *Bifurcation study of smectic A liquid crystals in three dimensions.*

We study the Landau-de Gennes free energy to describe the undulatory instability in smectic A liquid crystals subjected to magnetic fields. We prove this phenomena by the bifurcation theory to the nonlinear system of Landau-de Gennes model. The bifurcation at the onset of undulation in 3D is not simple. We identify the irreducible representations for natural actions on the functional that take into account the invariances of the problem thus allowing for reducing the bifurcation analysis to a subspace with symmetries. A reduced 2D model provides a qualitative structure of the minimizer. We also perform numerical simulations to illustrate the results of our analysis. (Received September 19, 2016)

## 83 ► *Relativity and gravitational theory*

1125-83-2830 **Rachel Maitra\*** ([maitrar@wit.edu](mailto:maitrar@wit.edu)), 550 Huntington Ave, Boston, MA 02115, and **John Haga** ([hagaj@wit.edu](mailto:hagaj@wit.edu)), 550 Huntington Ave, Boston, MA 02115. *Factor ordering and path integral measure for quantum gravity in (1+1) dimensions.*

We develop a mathematically rigorous path integral representation of the time evolution operator for a model of (1+1)d quantum gravity that incorporates factor ordering ambiguity. In obtaining a suitable propagator, one requires that the corresponding Hamiltonian is self adjoint; this issue is subtle for a particular category of factor orderings. We present a method of identifying and parametrizing a complete set of self-adjoint extensions and provide a canonical description of these extensions in terms of boundary conditions; our technique is quite general and can likely be extended to higher dimensional models. (Received September 20, 2016)

1125-83-2929 **R. R. Ahangar\*** ([reza.ahangar@tamuk.edu](mailto:reza.ahangar@tamuk.edu)), P.O. Box 790, Kingsville, TX 78364. *The Geometry of the Complex Matter Space in Relativistic Quantum Mechanics.*

Duality behavior of Photons in wave-particle property have posed challenges and opportunities to discover other frontiers of fundamental particles leading to the relativistic and quantum description of matter. The speed of particles faster than the speed of light could not be recognized, and matter was always described as a real number. A new fundamental view on matter as a complex value has been introduced by many authors who present a paradigm that is shifted from real or pure imaginary particles to Complex Matter Space. A new assumption will be imposed that matter has two intrinsic components, i) mass, and ii) charge. The mass will be measured by real number systems and charge by an imaginary unit. The relativistic concept of Complex Matter Space on energy and momentum is investigated and can conclude that the new Complex Matter Space (CMS) theory will help get one step closer to a better understanding toward, 1) Un-Euclidean description of Minkowski Geometry in the context of the Complex Matter Space, 2) the conversion of mass and energy 3) unifying the forces. Finally, geometrical foundations are essential to have a real picture of space, matter, and the universe. (Received September 20, 2016)

## 85 ► *Astronomy and astrophysics*

1125-85-1120 **Perry Bialek Vargas\*** ([perrybvargas@gmail.com](mailto:perrybvargas@gmail.com)), **Carl Gardner** and **Jeremiah Jones**. *Astrophysical Jet HH30: Hot, Dense, and Colorful.*

This talk describes a gas dynamical model of the supersonic protostellar jet HH 30. To simulate the equations of gas dynamics, we use a positivity preserving third-order WENO scheme, parallelized with OpenMP and MPI. We are able to model the temperature and Density of this jet, based on observations by Bacciotti et al. (1999), as well as the surface brightness in the brightest spectral lines for the jet [S II] 6717 Å, 6731Å, [O I] 6563Å, and H $\alpha$  using Cloudy (Ferland et al. 2013). In reproducing the Hubble Space Telescope observations, we validate a new emission map capability for the WENO3 jet code of C. Gardner and J. Jones ultimately enhancing our understanding of the processes and dynamics of astrophysical jets. (Received September 20, 2016)

## 86 ► *Geophysics*

1125-86-819 **Christina A Frederick\***, [cfrederick6@gatech.edu](mailto:cfrederick6@gatech.edu), and **Quyeh Hyunh, Bjorn Engquist** and **Haomin Zhou**. *Seafloor identification in sonar imagery via simulations of Helmholtz equations.* Preliminary report.

We present a multiscale approach for identifying objects submerged in ocean beds by solving inverse problems in high frequency seafloor acoustics. The setting is based on Sound Navigation And Ranging (SONAR) imaging used in military, engineering, and scientific applications. The forward model incorporates simulations, by solving Helmholtz equations, on a wide range of spatial scales, allowing for detailed recovery of seafloor parameters including the material type. In order to lower the computational cost of large-scale simulations, we take advantage of a library of representative acoustic responses from various seafloor parametrizations. (Received September 12, 2016)

## 90 ► Operations research, mathematical programming

1125-90-144

**Maryam Yashtini\*** ([myashtini3@math.gatech.edu](mailto:myashtini3@math.gatech.edu)), School of Mathematics, Georgia Institute of Technology, 686 Cherry Street, Atlanta, GA 30332-0160, and **Sung Ha Kang**. *A fast relaxed normal two split method and an effective weighted TV approach for Euler's elastica image inpainting.*

In this talk, I present two numerical algorithms for solving Euler's elastica-based inpainting model. The minimizing functional is non-smooth, non-convex, and involves high-order derivatives, that traditional gradient descent based methods converges very slowly. Recent alternating minimization methods show fast convergence when a good choice of parameters is used. The objective of this talk is to introduce efficient algorithms which have simple structures with a fewer parameters. These methods are based on operator splitting and alternating direction method of multipliers, and sub-problems can be solved efficiently by Fourier transforms and shrinkage operators. For the first method, we relax the normal vector in the curvature term of the Euler's elastica model and exploit two operator splitting techniques to propose a Relaxed Normal two Split (RN2Split) method. The second method,  $\kappa$ -weighted Total Variation ( $\kappa$ TV), solves the Euler's elastica minimization problem as a weighted total variation. We present the analytical properties of each algorithm. Various numerical experiments, including comparison with some existing state-of-art algorithms, are presented to show the efficiency and the effectiveness of the proposed RN2Split and  $\kappa$ TV methods. (Received August 03, 2016)

1125-90-338

**Gayatri Pany\*** ([gayatripany@gmail.com](mailto:gayatripany@gmail.com)), Indian Institute of Technology Bhubaneswar, Bhubaneswar, Odisha 751007, India. *A study on vector invex equilibrium problems with convexificator approach.*

In this work, we study vector invex equilibrium problems using convexificators. Stampacchia and Minty type mixed vector invex equilibrium problems (SMVIEP and MMVIEP) in terms of convexificators are formulated. Relationships between them are derived. Minty vector VI and its generalizations are used to obtain criteria for a point to be vector minimal of a vector optimization problem (VOP). Recently some existence results are obtained for weakly efficient solutions of convex VOPs under relaxed compact assumptions in Hadamard manifolds using subdifferentials. The results can be improved using convexificators, a weaker version to subdifferentials. We study equilibrium problems using convexificator. We consider the following MMVIEP. Let  $X, Y$  be real Banach spaces,  $L(X, Y)$  denote the set of all continuous linear mappings from  $X$  to  $Y$ . Suppose  $K$  is a nonempty closed convex subset of  $E$ ,  $P \subset Y$  is a closed convex pointed cone of  $Y$ ,  $\phi : K \times K \rightarrow 2^L(X, Y)$ ,  $b : K \times K \rightarrow Y$  and  $\eta : K \times K \rightarrow X$ , then MMVIEP is defined as,

$$\text{find } w \in K : \sup_{w^* \in \phi(w)} (w^*, \eta(v, w)) + b(w, w) - b(w, v) \leq_{\text{int}P} 0, \text{ for all } v \in K. \quad (1)$$

We formulate both SMVIEP and MMVIEP in terms of convexificators and discuss some results in connection with VOP. (Received August 28, 2016)

1125-90-394

**Diego Cifuentes\*** ([diegcif@mit.edu](mailto:diegcif@mit.edu)), 77 Massachussets Avenue, Office 32-D760, Cambridge, MA 02139, and **Pablo A Parrilo**. *Sampling Algebraic Varieties for Sum of Squares Programs.*

We study sum of squares (SOS) relaxations to optimize polynomial functions over a set  $\mathcal{V} \cap \mathbb{R}^p$ , where  $\mathcal{V}$  is a complex algebraic variety. We propose a new methodology that, rather than relying on some algebraic description, represents  $\mathcal{V}$  with a generic set of complex samples. This approach depends only on the geometry of  $\mathcal{V}$ , avoiding representation issues such as multiplicity and choice of generators. It also takes advantage of the dependencies in the coordinate ring of  $\mathcal{V}$  to reduce the size of the corresponding semidefinite program (SDP). In addition, the input can be given as a straight-line program. Our methods are particularly appealing for varieties which are easy to sample from but for which the defining equations are complicated, such as  $SO(n)$ , Grassmannians or rank  $k$  tensors. Nonetheless, for arbitrary varieties we can obtain the samples by using the tools of numerical algebraic geometry. In this way we connect the areas of SOS optimization and numerical algebraic geometry. (Received August 31, 2016)

1125-90-701

**Balendu Bhooshan Upadhyay\*** ([bhooshan@nitmanipur.ac.in](mailto:bhooshan@nitmanipur.ac.in)), Assistant Professor, Department of Mathematics, National Institute of Technology Manipur, Imphal, 795004, India. *On relations between vector variational inequality and nonsmooth vector optimization problems via strict minimizers.*

This paper deals with the relations between vector variational inequality problems and nonsmooth vector optimization problems using the concepts of efficient and strict minimizers of order  $m$  in terms of Clarke subdifferential. Gordan's theorem of alternative is employed to identify the vector critical points, the strict minimizers of

order  $m$  and the solutions of the weak vector variational inequality problems under generalized strong convexity assumptions. The results presented in this paper are more general than those existing in the literature. In particular, the results of the paper extend and unify some earlier results to the nonsmooth case as well as to a more general class of functions. (Received September 09, 2016)

1125-90-1009      **Thang Huynh\*** (tlh007@ucsd.edu), tlh007@ucsd.edu. *Phase Retrieval: under Corruption or with Quantization.*

Phase Retrieval refers to a problem of recovering a signal from its phaseless measurements. It is a very challenging problem arising in many real-life applications, e.g. X-ray crystallography. The problem is even more difficult in the presence of corrupted measurements due to sensor failures or sensor saturation. In the first part of the talk, I will show that an  $\ell_1$ -version of the PhaseLift algorithm can successfully solve the phase retrieval problem with high probability despite an overwhelming number of outliers, provided that there are enough measurements. This is joint work with Paul Hand (Rice University).

Another of interest problem is how to quantize the phaseless measurements. For the second part of the talk, I will discuss how the distributed noise-shaping method of Chou and Gunturk can be extended to the quantization problem of phaseless measurements and will show that a suitably modified version of the PhaseLift algorithm guarantees near-optimal error performance. This is joint work with Sinan Gunturk (NYU) and Halyun Jeong (NYU). (Received September 15, 2016)

1125-90-1555      **Clarissa Tahnise McMillar\*** (mcmillar\_c06478@utpb.edu). *Studying the Traveling Salesman Problem.*

In 1971, Stephen Cook and Leonid Levin described two classes of problem known as “polynomial” (P) and “non-deterministic polynomial” (NP). The vast majority of problems are P. The Clay Mathematics Institute describes the solutions to these problems as “easy to find”, meaning that an answer exists and can be found within a “reasonable” amount of time. The other set of problems known as NP are described as “easy to check”, but due to the amount of possible solutions, the answer is hard to find. The question that plagues mathematicians and computer scientists is whether  $P=NP$ . If so, an algorithm exists that can solve any NP problem, but it has yet to be found.

The “Traveling Salesman Problem” is a prime example of this debate. The TSP asks for the best possible Hamiltonian circuit i.e. tour to be found among a large number of possible routes. Since it is a Hamiltonian circuit each city can only be visited once with exception to the origin. This research project studies the different approaches used to find the optimal solution to a TSP using methods such as brute force and other algorithms. (Received September 18, 2016)

1125-90-1566      **Sabyasachi Pani\*** (spani@iitbbs.ac.in), Indian Institute of Technology Bhubaneswar, Bhubaneswar, Odisha 751024, India, and **Bijaya Kumar Sahu** (bks10@iitbbs.ac.in), Indian Institute of Technology Bhubaneswar, Bhubaneswar, Odisha 751024, India. *Extended Complementarity Problems.*

In this paper we define an extended generalized complementarity problem in a locally convex, Hausdorff, vector-topological space and show that, the extended generalized complementarity problem is equivalent to a generalized variational inequality problem. We prove an existence result of complementarity problem in this setting and observed that many of the known results follow from it. Finally we give an existence result for this problem in finite dimensional case under the assumption that the function is strongly K-copositive. (Received September 18, 2016)

1125-90-1758      **Pankaj Gupta\***, pankajgpta@gmail.com. *Possibilistic Optimization: Some Recent Developments.* Preliminary report.

Zadeh (Fuzzy Sets and Systems, 1978) presented the theory of possibility related to the theory of fuzzy sets by defining the concept of a possibility distribution as a fuzzy restriction, which acts as an elastic constraint on the values that can be assigned to a variable. We discuss the widely used possibilistic approach given by Lai and Hwang (Fuzzy Sets and Systems, 1992) that convert the fuzzy objective with a triangular possibility distribution into three crisp objectives corresponding to the three critical values (the most possible, the pessimistic, and the optimistic values). The approach has many advantages over other approaches from the literature. In this talk, we also report some new developments in possibilistic programming approach given by Gupta and Mehlaawat (IEEE Transactions on Fuzzy Systems, 2014) for solving optimization problems wherein the solution approach simultaneously minimizes the best scenario, the likeliest scenario, and the worst scenario for the imprecise objective functions using  $\alpha$ -level sets that define confidence level of the fuzzy judgements of the decision maker. (Received September 19, 2016)

1125-90-1764 **Mukesh Kumar Mehlawat\***, mukesh0980@yahoo.com. *Portfolio Optimization: Some Recent Developments*. Preliminary report.

We discuss some classes of portfolio optimization problems treated through various portfolio optimization models. Most basic portfolio optimization problems are based on mean-variance optimization models corresponding to return and risk preferences of the investor. These mathematical portfolio optimization problems are either the quadratic programming or linear parametric programming problems. Besides the return and risk preferences there could be other preferences of the investor based on more important criteria such as transaction costs. The extensions of the classical mean-variance portfolio optimization model have been proposed by considering alternative measures of risk and many realistic criteria other than the return and risk in order to arrive at better financial decisions. Furthermore, to deal with uncertainty, the major emphasis has been given to fuzzy set theory concepts for building portfolio optimization models using fuzzy variables. We will focus on certain issues highlighted as above to present some major contributions of the recent literature from the field. (Received September 19, 2016)

1125-90-2700 **Thy M. Tran, James P. Ferry, John D. Foley\*** (foley@metsci.com) and **Gregory A. Godfrey**. *Multi-commodity flow applied to tracking multiple targets*.

Tracking multiple objects of interest (targets) over time is a basic problem in automated image processing and surveillance. With the increasing availability of data, the problem of stitching together short-history tracks (known as tracklets) into longer-history tracks has emerged. Castañón and Finn reframe multi-target tracklet stitching as a minimum cost flow problem over a graph by introducing a memoryless assumption for tracklet sequences. In particular, a graph flow of minimal cost corresponds to an assignment of short-history tracklets to long-history tracks of greatest likelihood. To leverage both persistent target data and memoryless tracklets, we formulate a multi-target tracking problem that corresponds to a minimum cost multi-commodity flow problem under appropriate assumptions. (Received September 20, 2016)

1125-90-2848 **Rodrigue M Djikeuchi\*** (rodji2@morgan.edu), 1638 Hopewell Ave, Essex, MD 21221, and **Isabelle Kemajou-Brown**, 1700 E. Cold Spring Lane, Canergie Hall suite 152, Baltimore, MD 21251. *Modeling Portfolios using Big Data: Histories and Weight Presentation*.

Every investment has a risk associated with it. Depositing money at the bank has a lower risk than hiding cash under a mattress. However, a FDIC (Federal Deposit Insurance Corporation) insured bank account generally offer a lower return than any other investment such as equities (stocks), debt (bonds) or commodities (corn, oil, etc.). This paper addresses the optimization of a market risk management given an exposure by modeling asset portfolios. The asset portfolio modeling will consist of using Markowitz Frontiers, Efficient Frontiers and Long Frontiers. A frontier is a group of optimal portfolios offering the highest return for a well-defined level of risk or the lowest risk for a certain level of expected return. This paper will use real asset data, including big data, to compute the long frontier and observe how it evolves within a defined time period. Furthermore, with the consideration of a group of assets, we will examine how adding more asset data influences the frontier curve. Furthermore, we will explore how some analytical factors can influence the choice of the risk aversion coefficient. (Received September 20, 2016)

## 91 ► *Game theory, economics, social and behavioral sciences*

1125-91-452 **Jan Rychtar\*** (rychtar@uncg.edu). *Territorial movement game*.

The important biological problem of how groups of animals should allocate themselves between different habitats, has been modeled extensively. Such habitat selection models have usually involved infinite well mixed populations. In particular the problem of allocation over a number of food patches when movement is not costly, the ideal free distribution (IFD) model is well-developed. Here we generalize (and solve) a habitat selection game for a finite structured population. We show that habitat selection in such a structured population can have multiple stable distributions (in contrast to the equivalent IFD model that is practically unique). We also define and study a "predator dilution game" where unlike in the habitat selection game, individuals prefer to aggregate (to avoid being caught by predators due to the dilution effect) and show that this model has a unique solution when movement is unrestricted. (Received September 02, 2016)

1125-91-492 **Ngoc Mai Tran** and **Josephine Yu\***, [jyu@math.gatech.edu](mailto:jyu@math.gatech.edu). *Product-Mix Auctions and Tropical Geometry*.

In a recent and ongoing work, Baldwin and Klemperer explored a connection between tropical geometry and economics. They gave a sufficient condition for the existence of competitive equilibrium in product-mix auctions of indivisible goods. This result, which we call the Unimodularity Theorem, can also be traced back to the work of Danilov, Koshevoy, and Murota. We will introduce auction theory, prove of the Unimodularity Theorem, and discuss special cases such as stable matching with transferable utility . (Received September 03, 2016)

1125-91-650 **Yuanying Guan\*** ([guany@iun.edu](mailto:guany@iun.edu)), Department of Mathematics, Indiana University Northwest, 3400 Broadway, Gary, IN 46408. *Asset Pricing Models with Heterogeneous Behavioral Investors*. Preliminary report.

In the standard framework of classic asset pricing models, investors in financial markets are often assumed to be rational: individuals always try to optimize their expected utility functions based on complete information. In addition, individuals' decisions are not affected by change of descriptions of decision problems. This traditional setting in asset pricing models provides us convenience to study the mechanics of investment markets and solve the equilibrium prices. However, the perspective of markets with only rational individuals is far from empirical evidences such as high volatility of stock markets, high trading volumes, and pricing bubbles, etc.

Behavioral economics is a relatively new field that addresses these sorts of issue. Rather than assume that people generally know what is best for them and make decisions consistent with that knowledge, it acknowledges that people often do not act rationally in the economic sense. Inspired by some recent research (Barberis and Huang, 2009; He and Zhou, 2014), we incorporate heterogeneous behavioral investors into the traditional Lucas asset-pricing model and analyze how that would affect the market dynamics and the volatility of stock prices. (Received September 08, 2016)

1125-91-856 **Jason Callahan\*** ([jasonc@stedwards.edu](mailto:jasonc@stedwards.edu)), St. Edward's University, 3001 South Congress Ave, Austin, TX 78704. *Chutes, Ladders, and Markov Chains*.

Using Markov chains, we prove a conjecture that on any *Chutes and Ladders* board with  $n$  squares, uniformly distributed spinners of range  $n - 1$  and  $n$  always yield equal expected game lengths. We then show that non-uniformly distributed spinners can yield shorter games than uniformly distributed spinners but can also lead to seemingly paradoxical results when the related Markov chain is not absorbing. (Received September 12, 2016)

1125-91-858 **Jason Callahan\*** ([jasonc@stedwards.edu](mailto:jasonc@stedwards.edu)), St. Edward's University, 3001 South Congress Ave, Austin, TX 78704. *Analysis of a Coupled,  $n$ -Patch Population Model with Ceiling Density Dependence*.

We consider a system of difference equations with ceiling density dependence to model the dynamics of a coupled population on an arbitrary, finite number of distinct patches where migration between all patches is possible. In this model, each patch possesses a separate carrying capacity, and the dynamics of the coupled population is governed by a linear model until the population of a patch reaches its capacity after which it remains at this maximum value. Further, we analyze the global attractors of this model and apply these results to an Arabian oryx metapopulation model with some patches protected and others unprotected from poaching. (Received September 12, 2016)

1125-91-1038 **Kristina B. Hilton\*** ([khilton@mail.usf.edu](mailto:khilton@mail.usf.edu)), Department of Mathematics and Statistics, University of South Florida, 4202 E. Fowler Ave., CMC 342, Tampa, FL 33620-5700, and **G. S. Ladde** ([gladde@usf.edu](mailto:gladde@usf.edu)), Department of Mathematics and Statistics, University of South Florida, 4202 E. Fowler Ave., CMC 342, Tampa, FL 33620-5700. *Mutli-Cultural Networks Under Internal and External Stochastic Perturbations*. Preliminary report.

Using Lyapunov type functions and comparison theorems, we explore the dynamics of a multi-cultural network under random environmental and structural perturbations. We examine the long-term behavior of members of the network. Moreover, we characterize various cultural state invariant sets within the network. Further, we present numerical models as a means of comparison between simulation results and various type of computational bounds. (Received September 14, 2016)

1125-91-1157 **Robert Ghrist\*** ([ghrist@math.upenn.edu](mailto:ghrist@math.upenn.edu)) and **Sanjeevi Krishnan** ([sanjeevi@math.osu.edu](mailto:sanjeevi@math.osu.edu)). *Directed sheaf co/homology for pursuit-evasion games*.

Considered is a class of pursuit-evasion games, in which an evader tries to avoid detection. Such games can be formulated as the search for sections to the complement of a coverage region in a Euclidean space over a timeline. Prior results give homological criteria for evasion in the general case that are not necessary and sufficient. This will detail a new necessary and sufficient positive cohomological criterion for evasion in a general case. The

principal tools are a cone-valued positive sheaf co/homology theory, combined with Alexander duality. We show how a cellular version of this sheaf over the time axis makes positive cohomology (and the evasion criterion) computable as a linear program. (Received September 15, 2016)

1125-91-2560 **David P Ebert\*** ([david.ebert@go.tarleton.edu](mailto:david.ebert@go.tarleton.edu)). *A genetic algorithm approach to finding an optimal strategy for a folk dice game.* Preliminary report.

This research discusses steps to formulating a genetic algorithm for optimizing strategy for Fargo, a multiplayer folk dice game. After introducing Fargo and describing its similarities to other multiplayer dice games, the research discusses the space of strategy vectors in the one-player Fargo game. Next, a recursive algorithm is developed for finding the expected value of a strategy vector. Finally, a genetic algorithm searches through the strategy vector space to find strategies that maximize the expected value. The results of the genetic algorithm clearly point toward an optimal strategy in the simplified version of the game, with likely future extensions to the Fargo game ending and multiplayer Fargo. (Received September 20, 2016)

## 92 ► *Biology and other natural sciences*

1125-92-78 **Tufail M Malik\***, [tufail.malik@kustar.ac.ae](mailto:tufail.malik@kustar.ac.ae), and **M Imran** and **R Jayaraman**. *Optimal control with multiple human papillomavirus vaccines.*

Many hpv vaccination programs now administer three vaccines: bivalent, quadrivalent and the recently marketed nonavalent vaccine. A model will be presented to explore optimal vaccination strategies using the three vaccines, which differ in protection breadth, cross-protection, and type-specific efficacy. Assuming the HPV infection prevalence in the population under the constant vaccination regime, optimal control theory will be used to discuss optimal vaccination strategies for the associated non-autonomous model when the vaccination rates are functions of time. The impact of these strategies on the number of infected individuals and the accumulated cost will be assessed and compared with the constant control case. Switch times from one vaccine combination to a different combination including the nonavalent vaccine will be assessed during an optimally designed HPV immunization program. (Received July 12, 2016)

1125-92-98 **Jean MS Lubuma\*** ([jean.lubuma@up.ac.za](mailto:jean.lubuma@up.ac.za)), Dept of Mathematics and Applied Mathematics, Faculty of Natural and Agricultural Sciences, University of Pretoria, Pretoria, Gauteng 0002, South Africa, **Mataeli B Lerata** ([mataelilerata@gmail.com](mailto:mataelilerata@gmail.com)), Dept of Mathematics and Applied Mathematics, University of Pretoria, Pretoria, Gauteng 0002, South Africa, and **Abdullahi Yusuf** ([aayusuf@zoology.up.ac.za](mailto:aayusuf@zoology.up.ac.za)), Dept of Zoology and Entomology, University of Pretoria, Pretoria, Gauteng 0002, South Africa. *Continuous and discrete models for the declines of honeybee colonies.*

The alarming declines of the population of managed honeybees, specifically the Colony Collapse Disorder (CCD), constitute a serious threat to agriculture. To Khoury, Myerscough and Barron (KMB) model [PlosOne, 6(4), e18491, (2011)], we add two models. The first one is a limit of the KMB model in that the recruitment rate is a constant representing either a positive lower bound or a negative upper bound of the recruitment rate. In the second model, motivated by the Capensis calamity in South Africa, we describe the parasitic scenario by a lower recruitment rate. We show, for the KMB model and its limit, that there exists a critical value of the foragers' death rate, which is a transcritical bifurcation. In particular, above this value, the trivial equilibrium point is globally asymptotically stable, i.e. there is CCD. When the recruitment rate is negative, the CCD arises irrespective of the death rate of foragers. Equally, for the social parasite model, we prove the unconditional occurrence of the CCD. The underlying nonlinearity in the three models is a Holling function of type 2. Inspired by our earlier work on the Michaelis-Menten equation, we construct nonstandard finite difference schemes that are dynamically consistent with the continuous models. (Received July 25, 2016)

1125-92-156 **Michael Malisoff\*** ([malisoff@lsu.edu](mailto:malisoff@lsu.edu)), 301 Lockett Hall, Department of Mathematics, Louisiana State University, Baton Rouge, LA 70803-4918. *Stabilization in a Chemostat with Sampled and Delayed Measurements.*

We study chemostat models with constant substrate input concentrations. We allow growth functions that are not necessarily monotone. The measurement is the substrate concentration, which is piecewise constant with a nonconstant delay, so only sampled observations are available. Under new conditions on the size of the delay and on the largest sampling interval, we solve the problem of asymptotically stabilizing a componentwise positive equilibrium point with the dilution rate as the control. We use a new Lyapunov approach. This talk is based

on the speaker's joint research with Jerome Harmand and Frederic Mazenc, and was sponsored in part by the speaker's NSF Directorate for Engineering Grant 1408295. (Received August 05, 2016)

1125-92-160 **Arni S.R. Srinivasa Rao\*** (arrao@augusta.edu), Augusta, GA 30909, and **James R. Carey**, Davis, CA. *From Fibonacci to Alfred Lotka and beyond: Modeling the dynamics of population and age-structures.*

The subject of population dynamics is hundreds of years old and is been studied by famous mathematicians such as Fibonacci, d'Alambert Daniel Bernoulli, Euler, etc, Concepts such as stability and stationarity of population are essential pillars of population dynamics. In the last century the works by Alfred Lotka laid the foundation for the population stability theory, which was developed further by William Feller through renewal equations. Ansley Coale and Norman Ryder (during 1960s and 1970s) brought several properties of stationary populations from the Life Table perspective. During last decade new identities of stationary population have emerged due to Carey's Equality (early 2000s). James Carey's experimental work and deeper insights helped to discover newer perspectives of stationary populations by Vaupel (2009) and Goldstein (2009). Rao and Carey (2013/2015) have proved a fundamental theorem in stationary population using insights from Carey's equality by blending with algebraic and combinatory principles. These newer results bring similar patterns that are comparable to renewal type of theory due to Lotka, Feller and others. (Received August 05, 2016)

1125-92-183 **Laura Waller\***, waller@berkeley.edu. *Computational microscopy for phase retrieval.*

Computational imaging is the joint design of imaging system hardware and software, optimizing across the entire pipeline from acquisition to reconstruction. This talk will describe computational imaging methods for fast capture and reconstruction of Gigapixel-scale phase reconstructions. We use coded illumination and large-scale nonlinear non-convex optimization procedures to solve large-scale inverse algorithms that reconstruct phase with resolution beyond the diffraction limit of the microscope. Beyond theory, we discuss the implications of developing practical algorithms for real-world imaging systems, where forward models are inexact and experimental misalignment and errors can cause severe artifacts. In this light, we review the relative robustness of various proposed algorithms. Finally, we describe extensions that implement algorithmic self-calibration - joint estimation of both the object of interest and parametric aberration and mis-alignment corrections. (Received August 09, 2016)

1125-92-237 **Robert Stephen Cantrell\*** (rsc@math.miami.edu), Department of Mathematics, The University of Miami, Coral Gables, FL 33124, and **King-Yeung Lam**, **Xinru Cao** and **Tian Xiang**. *Fitness based prey dispersal and prey persistence in intraguild predation systems.*

We establish prey persistence in intraguild predation systems in bounded habitats under mild conditions when the prey disperses using its fitness as a surrogate for the balance between resource acquisition and predator avoidance. The model is realized as a quasilinear parabolic system where the dimension of the underlying spatial habitat is arbitrary. (Received August 16, 2016)

1125-92-246 **Nida K. Obatake\*** (nobatake@tamu.edu), **Elizabeth Gross** (nobatake@tamu.edu) and **Nora E. Youngs** (nobatake@tamu.edu). *Rat GPS: Drawing Place Field Diagrams of Neural Codes Using Toric Ideals.*

A rat has special neurons that encode its geographic location. These neurons are called place cells and each place cell points to a region in the space, called a place field. Neural codes are collections of the firing patterns of place cells. In this talk, we investigate how to algorithmically draw a place field diagram of a neural code, building on existing work investigating neural codes, ideas developed in the field of information visualization, and the toric ideal of a neural code. (Received August 18, 2016)

1125-92-331 **Suzanne Lenhart\*** (lenhart@math.utk.edu), NIMBioS, University of Tennessee, Knoxville, TN 37996-3410. *Modeling Hantavirus Among Rodents in Paraguay.* Preliminary report.

Hantavirus in rodents are zoonotic pathogens that can cause disease in humans through inhalation of rodent excreta. Using data collected from a survey of rodents in a reserve in Paraguay to formulate and parameterize a mathematical model, we investigate the prevalence of the Jabora virus over time within its rodent reservoir by using multiple age classes and a unique infection class progression feature. This model incorporates three types of infection over the lifetime of the rodent as well as a recovered class. A new feature of the model allows regression from the latent to the persistently infected class. This project was a part of the Summer Research Experiences for Undergraduates program at the National Institute for Mathematical and Biological Synthesis. (Received August 27, 2016)

1125-92-410 **Garri Davydyan\*** ([garri.davydyan@gmail.com](mailto:garri.davydyan@gmail.com)), Ottawa Hospital, Ottawa, Ontario K1H 8L6, Canada. *Split quaternions and carcinogenesis.*

Basis elements of the imaginary part of split quaternions comprise a Lie algebra  $\mathfrak{sl}(2, \mathbb{R})$ . There is a direct correspondence between the three basis elements of  $\mathfrak{sl}(2, \mathbb{R})$  and functional patterns of biologic systems. The patterns represented by second order matrices are negative feedback, positive feedback and reciprocal links. This is supported by the cardinal property of biologic systems- functional stability. Indeed, each of the basis matrices determines an evolution of subsystem, that occurs on the surface of the same energy level. There are numerous pathological conditions making biological systems unstable. Among them a cancer characterizes by invasive cells proliferation. Cell proliferation which lost reciprocity with apoptosis results in the deterioration of reciprocal relations between systems components. Formally it can be described as a transformation of the operator of reciprocal links to the unit operator 1, where -1 entry is replaced by +1. The unit element 1 considered as an operator of ODE determines an evolution of unsteady systems which escape the constant energy level. If added to the basis of  $\mathfrak{sl}(2, \mathbb{R})$  the obtained split quaternion structure will provide a functional frame for describing deteriorating behaviour of biological systems. (Received August 31, 2016)

1125-92-440 **Payson Merrit Lippert\***, [lippertpayson@rocketmail.com](mailto:lippertpayson@rocketmail.com), and **Laurie Battle**. *Impact of Light and Temperature on the Transpiration Ratio of Plants.*

A plant's efficiency is affected by different factors including light, temperature, water, and carbon dioxide uptake. When the stomata open to allow uptake of carbon dioxide for photosynthesis, water is lost through transpiration, and plants control this balance by opening and closing the stomata based on environmental conditions. While there are many factors that contribute to the loss of water and carbon dioxide uptake, the only factors used here are light and temperature. All other environmental factors were constant for this study, including soil moisture and relative humidity. One measure of efficiency is the transpiration ratio, defined as the rate of water loss through transpiration divided by the rate of carbon dioxide uptake. This balance is most efficient when the transpiration ratio is at its lowest. Three models were produced for the transpiration ratio as a function of light, temperature, and both light and temperature. Applying these models to potatoes, for instance, yielded a method to estimate the transpiration ratio according to the temperature of the leaf and the amount of radiance. Using these models, one can discover the ideal light and temperature conditions for a plant to be most efficient. (Received September 01, 2016)

1125-92-451 **Suzanne Lenhart\*** ([lenhart@math.utk.edu](mailto:lenhart@math.utk.edu)), National Institute for Math and Bio Synthesis, Knoxville, TN 37996-3410, and **Kokum DeSilva** and **Shigetoshi Eda**. *Modeling environmental transmission of MAP infection.*

Johne's disease is a progressive bacterial disease in dairy cows, that reduces milk production in infected cows. We develop a system of ordinary differential equations to describe the dynamics of this disease in a dairy farm. This model includes the progression of the disease and the age structure of the cows. To investigate the effect of persistence of this bacteria on the farm on transmission, we include an environmental compartment and representing the pathogen input in an explicit way. The effect on the environmental transmission and the culling of high-shedding adults can be seen in the basic reproduction number and in the numerical simulations. Since culling usually only happens once a year, we included a novel feature in the simulations with a discrete action of removing high-shedding adults once a year. (Received September 02, 2016)

1125-92-457 **Amber M Smith\*** ([amber.smith@stjude.org](mailto:amber.smith@stjude.org)). *Modeling Host-Pathogen Interactions During Influenza-Pneumococcal Coinfection.*

Influenza virus infections are often complicated by coinfection with bacterial pathogens like pneumococcus. Bacterial coinfections significantly increase influenza-associated morbidity and mortality. The host immune response plays a large role in driving bacterial establishment and the progression to pneumonia. However, how the host response is regulated and works to modulate pathogen growth is not well understood. To better characterize the regulatory mechanisms driving influenza-pneumococcal coinfection, we use an integrative analysis that combines data-driven mathematical models with model-driven experiments. Through this approach, we identified and detailed how virus-induced depletion of alveolar macrophages regulates bacterial invasion and leads to differential dynamics dependent on the time of bacterial acquisition. We also show how bacterial infection alters viral kinetics and the role of interferon in this process. Together, our models and data provide insight into the kinetics and mechanisms of coinfection and identify potential treatment strategies that abrogate the secondary infection. (Received September 02, 2016)

1125-92-464 **Ronald E. Mickens\*** ([rmickens@cau.edu](mailto:rmickens@cau.edu)), Department of Physics, Clark Atlanta University, Atlanta, GA 30314. *Analysis of an SIR Model with Population Dynamics.*

We construct an SIR model for which the susceptible population can grow (in and of itself), and the removed population can die off. This model also has the feature that the infective population (in and of itself) reaches zero in a finite time. There is one nontrivial fixed-point and we determine both its local and global stability properties. Of significance is that with our particular selection for the transition/interaction terms, the mathematical model can be solved exactly. For purposes of calculating numerical solutions, in the absence of the analytical solution, a NSFD discretization is formulated. (Received September 02, 2016)

1125-92-490 **Abba Gumell\***, School of Mathematical and Stat. Sciences, Arizona State University, Tempe, AZ 85287. *Construction of Dynamically-consistent Discrete-time Models Arising in the Natural and Engineering Sciences.*

Continuous-time models associated with the mathematical modeling of phenomena arising in the natural and engineering sciences often require the preservation of certain intrinsic physical properties, such as the positivity, boundedness and asymptotic properties of the associated solutions. This talk will discuss some of the new results and challenges pertaining to the design of Mickens-type dynamically-consistent, nonstandard finite-different discrete models for approximating the solutions of some continuous-time models arising in the natural and engineering sciences. (Received September 03, 2016)

1125-92-508 **Mitchell J Eithun\*** ([eithunm@ripon.edu](mailto:eithunm@ripon.edu)) and **Anne Shiu** ([annejls@math.tamu.edu](mailto:annejls@math.tamu.edu)). *An All-Encompassing Global Convergence Result for Processive Multisite Phosphorylation Systems.*

Phosphorylation, the enzyme-mediated addition of a phosphate group to a molecule, is a ubiquitous chemical mechanism in biology. Multisite phosphorylation, the addition of phosphate groups to multiple sites of a single molecule, may be distributive or processive. Distributive systems can be bistable, while processive systems were recently shown to be globally stable. However, this global convergence result was proven only for a specific mechanism of processive phosphorylation/dephosphorylation (namely, all catalytic reactions are reversible). Accordingly, we generalize this result to allow for processive phosphorylation networks in which each reaction may be irreversible, and also to account for possible product inhibition. We accomplish this by defining an all-encompassing processive network that encapsulates all of these schemes, and then appealing to recent results of Marcondes de Freitas, Wiuf, and Feliu that assert global convergence by way of monotone systems theory and network/graph reductions (which correspond to removal of intermediate complexes). Our results form a case study into the question of when global convergence is preserved when reactions and/or intermediate complexes are added to or removed from a network. (Received September 04, 2016)

1125-92-544 **Pengcheng Xiao\*** ([px3@evansville.edu](mailto:px3@evansville.edu)), 1800 Lincoln Ave, Evansville, IN 47630, and **Jianzhong Su** ([su@uta.edu](mailto:su@uta.edu)), 701 South Nedderman Dr, Arlington, TX 76019. *A calcium time course based computational model for acute stress related cortisol dynamics and synaptic plasticity.* Preliminary report.

How acute stress can affect human cognitive functions has been a very popular topic for researchers from different fields including physiology, psychology, biology, neuroscience, and applied mathematics. Hypothalamic-pituitary-adrenal (HPA) axis plays an important role in response to stress by releasing hormones, and level of glucocorticoid has been widely considered to be one key factor to distinguish people with different stress disorder. Emerging evidence has shown that glucocorticoid act on glutamate neurotransmission system and consequently influences neuronal activities's cognitive function. It changes in the glutamate release and induces synapse plasticity change. Spike-Timing Dependent Plasticity (STDP) is one of the important neuroscience foundations for cognitive function. In this paper, we incorporate the HPA axis and CA1 neuron models to explore the plasticity outcome based on divergent type of acute stress. Various of spikes will be applied to test the Spike-Timing Dependent Plasticity in different durations. The results in different facets show that CA1 neuron potentiation changes due to different stress input. (Received September 06, 2016)

1125-92-573 **Deborah Shutt\***, 1500 Illinois, Golden, CO 80232, and **Carrie Manore, Sara Y. Del Valle, Aaron Porter** and **Stephen Pankavich.** *Determining important Zika outbreak characteristics via Approximate Bayesian Computation for epidemics in South and Central America.* Preliminary report.

Mosquito-borne diseases contribute significantly to the overall morbidity and mortality caused by infectious diseases in Central and South America. Newly emergent pathogens such as Zika virus in 2015 highlight the need for data and models to help understand the public health impact and develop mitigation strategies. Since

Zika virus is newly emerging in the Americas, its impact on the naive population is unknown. We developed a mathematical model for Zika dynamics in Colombia, El Salvador, and Suriname. The number of people to contract the disease depends on factors such as climate, elevation, population density, sanitation and other socioeconomic and environmental factors. Therefore, the size of the susceptible at-risk demographic of a country may not be the entire population. Since dengue and Zika overlap geographically and share a common vector, *Aedes aegypti*, we used a calculated at-risk population size based on historically reported incidence rates for dengue from Colombia, El Salvador, and Suriname. We then used the Approximate Bayesian Computation method to estimate parameter distributions and calculate the reproductive number based on the Pan-American Health Organization's publicly available data. (Received September 06, 2016)

1125-92-633 **T.Mihiri M. De Silva\*** ([mihiri.de-silva@ttu.edu](mailto:mihiri.de-silva@ttu.edu)), Texas Tech University, Department of Mathematics and Statistics, Broadway and Boston, Lubbock, TX 79409, and **Sophia R Jang** ([sophia.jang@ttu.edu](mailto:sophia.jang@ttu.edu)), Texas Tech University, Department of Mathematics and Statistics, Broadway and Boston, Lubbock, TX 79409. *Stochastic Modeling of Phytoplankton-Zooplankton Interactions.*

To investigate random biological effects on the plankton interactions, we extend a previous well studied deterministic phytoplankton-zooplankton model by considering demographic stochasticity. Specifically, continuous-time Markov chain models and stochastic models of Itô differential equations are investigated. There are two phytoplankton populations where one population is toxin producing (TPP) while the other is not (NTP). The zooplankton prey on both the phytoplankton populations with the growth rate of the zooplankton inhibited by the TPP but not by the NTP. Mutual interference among the predator zooplankton and avoidance of zooplankton on TPP are incorporated into the model. We compare both types of stochastic models with the deterministic system using numerical simulations. It is found that the theoretical results established in the deterministic setting may not be valid in the stochastic models due to random effects of the birth and death process imbedded in the populations. (Received September 08, 2016)

1125-92-642 **Adrienne Kinney, Abby Quirk-Royal and Ellen Swanson\*** ([ellen.swanson@centre.edu](mailto:ellen.swanson@centre.edu)). *Modeling the Progression of Alzheimer's Disease.*

Alzheimer's disease is a progressive brain disorder that affects over 5 million people in the United States every year. The disease causes neuron connections to deteriorate within the brain and eventually death of the brain cells. Researchers have identified abnormalities in the brain of people with Alzheimer's, including amyloid-beta plaques and tau protein tangles. We will present a mathematical model of seven coupled ordinary differential equations that describes the effect of amyloid-beta in the brain. We will further explore this model to better understand the degeneration of the brain from this disease. (Received September 08, 2016)

1125-92-684 **Eva Marie Strawbridge\*** ([strawbem@jmu.edu](mailto:strawbem@jmu.edu)), MSC 1911, James Madison University, Harrisonburg, VA 22807. *Past, Current, and Potential Interdisciplinary Undergraduate Research Problems in Math Biology.*

In this talk I will describe how we have incorporated experiment and mathematical theory together using theoretical, computational and experimentally driven questions in the JMU WORM Lab as well as some of the past successful and current projects which have come out of this group. I will also discuss some potential open and accessible avenues of undergraduate research in this area and how I would approach them as an undergraduate research mentor. (Received September 09, 2016)

1125-92-737 **Maria-Veronica Ciocanel\*** ([veronica\\_ciocanel@brown.edu](mailto:veronica_ciocanel@brown.edu)), Brown University, Box F, Providence, RI 02912, and **Bjorn Sandstede** and **Kimberly Mowry**. *PDE models for messenger RNA localization in *Xenopus* (frog) oocytes.*

Messenger RNA (mRNA) localization is essential for *Xenopus* (frog) egg cell development and embryo patterning. This accumulation of RNA at the cell periphery is not well understood, but is thought to depend on diffusion, bidirectional movement and anchoring mechanisms. Our goal is to test these proposed mechanisms using dynamical systems analysis of linear and nonlinear PDE systems, informed by numerical parameter estimation. The model analysis allows us to extract asymptotic quantities such as effective velocity and diffusion, and yields approximate traveling wave solutions. The different parameter estimates in various regions of the cell cytoplasm are used to predict localization patterns and timescales for both healthy and mutant egg cells. Our results confirm the hypothesis of bidirectional transport, and suggest new experimental directions. (Received September 11, 2016)

1125-92-761 **Michael Barfield** and **Maia Martcheva\***, Department of Mathematics, University of Florida, Gainesville, FL 32611, and **Necibe Tuncer**, **Robert Holt** and **Yena Kim**. *An Immuno-Eco-Epidemiological Model of Competition*. Preliminary report.

We introduce a novel immuno-eco-epidemiological model where two species compete through interference. Furthermore, one of the species is affected by a pathogen and the competition from the other species diminishes the ability of the first species to mount effective immune response. The infected species is structured by its immune response which is general enough to capture chronic infection, recovery with permanent immunity and recovery with temporary immunity. This is coupled with appropriate between-host model with the same ability. The complete immuno-eco-epidemiological model is of nested type with bidirectional coupling. The system has seven types of equilibria whose existence and stability is governed by the several threshold conditions. We also investigate an ODE version of the chronic model obtained from the assumption that the within-host system equilibrates much faster than the between-host system. We find that the impact of species two on the immune response of species one destabilizes the dynamics and oscillations are possible. (Received September 11, 2016)

1125-92-782 **Erica M Rutter\*** ([erutter@ncsu.edu](mailto:erutter@ncsu.edu)) and **Adam Mahdi**. *Influence of Non-Physiological Blood Pressure Artifacts on Cerebral Autoregulation*. Preliminary report.

Cerebral Auto-regulation (CA) is the mechanism by which cerebral blood flow velocity (CBFV) remains constant despite an abrupt change in arterial blood pressure (ABP). The cerebral autoregulation index (ARI), which is a common assessment tool, uses ABP data as an input to the Tiecks' model [1] to generate a value between 0 (poor autoregulation) and 9 (best autoregulation). ARI are used by physicians to assess patient health.

However, there are many types of non-physiological blood pressure artifacts that can arise during measurement of ABP, such as crimping of the tube measuring ABP or a thrombus in the arterial line. We explore inserting these artifacts into patient ABP measurements and observing the resulting change in ARI values. For each type of artifact, we gradually increase the severity of the artifact and quantify when measured ARI is no longer reliable. We rank the artifacts in terms of their sensitivity to ARI. Lastly, we propose methods to detect and remove these artifacts in ABP.

[1] FP Tiecks, AM Lam, R Aaslid, and DW Newell. Comparison of static and dynamic cerebral autoregulation measurements. *Stroke.*, 26:1014–1019, 1995. (Received September 11, 2016)

1125-92-790 **F. B. Augusto\*** ([fbagusto@gmail.com](mailto:fbagusto@gmail.com)), Ecology and Evolutionary Biology, University of Kansas, Lawrence, KS 66045. *The Transmission Dynamics of a Within-and Between-Hosts Malaria Model*. Preliminary report.

In this paper, we developed a novel deterministic coupled model tying together the effects of within-host and population level dynamics on malaria transmission dynamics. The unique feature of this model is the way the coupling and feedback are done using the various life-history stages of the malaria parasite both in the human host and the mosquito vector. The coupled and the within-human host models have locally asymptotically stable infection- and parasite-free equilibria when the associated reproduction numbers are less than unity; while the population-level model, exhibits backward bifurcation, where the stable disease-free equilibrium coexists with a stable endemic equilibrium. Numerical exploration of the coupled model using a linear function of the mosquito biting rate as feedback functions reveal oscillations which dampen with increasing biting rate amidst the parasite populations within a human host in the presence of the host immune response. We also observed that the oscillations and damping effect observed in the within-human host dynamics feedback into the population level dynamics; which in turn amplifies the oscillations in the parasite population within the mosquito host. (Received September 11, 2016)

1125-92-793 **Cameron J Browne\*** ([cambrowne@louisiana.edu](mailto:cambrowne@louisiana.edu)), Mathematics Department, University of Louisiana at Lafayette, P.O. Box 43568, Lafayette, LA 70504. *Modeling Multi-Epitope HIV/CTL Immune Response Dynamics and Evolution*.

The CTL (Cytotoxic T Lymphocyte) immune response plays a large role in controlling HIV infection. CTL immune effectors recognize epitopes (viral proteins) presented on the surface of infected cells to mediate their killing. The immune system has an extensive repertoire of CTLs, however HIV can evolve resistance to attack at different epitopes. The ensuing arms race creates an evolving network of viral strains and CTL populations with variable levels of cross-reactivity (epitopes shared between virus strains). Motivated by this, we formulate a general ODE model of multi-epitope virus-immune response dynamics. Some special cases for the HIV/CTL interaction network are considered. We characterize the persistent viral strains and immune response in terms of system parameters, along with obtaining stability and uniform persistence results via Lyapunov functions. The results are interpreted in the context of within-host HIV/CTL evolution and numerical simulations are provided.

To conclude, we discuss extensions of the model to a PDE system which incorporates cell-infection age structure. (Received September 11, 2016)

1125-92-828 **Eva Marie Strawbridge\*** ([strawbem@jmu.edu](mailto:strawbem@jmu.edu)), MSC 1911, James Madison University, Harrisonburg, VA 22807. *Development and Analysis of a Worm-like Model*.

The past forty years have witnessed an ever-increasing interest in applications of slender-body dynamics (such as Kirchhoff rod theory), in particular with regard to the shape, movement, or material parameters of biomolecules or materials. In most applications, hydrodynamic interactions (i.e. surface traction often approximated by resistive force theory) have been of utmost importance since the biologically relevant scales usually result in very small Reynolds number. However, the formulation of classical Kirchhoff slender-body assumes no surface traction in the development of the constitutive relation. We will discuss an asymptotic approach to reconciling this apparent inconsistency. We will also discuss the addition of alternative constitutive relations to make this model more applicable to the study of swimming worms. (Received September 12, 2016)

1125-92-866 **Nessy Tania\*** ([ntania@smith.edu](mailto:ntania@smith.edu)), Department of Mathematics and Statistics, 44 College Lane, Northampton, MA 01063. *Modeling Actin Regulations in Motility Structures of Cancer Cells*. Preliminary report.

The actin cytoskeleton is an important part of cellular motility. Expressions of actin regulators are altered in metastatic carcinomas. In this talk, I will discuss an ongoing research on modeling two distinct motility structures involved in cancer metastasis, namely lamellipodia and invadopodia. Actin growth shows distinct dynamics during the formation of the two structures, which suggests that key regulators may be playing different roles. One regulator called cofilin severs capped actin-filaments spurring further growth in the lamellipodia. However, its role in the formation of invadopodia is less clear, and may be highly dependent on the availability of actin monomers. (Received September 12, 2016)

1125-92-874 **Rachelle Bouchat\*** ([rbouchat@iup.edu](mailto:rbouchat@iup.edu)), 210 South Tenth Street, Stright Hall, Room 208, Indian, PA 15705, and **G. Ledder, D. Sylvester** and **J. Thiel**. *Modeling the Sub-Saharan African Disease Onchocerciasis, a Commutative Algebraist's Perspective*. Preliminary report.

In graduate school, we often choose our academic path to study either pure or to study applied mathematics. I chose to focus my career exclusively on pure mathematics; that is, until I participated in the Research Experience for Undergraduate Faculty program in the summer of 2015. During this program, I began working with a group on an applied problem, the mathematical modeling of a disease common to many Sub-Saharan African countries called Onchocerciasis (also known as "River Blindness"). In this talk, I will present some of the work my research group has done thus far and talk about the experience during the transition from being a commutative algebraists to working on an applied problem in mathematical modeling. (Received September 12, 2016)

1125-92-886 **Ami Radunskaya\***, [aer04747@pomona.edu](mailto:aer04747@pomona.edu). *Blocking the immune blockers: helping the immune system fight cancer*.

The immune system attacks cells that are foreign, so it must know how to distinguish "self" from "non-self". One component of this recognition system consists of molecules, or *checkpoints* on certain immune cells that need to be switched on in order to initiate an immune response. Cancer cells can use these checkpoints to evade immune cells. When a checkpoint on an immune cell is bound to its partner protein on another cell, the checkpoint signals to the immune cell that it should leave the bound cell alone. Cancer cells masquerade as "normal" by binding to these checkpoints. In a recent breakthrough, drugs have been developed that target these checkpoints, blocking the cancer cells from binding to them, and allowing the immune cells to attack. However, the immune cells will also fail to recognize normal cells, sometimes causing severe side effects.

This is where mathematics can be useful: how should the immune suppressor blockers be administered so that their effect can be most beneficial and least toxic? In this talk we will discuss how mathematical models of the tumor-immune response can be used to answer this question. The underlying model was developed as a REUF project, and results are due to a large group of collaborators of all ages. (Received September 12, 2016)

1125-92-917 **Ivan Ramirez-Zuniga\*** ([ivr3@pitt.edu](mailto:ivr3@pitt.edu)), **Renee Brady, Charles Puelz, Kamila Larripa, Elisabeth Bangsgaard** and **Mette S. Olufsen**. *The interaction between inflammation and cardiovascular dynamics*.

The acute inflammatory response triggered by infection, trauma or surgery is a delicate system aimed at promoting healing and restoring homeostasis. However, inflammation can result in undesirable effects, such as tissue damage or changes in blood pressure and cardiovascular dynamics. It has been shown that the inflammatory response alters the levels of nitric oxide, a vasodilator. Here, we present a coupled model of the inflammatory

response and the cardiovascular system, with nitric oxide as the connecting link. Our model was calibrated to experimental data that was obtained measuring pro-inflammatory mediators (IL-6, CXCL8, and TNF) and the anti-inflammatory cytokine IL-10, in addition to blood pressure and heart rate, over 8 hours in 20 healthy men, given a low dose of lipopolysaccharide (LPS), an endotoxin stimulating inflammation. (Received September 13, 2016)

1125-92-970 **Matthew A Moreno\*** (mamoreno@pugetsound.edu), 4029 Wheelock Student Center, University of Puget Sound, Tacoma, WA 98416, and **Jason M Graham** and **S Garnier**. *Modeling the Collective Behavior of Ants on Uneven Terrain*. Preliminary report.

Ant foraging behavior is a collective decision making process in which, through individual interactions between ants and pheromone deposition, a colony of ants selects and exploits a path to follow between their nest and a food source. Research into the collective decision making strategies of ants, in addition to characterizing the biological mechanisms and emergent properties of the foraging process, has the potential to be leveraged into applications such as swarm robotics and commercial logistics management. Although ant foraging behavior has been extensively studied on flat terrains, ant foraging over uneven terrains is not well studied. This research presents an individual-based set of differential equations to model ant foraging behavior over uneven terrain in an enclosed arena. This model is employed to investigate the characteristics of foraging paths that ants tend towards when foraging over simple inclines of varying magnitudes. Numerical solutions of the model predict that, over most inclines, ants tend to favor the direct path between nest and food. (Received September 13, 2016)

1125-92-996 **William Sulis\*** (sulisw@mcmaster.ca). *Transient Induced Global Response Synchronization: Dispositional Cellular Automata*.

The relationship between neural activity and the behaviour of an organism is complex and still poorly understood. There have been attempts to model this connection using the notion of synchronization, but the participating neurons are fungible, their activity transient and stochastic, their dynamics highly variable. In spite of this, the behaviour of the organism may be quite robust. The phenomenon of transient induced global response synchronization (TIGoRS) has been used to explain the emergence of stable responses at the global level in spite of marked variability at the local level. TIGoRS is present when an external stimulus to a complex system causes a non-linear increase in concordance of the resulting driven trajectories over a broad range of initial conditions. A 10% input sample can result in a concordance of outputs of more than 90%, even when the underlying system dynamics is time varying and inhomogeneous across the system. This talk reviews the notion of TIGoRS and its expression in several complex systems models including driven cellular automata, cocktail party and in particular dispositional cellular automata. (Received September 13, 2016)

1125-92-1012 **Kamuela E Yong\*** (kamuela.yong@hawaii.edu). *Agent-based mathematical modeling as a tool for estimating Trypanosoma cruzi vector–host contact rates*.

The parasite *Trypanosoma cruzi*, spread by triatomine vectors, affects over 100 mammalian species throughout the Americas, including humans, in whom it causes Chagas' disease. In the U.S., only a few autochthonous cases have been documented in humans, but prevalence is high in sylvatic hosts (primarily raccoons in the southeast and woodrats in Texas). The sylvatic transmission of *T. cruzi* is spread by the vector species *Triatoma sanguisuga* and *Triatoma gerstaeckeri* biting their preferred hosts and thus creating multiple interacting vector–host cycles. The goal of this study is to quantify the rate of contacts between different host and vector species native to Texas using an agent-based model framework. The contact rates, which represent bites, are required to estimate transmission coefficients, which can be applied to models of infection dynamics. In addition to quantitative estimates, results confirm host irritability (in conjunction with host density) and vector starvation thresholds and dispersal as determining factors for vector density as well as host–vector contact rates. (Received September 14, 2016)

1125-92-1030 **Antonio Mastroberardino\*** (axm62@psu.edu), **Javed I Siddique**, **Richard J Braun** and **Daniel M Anderson**. *Tear Film Dynamics: Modeling the Glycocalyx as a Poroelastic Region*.

The human tear film is a complex fluid structure composed of an aqueous layer, an outermost lipid layer, and the glycocalyx, a forest of large transmembrane mucins that provide stability to the ocular surface. We formulate a thin film model based on lubrication theory and mixture theory in order to understand the dynamics between the aqueous layer and the glycocalyx, which we treat as a poroelastic region. (Received September 14, 2016)

1125-92-1039 **Zhilan Feng\*** ([fengz@purdue.edu](mailto:fengz@purdue.edu)), 150 N. University St., Department of Mathematics, Purdue University, West Lafayette, IN 47907, and **Andrew N Hill, Aaron T Curns and John W Glasser**. *Using the partial derivatives of effective reproduction numbers to guide public health policy.*

Because the average number of secondary infections per infectious primary,  $R_e$ , must exceed one for an outbreak, health authorities endeavor to reduce  $R_e > 1$ . Depending on one's model, it may be possible to derive an explicit expression for  $R_e$  via the next-generation matrix approach. If so, one can take its partial derivatives with respect to control parameters. If not, one can determine their respective effects numerically. These results describe not only alternative routes by which policymaking goals might be attained, but magnitudes of their respective effects and potential for effect modification. In meta-population models, the corresponding multivariate partial derivative is the gradient, a vector-valued function. Using an SEIR model of pandemic influenza with 7 age groups, 51 spatial strata (states plus the District of Columbia) and a suitable two-level mixing function, we use the gradient to compare optimal and actual vaccination during the 2009-10 influenza pandemic in the United States. We also show that vaccination efforts could have been adjusted month-to-month during the fall of 2009 to ensure maximum impact. Together with colleagues at the China CDC, we are using this model to identify the optimal strategies for eliminating measles and controlling rubella in China. (Received September 14, 2016)

1125-92-1041 **Zhilan Feng, Andrew N Hill, Aaron T Curns and John W Glasser\*** ([jglasser@cdc.gov](mailto:jglasser@cdc.gov)), National Center for Immunization and, Respiratory Diseases, Centers for Disease Control and Prevention, Atlanta, GA 30333. *Realistic mixing functions for meta-population modeling to support public health policymaking.*

When model populations are stratified, contacts among their respective sub-populations must be described. Using a simple meta-population model, we showed that mixing among sub-populations, as well as heterogeneity in characteristics affecting their reproduction numbers, must be considered when evaluating interventions to prevent or control infectious disease outbreaks. We employed the convex combination of preferential within- and proportional among-group contacts first described by Nold and subsequently generalized by Jacquez and colleagues. As the utility of meta-population modeling in support of public policymaking depends on more realistic mixing functions, we included preferential contacts between parents and children and among co-workers as well as contemporaries. Here we add preferential contacts between grandparents and grandchildren, but omit workplace contacts. We also describe a general multi-level mixing scheme and provide several two-level examples. We describe age- and gender-specific patterns in face-to-face conversations, proxies for contacts by which respiratory pathogens might be transmitted. And we discuss how meta-population models with multi-generational mixing could be employed to reevaluate prolonged school-closures, a proposed pandemic mitigation measure. (Received September 14, 2016)

1125-92-1059 **D. I. Wallace\*** ([dwallace@math.dartmouth.edu](mailto:dwallace@math.dartmouth.edu)), Department of Mathematics, Dartmouth College, Hanover, NH 03755. *A nested model of tumor growth tuned across multiple experiments.*

Cancer cell lines that have the capacity to form solid tumors can be cultured, studied and experimented upon in three modalities outside the human body. Monolayer in vitro cultures represent unrestrained exponential growth of cells in the presence of abundant nutrient. Three dimensional in vitro result in a growing ball of cells that develop a tripartite structure with a central necrotic core, an actively proliferating rim, and an intermediate zone of hypoxic quiescent cells. Spheroids remain alive but cease growth due to the presence of a by-product of necrosis, TNF-alpha, that induces apoptosis. Xenograft experiments in vivo include the growth of vasculature in response to VEGF signaling by tumor cells which are either hypoxic or which, although proliferating, produce VEGF in response to TNF-alpha. In the presence of growing vasculature, the process that restrains growth of an in vitro spheroid is overcome, and the tumor grows again. A tumor cell is the same organism whether grown in monolayer, 3-D spheroid, or xenograft culture. A model for such an organism should have the capability of describing growth in all three scenarios. Here we present a system of nonlinear ODE's tuned to a series of results in monolayer, spheroid, and xenograft experiments. (Received September 14, 2016)

1125-92-1062 **Marjorie T Jones\*** ([marjorie.jones@pepperdine.edu](mailto:marjorie.jones@pepperdine.edu)). *A Discrete Stage-Structured Model of Newt Population Declines Due to Severe Drought.*

We introduce a discrete mathematical model for studying the population dynamics under drought of the California newt (*Taricha torosa*), a species of special concern in California. Multiple studies predict California's severe drought conditions will increase in duration and severity. Recent declines and local extinctions of California newt populations in Santa Monica Mountain (SMM) streams motivate our study of the impact of drought on newt population sizes. A precipitation deficit reduces the space for newt egg-laying in streams. To forecast newt

population dynamics, we develop a nonlinear system of discrete equations that includes demographic parameters such as survival rates for new life stages and egg production, which depend on habitat availability and rainfall. We estimate parameters using 15 years of stream survey data collected from the SMM, and our model captures the observed decline of a SMM newt population. We make predictions about how the length and severity of drought can affect available newt egg-laying sites as well as the newt population's likelihood of persistence or time to critical endangerment. We predict that sustained severe drought will critically endanger the newt population but that the population can rebound if a drought is sufficiently short. (Received September 14, 2016)

1125-92-1063 **D. I. Wallace\*** ([dwallace@math.dartmouth.edu](mailto:dwallace@math.dartmouth.edu)), Department of Mathematics, Dartmouth College, Hanover, NH 03755. *Malaria and vector dynamics in the Kenyan highlands.*

A dynamic model of malaria transmission in response to mosquito habitat changes is constructed based on the classic Ross model using a system of ordinary differential equations. Such a model takes the parameters controlling the spread of disease as input and gives predicted incidence of malaria cases as output. A dynamic model has an advantage over models based on statistical correlations or data fitting because causality is clearer in dynamic models than in data-driven correlations.

Our model includes not only the dynamics of infected mosquitos and infected humans, but also the abundance of mosquitos and larvae at each stage of development. Therefore, the first stage of the development of this model involves modeling *Anopheles gambiae* larval and adult populations that matches temperature dependent maturation times and mortality measured experimentally as well as larval instar and adult mosquito emergence data, which is provided from field studies in the Kenyan Highlands. This allows us to investigate the impact of habitat changes on total mosquito population, which then affects the population of infected mosquitos and the population of infected human beings. The objective of this project is to predict the degree of malaria endemicity in response to mosquito habitat changes. (Received September 14, 2016)

1125-92-1071 **Zhilan Feng\*** ([fengz@purdue.edu](mailto:fengz@purdue.edu)), 150 N. University St., Department of Mathematics, Purdue University, West Lafayette, IN 4, and **Yiqiang Zheng, Nancy Hernandez-Ceron, Henry Zhao, John W Glasser and Andrew N Hill.** *Mathematical models of Ebola—Consequences of underlying assumptions.*

Mathematical models have been used to study Ebola disease transmission dynamics and control for the recent epidemics in West Africa. Many of the models used in these studies are based on the model of Legrand et al. (*Epidemiol. Infect.*, 2007), and most failed to accurately project the outbreak's course. Although there could be many reasons for this, including incomplete and unreliable data on Ebola epidemiology and lack of empirical data on how disease-control measures quantitatively affect Ebola transmission, we examine the underlying assumptions of the Legrand model, and provide alternate formulations that are simpler and provide additional information regarding the epidemiology of Ebola during an outbreak. We developed three models with different assumptions about disease stage durations, one of which simplifies to the Legrand model while the others have more realistic distributions. Control and basic reproduction numbers for all three models are derived and shown to provide threshold conditions for outbreak control and prevention. (Received September 14, 2016)

1125-92-1093 **Seema Nanda\*** ([seema.nanda@dartmouth.edu](mailto:seema.nanda@dartmouth.edu)), **Vardayani Ratti and Dorothy Wallace.** *On the road to eliminating cART for HIV.* Preliminary report.

We present treatment for HIV using a proposed new CRISPR/CAS gene therapy which could eliminate the need for the current combined anti-retroviral therapy (cART). We use a dynamical system model that takes into account the hard to eliminate latent reservoir of infected HIV cells, which are a barrier to elimination of the disease with current treatments. We show the possibility of replacing cART therapy with gene edited cells which could replace the infected cells over a period of time. (Received September 14, 2016)

1125-92-1096 **Shandelle M Henson\*** ([henson@andrews.edu](mailto:henson@andrews.edu)). *Models of Reproductive Synchrony in Colonial Seabirds.* Preliminary report.

Decreased resource availability associated with high sea surface temperatures is correlated with a rise in egg cannibalism and reproductive synchrony in a seabird colony. Proof-of-concept models illustrate the onset of synchrony in two-cycle bifurcations. We compare the average of the two-cycles to the equilibrium level in the absence of synchrony. (Received September 14, 2016)

1125-92-1102 **Alex Farrell, James P. Collins, Amy L. Greer and Horst R. Thieme\*** (hthieme@asu.edu). *Do fatal infectious diseases eradicate host species? Epidemic perspective*. Preliminary report.

In deterministic epidemic models, density-dependent incidence  $f(S, I) = SI$  does not lead to host extinction unless the population is subject to an Allee effect or there is a reservoir for the disease. For frequency dependent incidence  $f(S, I) = \frac{SI}{S+I}$ , it depends on the parameters whether the host just declines or goes extinct.

Infection experiments involving tiger salamanders and ranavirus indicate that other forms of incidence may be more appropriate to model this particular host-parasite system (Greer et al., 2008). We show that incidence functions satisfying  $f(S, I) \leq cSI$ , never lead to host extinction, while incidence functions  $S^q I^p$  with  $0 < p < 1$ ,  $q \geq 0$ , always do.

As for the tiger salamander/ranavirus system, the case remains undecided: The incidences that fit best have the form  $S I^p$  with  $0 < p < 1$  which always leads to extinction and  $S k \ln(1 + I/k)$  which never leads to extinction.

#### Reference

A.L. Greer, C.J. Briggs, J.P. Collins, Testing a key assumption of host-pathogen theory: density and disease transmission, *Oikos* 117 (2008), 1667-1673 (Received September 14, 2016)

1125-92-1104 **Alex Farrell, James P. Collins and Horst R. Thieme\*** (hthieme@asu.edu). *Prey-predator-parasite: an ecosystem model with fragile persistence*. Preliminary report.

A prey-predator-parasite model is considered where a species serves as prey to a predator and as host to a microparasite.

We choose asymmetric frequency-dependent incidence for disease transmission,  $\sigma SI/(pS + qI)$  with  $0 < p < q < 1$  and  $p + q = 1$ . This choice is motivated by infection experiments involving tiger salamanders and ranavirus in which such an incidence is a better fit than density- or frequency-dependent incidence.

Among others, the following phenomena are observed in various parameter regions of the model.

- (i) Predator-mediated survival of all three species at high initial predator levels and parasite-mediated extinction of all three species at low initial predator levels
- (ii) Survival of the prey and predator at high initial predator levels and parasite-mediated extinction of all three species at low initial predator levels
- (iii) Parasite-mediated extinction of all three species at all initial predator levels
- (iv) Persistence of all three species.

#### Reference

[GBC] Greer, A.L., C.J. Briggs, J.P. Collins, Testing a key assumption of host-pathogen theory: density and disease transmission, *Oikos* 117 (2008), 1667-1673 (Received September 14, 2016)

1125-92-1119 **Hayriye Gulbudak\*** (hayriye.gulbudak@asu.edu), Hayriye.Gulbudak@asu.edu, and **Vincent Cannataro, Necibe Tuncer and Maia Martcheva**. *A Multi-Scale Model of Vector-Host Infection Dynamics and Evolution*. Preliminary report.

The within-host interaction of a pathogen and a host's immune system governs the pathogen's transmission potential between hosts. After a host is infected, the pathogen population grows inside the host, triggering an immune response of pathogen-specific antibodies, which help clear the infection. Pathogen and antibody dynamics are often monitored in laboratory experiments and modeling their interaction may inform our understanding of disease spread. In this study, we formulate a novel immunological model to capture the within-host dynamics of arboviral vector-borne diseases and link it to a vector-host age-since-infection structured epidemiological model, by using a nested approach. By incorporating within-host pathogen and immune response dynamics, we are able to capture the heterogeneity that exists among infected individuals and analyze how this heterogeneity scales up and influences population-level dynamics. On the evolutionary scale, we analytically derive host and parasite fitness functions depending on intra-host pathogen-immune response antibody dynamics and by using numerical simulations, study host and pathogen evolutionary trajectories and the effect of tradeoff functions and vector initial inoculum on the coevolutionary attractor. (Received September 15, 2016)

1125-92-1208 **Eric Benjamin Randall\*** (ebrandal@ncsu.edu), **Jesper Mehlsen** and **Mette Olufsen**. *Mathematical modeling of blood pressure and heart rate in response to the Valsalva Maneuver*.

To elucidate the effects of the baroreceptor reflex (baroreflex) control mechanism on a healthy subject, we developed a cardiovascular model that predicts the dynamics of blood pressure (BP) and heart rate (HR) in response to the Valsalva Maneuver (VM). The VM is the action of forced exhalation against a closed airway and

elicits a distinctive response when held for 15 seconds. This assay is used by clinicians to determine autonomic nervous system (ANS) function, and deviations from the VM response, due to a system-wide inflammatory response or adrenergic receptor blockage, may signify ANS dysfunction or failure. This study focuses on modeling various healthy subjects, employing a closed-loop model that uses the thoracic pressure as an input. The goal of this study is for this model to be used for multiple subjects and eventually to change parameters in order to accommodate various anomalies in ANS function. (Received September 15, 2016)

1125-92-1249 **Kang-Ling Liao\*** (kangling325@gmail.com), NC, and **Roger D. Jones** (rogerdjonesphd@gmail.com), **Patrick McCarter** (pccart@email.unc.edu), **Meral Tunc-Ozdemir** (meraltuncozdemir@gmail.com), **James A. Draper** (drapja@live.unc.edu), **Timothy C. Elston** (timothy\_elston@med.unc.edu), **David Kramer** (kramer8@cns.msu.edu) and **Alan M. Jones** (alanjones@bio.unc.edu). *A shadow detector for photosynthesis efficiency.*

Plants tolerate large variations in the intensity of the light environment by controlling the efficiency of solar to chemical energy conversion. To do this, plants have a mechanism to detect the intensity, duration, and rapid change in light as they experience moving shadows, flickering light, and cloud cover. Our previous work used experiments and mathematical model to show that the heterotrimeric G protein complex in plant including its receptor-like Regulator of G signaling protein, AtRGS1, detects both the concentration and the exposure time of sugars. In this talk, I will use numerical simulation and experiments to show that another property of the signaling system is to detect large changes in light while at the same time, filtering types of fluctuation in light that do not affect photosynthesis efficiency. When AtRGS1 is genetically ablated, photosynthesis efficiency is reduced in a changing- but not a constant-light environment. Our mathematical modeling revealed that information about changes in the light environment is encoded in the amount of free AtRGS1 that becomes compartmentalized following stimulation. We propose that this property determines when to adjust photosynthetic efficiency in an environment where light intensity changes abruptly caused by moving shadows. (Received September 15, 2016)

1125-92-1251 **Kang-Ling Liao\*** (kangling325@gmail.com), **Xue-Feng Bai** (xue-feng.bai@osumc.edu) and **Avner Friedman** (afriedman@math.osu.edu). *Mathematical modeling of anti-PD-1 and IL-27 synergy in cancer immunotherapy.*

Anti-PD-1 drug is showing promise in clinical trials of cancer treatment, and some have already received FDA approval. There is the expectation that these drugs will transform the landscape of cancer treatment. Combining the anti-PD-1 with an immunotherapeutic drug achieve significantly better results than by the immunotherapeutic drug alone. An important question is how much of an improvement is actually achieved by anti-PD-1. In this talk, I will talk about this question by a mathematical model, where we focus on a specific immunotherapeutic drug, IL-27. Our simulations show that in combination therapy, the efficacy of the treatment depends nonlinearly on IL-27 and anti-PD-1. For any amount of IL-27, increasing anti-PD-1 always increases the tumor reduction. On the other hand, it is not always true that increasing IL-27 increases the tumor reduction. If the amount of anti-PD-1 is smaller than a certain value, there exists a critical value of IL-27 such that the following holds: (i) If the amount of IL-27 is smaller than the critical value, then increasing IL-27 increases the tumor reduction. (ii) If IL-27 is larger than the critical value, then increasing IL-27 decreases the tumor reduction; this occurrence is result of the complex role that IL-27 plays as an anti-tumor drug. (Received September 15, 2016)

1125-92-1258 **Fan Bai\*** (baifan.ubc@gmail.com), Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX 79409. *Dynamical analysis of malaria transmission model includes both extrinsic incubation period and intrinsic incubation period.*

We formulate a delayed malaria transmission model, which includes both extrinsic incubation period and intrinsic incubation period. We calculate the basic reproduction number  $\mathcal{R}_0$ , the disease-free equilibrium and the possible endemic equilibrium (exists if and only if  $\mathcal{R}_0$  is larger than the unity). Then we analyze the equilibria of this model. We prove that if  $\mathcal{R}_0 < 1$ , the disease-free equilibrium is asymptotically stable; while if  $\mathcal{R}_0 > 1$ , the endemic equilibrium is asymptotically stable for any lengths of delay periods. Several numerical simulations are provided to verify our theoretical results. (Received September 15, 2016)

1125-92-1288 **Ellie Mainou\*** (emainou@smith.edu), Box 7221, 1 Chapin Way, Northampton, MA 01063. *Incorporating Antibiotic Resistance in the Modeling of TB in the US.* Preliminary report.

According to WHO, one third of the world's population is infected with tuberculosis (TB), with drug resistance posing another challenge to the management of TB. Strains resistant to the most common drugs for treatment (e.g. isoniazid, rifampin) are already widespread. To address the issue, we constructed a mathematical compartmental model described by differential equations showing the transmission of TB in the US. The model

encompasses four strains of different resistance to antibiotics, while taking into consideration immigration, which greatly contributes to the excessive transmission of latent TB. We fitted parameters to recent CDC data on TB morbidity and mortality, producing reliable fits, by generating random parameter values within the ranges obtained from real-world data. We then algorithmically adjusted parameter values to better fit CDC and census data in three categories (total US population, active cases of TB, total TB deaths). Our focus now is improving the algorithm by understanding the its behavior of through tests, which show that the order in which the parameter space is investigated may affect the results of the simulation, and that the stop rule of the algorithm is effective. The final goal is to use the model to identify effective strategies against TB. (Received September 15, 2016)

1125-92-1291 **Ruijun Zhao\*** ([ruijun.zhao@mnsu.edu](mailto:ruijun.zhao@mnsu.edu)), Department of Mathematics and Statistics, Minnesota State University, Mankato, MN 56001. *Mathematical Models of Malaria Control Using ITNs.*

Malaria is the most prevalent tropical parasitic disease in the world. An estimated 214 million cases of malaria occurred and 438,000 people died in 2015. Together with medical treatment, the most commonly used control strategies include insecticide-treated bed-nets (ITNs) and indoor/outdoor residual chemical sprays (IRS).

In this talk, we will discuss a few mathematical models that we recently developed, which investigate the effectiveness of ITNs. (Received September 15, 2016)

1125-92-1316 **Rebecca Pettit\*** ([rpettit@vols.utk.edu](mailto:rpettit@vols.utk.edu)) and **Suzanne Lenhart.** *Optimal Control in a Discrete Model for Invasive Species Integrating Surveillance and Removal.*

Managing an invasive species can involve allocating resources to surveillance of the area and removal of the detected invaders. We formulate a model discrete in time, including these two features as control actions. The area is divided into three compartments: absent (no invaders present), detected, and undetected. We use optimal control theory to aid in the decision of allocation resources to these two controls. Numerical illustrations will be presented. (Received September 16, 2016)

1125-92-1328 **Leili Shahriyari\*** ([shahriyari.1@mbi.osu.edu](mailto:shahriyari.1@mbi.osu.edu)), Mathematical Biosciences Institute, 1735 Neil Ave, 3rd floor Jennings hall, Columbus, OH 43210. *Mathematical Models for Cells' Interactions in Tumor Micro-environment.*

In order to obtain optimal cancer therapies, the knowledge of cells behavior and their interactions is crucial. Each tumor has its own special characteristics. Many computational models have been designed to investigate the cells' cross talks in various tumors. In this talk, I will present and compare several non-linear models, which have been developed to study cells' interactions in tumor micro-environment. (Received September 16, 2016)

1125-92-1348 **Ruth Davidson\*** ([redavid2@illinois.edu](mailto:redavid2@illinois.edu)) and **James H. Degnan.** *When is it surprising that all gene trees are unique? An application of the generalized birthday problem.*

Invariants corresponding to probabilities of sequence patterns in the leaves of phylogenetic trees are a major area of research in algebraic statistics. In 2011, Allman, Degnan, and Rhodes derived invariants for probabilities of gene tree topologies evolving within a species tree under the multi-species coalescent (MSC) model. Yet gene invariants arise from a mathematical object—a probability distribution—instead of from frequencies of gene tree topologies in finite datasets. So it is essential to compare expected gene frequencies to phylogenomic datasets. In 2013 Salichos and Rokas recovered 1,070 topologically distinct gene trees from 23 yeast genomes, a result considered surprising by many. Through a large-scale data simulation and by deriving bounds on probabilities for gene tree discord, we show that this result is not surprising under the MSC model. This is joint work with James Degnan. (Received September 16, 2016)

1125-92-1369 **Yi Jiang\*** ([yjiang12@gsu.edu](mailto:yjiang12@gsu.edu)), 30 Pryor Street, Department of Mathematics and Statistics, Georgia State University, Atlanta, GA 30303. *Modeling Cell-ECM Interactions in Cancer Invasion.*

The extracellular matrix (ECM), a fibrous network in tissues, significantly affects many aspects of cellular behavior, including cell movement and proliferation. Transgenic mouse tumor studies indicate that excess collagen, a major component of ECM, enhances tumor formation and invasiveness. Clinically, tumor associated collagen signatures are strong markers for breast cancer survival. However, the underlying mechanisms are unclear since the properties of ECM are complex, with diverse structural and mechanical properties depending on various biophysical parameters. We have developed a three-dimensional elastic fiber network model, studied the structure of 3D networks, and calibrated the mechanical properties using in vitro collagen mechanical measurements. Using this model, we study ECM remodeling as a result of local deformation and cell migration through the

ECM as a network percolation problem. We are in the process of developing a three-dimensional, multiscale model of cell migration and interaction with ECM. Preliminary results reproduced quantitative single cell migration experiments. This model is a first step toward a fully biomechanical cell-matrix interaction model, which may shed light on tumor associated collagen signatures in breast cancer development. (Received September 16, 2016)

1125-92-1378 **Mariel Vazquez\*** ([mariel@math.ucdavis.edu](mailto:mariel@math.ucdavis.edu)), Department of Mathematics, University of California, Davis, One Shields Ave, Davis, CA 95616. *Packing, folding and simplifying DNA topology.*

Cellular processes such as replication, recombination, and packing change the topology of DNA. Controlling these changes is key to ensuring genome stability. Techniques from knot theory and low-dimensional topology, aided by computational tools, now make it possible for us to ask questions about the topological state of the genome and to study the specific action of enzymes that control and modify DNA topology. I will illustrate the use of these methods with examples drawn from our study of DNA unlinking in bacterial cells. (Received September 18, 2016)

1125-92-1415 **Linda J. S. Allen\*** ([linda.j.allen@ttu.edu](mailto:linda.j.allen@ttu.edu)) and **William Tritch.** *Duration of an Epidemic Near the Critical Threshold of  $\mathcal{R}_0 = 1$ .* Preliminary report.

If the value of the basic reproduction number  $\mathcal{R}_0 > 1$ , an increase in number of cases is expected while if  $\mathcal{R}_0 < 1$ , a decrease in number of cases is expected. When the critical threshold of  $\mathcal{R}_0 = 1$  is exceeded an outbreak occurs. The magnitude of  $\mathcal{R}_0$  is also an indication of the seriousness of the disease and of the duration of an outbreak. In continuous-time Markov chain epidemic models, we investigate the probability and the duration of an outbreak as a function of  $\mathcal{R}_0$  via branching process theory. Analytical formulas are obtained for the mean duration and higher-order moments from the branching process approximation, conditioned on disease extinction. The mean duration in the branching process approximation approaches infinity as  $\mathcal{R}_0$  approaches one. Although this approximation breaks down for models with finite population sizes, it is shown for sufficiently large population sizes that the mean duration generally increases near the critical threshold. The results are illustrated and discussed for vector-host models and their implications for disease control. (Received September 16, 2016)

1125-92-1423 **Xiaochuan Hu\*** ([xiaochuan.hu@ttu.edu](mailto:xiaochuan.hu@ttu.edu)), Texas Tech University, Department of Mathematics and Statistics, and **Sophia R.-J. Jang**, Texas Tech University, Department of Mathematics and Statistics. *Dynamics of tumor-CD4<sup>+</sup>-cytokine-host cells interactions with treatments*

Historically, CD4<sup>+</sup> T cells have been assumed to have only a helper role by activating CD8<sup>+</sup> T cells to kill cancer cells. Recent experiments have shown, however, that CD4<sup>+</sup> T cells actually play a more direct role in tumor eradication even in the absence of CD8<sup>+</sup> T cells by using the cytokines and chemokines that they produced. In this talk, we introduce several mathematical models of interactions between tumor cells, CD4<sup>+</sup> T cells, cytokines and host cells to investigate the role of CD4<sup>+</sup> T cells on tumor regression. We apply immunotherapies with either CD4<sup>+</sup> T cells or cytokines to study their effectiveness. It is concluded that doses of treatments along with initial tumor sizes are critical in determining the fate of a tumor. Tumor cells can be eliminated completely if doses of treatments by cytokines are large. The treatments are in general more effective if the tumor size is smaller. Bistability is observed in all models with or without the treatment strategies, which indicates that tumor cells can be successfully controlled by a carefully derived treatment strategy. (Received September 16, 2016)

1125-92-1435 **Eric Alan Eager\***, 1725 State St., La Crosse, WI 54601, and **James P Peirce, Gregory J Sandland, Richard A Erickson and Barbara Bennie.** *Undergraduate Research in Mathematical Biology at the University of Wisconsin - La Crosse.*

In this talk we provide an overview of undergraduate research undergone at the University of Wisconsin - La Crosse (UW-L) at the interface between mathematics and biology. Example undergraduate projects ranging from the modeling and analysis of swimmer's itch to the development of Boolean networks to explain the bistability found in the germ layer formation of salamanders and frogs will be discussed. The results of various externally-funded opportunities, such as the Collaboration on Riverine Ecology (CORE), Center for Undergraduate Research in Mathematics (CURM) and Research Experience for Undergraduates (REU) in Mathematical Ecology at UW-L will also be discussed. (Received September 16, 2016)

1125-92-1453 **Linh Huynh\*** ([huynh.134@osu.edu](mailto:huynh.134@osu.edu)). *Analyzing Sleep-Wake Transitions.*

Contrary to the common perception that sleep is continuous, sleep is actually fragmented by brief awakenings throughout the night—even in healthy people. This work analyzes transitions between sleep and wake states in rat

electromyography data with the aim of gaining understanding of fundamental mechanisms in sleep cycle dynamics. Types of transitions are identified by applying variants of hierarchical clustering methods, and probabilities of transition occurrences are determined using hidden Markov models. The identification of multiple types of transitions with distinct dynamical features, in comparison with activity in mathematical models of bistable neuronal networks, suggests hypotheses regarding activity in the neuronal network responsible for regulating sleep and wakefulness. (Received September 16, 2016)

1125-92-1512 **Amy Veprauskas\***, aveprauskas@louisiana.edu. *Synchrony and the dynamic dichotomy in a class of matrix population models.*

In this talk, I will discuss the dynamics of a class of discrete-time structured population models called synchrony models. Synchrony models are characterized by the simultaneous bifurcation of a branch of positive equilibria and a branch of synchronous 2-cycles from the extinction equilibrium. These models exhibit a dynamic dichotomy in which the two steady states have opposite stability properties that are determined by the relative levels of competition in the population. I will also present an application that is motivated by observations of a cannibalistic gull population. (Received September 17, 2016)

1125-92-1559 **Ruth Davidson\*** (redavid2@illinois.edu), 3526 North Broadway Street, Apartment 2W, Chicago, IL 60657. *SVDquartets and Numerical Algebraic Geometry.*

Phylogenies are mathematical models of the evolutionary history of a set of taxa. The SVDquartets method (Chifman-Kubatko 2014) assigns a numerical score to each of the three possible four-taxon (quartet) trees within a larger dataset. Lowest-scoring quartets are combined via quartet-agglomeration methods such as Quartets Max Cut (Snir-Rao 2012), producing a phylogeny on the entire dataset. The theory (Chifman-Kubatko 2015) behind this method is that the lowest-scoring quartet is generically identifiable assuming the multi-species coalescent model (MSC): the score represents the distance to an algebraic variety corresponding to matrices for time-reversible models of nucleotide substitution. This score enables the SVDquartets method to bypass the problem of gene tree estimation error that is a major issue in MSC-based methods. A comparative study showed that SVDquartets performed unevenly in comparison with other MSC-based methods on many datasets. In this talk we address (1) interaction of the SVDquartet score with quartet-agglomeration methods, (2) the distance computation to the algebraic variety representing the substitution model, and (3) strategies to boost the performance of this approach and expand the theory behind it. (Received September 18, 2016)

1125-92-1567 **Adnan A Khan\*** (adnan.khan@lums.edu.pk), 149 SSE Building, Opposite Sector U, DHA, Lahore, 54792, Pakistan, and **Mudassar Imran** (mimran@asu.edu). *Modeling the transmission dynamics of Congo Virus Disease with Control Strategies.*

Congo Virus Disease (CVD) is a, highly contagious, tick borne disease with a high mortality rate. Due to this even suspected cases should be dealt with cautiously and control measures be taken with no delay. The people who are most at risk the include farmers, livestock owners, butchers, and medical staff. In this talk, we model the transmission dynamics of CVD based on a three-fold SIR model that includes three different population groups (humans, cattle and ticks). Dynamical systems analysis of the model is performed, and it is consequently the stability of the disease free steady states are determined when the basic reproduction number,  $R_0 < 1$ , it is also shown that there exists an endemic equilibrium when  $R_0 > 1$ . Sensitivity analysis is performed in order to determine the most 'important' parameters that are primarily responsible for the transmission of the disease. Finally, using optimal control theory, we propose control strategies, which if adopted will help eliminate the disease. (Received September 18, 2016)

1125-92-1572 **Sainan Wu and Junping Shi\*** (jxshix@wm.edu), Department of Mathematics, College of William and Mary, Williamsburg, VA 23187, and **Boying Wu and Jinfeng Wang.** *Dynamics and pattern formation in diffusive predator-prey system with prey-taxis or predator-taxis.*

It has been recognized that in the spatial predator-prey interaction, in addition to the random diffusion of predator and prey, the spatiotemporal variations of the predator velocity are affected by the prey gradient. The global existence and global stability of solutions to diffusive predator-prey systems with prey-taxis and a similar problem with predator-taxis are proved, and the existence of non-constant positive steady state solutions (spatial patterns) is also studied. It is shown that the prey-taxis or predator-taxis usually increases the stability of the system, and makes the spatial pattern less likely to occur. (Received September 18, 2016)

1125-92-1598 **Fred Brauer\*** ([brauer@math.ubc.ca](mailto:brauer@math.ubc.ca)), Department of Mathematics, University of British Columbia, Vancouver, BC V6T 1Z2, Canada. *A Final Size Relation for Epidemic Models of Vector-Transmitted Diseases*. Preliminary report.

We formulate and analyze an age of infection model for epidemics of diseases transmitted by a vector, including the possibility of direct transmission as well. We show how to determine a basic reproduction number. While there is no explicit final size relation as for diseases transmitted directly, we are able to obtain estimates for the final size of the epidemic. (Received September 18, 2016)

1125-92-1655 **Suzannah R Tebon\*** ([tebonsr@beloit.edu](mailto:tebonsr@beloit.edu)), **Kiefer Green**, **Richard Noriega** and **Erin Munro**. *UP Phase Characterization*. Preliminary report.

UP phases are periods of high amplitude neural firing that are a characterizing aspect of slow wave sleep. In order to better understand the potential effects of UP phases in slow wave sleep we investigate where UP phases originate. UP phase origins are determined by latency calculations based on current source density data. By using Independent Component Analysis we are able to locate sources of neural activity and look at their contribution to the overall neural activity that is picked up by the recording electrode. This has led to the locating of two neural sources, BL5 and SUB. UP phases during sleep dominated by BL5 or SUB activity originated in different locations. This could lead to better understanding of what causes UP phases and their potential effects on sleep. (Received September 18, 2016)

1125-92-1696 **Samantha Erwin\*** ([sherwin@vt.edu](mailto:sherwin@vt.edu)), **Meghan Verma**, **Vida Abedi**, **Raquel Hontecillas**, **Stefan Hoops**, **Josep Bassaganya-Riera** and **Stanca Ciupe**. *Modeling the mechanisms by which HIV-associated immunosuppression influences HPV persistence at the oral mucosa*.

Human immunodeficiency virus (HIV)-infected patients are at an increased risk of co-infection with human papilloma virus (HPV), and subsequent malignancies such as oral cancer. To determine the role of HIV-associated immune suppression on HPV persistence and pathogenesis, we developed a mathematical model of HIV/HPV co-infection and used it to investigate the mechanisms underlying the modulation of HPV infection and oral cancer by HIV. Our model captures known immunological and molecular features such as impaired HPV-specific effector T helper 1 (Th1) cell responses, and enhanced HPV infection due to HIV. We used the model to determine HPV prognosis in the presence of HIV infection, and identified conditions under which HIV infection alters HPV persistence in the oral mucosal system. The model predicts that conditions leading to HPV persistence during HIV/HPV co-infection are the permissive immune environment created by HIV and molecular interactions between the two viruses. The model also determines when HPV infection continues to persist in the short run in a co-infected patient undergoing antiretroviral therapy. Lastly, the model predicts that under efficacious antiretroviral treatment HPV infections will decrease in the long run due to the restoration of CD4+ T cell levels. (Received September 18, 2016)

1125-92-1779 **Jorge X Velasco-Hernandez\*** ([jx.velasco@im.unam.mx](mailto:jx.velasco@im.unam.mx)), LIIGH Building, Boulevard Juriquilla No. 3001, 76230 Juriquilla, Queretaro, Mexico. *Transmission Dynamics of Two Dengue Serotypes with vaccination scenarios*. Preliminary report.

A mathematical model that incorporates two Dengue serotypes is presented. We consider two viral strains and temporary cross-immunity with one vector mosquito population. Results suggest that vaccination scenarios will not only reduce disease incidence but will also modify the transmission dynamics. Indeed, vaccination and cross immunity period are seen to decrease the frequency and magnitude of outbreaks but in a differentiated manner with varied effects depending upon the interaction vaccine and strain type. (Received September 19, 2016)

1125-92-1804 **Tae Jin Lee\*** ([talee@augusta.edu](mailto:talee@augusta.edu)) and **Arni S. R. Srinivasa Rao** ([arrao@augusta.edu](mailto:arrao@augusta.edu)). *Mathematical Modeling for understanding the the Impact of eCD4-Ig molecule within an HIV Infected Host*. Preliminary report.

Even though there are treatments that prolongs the life of HIV infected people for decades, complete removal of virus for cure is still not observed experimentally. The new molecule called eCD4-Ig (discovered by Gardner et al in 2015) may give some hope in search of possibilities to eliminate HIV. Experiments in this direction revealed following properties: It is not immunogenic, has very broad neutralizing ability against all HIV, works with lower concentrations. Moreover, it protected monkeys from multiple introduction of SHIV virus and the protection lasted for at least 40 weeks. However, we are interested in effect of eCD4-Ig in human. An eCD4-Ig molecule is differ from many other broadly neutralizing antibodies (bNabs) as it neutralizes HIV virus through binding to two spots on a viral spike of HIV and blocking the interaction of CD4 T-cell and HIV virus. To understand dynamics of viral spikes, eCD4-Ig, CD4 cells, we have developed a mathematical model by incorporating interactions between

this new molecule and other known immunological, virological information. We do model based speculations for management and elimination of virus. (Received September 19, 2016)

1125-92-1826 **James Peirce\*** (jpeirce@uwlax.edu), **Greg Sandland** (gsandland@uwlax.edu), **Kelly Buch** (kbuch@siue.edu) and **Rob Hendrickson** (hendr862@umn.edu). *Modeling the effects of varying treatment regimes on the control of an emerging disease in the Midwest of the United States.*

Swimmer's itch is an emerging disease caused by flatworm parasites that typically use water birds as definitive hosts. When parasite larvae mistakenly penetrate human skin they initiate localized inflammation that leads to intense discomfort. Concerns about this issue have been growing recently due to an apparent increase in the occurrence of swimmer's itch and its subsequent impacts on recreational activities and revenues. Past work has identified the common merganser as a key definitive host for these worms; a number of snail species serve as intermediate hosts. Although past attempts at controlling swimmer's itch have targeted snails, recent efforts have concentrated on treating water birds with the anti-parasitic drug, Praziquantel. In this talk, we will introduce a mathematical model that explores the effects of Praziquantel dose and treatment frequency on the occurrence of swimmer's itch in a typical Midwestern lake. We modeled susceptible and infected mergansers and snails using first order differential equations and introduced aspects of Praziquantel treatment into the system. Results from this model may help to identify treatment regimes that lower merganser infection rates and ultimately reduce the occurrence of swimmer's itch in water bodies around the Midwest. (Received September 19, 2016)

1125-92-1836 **Azmy S. Ackleh\***, Department of Mathematics, University of Louisiana at Lafayette, Lafayette, LA 70504-1010, and **Robert Miller**, 135 Regency Sq., Lafayette, LA 70508. *A Model for the Interaction of Phytoplankton Aggregates and the Environment: Approximation and Parameter Estimation.*

We present a model for the dynamics of phytoplankton aggregates and their interaction with the environment in a well-mixed reactor or surface layer of a water column. The biophysical problem yields a nonlinear partial differential equation coupled with a general system of ordinary differential equations. We first develop a finite difference scheme for approximating the solution of this model. Then convergence results are established for the finite difference method. A least-squares parameter estimation method is presented to illustrate the performance of the model against real data, and to highlight the effects of aggregate growth on algal bloom dynamics and nutrient consumption rates. (Received September 19, 2016)

1125-92-1838 **Jamie Prezioso\*** (jep127@case.edu), **Daniela Calvetti** (dxc57@case.edu) and **Erkki Somersalo** (ejs49@case.edu). *Inverse problem of cerebral hemodynamics with model uncertainty.*

Neuronal activity in brain is accompanied by a concomitant increase in cerebral blood flow, a phenomenon that is at the base of several functional imaging modalities, e.g., BOLD-fMRI and optical absorption and scattering tomography (OAST). Understanding the mechanism behind the neurovascular connection is important for the interpretation of the dynamic imaging results. Mathematical models of the brain hemodynamics assume a ballooning of the vessels to accommodate the additional blood, however many details of the model remain to be explained and several of the key parameters are poorly known. In this talk we address the question of identifying the vascular compartment accommodating the increase in blood volume and estimating the shape of the underlying vasodilatory stimulus from the cerebral blood flow data. Bayesian hierarchical models are used to find a physiologically meaningful stimulus as the solution of a nested sequence on inverse problems. Moreover, we introduce a statistical modeling error term to account for the uncertainty in the model parameters. Computed examples illustrate the effectiveness of the proposed approach and the need for uncertainty quantification techniques. (Received September 19, 2016)

1125-92-1863 **J T Lin\*** (jlin46@calpoly.edu), **J P Keener**, **S Poelzing**, **R Veeraraghavan**, **S A George**, **R G Gourdie**, **M E Salama**, **K J Sciuto** and **G S Hoeker**. *Mathematical Modeling of Cardiac Tissue: Theoretical, Numerical, and Experimental Study of Ephaptic Effects.*

Conduction failure of action potential propagation has been strongly linked to ventricular arrhythmia and sudden cardiac death. Recent experimental studies have shown that cellular geometry plays an important role in action potential propagation, but existing homogenized models cannot accurately capture the effects of microscale variations.

We have developed a multiscale model that incorporates the complex cellular geometry of cardiac tissue, while remaining numerically efficient. This model has shown that the mechanisms behind action potential propagation have still yet to be well understood. In particular, ephaptic effects occurring at the cardiomyocyte intercalated

disk can facilitate conduction in unexpected ways. We have used this model to reproduce results found by experimental modification of murine hearts, preferentially down-regulating gap junctional proteins, as well as sodium and potassium ion channels, that provide evidence of ephaptic effects. (Received September 19, 2016)

1125-92-1870 **Scott W Greenhalgh\*** ([scott.greenhalgh@queensu.ca](mailto:scott.greenhalgh@queensu.ca)), Department of Math & Stats, Jeffery Hall, University Ave., Kingston, Ontario K7L 3N8, Canada, and **Troy Day**, Department of Math & Stats, Jeffery Hall, University Ave., Kingston, Ontario K7L 3N8, Canada. *Recovery rates in epidemiological models.*

Constant recovery rates do not perfectly describe the dynamics of recovery. However, despite the many theoretical extensions that differential equation models of infectious disease have undergone, the generalization of recovery rates to physically more realistic formulations have only recently started to be developed.

In this talk, we provide a first principle derivation of state-dependent and time varying recovery rates in differential equation models of infectious disease. We justify our derivation through the connection between integral equations, differential equations, and stochastic processes. Finally, we apply our approach using measles transmission in Iceland, where we demonstrate the potential impact that uncertainty in an infectious period distribution's skewness and kurtosis has on predicting epidemic peaks. (Received September 19, 2016)

1125-92-1873 **Mahbubur Rahman\*** ([mrahman@unf.edu](mailto:mrahman@unf.edu)), Mathematics & Statistics Department, University of North Florida, Jacksonville, FL 32224. *Wave propagation in a noisy system near saddle node on limit cycle bifurcation.* Preliminary report.

We develop and apply a method of stochastic approximation to a canonical model arises in a circular process in mathematical neuroscience that has a parametric noise. We also investigate the role of noise in the circular process. (Received September 19, 2016)

1125-92-1905 **Rebecca K. Borchering\*** ([rborchering@unf.edu](mailto:rborchering@unf.edu)), **Steve E. Bellan**, **Jason M. Flynn**, **Juliet R.C. Pulliam** and **Scott A. McKinley**. *Resource-Driven Encounters and the Induction of Disease Among Consumers.* Preliminary report.

Territorial animals share a variety of common resources, which can be a major driver of conspecific encounter rates. We examine how changes in resource availability influence the rate of encounters among individuals in a consumer population by implementing a spatially explicit model for resource visitation behavior by consumers. Using data from 2009 and 2010 in Etosha National Park, we verify our model's prediction that there is a saturation effect in the expected number of jackals that visit a given carcass site as carcasses become abundant. However, this does not directly imply that the overall resource-driven encounter rate among jackals decreases. This is because the increase in available carcasses is accompanied by an increase in the number of jackals that detect and potentially visit carcasses. Using simulations and mathematical analysis of our consumer-resource interaction model, we characterize key features of the relationship between resource-driven encounter rate and model parameters. These results are used to investigate a standing hypothesis that the outbreak of a fatal disease among zebras can potentially lead to an outbreak of an entirely different disease in the jackal population, a process we refer to as indirect induction of disease. (Received September 19, 2016)

1125-92-1921 **Paul L Salceanu\*** ([salceanu@louisiana.edu](mailto:salceanu@louisiana.edu)), 217 Maxim Doucet Hall, 1401 Johnston Street, Lafayette, LA 70504, and **Azmy A Ackleh** and **Jim M Cushing**. *On the Dynamics of Evolutionary Competition Models.*

As exemplified by classic Lotka–Volterra theory, there are several canonical outcomes possible to a two species (interference) competitive interaction: coexistence, initial condition-dependent competitive exclusion of one species, or the global exclusion of one species. Evolutionary versions of Lotka–Volterra dynamics have been investigated in order to see the role that evolutionary adaptation can play in influencing the competitive outcome. For the most part, however, these investigations have been carried out by numerical simulations. We provide some rigorous mathematical criteria concerning the outcome of a competition between two species  $x_1$  and  $x_2$  when evolution is taken into account. Motivated by two classic experiments in which the outcome of two competing beetle species was observed to change due to phenotypic or genetic changes in one species, we consider the case when only the species  $x_2$  can evolutionarily adapt. Using methods from persistence theory, we obtain criteria under which  $x_2$  will persist and criteria under which  $x_1$  will persist. (Received September 19, 2016)

- 1125-92-1998     **Munsur Rahman\*** ([mmrhwb@mail.umkc.edu](mailto:mmrhwb@mail.umkc.edu)), 5110 Rockhill Road, 318 R.H Flarsheim Hall, Kansas City, MO 64110, and **Naveen K Vaidya** ([vaidyan@umkc.edu](mailto:vaidyan@umkc.edu)), 5100 Rockhill Road, 205C Manheim Hall, Kansas City, MO 64110. *Mathematical Model for ZIKA virus transmission dynamics.*
- Several outbreaks of the Zika virus, including the 2007 outbreak in Yap Island, the 2013-14 outbreak in French Polynesia, and the recent devastating spread of the virus across Americas, have become a big concern for public health. Currently, because of the limited studies on Zika virus, much about it remains unknown. In this talk I will present a mathematical model to investigate the Zika virus transmission dynamics between hosts. Our model agrees well with the data from French Polynesia and Yap Island. I will discuss how our model helps evaluate the key parameters that affect Zika infection and transmission, such as incubation period, infectious period, and basic reproduction number. Such estimates can provide better understanding of this less understood virus and can be used to evaluate potential treatment and prevention methods. (Received September 19, 2016)
- 1125-92-2003     **Mette S Olufsen\*** ([msolufse@ncsu.edu](mailto:msolufse@ncsu.edu)), NC State University, Raleigh, NC 27502. *Modeling control of cardiovascular dynamics during Head-up tilt.* Preliminary report.
- The Head-up tilt (HUT) test is commonly used to assess the body's ability to control blood pressure and heart rate. The test starts with the subject lying in supine position, after steady state heart rate and blood pressure signals have been acquired, the subject is tilted head up for about 20-40 min. This test is commonly applied to subjects who have problems controlling blood pressure resulting in syncope following the tilt. Most commonly, HUT data are analyzed manually to determine which tilt etiology best fits the given patient. In this study, we show how modeling can be used to distinguish the various syncope etiology, focus will be on predicting patterns from patients with postural tachycardia (POTS) and cardio-inhibition. The first disease type can be studied changing one parameter in the heart rate control equation causing a switch similar to the one found in a Hopf bifurcation. The second response is observed by flipping one control from negative to positive feedback representing mimicking by action caused by the Bezold Jarish reflex. We also include more general analysis discussing impact of inhibiting either sympathetic or parasympathetic controls. Finally we show that adding noise to predictions of heart rate makes the model resemble data closely. (Received September 19, 2016)
- 1125-92-2011     **Gerald W Baygents\*** ([baygentsg@umkc.edu](mailto:baygentsg@umkc.edu)), Kansas City, MO 64118. *Transmission dynamics of bluetongue and epizootic hemorrhagic diseases in a patchy environment.* Preliminary report.
- Epizootic hemorrhagic disease and bluetongue are two orbivirus-related diseases (HD) of white-tailed deer with multiple outbreaks in late July through November are spread by the small biting midge, *Culicoides Ceratopogonidae*. HD outbreak is a major concern in the wildlife and natural resource management, and using mathematical modeling, we explore efficacy of various control strategies. In particular, an epidemic model is proposed to analyze the HD dynamics between multiple patches due to deer population dispersal. The model embodies deer movements between the patches and the vector-borne infections within the patches. In the present work (1) using the harvest, disease mortality and dispersal data of deer population in Missouri, the host-related parameter values of the model are estimated, (2) the conditions for existence and stability of equilibria are established, and (3) the local and global basic reproduction numbers are respectively calculated for each patch and the entire environment. It is shown that increasing movements of susceptible deer from patches with reproduction numbers greater than one to those with reproduction number less than one can only be an effective control strategy when it is combined with suppression of the midges' population in all patches. (Received September 19, 2016)
- 1125-92-2022     **Yu-Min Chung\*** ([ychung@wm.edu](mailto:ychung@wm.edu)), Department of Mathematics, College of William and Mary, Jones Hall, 200 Ukrop Way, Williamsburg, VA 23185, and **Madalena Costa** and **Sarah Day**. *Topological Roughness of the Human Red Blood Cell.* Preliminary report.
- Human red blood cells (RBCs) exhibit spontaneous vibratory motions, referred to as flickering. Previous work using measurements of the cell roughness has indicated that older or diseased cells show significantly less roughness and temporal complexity than newly-formed and healthy cells. In our work, we use persistent homology to study this phenomena, and discover a topological version of the roughness. In addition, we show that this topological roughness represents the cells better, and can be utilized to classify different types of cells. (Received September 19, 2016)
- 1125-92-2029     **Naveen K. Vaidya\*** ([vaidyan@umkc.edu](mailto:vaidyan@umkc.edu)), Kansas City, MO 64110. *Mathematical models to evaluate morphine-altered antibody responses on HIV Dynamics.*
- Use of drugs of abuse among HIV infected individuals is rapidly increasing, and thus, is a major concern in HIV infections. In this talk, I will present mathematical models that incorporate the effects of morphine-altered

antibody responses on HIV dynamics within a host. The model is consistent with the experimental data from simian immunodeficiency virus (SIV) infections in morphine-addicted macaques. Using our model, we quantify how morphine alters the HIV-specific antibody responses and how this alteration affects the key components of virus dynamics such as the infection rate of target cells and the clearance rate of free viruses. Furthermore, we incorporate pharmacodynamics properties of morphine into the model and analyze how periodic morphine intake affects the global stability properties of host-virus dynamical system. (Received September 19, 2016)

1125-92-2059 **Emma J Talis\*** ([emma.talis1@marist.edu](mailto:emma.talis1@marist.edu)), **Ondrej Maxian** and **Anna Neufeld**.  
*Understanding Zika Dynamics: Sex, Mosquitoes, and Gender.*

The Zika virus (ZIKV) has captured worldwide attention with the ongoing epidemic in South America and its link to severe birth defects, most notably microcephaly. ZIKV is spread to humans through a combination of vector and sexual transmission, but the relative contribution of these transmission routes to the overall epidemic remains largely unknown. Furthermore, a disparity in the reported number of infections between males and females has been observed. We develop a mathematical model that describes the transmission dynamics of ZIKV to determine the processes driving the observed epidemic patterns. Our model reveals a 7% contribution of sexual transmission to the basic reproductive number,  $R_0$ . This contribution is too minor to independently sustain an outbreak but suggests that vector transmission is the main driver of the ongoing epidemic. We also find a minor, yet statistically significant, difference in the mean number of cases in males and females, both at the peak of the epidemic and at equilibrium. While this suggests an intrinsic disparity between males and females, the differences do not account for the vastly greater number of reported cases for females, indicative of a large reporting bias. (Received September 19, 2016)

1125-92-2093 **Irina N Trofimova\*** ([iratrofimov@gmail.com](mailto:iratrofimov@gmail.com)), 92 Bowman St., Hamilton, Ontario L8S 2T6, Canada. *Operators of diagonal evolution.*

The phenomena of vertical and horizontal emergence are analysed in terms of functional differentiation, the concept of fractal functionality, the concept of the zone of proximate development and an application to iterative map techniques. These theoretical components use three operators that could be linked to functionality of elements composing complex systems. These three operators act on at least three different levels of complexity, making a division into levels of organization rather conditional. (Received September 19, 2016)

1125-92-2131 **Jia Li\*** ([li@math.uah.edu](mailto:li@math.uah.edu)), Department of Mathematical Sciences, University of Alabama in Huntsville, Huntsville, AL 35899. *Mathematical modeling and dynamics of interactive wild and sterile mosquito populations and release strategies.*

In this talk, several mathematical models for the interactive wild and sterile mosquitoes will be presented. We incorporate different strategies for the releases of sterile mosquitoes in the models, including impulsive releases, and investigate the model dynamics. We illustrate how these different release strategies are applied in control of mosquito populations. We then compare the impacts of these control measures. (Received September 19, 2016)

1125-92-2138 **Meghan Stevens\*** ([meghan.stevens@drake.edu](mailto:meghan.stevens@drake.edu)), **Arden Baxter**, **Kristen Abernathy** and **Zachary Abernathy**. *Global Dynamics of a Breast Cancer Competition System.* Preliminary report.

In this talk we present a system of five ordinary differential equations to model the competition for space and resources between breast cancer cells and healthy cells. We include the cancer stem cell hypothesis, which states that there exist proliferating cancer stem cells that repopulate non-proliferating tumor cells and can lead to tumor recurrence. These cancer stem cells exist in a smaller population, making them harder to detect. Additionally, our system contains an equation for the immune system in order to show how the body naturally defends itself from invading tumors. Finally, because the majority of breast cancer cells are estrogen-receptor positive, we include the role of excess estrogen in the body introduced through birth control. Estrogen increases the amount of cancer cells while hindering the effectiveness of the immune system. Its presence also increases the likelihood that healthy cells will mutate. Through stability analysis of sub-models in addition to the full model, we are able to find states in which cancer is eradicated, as well as states in which cancer persists, given certain parameter values. (Received September 19, 2016)

1125-92-2147 **Dawit Denu\*** ([dbd0005@auburn.edu](mailto:dbd0005@auburn.edu)), Auburn University, Department of Mathematics and Statistics, 221 Parker hall, Auburn, AL 36849. *Vector-host epidemic model with direct transmission in random environment.* Preliminary report.

In this presentation, we consider a stochastic vector-host epidemic model with direct transmission under regime-switching. First we examine the existence of a unique positive global solution. Then we study some stability

conditions, such as almost sure exponential stability,  $p$ th moment exponential stability and stochastic asymptotic stability. These stabilities will help us to determine when the infection will die out. Additionally, we provide conditions for the existence and uniqueness of a stationary distribution. Finally, we present numerical simulations to illustrate some of the theoretical results. (Received September 19, 2016)

1125-92-2150 **Azmy S. Ackleh** (ackleh@louisiana.edu), **Baoling Ma\*** (baoling.ma@millersville.edu) and **Robert Miller** (robert@fenstermaker.com). *Finite Difference Approximations for a General Nonlinear Model for the Interaction of Structured Populations and the Environment.*

In this talk, I will present a general model for the interaction of a size-structured population with its environment. The vital rates of the individuals are assumed to depend on a number of variables including the total population and the environmental factors. Finite difference approximations for this general model has been developed. The convergence of the numerical method to the unique weak solution of the nonlinear system of partial differential equations coupled with ordinary differential equations has been proved. At the end some potential applications of this general model will be provided in various fields ranging from blood cell population dynamics to the study of invasive species. (Received September 19, 2016)

1125-92-2165 **Andre Archer, Eliza Matt\*** (ewm1@williams.edu) and **Colin Okasaki.** *Modeling Coral Reef Ecosystems: The Need for Models of Intermediate Complexity.* Preliminary report.

Coral reefs in the Caribbean have been in crisis since the mass die-off of keystone herbivore *Diadema antillarum* in 1983. Since then, reef coverage has declined dramatically, and macroalgae have proliferated to become the dominant occupier of space in many reefs. Though this system has been well studied, there is a need for models of intermediate complexity, that capture a small number of complex interactions and provide insight into the resulting system behavior. To respond to this need, we have constructed several models of Caribbean coral reefs. We focus in particular on parrotfish, whose life-cycle is complex and whose role as grazers of macroalgae is unclear; sea urchins, which are effective grazers but require careful conservation efforts if they are ever to reach their previous population levels; and the importance of spatial information and violations of the mean-field approximation commonly used in reef modeling.

(Received September 19, 2016)

1125-92-2176 **Mary Aronne, Samantha Clapp** and **Sohwan Jung\***, jusohwn@ucla.edu, and **Abigail Kramer, William Wang, Janita Patwardhan** and **Bradford Peercy.** *The Interaction of Calcium and Metabolic Oscillations in Pancreatic  $\beta$ -cells.*

Diabetes is a disease characterized by an excessive level of glucose in the bloodstream, which may be a result of improper insulin secretion. Insulin is secreted in a bursting behavior of pancreatic  $\beta$ -cells in the islets of Langerhans, which is affected by oscillations of cytosolic calcium concentration. We used the Dual Oscillator model to explore the role of calcium in calcium oscillation independent (CaI) versus calcium oscillation dependent (CaD) modes as well as the synchronization of metabolic oscillations in electrically coupled cells. We observed that voltage and calcium coupling result in increased synchronization and are more effective in CaD modes. Also increasing voltage coupling results in greater synchronization. Furthermore, we studied heterogeneous cellular bursting arrangements in the islets and their effects on synchronization. Calcium coupling has a larger impact on synchronization than voltage coupling, in the heterogeneous bursting scenarios. To better represent an entire islet, we altered previous code by further optimizing run-time and memory usage to allow for a greater number of cells. (Received September 19, 2016)

1125-92-2216 **Xiang-Sheng Wang\*** (xswang@louisiana.edu). *Age-structure model with periodic mature probability.* Preliminary report.

In this talk, we will study an age-structure model with periodic mature probability. By using characteristic line method and linear chain trick, we obtain an infinite dimensional differential system. Asymptotic analysis of the steady state solution for this system will be provided. We will also discuss computation of the basic reproduction number and dynamical behavior of the model system. (Received September 19, 2016)

1125-92-2220 **Yanyu Xiao\*** (yanyu.xiao@uc.edu), Pobox 210025, Cincinnati, OH 45221. *A computational model to evaluate outcomes of various vaccine strategies.*

In this work, we investigated the impact of early vaccination on age-specific attack rates and evaluate the outcomes of different vaccination strategies that are influenced by the level of single or two-dose vaccine-induced protections. We developed and parameterized an individual-based model for two population demographics of urban and remote areas in Canada. Our results demonstrate that there is a time period before and after the

onset of epidemic, during which the outcomes of vaccination strategies may differ significantly and are highly influenced by demographic characteristics. (Received September 19, 2016)

1125-92-2243 **Laura F. Strube\*** (strube@math.utah.edu) and **Frederick R. Adler**. *A Mathematical Model of Translational Regulation by the Integrated Stress Response*. Preliminary report.

The Integrated Stress Response (ISR) is a protective mechanism that is activated in response to a wide variety of intracellular stresses. Cells use the ISR to temporarily attenuate canonical translation while simultaneously upregulating the translation of stress response genes via a non-canonical pathway. The key proteins in this system are the eukaryotic initiation factor eIF2 $\alpha$ , its recycler eIF2B, a stress-detecting eIF2 $\alpha$  kinase, and the transcription factor ATF4. We describe a non-linear ODE model of ISR-induced translation regulation incorporating stochastically derived reaction rates that describes translation as a function of stress level. We show that the model exhibits three qualitative behaviors corresponding to degree of stress. When stress levels are low, the system acts as a filter and maintains general translation while exhibiting minimal translation of ATF4. Under intermediate levels of stress, the system produces ATF4 protein while reducing general translation. When stress levels are high both general translation and ATF4 translation fail. This model demonstrates that the stochastic mechanism underlying ATF4 translation allows the cell to differentially regulate two translation mechanisms despite their reliance on the same initiation factors. (Received September 20, 2016)

1125-92-2258 **Faina Berezovskaya\*** (fberezovskaya@howard.edu), Mathematics Department, Howard University, Washington, DC, 20059, USA, Washington, DC 20059. “Fast” and “Slow” *Traveling Impulses and Trains in FitzHugh Model with Diffusion and Cross-diffusion*.

FitzHugh (FH) model (Biophys. J., 1961) has been used as a caricature of the Hodgkin–Huxley equations of neuron firing and to capture, qualitatively, the general properties of an excitable membrane. The spatial propagation of neuron firing due to diffusion of the current potential was described by the FitzHugh–Nagumo (FHN) model (Proc. IRE, 1962). Assuming that such propagation is caused by not only diffusion but also cross-diffusion connection between the potential and recovery the cross-diffusion version of the FH model gives rise to the typical fast traveling impulses and trains, and additionally to the slow traveling waves exhibited in the diffusion FHN equations (Berezovskaya et al, MBE, 2008). In this paper we study the spatial generalization of FH model with both diffusion and cross-diffusion. Bifurcation approach which has been applied to the analysis of the wave system of the model allowed to find the parameter points and velocities of the wave propagation corresponding to the principal rearrangements of the model behaviors ; supposing one of diffusion or cross-diffusion coefficients be small and applying Tikhonov theorem we showed (by computation) that the model has “fast” and “slow” traveling waves that were observed in the model with only cross-diffusion. (Received September 20, 2016)

1125-92-2274 **Tim Reid\*** (treid5@gnu.edu), 12801 Middleton Ln, Fairfax, VA 22033, and **Daniel Anderson**, 4400 University Drive, MS3 F2, Fairfax, VA 22030. *Solving a Tear Film Model with a Spectral Method*. Preliminary report.

This research computes a numerical solution to a tear film thickness model. The model is a partial differential equation that approximates the thickness of a tear film on a contact lens in a blinking eye where the contact lens motion is governed by an ODE that depends on the blinking. The motion of the eyelid and contact lens affect the boundary conditions of the PDE. The numerical solution is calculated using a Chebyshev spectral method for the spatial derivatives and the method of lines for the time evolution. The numerical solution is then compared to similar models that measure tear film thickness in the absence of a contact lens and during an up/down saccade of the eye. (Received September 20, 2016)

1125-92-2285 **William Milligan\*** (william.milligan@emory.edu), 1201 Summit Pointe Way NE, Atlanta, GA 30329, and **Marjorie Jones, Timothy Lucas** and **Courtney Davis**. *Predicting the Effects of Manual Crayfish Removal on California Newt Persistence in Santa Monica Mountain Streams*.

We construct a hybrid, stage-structured mathematical model to study whether trapping of the invasive predatory crayfish *Procambarus clarkii* can prevent local extinctions of the California newt (*Taricha torosa*), a species of special concern native to Santa Monica Mountain streams. Specifically, we numerically and analytically determine under what conditions trapping can drive the crayfish population size to zero. We observe the persistence or the time to extinction for newt populations under corresponding trapping scenarios. No simulations allow for long-term coexistence of newts and crayfish. That is, in every simulation either the crayfish or newt population goes extinct, although several scenarios delay newt extinction by several years in the presence of crayfish. We

predict that crayfish extinction and prolonging newt persistence becomes more likely as the quantity of trapping resources, frequency of trapping implementation, and susceptibility of the crayfish population to trapping increases. Predictions made with our model inform restorative efforts and crayfish management. (Received September 20, 2016)

1125-92-2286 **Caleb L Adams\*** ([cadams5@radford.edu](mailto:cadams5@radford.edu)), PO Box 6942, Radford University, Radford, VA 24142. *Dynamics of a Two-Vector, Two-Pathogen, Two-Host Model*. Preliminary report.

In this talk, the speaker will present recent theoretical results from the dynamics of a two-vector, two-pathogen, two-host model. A system of ordinary differential equations is used to model the dynamics of two vector-borne pathogens (*Rickettsia parkeri* and *Rickettsia amblyommii*) that are increasingly found within tick populations of Virginia spread by two species of ticks (*Amblyomma maculatum* and *Dermacentor variabilis*), within a dual host system. Three methods of transmission are included in the model: vector-borne, transovarial, and co-feeding. Results of numerical simulations are presented and determine a range of parameter values which lead to coexistence of the two pathogens and values which lead to the extinction of one pathogen and persistence of the other. (Received September 20, 2016)

1125-92-2287 **Jia Li\*** ([li@math.uah.edu](mailto:li@math.uah.edu)), Department of Mathematical Sciences, University of Alabama in Huntsville, Huntsville, AL 35899, and **Yang Li**, Department of Mathematical Sciences, University of Alabama in Huntsville, Huntsville, AL 35899. *Discrete-time models for interactive wild and sterile mosquito populations and impact of releases of sterile mosquitoes on malaria transmission*. Preliminary report.

In this talk, we present discrete-time models for the interactive wild and sterile mosquitoes. The population dynamics of the mosquitoes are based on Beverton-Holt type of nonlinearity. We consider different strategies for the releases of sterile mosquitoes in the models and investigate the model dynamics. We then include the mosquito populations into malaria disease models and study the impact of releases of sterile mosquitoes on the disease transmissions. (Received September 20, 2016)

1125-92-2315 **Eli E Goldwyn\*** ([eli.goldwyn@trincoll.edu](mailto:eli.goldwyn@trincoll.edu)) and **Greg Dwyer**. *Density and Environmental-Dependent Transmission in the North American Gypsy and Its Two Pathogens*.

The North American gypsy moth is a non-native pest that undergoes episodic outbreaks. At low densities the larvae are nearly undetectable, while at high densities they cause widespread economic damage and forest defoliation. These population fluctuations are primarily driven by interactions with a viral pathogen and a fungal pathogen. The viral pathogen exhibits density-dependent transmission, while the fungal transmission is both density and weather-dependent. In order to better understand the roles of each pathogen and the effect of weather, we develop a stochastic SIR-type disease transmission model and, using maximum likelihood techniques and an MCMC routine, fit parameters to data collected from nineteen epizootic outbreaks in the upper Midwest. (Received September 20, 2016)

1125-92-2333 **Daniel A Beard\*** ([beardda@umich.edu](mailto:beardda@umich.edu)), 2800 Plymouth Road, Ann Arbor, MN 48105. *Mechanical-Energetic Coupling in Cardiac Contraction and Heart Failure*.

The energetic state of the myocardium is abnormal in decompensated heart failure: phosphate metabolite concentrations are altered and the ATP hydrolysis potential is diminished compared to normal healthy conditions. Although this metabolic dysfunction has been characterized in both patients and animal models, the potential functional consequences of the observed changes in energetic state on mechanical function are not known. We have conducted a series of modeling and experimental studies aimed at quantifying mechanical/energetic coupling in the heart, determining if/how energetic dysfunction contributes to mechanical failure, and testing the following predictions of computer simulations: (1.) Changes in phosphate metabolite concentrations (ATP, ADP, and Pi) that are observed to occur in decompensated cardiac hypertrophy impair force/tension development the heart through a direct effect on myofilament cross-bridge cycle kinetics; and (2.) The resulting impairment in wall tension development leads to systolic dysfunction. This talk will introduce the theoretical/computational basis of mechano-energetic coupling in cardiac contraction, describe our hypothesis on mechano-energetic dysfunction in heart failure, and present experimental tests of our theoretical model predictions. (Received September 20, 2016)

- 1125-92-2348 **Christina J Edholm\*** (cedholm@utk.edu), 236 Ayres Hall, 1403 Circle Drive., Knoxville, TN 37996-1320, and **Benjamin Levy, Orou G Gaoue, Roselyn Kaondera, Moathodi Kgosimore, Enock Koga and Suzanne Lenhart.** *Modeling the Role of Education in Preventing An Outbreak of the Ebola Virus Disease.* Preliminary report.

In light of the Ebola outbreak in 2014, we worked on an Ebola model during our South Mrica Mathematical Sciences Association Masmaw program in 2014 and 2015. Our model partitions the population into those who take precautions against contracting the disease and those who do not. We consider new infections arising in both hospital settings as well as in the community, and include transmission from dead bodies and the environment. Our goal is to illustrate role of education in limiting a potential future Ebola outbreaks in Sudan using data and modeling. We considered implications of a new strain with respect to different death rates and recovery rates. (Received September 20, 2016)

- 1125-92-2354 **Evan Milliken\*** (evmilliken@ufl.edu). *Critical population size for multitype branching process approximation.* Preliminary report.

The basic reproduction number,  $R_0$ , is a threshold for deterministic epidemic models. When  $R_0 < 1$  the disease cannot invade. When  $R_0 > 1$  the disease invades and in some cases, persists. Stochastic models are used to capture the dynamics in a random environment. In this setting,  $R_0 < 1$  implies the disease goes extinct with probability 1, but  $R_0 > 1$  implies only that the probability of extinction is strictly between 0 and 1. In the cases of multiple infectious classes, multitype branching process approximation (MBPA) is used to estimate the probability of extinction of the virus. For MBPA to be accurate, independent transitions and sufficiently large number of susceptible individuals at disease free equilibrium,  $S_0$  are necessary conditions. Two deterministic models of Infectious Salmon Anemia virus infection in a single population are presented with different forces of infection. Equilibria and  $R_0$  are determined and it is shown that  $R_0 > 1$  implies the virus persists. Related stochastic models are introduced and probability of extinction is determined via Monte-Carlo simulation and MBPA. The critical number of susceptibles  $\bar{S}$  is determined so that MBPA is accurate whenever  $S_0 > \bar{S}$ . (Received September 20, 2016)

- 1125-92-2412 **Calistus N Ngonghala\*** (calistusnn@ufl.edu), 1400 Stadium Rd, Department of Mathematics, University of Florida, Gainesville, FL 32611, and **Matthew H Bonds**, 641 Huntington Ave, Boston, MA 02115. *Mathematical exploration of virtuous versus vicious cycles of human health and economic development.*

The livelihoods of the rural poor are nested within ecological communities. The poor rely heavily on their immediate natural environment for subsistence and suffer high morbidity and mortality due to infectious diseases. We show how the dynamics of poverty can be modeled, focusing on infectious diseases. Interactions between infectious diseases and economics can create reinforcing feedbacks associated with persistent poverty, characterized by a stable, low level, equilibrium; or poverty traps characterized by multiple stable equilibria in ecological-economic space. The inherent complexity of these relationships, combined with the spatial/temporal scales at which they occur in the real world, create empirical challenges for estimating parameters and validating the models. We use numerical methods to evaluate the parameter spaces for various combinations of these coupled systems. We find that 13-20% of the feasible parameter space generate bistable outcomes, or poverty traps, and identify key parameters that are most influential to the outcomes of the systems. Combined, these models provide a general framework for deeper theoretical and empirical explorations of infectious diseases and other ecological driver of poverty. (Received September 20, 2016)

- 1125-92-2443 **Adam Mahdi\*** (adam.mahdi@eng.ox.ac.uk), Institute of Biomedical Engineering, University of Oxford, Oxford, OX37DQ, United Kingdom. *A mathematical model for generating synthetic blood pressure.*

A new model capable of simulating many important aspects of human arterial blood pressure is proposed. An effort has been made to reflect realistic morphology of the real ABP signal. Major static and dynamic features of the model can be prescribed by the user, including heart rate, mean systolic and diastolic pressure, and the corresponding physiological control quantities such as baroreflex sensitivity coefficient and Windkessel time constant. It is shown how the model can be used to compile a virtual database of ABP signals reflecting individuals with different clinical conditions and signals containing common artefacts. This, in turn, can be used as a platform for testing a wide range of biomedical signal processing approaches. Other applications of the model, including its use in conjunction with Kalman filters, are also discussed. (Received September 20, 2016)

1125-92-2475 **Anna Mummert\*** ([mummerta@marshall.edu](mailto:mummerta@marshall.edu)) and **Howard Weiss**. *Controlling Viral Outbreaks: Quantitative Strategies*. Preliminary report.

Infectiousness, transmissibility, and death rate of individuals for a viral infectious disease commonly depends on their viral load. We develop and analyze a viral titer structured transmission model that classifies infected individuals by viral load. The model allows for both direct transmission from infectious individuals and indirect transmission from an environmental reservoir. We incorporate a large variety of control measures onto the backbone of our basic viral titer structured transmission model, including vaccination, antivirals, isolation, and environmental disinfection. Mass culling remains the primary strategy for controlling livestock epidemics. The collective information presented here provides the ability to test the efficacy of many combinations of control strategies to end an outbreak, including forms of less indiscriminate culling. (Received September 20, 2016)

1125-92-2486 **Tyler Meadows, Marion Weederhann and Gail S. K. Wolkowicz\*** ([wolkowicz@mcmaster.ca](mailto:wolkowicz@mcmaster.ca)), Department of Mathematics & Statistics, 1280 Main Street West, Hamilton, Ontario L8S 4K1, Canada. *Analysis of a simplified model of anaerobic digestion*.

Anaerobic digestion is a complex naturally occurring process used for waste and wastewater treatment to produce biogas as a renewable source of energy. The so-called Anaerobic Digestion Model No. 1 (ADM1) includes 32 state variables and is not mathematically tractable. We analyze a simplified model proposed by Bornhoff, Hanke-Rauschenback, and Sundmacher, that seems to capture the qualitative dynamics of the ADM1 model, including the possibility of bi-stability and the bifurcation dynamics when substrate concentration is used as the bifurcation parameter. Our analysis applies to a general class of response functions. We then consider the effects of stochastic perturbations of the model using several different numerical schemes having different interpretations in an attempt to understand how to recognize and prevent imminent failure of the reactor. (Received September 20, 2016)

1125-92-2494 **Kiefer Hart Green\*** ([kieferhgreen@gmail.com](mailto:kieferhgreen@gmail.com)). *Finding Theta Oscillations Using Independent component analysis*. Preliminary report.

Prior research using independent component analysis observed theta type oscillations in slow-wave sleep of rats whereas before theta had only been observed in REM sleep and while awake, so this could simply be a false positive. In order to confirm that the observed presence of theta oscillations using independent component analysis means that theta is actually present, we need to know the probability of a false positive. We present an explorative mathematical model for recordings of the brain in order to determine the probability of observing a theta type signal using independent component analysis. We find the probability of observing theta that is not significant at the 97.5% level. (Received September 20, 2016)

1125-92-2544 **Patrick T Davis\*** ([davis1pt@cmich.edu](mailto:davis1pt@cmich.edu)). *Modeling an Infectious Disease in a Continuous Region with an Embedded Metapopulation*. Preliminary report.

The dynamics of an infectious disease are typically modeled through the well-known compartmental framework, where the population is partitioned according to the stage of a disease; and when a population is spatially heterogeneous, the number of compartments is expanded to include the appropriate patchy or metapopulation structure. While such models may adequately describe the overall disease dynamics, they generalize the spatial domain as a series of nodes, thus failing to capture the full detail of the disease's spread. In this talk, we consider the spread of an infectious disease in a continuous spatial domain with an embedded metapopulation structure. (Received September 20, 2016)

1125-92-2549 **Richard Schugart\*** ([richard.schugart@wku.edu](mailto:richard.schugart@wku.edu)), **Nitin Krishna, Hannah Pennington, Ayush Prasad, Marisa Eisenberg and Canaan Coppola**. *Connecting Local and Global Sensitivities in a Mathematical Model for Wound Healing*.

A mathematical model for proteolytic enzyme interactions and their effects on the healing response of a wound will be presented. The differential-equation model was curve fit to averaged data of patients with diabetic foot ulcers. Multiple sensitivity analyses were conducted. Results from a Latin hypercube sampling combined with a partial correlation coefficient analysis will be shown and compared to a local classical sensitivity analysis leading to a local-to-global analysis of our parameter space. Extensions of this work, including work with individual patient data, will be discussed. (Received September 20, 2016)

- 1125-92-2562 **Brian E Carlson\*** (bcarl@umich.edu), 2800 Plymouth Road, NCRC B10/ A126, University of Michigan, Ann Arbor, MI 48109, and **Daniel A Beard, Brendan M McCracken, Kevin R Ward, M Hakam Tiba** and **Jae Hyuk Lee**. *A Combined Experimental and Computational Approach to Characterize Differential Response to Hemorrhagic Shock*.

Treatment of hemorrhagic shock centers around halting bleeding and replacing lost blood volume. However, the differential physiological response of organs during hemorrhage and resuscitation are poorly understood. We used a combined porcine experimental and computational cardiovascular systems model to identify which cardiovascular states relates to a successful resuscitation. In this experimental study eleven animals were hemorrhaged removing 50-55% of blood volume then resuscited with whole blood in amount equal to shed blood. It was found from the experimental observations that baseline blood oxygen extraction (before hemorrhage) correlated well with a full return of cardiovascular function post resuscitation. In the eleven animals, six were found to have an oxygen extraction ration (OER) of less than 30% and five were greater than 30% at baseline. The high OER group showed low peak lactate in the blood and heart rates returning to normal in resuscitation which was indicative of successful resuscitation. The reason for this differential response could not be determined from the measurements alone therefore a lumped parameter cardiovascular system model consisting of a system of 50 ODEs was used to understand the underlying physiological differences between these two groups. (Received September 20, 2016)

- 1125-92-2690 **Amin Oroji, Mohd Omar, Ivy Chung** and **Shantia Yarahmadian\*** (syarahmadian@math.msstate.edu), Department of Mathematics and Statistics, Mississippi State, MS 39762. *An Ito stochastic differential equations model for the dynamics of the MCF-7 breast cancer cell line treated by radiotherapy with the experimental results*. Preliminary report.

We introduce a new mathematical model for studying the population dynamics of breast cancer cells treated by radiotherapy by using a system of stochastic differential equations. As a novel approach, the model captures the concept of the cell cycle in the modeling to be able to evaluate the tumor lifespan. The simulation and experimental results will be presented to show the effectiveness of the radiation under the condition of stability. (Received September 20, 2016)

- 1125-92-2698 **Zhisheng Shuai\*** (shuai@ucf.edu), University of Central Florida, Orlando, FL 32765. *Target reproduction numbers with applications in ecology and epidemiology*. Preliminary report.

There are several important threshold parameters in the literature on population dynamics that determine the population persistence and disease invasibility. For example, the population growth rate and the net reproductive ratio are often derived in matrix population models to determine whether the population persists or goes to extinction. In mathematical epidemiology, the basic reproduction number often serves as a sharp threshold parameter determining whether or not a disease dies out; type reproduction numbers and target reproduction numbers recently are introduced for measuring disease control strategies in a heterogeneous host population. In this talk the concept of the target reproduction number is extended to a more general setting so that it unifies all the above threshold parameters and provides a new measure for population/disease control. (Received September 20, 2016)

- 1125-92-2725 **Turayo Tijani\*** (tutij1@morgan.edu), 1700 E Cold Spring Ln, Department of Mathematics - Actuarial Science, Morgan State University, Baltimore, MD 21251, and **Isabelle Kemajou-Brown** (elisabeth.brown@morgan.edu), 1700 E Cold Spring Ln, Department of Mathematics - Actuarial Science, Morgan State University, Baltimore, MD 21251. *Modeling Epidemics: An Application Using Big Data*.

The human and economic loss epidemics leave in their wake has led to improved methods to model and study these occurrences. The increase in global travel, and the mass movement of people and things has not only improved nation's economies but also the spread of diseases. The use of big data in modeling epidemics can prove to be instrumental in studying the spread of this diseases and how to best combat it while saving time, money, and lives. Considering large volumes, variety, and velocity of data on human interaction can help make informed predictions about the spread of diseases. This paper proposes a modified SIR model, using big data, in order to predict future diagnoses of a disease, to assess the quality of the model, to evaluate the parameters of the model, and to suggest recommendations to improve the quality of the model and its parameters. The paper introduces real-life scenarios by introducing quantifiable intervention strategies and a budget constraint in order to develop a model which minimizes newly infected individuals. The use of predictive analysis in the form of

big data in responding to epidemics can help better allocate resources to specific populations in order to reduce mortality. (Received September 20, 2016)

1125-92-2744 **Seema Nanda\***, 6188 Kemeny Hall, Dept. of Mathematics, Dartmouth College, Hanover, NH 03755, **L G DePillis**, Claremont, CA, and **Ami Radunskaya**, Claremont, CA.

*B-CLL progression considering immune response.*

We propose a model for B cell Chronic Lymphocytic Leukemia (B-CLL) which captures the clinical heterogeneity the disease is known to have. We study disease progression across different patients via a unified yet flexible approach, using a mathematical model of B-CLL with immune response, that can exhibit both rapid and slow disease progression. We present analysis of existing data in the medical literature, determine ranges of values for parameters of the model, and compare our model outcomes to clinical patient data. This work provides a tool that could shed light on factors affecting the course of disease progression in patients. (Received September 20, 2016)

1125-92-2749 **Parisa Florence Samareh\*** (psamareh@vt.edu), 1628 Patrick Henry Drive, Apt 247, Blacksburg, VA 24060. *Accelerating Thermoacoustic Tomography*. Preliminary report.

Thermoacoustic Tomography (TAT) is an imaging technique used to display a cross section through a solid object. TAT exceeds the health benefits of other tomographic methods such as x-ray and ultrasound because it does not require as much radiation, making it a safer alternative. However, like many inverse problems, the process of TAT is extremely involved and subsequently still under development. This research investigates computational improvements to TAT that increase the method's efficiency as well as overall accuracy. (Received September 20, 2016)

1125-92-2772 **Timothy Wessler\***, The Department of Mathematics, Phillips Hall, CB#3250 UNC-CH, Chapel Hill, NC 27599-3250, and **Alex Chen**, **Scott A McKinley**, **Richard Cone**, **M Gregory Forest** and **Samuel K Lai**. *Modeling HIV-Antibody-Mucin Kinetics*.

A promising strategy to reduce vaginal HIV transmission is to design antibodies (Ab) that can crosslink HIV viruses to mucins mesh comprising cervicovaginal mucus (CVM), thereby blocking viruses from reaching and infecting target cells. Here, we modeled the kinetics of HIV in semen that diffuse into a layer of CVM populated by Ab with distinct affinities to mucins. We compared a continuum approach using partial differential equations for the diffusion of Ab and virion concentrations with stochastic simulations of individual virions. Our model has generated many unexpected yet important insights into the design of antibodies that can effectively reduce the flux of HIV arriving at target cells, offering a promising approach to reducing vaginal transmission of HIV and other common sexually transmitted infections. (Received September 20, 2016)

1125-92-2775 **Katrina Johnson\*** (kjohnson@math.utah.edu), 155 S 1400 E ROOM 233, Salt Lake City, UT 84112-0090, and **Frederick R. Adler**. *Understanding Adipocyte Dynamics through Mathematical Modeling*. Preliminary report.

Adipocytes, or fat cells, are responsible for limiting the exposure of other tissues to lipid accumulation by sequestering and storing lipids during the feeding state and providing energy by releasing lipids during the fasting state. Obesity is the expansion of fat mass in response to positive energy imbalance. Fat tissue expands by increasing the size of adipocytes, hypertrophic expansion, and/or the number of adipocytes, hyperplastic expansion. I developed a size structured model to explore the population dynamics of adipocytes. This model is composed of three equations: a logistic ordinary differential equation, a variable coefficient advection-reaction equation, and an integral differential equation. These ordinary and partial differential equations are coupled through the boundary condition, which represents the production of new fat cells, and the variable advection term. The model highlights the dynamics of two potential problems with fat mass expansion that may lead to metabolic syndrome: insufficient production of new fat cells and fat cells with inadequate potential to sequester lipids.

Keywords: advection-reaction equations, coupled ODEs and PDEs, mathematical modeling, adipocyte dynamics, obesity (Received September 20, 2016)

1125-92-2792 **Joe Latulippe\*** (jlatulip@norwich.edu), **Derek Lotito** and **Donovan Murby**. *A mathematical model for calcium signaling in neurons in the presence of Amyloid Beta*. Preliminary report.

Alzheimer's Disease (AD) is the leading cause of Dementia and can ultimately lead to death. It is believed that the onset of AD occurs due to intermittent interruption of synaptic transmission between neurons. Regulation of intracellular calcium signaling plays a critical role in neuronal signal transmission. The accumulation of Amyloid-Beta proteins may trigger an increase in intracellular calcium levels by disrupting the regulatory

mechanisms within the neuron. Steadily elevated intracellular calcium levels disrupt normal calcium signaling leading to synaptic dysfunction, synaptic loss, memory loss, and neurodegeneration as seen in AD. We present a mathematical model for calcium regulation in neurons in the presence of Amyloid-Beta. A detailed description of the model components is given and compared to experimental data. By investigating the mathematical structure of the model, we systematically determine the key parameters and how they affect calcium regulation. We use numerical simulations to further investigate the model and to determine what it tells us about the biological mechanisms involved in AD. (Received September 20, 2016)

1125-92-2793 **Janet A. Best\*** (best.82@osu.edu), **Michael C. Reed**, **Sean D. Lawley** and **H. Fred Nijhout**. *Modeling volume transmission and neurotransmitter homeostasis.*

In volume transmission, neurons in one brain nucleus send their axons to another nucleus where neurotransmitter is released into the extracellular space, modulating the activity of the electrophysiological circuitry. Here we present a number of mathematical issues that arise in calculating the neurotransmitter concentration in the extracellular space and its homeostatic regulation. (Received September 20, 2016)

1125-92-2807 **Markus H. Schmidt** (mschmidt@sleepmedicine.com), **Theodore W. Swang\*** (swang.1@osu.edu), **Ian M. Hamilton** (hamilton.598@osu.edu) and **Janet A. Best** (jbest@math.ohio-state.edu). *Energy Conservation and the Function of Sleep: A Mathematical Model.*

Sleep has long been considered an energy conservation strategy. Metabolic rate reduction has been considered the mechanism by which sleep conserves energy, although modest computed savings and diverse upregulated functions during sleep have led to skepticism of this mechanism. Using a mathematical model, we argue instead that the mechanism by which sleep saves energy is a state-dependent repartitioning of metabolic processes. (Received September 20, 2016)

1125-92-2861 **Dan Ryan** and **Carl Toews\***, toewsc@gmail.com, and **James Sanchirico** and **Paul Armstrong**. *Implications of policy adjustment costs for fisheries management.*

Optimal control applied to natural resource management models often results in fast moving policies. However, many management policies change slowly, suggesting mis-modeled policy adjustment costs. This talk presents the results of a study that compares two methods for incorporating such costs into fisheries management models. Both methods involve a quadratic penalty term, but while one method attaches this term directly to harvesting effort, the other attaches it to the rate at which effort changes. Using a combination of numerical and analytical methods, and focusing on cyclically changing environments, this work compares features of equilibrium and approach paths. Both cost models decrease variation in effort but increase variation in optimal stock size, and produce smoother approach paths to long term equilibrium targets. The work includes an analysis of model and parameter errors, and shows that mis-specifying the rate-of-change costs can result in particularly complicated optimal approach paths and poor tracking of recruitment variation. (Received September 20, 2016)

1125-92-2868 **Oliver Matthew Sale\*** (saleo@southwestern.edu), 1400 Rivery Blvd., APT 4230, Georgetown, TX 78628. *Investigation of Central Texas Surface Ozone Concentrations 1980-2015.* Preliminary report.

We aim to investigate the effect of increasing population on the Austin region's surface ozone concentrations. To understand this effect, we need to understand the effect El Nino Southern Oscillation (ENSO) has on the region's surface ozone concentrations. In our preliminary comparative data analysis on the effects of "very strong" ENSO events (Southern Oscillation Index, or SOI, of 4) on the surface ozone concentrations of Site 484530014 (Austin Northwest). Stratifying by precipitation, the mean monthly ozone concentrations for very strong ENSO events were compared to the neutral (SOI of 0) or very weak (SOI of 1-3) ENSO events. Suppression of ozone concentrations was observed for the very strong ENSO years relative to ENSO neutral years for April and May, no significant change was observed in other months. Our current investigation includes examining, wind speed and direction, temperature, and surface reflectivity in order to identify natural drivers. The remaining surface ozone concentration could be attributed to the local anthropogenic production of ozone precursors. By having reliable surface ozone concentrations with robust knowledge of regional background ozone and ENSO effect events, we could study anthropogenic trends in Austin's surface ozone concentrations. (Received September 20, 2016)

1125-92-2896 **Jemal Mohammed-Awel**, **Ruijun Zhao**, **Eric Numfor\*** (enumfor@augusta.edu) and **Suzanne Lenhart**. *Management Strategies in a Malaria Model Combining Human and Transmission-blocking Vaccines.*

Concurrent use of multiple strategies has been recommended as an effective strategy to reduce malaria and its burden. In this talk, we present a new mathematical model studying control strategies of malaria transmission,

where the control is a combination of human and transmission-blocking vaccines, and larvicide (vector control). The existence of a backward bifurcation is established analytically in the absence of vaccination, and numerically in the presence of vaccination. Optimal strategies, using vaccination and vector control are investigated to gain qualitative understanding on how different combinations of these controls should be used to reduce disease prevalence in a malaria endemic setting. Our results suggest that the combination of the two vaccination controls integrated with vector control has the highest impact on reducing the number of infected humans and mosquitoes. (Received September 20, 2016)

1125-92-2905 **Zeynep Akcay\*** (zakcay@qcc.cuny.edu), zakcay@qcc.cuny.edu, and **Xinxian Huang, Farzan Nadim and Amitabha Bose.** *Phase resetting and bistability in a neuronal network.*

The behavior generated by neuronal networks depends on the phase relationships of its individual neurons. Observed phases result from the combined effects of individual cells and synaptic connections whose properties change dynamically. We consider a recurrent network of two oscillatory neurons that are coupled with reciprocal synapses. We use feed-forward descriptions of the phase response curves of the neurons and the short-term synaptic plasticity properties to define Poincare maps for the activity of the network. The fixed points of these maps correspond to the phase locked modes of the network. Using these maps, we analyze the creation of bistable phase locking solutions in neuronal networks when there is short-term synaptic depression from one neuron to another. We discuss how achievement of bistable solutions depend on the phase response curves of the neurons and the dynamics of the short-term synaptic plasticity. We illustrate our findings in Quadratic Integrate-and-Fire and Morris-Lecar neuron models. (Received September 21, 2016)

1125-92-2974 **Matthew Glomski\*** (matthew.glomski@marist.edu) and **Olivia Brozek.** *Conditions for endemicity in a compartmental model with deceased-infectious class.* Preliminary report.

We describe the transmission of certain filoviral diseases via a compartmental model with deceased-infectious class. We then consider the effects of background population dynamics under this model, and find that under specific assumptions on the parameter space the model predicts the existence of an asymptotically stable endemic equilibrium. Our report draws on the analysis of similar models appearing in the literature. (Received September 20, 2016)

1125-92-3003 **Balamurugan Pandiyan\*** (pandiyab@uww.edu), 800 West Main Street, Whitewater, WI 53190. *Modeling slow growth of thyroid nodules and initiation of cancer.*

Hashimoto's thyroiditis is an autoimmune disorder of the thyroid gland in which the immune system produces antibodies and attacks the tissue of thyroid gland and disrupts normal function of the HPT axis. As a result, thyroid stimulating hormone (TSH) increases, thyroxine (FT4) decreases and the functional size of thyroid gland decreases over time. Thyroid nodules coexist with Hashimoto's thyroiditis and grow slowly as TSH increases above normal value ( $> 4$  mU/L). About 5-15% of thyroid nodules becomes cancerous. In this talk, we present a stochastic model for the formation of thyroid nodules and initiation of cancer in nodules for patients with Hashimoto's thyroiditis. (Received September 20, 2016)

1125-92-3035 **Shelby Nicole Wilson\*** (shelby.wilson@morehouse.edu), Department of Mathematics, 830 Westview Dr., Atlanta, GA 30314. *Mathematical Model of Temperature Effects on Human Sleep Regulation.*

Sleep is a behavioral state in which we spend nearly one third of our lives. This biological phenomenon clearly serves an important role in the lives of most species. Here, we present a mathematical model of human sleep-wake regulation with thermoregulation and temperature effects. Simulations of this model show features demonstrated in experimental data such as elongation of duration and number of REM bouts across the night as well as the appearance of awakenings due to deviations in body temperature from thermoneutrality. The model highlights how temperature effects interact with sleep history to effect sleep regulation. This model serves to insights into the role that temperature plays in abnormal sleep behavior. (Received September 20, 2016)

1125-92-3038 **Caitlin S Hult\*** (chult@live.unc.edu), **Paula A Vasquez, David Adalsteinsson, Josh Lawrimore, M Gregory Forest and Kerry Bloom.** *Modeling nucleosomal DNA in living yeast.*

The genome in living yeast cells is a highly dynamic system where entropic interactions and nuclear confinement drive the formation of domains of high chromosomal interaction, known as topologically associating domains. We investigate dynamic organization and territory formation of all 16 chromosomes in living yeast cells during inter-phase, using coarse-grained, entropic polymer chain models. We are interested in determining the mechanisms, such as packaging molecules that create loops within chromatin fibers, governing inter- and intra-chromatin

fluctuations, and inducing global features of the entire genome. The Bloom lab measures specific DNA sites in specific chromosomes using live cell fluorescence microscopy. Our goal is to identify the sufficient biological and biophysical assumptions necessary to reproduce the experimental data, from which we aim to shed insights into dynamics and structure that are beyond current experimental resolution. (Received September 20, 2016)

1125-92-3084 **Heidi E Brown\*** ([heidibrown@email.arizona.edu](mailto:heidibrown@email.arizona.edu)), 1295 N. Martin Ave., Tucson, AZ 85724, and **Joceline Lega, Daoqin Tong** and **Wangshu Mu**. *Network-based modeling for chikungunya spread in Dominica*.

In 2013/14 chikungunya spread through the Caribbean infecting more than 3 million individuals. The following year, Zika similarly spread throughout the region. Between island spread of chikungunya was shown to be associated with geographical proximity to an infected island. An analysis of pandemic spread found that diseases propagated at uniform speeds when Euclidean distance was replaced by a flow based network between cities. We were able to downscale a pandemic model, to describe the propagation of infection across the island nation, Dominica. Similar to the pandemic model, we found the effective speed at which chikungunya spread across the 750 sqkm island. This suggests that the spread of infectious diseases can be estimated based on an understanding of the flow between transportation hubs (in this case major cities). (Received September 21, 2016)

1125-92-3141 **Ariel Cintron-Arias\*** ([cintronarias@etsu.edu](mailto:cintronarias@etsu.edu)), Department of Mathematics and Statistics, Box 70663, East Tennessee State University, Johnson City, TN 37614-0663. *Student Life Tables: Case Study for a Regional University*. Preliminary report.

Longitudinal datasets of enrollment for a regional public university, together with records at the state level, are employed here. We implement matrix population models (e.g. Leslie) in the context of mathematical demography, while inspired by four measurements of enrollment corresponding to student classification: freshman, sophomore, junior, and senior.

Matrix model parameters are estimated with ordinary least squares and bootstrap sampling methods. Then, the parameter estimates are transformed to approximate student life table functions that include: student life expectancy, probability of school departure, average number of years in each school group (e.g. sophomore). (Received September 21, 2016)

## 93 ► *Systems theory; control*

1125-93-568 **Ozkan Ozturk\*** ([ozturko@mst.edu](mailto:ozturko@mst.edu)), 2024 Highway E, Apt B, Rolla, MO 65401, and **Haci Mehmet Guzey**. *Optimal Control of Quadrotor Unmanned Aerial Vehicles on Time Scales*. Preliminary report.

In this paper, an optimal controller is developed for quadrotor unmanned aerial vehicles (UAVs) on time scales. The UAVs are assumed to have desired positions and orientations and the proposed controller is used to bring the UAVs to the desired positions and orientations by minimizing a cost function on time scales. The proposed controller will be able to work for generalized time scales such as the discrete time intervals with time varying sampling interval. This will provide several benefits such as computational cost reduction in real time applications. The effectiveness for our optimal controller of quadrotor UAVs is demonstrated in a simulation, which validates our theoretical claims. (Received September 06, 2016)

1125-93-837 **Dimplekumar N Chalisahar\*** ([dipu17370@gmail.com](mailto:dipu17370@gmail.com)), 435 Mallory Hall, Department of Applied Mathematics, Letcher Avenue, Lexington, VA 24450, and **C Ravichandran** and **S Dhanalakshmi**. *Controllability of fractional neutral impulsive integro-differential equations in Banach spaces*.

In this paper, we prove the controllability for impulsive neutral fractional integrodifferential equations in Banach spaces using Banach fixed point theorem, semigroup theory and fractional calculus. We present the controllability result by introducing a class of distributed controls which are highly useful for the computational purpose also. The controversy on the solution operator is discussed here and we emphasize that we use the generalized Caputo derivative with the lower bound at zero for the system considered. An example is given to illustrate the abstract results.

(Received September 12, 2016)

1125-93-1028 **Sergei Avdonin\*** (s.avdonin@alaska.edu), 2534 Bridlewood Lane, Atlanta, GA 30339, and **Nina Avdonina** and **Julian Edward**. *Inverse Problems for the Wave Equation on Graphs*.

Quantum graphs are metric graphs with differential equations defined on the edges. Recent interest in control and inverse problems for quantum graphs is motivated by applications to important problems of classical and quantum physics, chemistry, biology, and engineering.

In this talk we describe exact controllability and identifiability results for the wave equation on compact graphs. We consider inverse problems for graph-like networks of inhomogeneous strings with masses attached at the interior vertices. For graphs without cycles, we demonstrate that the unknown densities of the strings, lengths of the edges, attached masses, and the topology of the graph can be recovered using observations associated with all but one boundary vertices. For graphs with cycles additional observations at the internal vertices are required for constructing stable identification algorithms. The proofs are based on the boundary control method and leaf peeling method developed in our previous papers.

The exact controllability results in non-symmetric Sobolev spaces related to described identification problems are proved. We will also discuss the corresponding multivariate sampling and interpolation problems. (Received September 14, 2016)

1125-93-1083 **Ozkan A Ozer\*** (ozkan.ozero@wku.edu), Department of Mathematics, Western Kentucky University, Bowling Green, KY 42101. *Dynamic modeling of some smart composites and the lack of controllability/stabilizability*. Preliminary report.

Traditionally, piezoelectric materials have been modeled by the electrostatic assumption and actuated by a voltage source. Recent studies show that different types of actuation (charge or current) are also possible, and they have their own pros and cons over voltage actuation. Due to the electrostatic approach in each type of actuation, the corresponding models can be stabilized by appropriate "mechanical" feedback controllers. In this talk, a smart composite involving piezoelectric layers is modeled through the variational approach with/without magnetic effects (electrostatic vs. dynamic). All types of actuation are considered. For each case, unlike the existing results in the literature, we show different types of stability failures. (Received September 14, 2016)

1125-93-1552 **Nak-seung Patrick Hyun\*** (nhyun3@gatech.edu) and **Erik I Verriest** (erik.verriest@ece.gatech.edu). *A Causal Impact Modeling in Impulsive Systems using Non-standard Analysis*.

A new approach based on nonstandard analysis (NSA) for modeling nonlinear impulsive system is proposed. An extended real space is constructed as a proper subspace of hyperreals in NSA with a countable basis. The extended space is then decomposed into two parts, a *sensible* (reals) part and an *insensible* (infinitesimals) part, which insensible parts are further decomposed depends on the convergence rate of infinitesimals. Next, a hyperreal-valued Heaviside functions on the extended space is point-wise well defined via a sequence approach, where its restriction function to insensible part is denoted as a *shape* function. Depending on the shape function and its domain on insensible part, a class of distinct Heaviside functions is proposed. In addition, a generalization of the differential operator in NSA is proposed, and used to represent singular functions as regular ones on the extended space. The key idea is that a discontinuous function can be not only continuized in the insensible part but also differentiated in the infinitesimal space. A causal way for characterizing jumps in discontinuous system follows by converting the classical impulsive differential equation (IDE) with a generalized IDE that involves the newly defined singular function. (Received September 17, 2016)

1125-93-2674 **Scott Hansen\*** (shansen@iastate.edu), Department of Mathematics, Iowa State University, Ames, IA 50011, and **Marius Tucsnak**. *Exact Boundary Controllability as a Limit of Distributed Control Problems in Wave Systems*.

As is well known, for wave type systems, exact controllability and exponential stabilizability are closely related. In particular, Russell's principle (RP) asserts that for time reversible systems, exponential stabilizability implies exact controllability. Moreover RP provides a construction of the corresponding controls. In this work utilize RP-based controls to study the behavior as  $\epsilon \rightarrow 0$  of solutions corresponding to locally distributed controls supported in an  $\epsilon$  neighborhood of an endpoint of an interval. Examples focus on the 1-d wave equation, but we also describe a general abstract result that may apply to a variety of similar problems. (Received September 20, 2016)

1125-93-3103 **Roby Poteau\*** (rpoteau2010@my.fit.edu) and **Ugur Abdulla**. *On the inverse problems for the nonlinear dynamical systems arising in mathematical biology*. Preliminary report.

We consider the inverse problem for the identification of constant parameters for systems of nonlinear ODEs arising in mathematical biology. We implement a numerical method suggested in U.G.Abdulla, *Journal of Optimization Theory and Applications*, 85, 3(1995), 509-526. The method combines Bellman's quasilinearization with sensitivity analysis and Tikhonov's regularization. We apply the method to various biological models such as the classical Lotka-Volterra system, bistable switch model in genetic regulatory networks, gene regulation and repressilator models from synthetic biology. The numerical results and application to real data demonstrate the quadratic convergence. We discuss the application of the method for the identification of functional parameters in the Hilbert spaces setting. (Received September 21, 2016)

## 94 ► *Information and communication, circuits*

1125-94-269 **Ingrid Daubechies\*** (ingrid@math.duke.edu), Math Department, Campus Box 90320, Durham, NC 27708. *Phase retrieval on infinite dimensions*.

Retrieving an arbitrary signal from the magnitudes of its inner products with the elements of a frame is not possible in infinite dimensions. Under certain conditions, signals can be retrieved satisfactorily however. (Received August 22, 2016)

1125-94-389 **Joseph H Silverman\*** (jhs@math.brown.edu), Mathematics Department - Box 1917, Brown University, Providence, RI 02912. *Cryptographic constructions via secret finite field isomorphisms*. Preliminary report.

We describe how to use secret finite field isomorphisms to construct cryptographic primitives such as partially homomorphic encryption schemes and digital signature schemes. This is joint work with Jeff Hoffstein, Jill Pipher, and others. (Received August 30, 2016)

1125-94-546 **Jill Pipher\*** (jill\_pipher@brown.edu), **Jeffrey Hoffstein** (jhoff@math.brown.edu) and **Joseph Silverman** (jhs@math.brown.edu). *Cryptographic constructions via secret finite field isomorphisms*.

We describe how to use secret finite field isomorphisms to construct cryptographic primitives such as partially homomorphic encryption schemes and digital signature schemes. This is joint work with Jeff Hoffstein, Joseph Silverman, and others. Primary MSC(s): 94A60, 11T71 (Received September 06, 2016)

1125-94-653 **Wei-Hsuan Yu\*** (u690604@gmail.com), okemos. *New bounds for equiangular lines and spherical two-distance sets*.

A set of lines in  $\mathbb{R}^n$  is called equiangular if the angle between each pair of lines is the same. We derive new upper bounds on the cardinality of equiangular lines. Let us denote the maximum cardinality of equiangular lines in  $\mathbb{R}^n$  with the common angle  $\arccos \alpha$  by  $M_\alpha(n)$ . We prove that  $M_{\frac{1}{a}}(n) \leq \frac{1}{2}(a^2 - 2)(a^2 - 1)$  for any  $n \in \mathbb{N}$  in the interval  $a^2 - 2 \leq n \leq 3a^2 - 16$  and  $a \geq 3$ . Moreover, we discuss the relation between equiangular lines and spherical two-distance sets and we obtain the new results on the maximum spherical two-distance sets in  $\mathbb{R}^n$  up to  $n \leq 417$ . (Received September 08, 2016)

1125-94-658 **Michelle R DeDeo\*** (mdedeo@unf.edu), 1 UNF Dr., Dept of Math & Stat, Jacksonville, FL 32224. *An alternative perspective for EEG data analysis*. Preliminary report.

Methods from graph theory have been valuable in analyzing connectivity of real world networks such as EEGs and fMRIs. In this talk we compare different how parameters such as characteristic path length, global efficiency, local efficiency, and the clustering coefficient are used to analyze brain connectivity as well as propose an alternative way of thinking about the data. (Received September 08, 2016)

1125-94-827 **Alice Silverberg\*** (asilverb@math.uci.edu). *Through the Cryptographer's Looking-Glass, and what Alice found there*.

I will give an account of my adventures in the wonderlands of mathematics and cryptography. I'll offer some food for thought on how mathematics can be useful in cryptography, how cryptography can motivate research of mathematical interest, and how mathematicians and cryptographers can learn to play well together. A primary focus will be on the quest to find cryptographically useful multilinear maps. Along the way I will share some thoughts that I hope will be helpful on what I learned about community. (Received September 12, 2016)

1125-94-853 **Navid Ghadermarzy\*** (navidgh@math.ubc.ca), UBC Math Department, 1984 Mathematics Road, Vancouver, BC V6T1Z2, Canada, and **Yaniv Plan** and **Ozgur Yilmaz**. *1-Bit Tensor Completion*. Preliminary report.

We consider the problem of 1-bit tensor completion where a multi-dimensional array (tensor) is estimated from 1-bit measurements of a subset of its entries. These measurements are generated according to a probabilistic model. In order to make the problem feasible, further restrictions must be imposed on the underlying tensor. Specifically, we study "low rank" tensors whose indices have bounded infinity norm.

In order to solve this problem, we study low rank tensors and different norms that can be used to impose low-rank structure on a tensor. In particular, we focus on three classes of tensors: exact low rank tensors, tensors with bounded nuclear norm, and tensors with bounded max-norm and provide theoretical bounds on reconstruction error.

A common practice in solving general low rank tensor completion problems is matricizing the tensor and solving the matricized problem. Our numerical results show that we get significantly better results when we factorize the low rank tensor and employ an alternating method for minimizing the nuclear norm or the max norm. Although these algorithms just approximate the optimal solution, our empirical results confirm the theoretical advantages of solving the 1-bit tensor completion problem instead of matricizing the problem. (Received September 12, 2016)

1125-94-1014 **Lara Dolecek\***, Engineering IV, 56-147B, Los Angeles, CA 90095. *Coding-Theoretic Techniques for Modern Memories*.

Driven by the needs of emerging data-centric applications, modern memories must be reliable, cheap, and dense. In this talk, we overview some of our recent research on novel channel coding methods for modern memories, including context-aware coding for on-chip memories and graph-based methods for dense non-volatile memories. We demonstrate several new constructions and show how carefully designed channel codes can substantially improve the performance of memories. We also discuss several new research directions in this area. (Received September 14, 2016)

1125-94-1033 **Alexander T Drumm** and **Hieu D Nguyen\*** (nguyen@rowan.edu), Rowan University, Department of Mathematics, 201 Mullica Hill Rd., Glassboro, NJ 08028. *Filtering and Decoding Non-binary Tenengolts Codes Capable of Correcting Multiple Insertion/Deletion Errors*.

We present a filtering algorithm to remove unwanted twin codewords in the construction of non-binary Tenengolts codes (due to Palunčić et al.) whose codewords generate run sequences consisting of Helberg codewords. This leads to new codes with good rates that are capable of correcting multiple insertion/deletion errors. We also describe an efficient decoding algorithm to correct for such errors. (Received September 14, 2016)

1125-94-1037 **Carolyn Mayer\*** (cmayer@huskers.unl.edu) and **Christine A. Kelley**. *Fountain Codes on Partial Erasure Channels*. Preliminary report.

Fountain codes are a class of erasure codes in which encoding symbols are generated and transmitted until enough symbols are received to recover the original message. Since the number of encoding symbols is unlimited, fountain codes are rateless. Moreover, the structure of the dynamically-constructed encoding graph greatly affects a fountain code's performance. Recently, partial erasure channels have been introduced to model applications in which some information may remain after an erasure event. In this talk, we will discuss fountain codes on partial erasure channels. (Received September 14, 2016)

1125-94-1143 **Justin Romberg\*** (jrom@ece.gatech.edu), **Kiryung Lee** and **Ning Tian**. *Multichannel Blind Deconvolution: New models, Fast algorithms, and Mathematical Performance Guarantees*.

We consider a classic problem in signal processing. We observe the convolution of a signal unknown source vector with  $M$  different channel response vectors; from these observations, our task is to estimate the  $M$  responses. Traditionally, this is formulated as a null space approximation problem; given that the channel responses have limited length, we can form a matrix from the observations that is guaranteed to have a null space of dimension 1, and knowledge of this null space immediately reveals the channel responses up to a global constant. When the observations are noisy, there are classical guarantees about the asymptotic consistency of this process, but numerically it is very ill posed.

In this talk, we report progress on this problem on several fronts. First, we demonstrate that with a different kind of channel model, where the responses live in generic subspaces located on a small number of intervals, this problem becomes much more well-posed. Second, we introduce a realistic joint channel model, and show

that estimating the parameters for this model amounts to estimating a rank-1 tensor from noisy linear observations. Finally, we give performance guarantees for estimating this tensor using a simple, efficient alternating minimization procedure. (Received September 15, 2016)

1125-94-1462      **Arya Mazumdar\*** ([arya@cs.umass.edu](mailto:arya@cs.umass.edu)), College of Information and Computer Sciences, 140 Governors Dr, Amherst, MA 01003. *Algorithms for Storage Capacity: An Information Theoretic Analogue of Vertex Cover.*

Motivated by applications in distributed storage, we have recently defined the storage capacity of a graph as the maximum amount of information that can be stored across the vertices of a graph such that the content at any vertex can be recovered from the information stored at the neighboring vertices. Computing the storage capacity is a fundamental problem in network coding and is related, or equivalent, to some well-studied problems such as index coding and generalized guessing games. In this talk, we consider storage capacity as a natural information-theoretic analogue of the minimum vertex cover of a graph. Indeed, while it was known that storage capacity is upper bounded by minimum vertex cover, we show that by treating it as such we can get a  $3/2$  approximation for planar graphs, and a  $4/3$  approximation for triangle-free planar graphs. Since the storage capacity is intimately related to the index coding rate, we get a 1.923 approximation of index coding for planar graphs and  $3/2$  approximation for triangle-free planar graphs. Previously only a trivial 4 approximation of the index coding rate was known for planar graphs. We then develop a general method of “gadget covering” to upper bound the storage capacity in terms of the average of a set of vertex covers. (Received September 16, 2016)

1125-94-1508      **Wentao Huang, Michael Langberg and Joerg Kliewer\*** ([jkliewer@njit.edu](mailto:jkliewer@njit.edu)), New Jersey Institute of Technology, Department of Electrical and Computer Eng., Newark, NJ 07102, and **Jehoshua Bruck.** *Communication Efficient Secret Sharing.*

A secret sharing scheme is a method to store information securely and reliably. Particularly, in a threshold secret sharing scheme, a secret is encoded into  $n$  shares, such that any set of at least  $t_1$  shares suffice to decode the secret, and any set of at most  $t_2 < t_1$  shares reveal no information about the secret. Assuming that each party holds a share and a user wishes to decode the secret by receiving information from a set of parties; the question we study is how to minimize the amount of communication between the user and the parties. We show that the necessary amount of communication, termed “decoding bandwidth”, decreases as the number of parties that participate in decoding increases. We prove a tight lower bound on the decoding bandwidth, and construct secret sharing schemes achieving the bound. Particularly, we present a scheme that achieves the optimal decoding bandwidth when  $d$  parties participate in decoding, universally for all  $t_1 \leq d \leq n$ . (Received September 17, 2016)

1125-94-1684      **Cheng Cheng\*** ([cheng.cheng@knights.ucf.edu](mailto:cheng.cheng@knights.ucf.edu)), Department of Mathematics, University of Central Florida, Orlando, FL 32816, **Junzheng Jiang** ([jzjiang@guet.edu.cn](mailto:jzjiang@guet.edu.cn)), School of Information and Communication, Guilin University of Electronic Technology, Guilin, Peoples Rep of China, and **Qiyu Sun** ([qiyu.sun@ucf.edu](mailto:qiyu.sun@ucf.edu)), Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Graph filter banks and signal reconstruction.* Preliminary report.

A spatially distributed network contains a large amount of agents with limited sensing, data processing, and communication capabilities. Its topology could be described by a graph with a vertex representing an agent and an edge meaning a direct communication link between two vertices. In this talk, we consider filter banks on spatially distributed network and use them for signal analysis and reconstruction, (Received September 18, 2016)

1125-94-1711      **Abdul J. Jerri\*** ([jerria12@yahoo.com](mailto:jerria12@yahoo.com)), 69 Leroy Street, Potsdam, NY 13676. *Multivariate Sampling in Signal Processing: A Tutorial Review with Brief Analysis and Applications.*

The well known Shannon sampling theory is mainly for one variable signals. For its various aspects, to date we have many excellent overview treatments as tutorials and books. However, for the multivariate sampling theory, we have a good number of papers, but its overview is limited to only a couple of chapters in several books on sampling theory. The present talk centers on a tutorial review of the various aspects of the multivariate sampling for the past sixty years, since it was suggested by Parzen in 1956. (Received September 18, 2016)

1125-94-1793 **Hiram Habid Lopez Valdez\*** (hlopezv@clemson.edu), Clemson University, Martin Hall O-2, Clemson, SC 29634. *Evaluation codes: codes using commutative algebra.*

In this talk we introduce the linear codes known as evaluation codes. We are interested in this sort of codes because its parameters can be computed using basic tools of commutative algebra, and we will see how to do it.

The family of evaluation codes mainly depends on an affine or a projective set, thus we are going present some examples using different families of affine and projective points.

Finally, we are going to use the ideas of evaluation codes to define new algebraic concepts. (Received September 19, 2016)

1125-94-2922 **Kashi Neupane\*** (knneupane@ung.edu). *One-Round Authenticated Group Key Establishment from Multilinear Mappings.*

In this paper, we propose a one-round authenticated group key establishment protocol. Our protocol is based on Graded Decisional Diffie-Hellman assumption, and it requires timestamps. The resulting solution is in the random oracle model, builds on a multi-linear map, and offers integrity as well as strong entity authentication. (Received September 20, 2016)

1125-94-3102 **Michael E O’Sullivan\*** (mosullivan@mail.sdsu.edu) and **John Mackenzie**. *Violations of The Ingleton inequality and the inclusion-exclusion ratio.* Preliminary report.

The Ingleton inequality gives an inner bound for the space of entropy vectors associated to random  $n$ -vectors. Entropy vectors that are abelian group characterizable all satisfy the Ingleton inequality, and a natural question is to study constructions, using non-abelian groups, that violate the Ingleton inequality. We decompose the Ingleton ratio into the product of three simpler terms—two *inclusion-exclusion ratios*, and one *mixing ratio*. We show that each of these ratios is larger than 1 for an abelian group and we look for examples where these are smaller than 1. The smallest example is  $A_5$ , the smallest non-abelian simple group. (Received September 21, 2016)

## 97 ► Mathematics education

1125-97-1 **Arsalan Wares\*** (awares@valdosta.edu). *A Twist on an Old Mathematical Puzzle.*

The emphasis on problem solving has always been a hallmark of a reformed mathematics classroom. However, it is not always easy for teachers to find problems that are challenging enough that their students can persevere in solving. Traditional textbooks are full of bite-size problems- that would be more accurately called “tasks.” Rich mathematical problems are still rare in US contemporary textbooks of mathematics. It is always a challenge for teachers to find mathematical problems that absolutely refuse to leave one’s head.

The purpose of this talk is to introduce and discuss a large-scale mathematical problem in the area of number theory. The problem that we will discuss was inspired by a well-known problem commonly known as the Monkey and the Coconuts problem. Martin Gardner, the American puzzle enthusiast, described the Monkey and the Coconuts problem as “the most tried and least solved problem in the world.” (p. 3, Gardner, 2001). The problem discussed in this talk is like (but not parallel to) the Monkey and the Coconuts problem. Knowledge of high-school mathematics can be employed to solve the problem.

Gardner, M. (2001). *The Colossal Book of Mathematics: Classic Puzzles, Paradoxes, and Problems*. New York : W. W. Norton & Company. (Received April 12, 2016)

1125-97-6 **Tanvir Prince** (tprince@hostos.cuny.edu), 500 Grand Concourse, B 408, New York City, NY 10451, and **Nieves Angulo\***, 500 Grand Concourse, B 408, New York City, NY 10451. *An Experiment with Module Based Instruction in Linear Algebra.*

The Hostos Community College, as part of the City University of New York, developed some modules in Linear Algebra to enhance the curriculum of the linear algebra and also introduce students to research environment early in their student’s life. This is possible through a collaborative research grant with City College of New York. The presentation will discuss how this module is implemented in the classroom setting to improve critical thinking and creativity skills, hands-on, team oriented, and interdisciplinary learning via collaborative research projects. (Received May 02, 2016)

1125-97-46 **Sean P Yee\*** ([yee@math.sc.edu](mailto:yee@math.sc.edu)), University of South Carolina, LeConte College, 317K, 1523 Greene Street, Columbia, SC 29208, and **Kimberly Rogers** ([kcroger@bgsu.edu](mailto:kcroger@bgsu.edu)), Bowling Green State University, 408 Mathematical Sciences Building, Bowling Green, OH 43403. *Graduate Student Instructor Mentorship Model: A professional development that trains experienced graduate students to pedagogically mentor novice mathematics graduate student instructors.* Preliminary report.

Collegiate mathematics departments rely on graduate students instructors (GSIs) to teach foundational undergraduate mathematics courses. Although GSIs are positioned to significantly impact the quality of instruction for these courses, they lack the infrastructure and pedagogical training about collegiate mathematics teaching. Through a collaborative NSF grant, Bowling Green State University and the University of South Carolina have created and implemented the Graduate Student Instructor Mentorship Model (GSIMM). The GSIMM offers professional development and training to experienced graduate students (mentors) to support less experienced graduate student instructors (protégés). Mentors conduct iterative teaching observations of protégés, one-on-one post-observation discussions, and biweekly meetings. The GSIMM allows for a sustainable, cost-effective, pay-it-forward design, bridging education and mathematics departments. Our presentation will share results from implementing the mentor professional development at both universities including first-hand discussion by a few of the mentors. Data collection, analysis, and results will be shared along with criteria for implementing the GSIMM at other universities. (Received June 20, 2016)

1125-97-372 **Jessica Libertini\*** ([libertinijm@vmi.edu](mailto:libertinijm@vmi.edu)), 407 Mallory Hall, VMI, Lexington, VA 24450, and **Karen Bliss** ([kmlbliss@ncsu.edu](mailto:kmlbliss@ncsu.edu)), 407 Mallory Hall, VMI, Lexington, VA 24450. *Modeling Improvisation: Taking Classroom Materials & Making Them Your Own.*

From SIMIODE to GAIMME, modeling activities for the classroom are popping up everywhere. However, even the most-detailed, best-intentioned classroom materials suffer from a fatal flaw that almost always prevents a simple as-is adoption: every classroom is unique. Each group of students brings their own personalities, each instructor has his or her own style and comfort zone, and each syllabus has its own learning objectives unique to the needs of the institution. In this presentation, we, as two instructors, will discuss how we've been able to take the same modeling activity and adapt it to best fit our individual teaching styles and our courses' learning objectives. During the presentation, we will also guide the audience through a thought-process designed to aid in selecting and adapting materials. (Received August 30, 2016)

1125-97-400 **Denise Natasha Brewley\*** ([dbrewley@ggc.edu](mailto:dbrewley@ggc.edu)), 1000 University Center Ln, Georgia Gwinnett College, Building A- 1601, Lawrenceville, GA 30043, and **Priya Shilpa Boindala** ([pboindal@ggc.edu](mailto:pboindal@ggc.edu)), 1000 University Center Ln, Georgia Gwinnett College, Building D-1470, Lawrenceville, GA 30043. *Using LearningCatalytics<sup>TM</sup> Technology as a Tool to Enhance Student Engagement in Pre-Calculus & Calculus.*

*LearningCatalytics<sup>TM</sup>* (LC) is an interactive student response technology which allows students to participate in class using a dynamic polling feature. Students are able to use their own mobile devices, to participate in class sessions. This technology allows students to respond to a variety of question types, not limited to multiple choice or fill in the blank questions. With its unique grouping feature, LC can also group students together based on their responses in order for them to engage in meaningful discussion about their answers. In this digital poster session, the presenters will give a live demonstration on how LC is used in their Precalculus and Calculus courses. Audience participants will have the opportunity to experience first hand, as students, the power of this technology and how it may be used to transform a Pre-Calculus, Calculus, or other mathematics course in order to create significant learning experiences through enhanced student engagement. The presenters will (1) provide a brief description of LC; (2) demonstrate how to use LC, highlighting its dynamic polling feature; (3) show how LC aides in peer instruction. Finally, presenters will close the poster session with time for question and answer from audience participants. (Received August 31, 2016)

1125-97-416 **Virgil U Pierce\*** ([virgil.pierce@utrgv.edu](mailto:virgil.pierce@utrgv.edu)), 1201 University Dr., Edinburg, TX 78542. *WebWork for High School College Preparatory Mathematics Courses in Texas.*

We are engaged in a collaboration of the community colleges, a technical college, and our university serving four counties of South Texas. In response to 2014 legislation in the State of Texas we joined with a local non-profit agency and the public school districts to develop and deliver College Prep Courses in English language arts and mathematics. The goal is for students to place into, and ultimately be successful in, college level writing intensive and mathematics intensive courses (for us this is mostly College Algebra and Statistics courses). While this is a state level mandate without funding our implementation has been unique in a very important way: our course,

using WebWork, can be implemented by a school district for no additional cost beyond that for the teacher and the classroom.

We are hosting the mathematics courses on the online homework server WebWork being managed at UTRGV, and now are offering the use of our server for all high school level mathematics courses in our local region. This presentation will focus on our development process and the product we have put together. We will try to address how this effort could be replicated in other regions. (Received September 01, 2016)

1125-97-449 **Michael Hvidsten\*** ([hvidsten@gac.edu](mailto:hvidsten@gac.edu)), 1271 Raymond Avenue, St. Paul, MN 55108.  
*Gesture-Based Geometry on Mobile Platforms.*

A new geometry application for mobile devices will be presented. This application uses gesture-based pattern recognition to allow students to create circles, lines, perpendiculars, etc, by simply drawing on the screen with their finger, much as they would on paper. This application permits exploration of Euclidean and Non-Euclidean geometries and is ideally suited for use in K-12 and college level geometry. (Received September 02, 2016)

1125-97-560 **Hiroko Kawaguchi Warshauer\*** ([hw02@txstate.edu](mailto:hw02@txstate.edu)), 601 University Drive, San Marcos, TX 78666, **Max Leon Warshauer** ([max@txstate.edu](mailto:max@txstate.edu)), 601 University Drive, San Marcos, TX 78666, and **Terence William McCabe** ([tm04@txstate.edu](mailto:tm04@txstate.edu)), 601 University Drive, San Marcos, TX 78666. *University and Public School Partnerships: Camps, curriculum, and teacher training.*

Mathworks, a center for mathematics education at Texas State University, has 3 pillars—summer math programs for students, teacher training, and curriculum development. In this talk, we describe the role of partnerships with K-12 schools in supporting each of the pillars. We describe how we set up collaborations and how research is integrated into our graduate mathematics education program. This includes a collection of current and ongoing research projects, problems and questions. The Mathworks Junior Summer Math Camp (JSMC) is a 2-week multi-tiered program for 200 students in grades 4-8, organized in partnership with the San Marcos Consolidated Independent School District (SMCISD). SMCISD provide classrooms, transportation for students and a free breakfast and lunch for our participants. Mathworks provides master teachers from local schools and 100 scholarships for students with financial need. Ten inservice teachers observe the camp in the morning and receive “continuing education credits” for participating in the afternoon teacher professional development. Thirty undergraduates assist in the camp and participate in the professional development along with the master and inservice teachers. (Received September 06, 2016)

1125-97-584 **Emma Previato\*** ([ep@bu.edu](mailto:ep@bu.edu)), Department of Mathematics and Statistics, Boston University, Boston, MA 02215-2411. *Funding and Mentoring Projects at Boston University.*

The talk will describe the Undergraduate Research Opportunities Program at Boston University, created in 1997. Awarded on a competitive basis, it provides a work-study research stipend for students to experience research, in partnership with a faculty member. Students present a poster at the annual symposium. The experience is quite different from a standard Research Experience for Undergraduates, in that the mentoring relationship begins at the application process, is one-on-one throughout the experience, and typically also includes professional focus and professional development. The presenter has been an above-average active participant (two students per term, including the summer) since inception and will highlight general strategies to design, conduct, and report a research project, and follow through with professional development. She will illustrate specific projects, with the goal of showcasing the diversity of techniques, student skills and styles that students have exhibited, ranging from areas such as mathematical finance, to cryptography, quantum mechanics to climate change, computer simulation to projective geometry. Both strengths of, and challenges encountered by, different students at different stages of learning will be exemplified. (Received September 09, 2016)

1125-97-603 **Bariaa Ghazi Shatila\*** ([bshatila@flagler.edu](mailto:bshatila@flagler.edu)), Saint Augustine, FL 32084. *Using MyMathLab for teaching undergraduate Mathematics courses.*

Previous studies have shown that MyMathLab is a valuable online learning tool for college students. As part of their college curriculum, students are required to take math courses. However, some students need additional help to succeed in their Mathematics courses. In my presentation, I will show how MyMathLab helps students do well in this subject.

: (Received September 07, 2016)

1125-97-638 **Daniel J Katz\*** ([dkatz@math.brown.edu](mailto:dkatz@math.brown.edu)). *Thirteen Hours: Graduate Student Teacher Training Under Time Constraints.*

At Brown University, graduate students have the luxury of a first year free of teaching responsibilities, but they have a heavy courseload and many demands on their time during that year. As a result, we are able to deliver

a teacher training program for first-years during the spring semester (which I have overseen for four years) but our time with these students is limited to one hour per week.

In this talk, I will discuss the structure of this program and how I aim to accomplish as much as possible in a total of thirteen hours, hoping that this will help other schools with similar time constraints to introduce or modify programs of their own. I will also discuss changes I am considering, and what I might do with the program if we had more time available. (Received September 08, 2016)

1125-97-639 **Judith Quander\*** ([quanderr@uhd.edu](mailto:quanderr@uhd.edu)), The University of Houston-Downtown, Department of Mathematics and Statistics, One Main Street, S705, Houston, TX 77002. *Building a Graduate Certificate Program for Dual-Credit Mathematics Teaching For High School Mathematics Teachers*. Preliminary report.

In this session, I will talk about a collaborative project between mathematics and mathematics education faculty at the University of Houston-Downtown and a Houston-area school district. The idea to create a certificate program that would provide current high school mathematics teachers with enough graduate mathematics coursework so that they would be eligible to teach dual-credit mathematics came from a direct need of the school district. The planning for this program was a highly collaborative effort between UHD, the school district and neighboring community colleges. I will discuss the implications and considerations for a small teaching university trying to reach the needs of the surrounding school districts including creating appropriate courses, finding willing faculty, providing meaningful support to certificate students and working with the school districts to create a successful program. I will detail the 18-month collaborative planning process of this program; successes that we are proud of and pitfalls that we grappled with and continue to grapple with as we move forward; an overview of the graduate certificate program itself; and some preliminary data collected on the success of the teachers. (Received September 08, 2016)

1125-97-681 **Robin J. Gottlieb\*** ([gottlieb@math.harvard.edu](mailto:gottlieb@math.harvard.edu)), 1 Oxford Street, Department of Mathematics, Cambridge, MA 02138, and **Jameel Habib Al-Aidroos** ([jameel@math.harvard.edu](mailto:jameel@math.harvard.edu)), 1 Oxford Street, Department of Mathematics, Cambridge, MA 02138. *From Graduate Student to Classroom Teacher: Constructing Pathways for Success*.

We have a department full of mathematically talented graduate students. This exceptional pool of talent has not always translated neatly into a department full of exceptional Teaching Fellows. As our program of support for developing teachers has evolved we have kept our sights focused on excellence in pedagogy; on engaging, student-centered instruction. Over the years we have developed a collection of paths that lead much more reliably to excellence in the classroom. We will tell you how our program evolved, how we assessed our challenges, and how we came up with ways to conquer them. (Received September 09, 2016)

1125-97-719 **Juliana V Belding\*** ([juliana.belding@bc.edu](mailto:juliana.belding@bc.edu)). *The Teaching Seminar and Beyond: What Do Graduate Students Find Valuable as They Learn to Teach?*

We'll start with a brief overview of graduate student teaching and teaching development programs at Boston College, which includes both Teaching Assistants (TAs) and Teaching Fellows (TFs) who teach their own classes. Then we will discuss results from a recent survey of graduate students at BC about their teaching and training experience, with an emphasis on comments that might generalize to other departments and settings.

In particular, we'll focus on what graduate students value most about the teaching seminars, as well as what they find missing in their training. We'll also address balancing autonomy in the classroom (which many TFs value) with a need for consistency in courses, both logistically and mathematically, and the fact that novice TAs and TFs often "don't know what they don't know" about teaching.

We'll conclude with some ideas that can improve teaching assistant development, including creating more formal mentoring and observation structures, and holding departmental discussions about the goals and expectations of the courses being taught. (Received September 09, 2016)

1125-97-766 **David Manderscheid\*** ([manderscheid.1@osu.edu](mailto:manderscheid.1@osu.edu)), Office of the Exec. Dean, College of Arts and Sciences, 186 Univ. Hall, 230 North Oval Mall, Columbus, OH 43210. *A Dean's Perspective*.

I will present my perspective on teaching assistant development programs. In particular I will address their importance for student learning, graduate student development, and for the college and university. (Received September 11, 2016)

1125-97-813 **Eliza Dargan Gallagher\***, [egallag@clemson.edu](mailto:egallag@clemson.edu). *'I teach' versus 'I am a teacher': The Role of Identity in Graduate Student Professional Development.*

The K-12 community has long wrestled with many of the issues we now face in preparing graduate students to teach. Elements of current TA professional development programs such as peer mentoring, supervised early teaching, and use of case studies and lesson study all have their roots in best practices for K-12 teacher preparation. These best practices have a short-term impact on student success and participant satisfaction, and also strengthen *teacher identity*, the extent to which participants think of themselves as teachers. K-12 education research has established teacher identity as a strong long-term predictor of retention in the field, of reflection on and adaptation of instructional practice, and of student success in mathematics.

Translating research on teacher identity to the graduate student population is complex and challenging. We present models of teacher identity development for secondary mathematics and discuss how these translate - or fail to translate - to graduate students. We discuss implications for designing and assessing professional development for graduate students, and provide recommendations for practice to strengthen teacher identity as a central facet of professional identity as a mathematician. (Received September 12, 2016)

1125-97-814 **David Manuel and Tracy Weyand\*** ([tracy\\_veyand@baylor.edu](mailto:tracy_veyand@baylor.edu)). *The Evolution of TA Training at Texas A&M.* Preliminary report.

In 2012, Tracy Weyand developed, implemented, and ran the GREAT (Graduate students Reaching for Excellence in Academics and Teaching) Program while a graduate student at Texas A&M University. This voluntary seminar and mentoring program provided knowledge about university policies and course content, as well as time-saving methods. It also provided an introduction to classroom technology and alternative pedagogical styles. This program has since evolved into the official required Graduate Teaching Assistant training program for the Department of Mathematics at Texas A&M under the direction of Instructional Assistant Professor David Manuel.

We will discuss this evolution and describe the ongoing training which graduate students receive from the time they begin our PhD program to their search for employment in academia as graduation approaches. We will also discuss the impact the program has had on both the culture within the department and on the graduate students themselves. (Received September 12, 2016)

1125-97-882 **Jacqueline A Jensen-Vallin\*** ([jacqueline.jensen@lamar.edu](mailto:jacqueline.jensen@lamar.edu)) and **W. Ted Mahavier** ([mahavierwt@lamar.edu](mailto:mahavierwt@lamar.edu)). *A Moore Method Apprenticeship Program.*

At Lamar University, incoming Master's students with an interest in teaching are assigned a mentor through the Moore Method Apprenticeship program. During the four-semester program, the student works closely with the mentor to master the skills required to conduct an inquiry-based course independently. The student observes the class during the first semester and team-teaches during the second and third semesters. The program culminates with the student teaching independently during the last semester. We will discuss this program, providing hints for success and pitfalls to avoid in this type of mentoring relationship. (Received September 12, 2016)

1125-97-982 **Fabiana Cardetti\*** ([fabiana.cardetti@uconn.edu](mailto:fabiana.cardetti@uconn.edu)). *Identifying Opportunities to Renovate the Professional Development of Teaching Assistants.*

In this presentation, I will talk about the efforts that our department has gone through to renovate the support we offer our teaching assistants. Our focus has been on finding opportunities based on research, recommendations, and best practices that effectively and efficiently address the current expectations for excellence in mathematics teaching and the demand for improved professional development for the next generation of mathematics faculty. I will share specific resources, activities, and materials we have incorporated successfully, as well as some core challenges we have encountered thus far. (Received September 13, 2016)

1125-97-1005 **Tina Deemer\*** ([deemer@math.arizona.edu](mailto:deemer@math.arizona.edu)), 6021 E. Eastland St., Tucson, AZ 85711. *Arizona Mathematics GTA Training: A Gentle Crash Course in Teaching.*

The University of Arizona Department of Mathematics first created a formal training program for first-year GTAs twenty years ago. Over these years, the program has been revamped and re-envisioned a number of times. The current model, which includes a three-day pre-semester training and a one-semester course, is centered on supporting the TAs in the development of the core competencies needed for teaching. We have come to believe that, given where our new TAs are coming from and what we ask of them to accomplish in their first year, we should expose them to a variety of techniques that can be effective and provide support and feedback as they try to implement different strategies. In this presentation, I will give a brief summary of the structure of our TA

Training Program, and provide an overview of the philosophy we espouse and the types of activities in which we and the TAs engage. (Received September 14, 2016)

1125-97-1022 **Sean D Lawton\*** (slawton3@gmu.edu), Department of Mathematical Sciences, 4400 University Dr, Exploratory Hall, 4400, Fairfax, VA 22030, and **Jack Love** (jack.eddie.love@gmail.com), Department of Mathematical Sciences, 4400 University Dr, Exploratory Hall, 4400, Fairfax, VA 22030. *Examples of Outreach Activities*. Preliminary report.

In this talk we discuss our point-of-view on university outreach in mathematics, in particular, outreach conducted in elementary, middle, and high schools. We then discuss and demonstrate some example activities that we have and are currently running. (Received September 14, 2016)

1125-97-1054 **D. I. Wallace\*** (dwallace@math.dartmouth.edu), Department of Mathematics, Dartmouth College, Hanover, NH 03755. *Preparing graduate students for 40 years of teaching*.

Mathematics graduate students who pursue a career in an academic department can expect a 40-year career doing research and teaching. In a typical graduate program they will spend 5 years preparing for the research part of their career, but little time preparing for 40 years of teaching. What would it mean to take this preparation seriously? In this presentation we look at some frameworks for thinking about teaching graduate students to become excellent teachers, using the teaching seminar at Dartmouth as a source of examples as they relate to these frameworks. (Received September 14, 2016)

1125-97-1123 **Judith M Arms\*** (arms@math.washington.edu), Department of Mathematics, University of Washington, Box 354350, Seattle, WA 98195-4350, and **Alexandra Nichifor** (nichifor@math.washington.edu), Department of Mathematics, University of Washington, Box 354350, Seattle, WA 98195-4350. *Career-long and Timely Training for TAs: An overview of the various components of the Math TA Training at the University of Washington, how the program evolved, and lessons learned*.

The departmental TA Training at UW started in the 1980's. The original version provided new TAs with an initial orientation and some teaching information before classes began. This has expanded to include hands-on activities, an online manual, guidance through the first quarter by experienced mentor TAs, and weekly prep workshops with a course TA coordinator.

After teaching quiz sections in freshmen classes for a year or longer, our TAs may choose to give calculus lectures for small summer classes, or to teach their own sections of some sophomore courses. For both types of assignments, a faculty mentor provides guidance both before teaching begins and throughout the term.

At both levels the goal is to provide people with information at the point in their teaching career when they are ready to hear it and use it. (Received September 15, 2016)

1125-97-1335 **PJ Couch\***, P.O. Box 10047, Beaumont, TX 77710, and **Braxton Carrigan, Steven Clontz** and **Zachary Sarver**. *Mathematical Puzzle Programs: Connecting with Students Using Puzzles and Games*.

**Mathematical Puzzle Programs** (MaPP) designs annual team-based problem-solving competitions for secondary students, available at no cost for colleges and universities to print and run at their own campus. Our goal is to challenge students to use discovery-based methods for solving mathematical problems while giving them experience with mathematics beyond the usual curriculum.

Based upon local outreach events developed at Auburn University and Lamar University, the MaPP High School Challenge '17 will be hosted by several universities and math camps in the United States and Japan during the 2016-17 school year. This year also debuted MaPP's first Middle School Challenge, hosted at Lamar University in November.

In this talk, we hope to give you a feel for the types of puzzles students encounter during our events, as well as provide information on how you can download MaPP's free open-source resources or host a MaPP event at your own campus. (Received September 16, 2016)

1125-97-1357 **Dan Teague\*** (teague@ncssm.edu), NCSSM, 1219 Broad Street, Durham, NC 27705. *Mathematical Modeling in High School Mathematics*.

The mathematical content of the high school curriculum offers a vast array of new tools appropriate for mathematical modeling. At the same time, the life experiences and interests of high school students are rapidly expanding, opening the door to a variety of real-world problems from every discipline for their investigation.

The High School section of the GAIMME Report focuses on these unique opportunities and some of the special issues that arise in teaching modeling in high school. This presentation will highlight some distinguishing features of modeling in the high school curriculum. (Received September 16, 2016)

1125-97-1364 **Peter Olszewski\*** (pto2@psu.edu), 4205 College Drive, Erie, PA 16563. *A Five-Year Update on the Academic Transition Program.*

Back in the spring of 2011, Penn State Behrend started the Academic Transition Program for older adult students. The program was a 6-week summer session, which provided future students a foundation on the mathematics, writing, and study skills needed for a fall semester start. For the past five years, the program has been overwhelmingly successful. This presentation will provide an overview of the program's beginnings, how it has evolved, and the success stories of our students. (Received September 16, 2016)

1125-97-1409 **John Boller\*** (boller@math.uchicago.edu), Mathematics Department, 5734 S. University Ave., Chicago, IL 60637. *The College Fellow Program at the University of Chicago.*

The Mathematics Department at the University of Chicago hires many different types of course assistant, depending upon the level and nature of the course, the experience of the instructor, and other factors. The one with the most rigorous level of preparation is the College Fellow Program. College Fellows are second-year graduate students who do a full-year apprenticeship with one or more senior faculty, usually in upper-level undergraduate courses. Their training involves practice lectures, coaching, and much conversation. Newly added to the program, there has been a recent collaboration with the Chicago Center for Teaching, with videotaped microteaching sessions as well as quarterly one-day seminars on teaching. (Received September 16, 2016)

1125-97-1428 **Lisa Townsley\*** (townsley@math.uga.edu), University of Georgia Dept. of Mathematics, Athens, GA 30602. *Mentoring GTAs as they teach: Providing tools to develop successful teachers.*

The University of Georgia Mathematics Department requires graduate students to complete 7 semester hours of training in teaching college level mathematics. When a graduate student is tapped to teach Precalculus (first) and Calculus I, s/he participates in a semester-long teaching seminar focused on the content and teaching issues relevant to that particular course. The seminars follow the recommended course content schedule, but also include activities and interactions to help graduate TAs grow as teaching professionals in the discipline. The primary goal of the seminars is to develop the teaching skills of our graduate students so as to ensure that the undergraduate students registered for Precalculus and Calculus receive competent instruction. This talk will provide details on these seminars as well as how we monitor the growth of our teachers. (Received September 16, 2016)

1125-97-1463 **Scott Baldrige\*** (sbaldrid@math.lsu.edu), Department of Mathematics, Baton Rouge, LA 70803. *Writing the PK-12 Eureka/EngageNY Math Curriculum.*

I will describe how we brought together a group of over 100 mathematicians, teachers, and educators to write the Eureka/EngageNY math curriculum. Why is this curriculum so special and so widely used today? (Over 50% of U.S. teachers use the curriculum in some way.) In this talk I will discuss one of the reasons: We created a check-your-egos-at-the-door, tough-love environment that motivated everyone on the writing team to put forward their best effort. (Received September 16, 2016)

1125-97-1526 **James S Tanton\*** (tanton.math@gmail.com), 5033 E Turquoise Ave, Paradise Valley, AZ 85253. *Math Circles: Circles for Students and Circles for Teachers. Circles Galore across the Nation!*

What is mathematics circle? What has caused their number to grow from two to well over two hundred in the past few decades? Who do they serve and what does one do in them? What mathematics is typically explored in a mathematics circle? James Tanton, co-founder of the MAA's Special Interest Group of Math Circles for Students and Teachers, and advisor for the American Institute of Mathematics' Math Teachers' Circle Network and for the National Association of Math Circles will present a brief overview of the Math Circles in the U.S. and the impact they have had. (Received September 17, 2016)

1125-97-1657 **J Todd Lee\*** (tlee@elon.edu), Campus Box 2320, Mathematics and Statistics, Elon, NC 27713. *Flipped Mastery Cycle Learning of Calculus II using a Plethora of Tools.*

The type and variety of content in a traditional American collegiate Calculus II course suggests a best-practice pedagogy like an emporium model or flipped-mastery-cycle (FMC) model. At a university where professor-student engagement is highly prized, a FMC based pedagogy is a natural choice to allow for just-in-time help and discussions between the professor and one or more students. Such a model can also be more robust in

“covering” material that still exists with high inertia against change, and but still include the almost necessary skills in scripting in a professional computational software environment. With the included required familiarity with the use of Mathematica as a computational, visualization and communication tool, the number of technology tools in this course is rather high. Presentation will include sample screencastings along with the equipment and e-tools used, as well as demonstrations of the LMS and WebAssign setups and the work flow used for the course. (Received September 18, 2016)

1125-97-1718      **Paul E Seeburger\*** (pseeburger@monroecc.edu), 1000 E. Henrietta Rd., Rochester, NY 14623. *Playing with Multivariable Calculus Concepts Wearing 3D Glasses*. Preliminary report.

A tour of an NSF-funded project that seeks to develop geometric intuition in students of multivariable calculus. CalcPlot3D, an online exploration environment, allows students (and instructors) to create and freely rotate the graphs of functions of two variables, contour plots, vectors, plane and space curves, regions of integration, vector fields, parametric surfaces, implicit surfaces, etc. 3D glasses can be used for a real 3D perspective! Come get a pair and try it out! This JavaScript web app works on smart phones, tablets, and regular computers. A series of concept explorations is also being created. Each allows students to “play” with the concepts visually to develop their geometric understanding. The grant project is titled, Improving Conceptual Understanding of Multivariable Calculus Through Visualization Using CalcPlot3D (NSF-DUE-IUSE # 1524968). See <http://web.monroecc.edu/calcNSF/>. (Received September 19, 2016)

1125-97-1756      **Laura R Lynch\*** (llynch@ccga.edu), One College Drive, Brunswick, GA 31520, and **Tanya Cofer** (tcofer@ccga.edu) and **German Vargas** (gvargas@ccga.edu). *Supporting Teachers and Engaging Students: The College of Coastal Georgia Math Department’s partnership with Glynn County Public Schools*.

Catalyzed by an institutional focus on service-learning and a successful faculty-teacher partnership initiative, CCGA faculty and Glynn County public school teachers have forged a strong and productive alliance with the common goal of bridging the mathematics achievement gap for students in the local school system. Our collaborative work spans the elementary, middle and high-school level while focusing jointly on teacher development and engaging K-12 students through STEM initiatives in their schools and on our campus. Strengthening our partnership, our new STEM Education Improvement Plan goes beyond individual outreach efforts and brings the integration of the two institutions one step further. STEM college faculty are working with lead middle school teachers in math and science to identify content areas and to collaboratively improve both content and pedagogy. Driven by the needs of the teachers and supported by administrators, faculty, and staff from the college and the school system, these projects align with our efforts to approach STEM education as an integrated and seamless growth process that starts at the elementary school level and culminates in satisfying the workforce needs of the state. (Received September 19, 2016)

1125-97-1776      **Meghan De Witt\*** (mdewitt@stac.edu). *They Really Can Do Research in Pure Math: Group-theoretic Problems Accessible to Underprepared Students*.

Applied and computational mathematics are not the only subjects in which undergraduate students can perform meaningful research. We describe several problems in group theory which undergraduate student have investigated with almost no algebraic background prior to the start of their research. These include pairings of group theory with topics such as the Fibonacci Numbers and the tesseract. (Received September 19, 2016)

1125-97-1806      **Sylvia T Bozeman\*** (sbozeman2@bellsouth.net), Atlanta, GA, and **Yewande Olubummo** (yolubumm@spelman.edu) and **Joycelyn Wilson** (jwilso20@spelman.edu). *Mathematics at Spelman College: Mission Possible*.

Spelman College has a long history of educating African-American women for leadership in the liberal arts tradition. In this talk we give attention to an unfolding story of how the College successfully intensified its efforts and achieved success over time in mathematics and the sciences. We explain how a classic mathematics curriculum combined with a series of special programs that address the particular needs of the College’s students, including the need for mentoring, financial support, access to scientific professions, and an early exposure to research, has led to success by large numbers of students. We make note of some common elements that have enhanced student development, including the critical role of scholars programs. Today, this undergraduate Historically Black College of approximately 2000 students celebrates a unique level of productivity in mathematics, the sciences and engineering at the undergraduate and graduate levels among its students and alumnae. Our goal is to share some strategies that may be adaptable in departments working to increase success and diversity among its mathematics graduates. (Received September 19, 2016)

- 1125-97-2053 **Jessica M Deshler\*** ([deshler@math.wvu.edu](mailto:deshler@math.wvu.edu)), West Virginia University, Dept. Of Mathematics, PO Box 6310, Morgantown, WV 26506. *Current Teaching Assistant Professional Development Program Structures: Findings from a National Survey.*

The design of many professional development programs for novice college mathematics instructors is or could be better informed by education research on teaching and learning. This is one of four “research sampler” talks where the presenters will discuss research findings relevant to graduate student professional development. Many mathematics departments across the US are currently providing professional development in teaching to their graduate teaching assistants. The formats and components of these programs, however, vary. In this talk, I will present the data from a national survey about the structure of existing TA professional development programs. (Received September 19, 2016)

- 1125-97-2177 **Brendan W Sullivan\*** ([sullivanb@emmanuel.edu](mailto:sullivanb@emmanuel.edu)), 400 The Fenway, Boston, MA 02115. *“You ask ME a question!” What happens when we let students “strike back” on online quizzes?*

Course management systems allow instructors to create a variety of quiz questions to assess student learning. To keep students engaged in my courses and let me consistently check on their progress, I give short quizzes once every couple of weeks. I have also started including a final “question” on each quiz: students must pose a question *to me!* By requiring them to self-reflect on a regular basis, I have found that students are more likely to ask interesting and insightful questions (or actually ask anything). Moreover, because students take these quizzes online at a time of their choosing, I find that they are less anxious and more honest with their queries.

In this talk, I will share some observations and recommendations based on my experiences with this approach over the last two years (via the Canvas LMS). I will also provide some rough statistics about what students tend to ask about, as well as some suggestions for how and why to implement this (or something like it) in your course. (Received September 19, 2016)

- 1125-97-2186 **Wendy M Smith** ([wsmith5@unl.edu](mailto:wsmith5@unl.edu)) and **Yvonne Lai\*** ([yvonnexlai@unl.edu](mailto:yvonnexlai@unl.edu)), Department of Mathematics, University of Nebraska-Lincoln, Avery 203, Lincoln, NE 68588, and **Lorraine Males** ([lmale2@unl.edu](mailto:lmale2@unl.edu)). *Mathematical knowledge for teaching and mathematical habits of mind in the NebraskaNOYCE program.* Preliminary report.

In this presentation, we discuss the NebraskaNOYCE partnership between the University of Nebraska-Lincoln and 3 large high-needs school districts. We discuss the engagement and perspectives of 24 experienced and 16 new teachers. We highlight particularly influential activities. We then describe their viewpoints on mathematical habits of mind and mathematical knowledge of teaching. We will compare and contrast these viewpoints with those of the mathematics instructors involved with the program. (Received September 19, 2016)

- 1125-97-2211 **Catherine Paolucci\*** ([paoluccic@gmail.com](mailto:paoluccic@gmail.com)) and **Evan Smith** ([mr.smith@fmhsnyc.org](mailto:mr.smith@fmhsnyc.org)). *Changing past perceptions and fostering future STEM ambitions through secondary and higher education partnerships.*

This presentation will share the exciting outcomes of an innovative partnership aimed at giving secondary students an opportunity to learn mathematics through the use of computer programming and 3-D printing. It will give the background of a partnership formed between mathematics faculty in a New York State university and a New York City public high school.

The project, funded by the SUNY Teacher and Leader Network, involved collaborative development of a short-course curriculum for high school students with a wide range of mathematical achievement levels. Students in the course learned, applied and reinforced algebraic and geometric concepts through the use of OpenSCAD programming software to design and print 3-D objects. In addition to notable outcomes involving mathematical learning and dispositions, the collaboratively developed and delivered course also impacted students’ attitudes and ambitions regarding further study of mathematics and computer programming. (Received September 19, 2016)

- 1125-97-2214 **Aaron T Wilson\*** ([aaron.wilson@utrgv.edu](mailto:aaron.wilson@utrgv.edu)). *A near-peer mathematical mentoring pipe-line from the secondary mathematics classroom to graduate studies in mathematics.*

We report a near-peer mentoring project aimed at increasing entry, especially by minority and low-income students, into math and science careers at the college-level and also of promoting the matriculation of undergraduate students into either math teaching and/or graduate studies in mathematics. Working with participating school districts, our project works to accomplish this goal by exploiting a key idea to reach out to young people: we recognize that many students seem to be more enthusiastic and to absorb information more readily when delivered to them by their near-peers, rather than traditional figures of authority such as teachers. Studies have shown

that peer and near-peer led activities have a strong impact on students. Moreover, such near-peer approaches not only affect the intended target audience, but also have a "feedback" effect on the group doing the presentations. We report the successful and highly sought after mathematical outreach work that we have completed over the last two years and show how we use this work as a cycle of near-peer interventions and mentoring that connects the high school mathematics classroom to the university mathematics classroom, facilitating students' pursuit of studies in mathematics. (Received September 19, 2016)

1125-97-2240 **Ben Galluzzo\*** ([bjgalluzzo@ship.edu](mailto:bjgalluzzo@ship.edu)), Shippensburg University, 1871 Old Main Drive, Department of Mathematics, Shippensburg, PA 17257. *Assessing Mathematical Modeling.*

Math modeling, at all levels, is often perceived to be difficult to assess. In this talk we will share approaches one can use to evaluate student work throughout the math modeling process. In particular we will highlight classroom experiences using the assessment tools found in the GAIMME Report. (Received September 20, 2016)

1125-97-2367 **Frank Wattenberg\***, Department of Mathematical Sciences, United States Military Academy, West Point, NY 10996. *Two gadgets – Simulations Using the Language Processing and Cell Phone Cameras.*

The computer language Processing was developed by the visual arts community. It is easy-to-learn and well-suited for high quality, interactive, immersive, three-dimensional simulations. We look at several examples. The gadget that is always with me is my cell phone with its insanely great camera. Analyzing photos taken with such cameras and, especially extracting three-dimensional data, involves a lot of cool mathematics. We look at several examples. (Received September 20, 2016)

1125-97-2383 **Natasha M. Speer\*** ([speer@math.umaine.edu](mailto:speer@math.umaine.edu)). *Using research on student learning and teachers' knowledge to design professional development for novice college mathematics instructors.*

The design of many professional development programs for novice college mathematics instructors is or could be better informed by education research. This is one of four "research sampler" talks where the presenters will discuss research relevant to graduate student professional development. The presentation will include discussion of how what we know about how people learn mathematics can be a focus in graduate student professional development. In addition, the presentation will address how activities and assignments can be designed to support graduate students' development of their mathematical knowledge for teaching. (Received September 20, 2016)

1125-97-2398 **Paul N Runion\*** ([prunion@mst.edu](mailto:prunion@mst.edu)) and **Barbara Wilkins** ([bwilkins@mst.edu](mailto:bwilkins@mst.edu)). *Success for Calculus: Is It a Success?* Preliminary report.

At Missouri University of Science and Technology, less than 10% of students with a D or F in Calculus I at midterm finished the course with a grade of C or better in recent semesters. To address this major concern, we introduced a new course, Success for Calculus, in Fall 2015. Success for Calculus uses the second half of the semester to improve the overall preparedness of students while reinforcing the calculus they saw during the first half. We will look at lessons learned from the first three semesters of this exciting new course and examine how students have progressed after completing Success for Calculus. (Received September 20, 2016)

1125-97-2413 **Joni J Schneider\*** ([js1824@txstate.edu](mailto:js1824@txstate.edu)), 1230 N. LBJ DR 712, San Marcos, TX 78666. *Developing Mathematical Identity in Post-Secondary Students in a Student Math Seminar.*

An individual's mathematical identity is how one would see one's self in the mathematical community. In this presentation, we will share how a student math seminar program is affecting undergraduate students' mathematical identity and their view of mathematics culture. This program is a weekly seminar that provides students with the opportunity to present research of mathematics, mathematics history, and/or interesting math ideas to an audience of their peers. The seminar is set in a casual environment for students to share how they view mathematics. In addition to affecting their mathematical identity, we will address how this seminar has also been increasing undergraduate students' awareness about how fun and rewarding mathematics research can be. (Received September 20, 2016)

1125-97-2421 **Fabiana Cardetti\*** ([fabiana.cardetti@uconn.edu](mailto:fabiana.cardetti@uconn.edu)), **Mary Truxaw** ([mary.truxaw@uconn.edu](mailto:mary.truxaw@uconn.edu)) and **Megan Staples** ([megan.staples@uconn.edu](mailto:megan.staples@uconn.edu)). *School and University Collaboration: Working Together to Provide Quality Support for Teachers.*

In this presentation we share collaborative efforts among school of education faculty, mathematics department faculty, and school district leaders to provide quality professional development activities for K-12 teachers. In

this work our focus has been on creating opportunities for professional learning communities and other forms of teacher collaboration, within and outside of school, that have resulted in increased content knowledge, as well as higher levels of confidence in different aspects of mathematics teaching and professional engagement. We will share our experiences, including successful activities and examples of milestones and challenges we have faced along the way. (Received September 20, 2016)

1125-97-2480 **Jack Bookman\*** ([bookman@math.duke.edu](mailto:bookman@math.duke.edu)), Mathematics Department, Box 90320 Duke University, Durham, NC 27708. *How can we help novice college mathematics instructors know what their students know? Questioning strategies and formative assessment for the college mathematics classroom.*

The design of many professional development programs for novice college mathematics instructors is or could be better informed by education research on teaching and learning. This is one of four “research sampler” talks where the presenters will discuss research relevant to graduate student professional development. In this session, I will discuss research on how teachers can learn to ask good questions to promote learning and to better carry out formative assessment of student learning. (Received September 20, 2016)

1125-97-2518 **Alicia Prieto Langarica\*** ([aprietolangarica@ysu.edu](mailto:aprietolangarica@ysu.edu)). *Undergraduate Research for Non Academic Careers.*

Students not seeking academic job or even planning on going to graduates school often are not exposed to research. Even when they are, many times is hard to make the connection of how this research experience can translate to their industry. In this talk I will discuss a few problems I have worked on with undergraduate students who either have a second major outside of mathematics or who are seeking to get a job outside academia right out of their undergraduate degree. Although specific examples of problems will be used, we will only briefly talk about the solutions to the problems. Instead, the talk will focus on how to come up with the problems as well as the modeling approach needed to address the problems and how to tailor the experience for a job outside academia. (Received September 20, 2016)

1125-97-2539 **Jess Ellis\*** ([ellis@math.colostate.edu](mailto:ellis@math.colostate.edu)), **Daniel Bragdon** ([dbragdon314@gmail.com](mailto:dbragdon314@gmail.com)) and **Jessica Gehrtz**. *Mathematics GTA PD: Where we are and where we are going.*

The design of many professional development programs for novice college mathematics instructors is or could be better informed by education research on teaching and learning. This is one of four “research sampler” talks where the presenters will discuss research relevant to graduate student professional development. In this research sampler session, we share a taxonomy of mathematics graduate student teaching assistant (GTA) professional development (PD) programs that currently exist in the US. This taxonomy is based off of the characterization of GTA PD programs from 120 mathematics departments. A cluster analysis revealed nine distinct models of GTA PD within the 120 programs that vary with respect to the amount of interaction the GTAs have through the PD, the amount of activities involved in the PD, and the level of feedback given to GTAs involved with the PD. Of these 120 programs, 33% have recently implemented changes to their GTA teaching preparation program or are discussing changes. We will discuss productive strategies for institutional change and consider how these strategies relate to GTA PD. (Received September 20, 2016)

1125-97-2625 **Vandana Sharma\*** ([vandanas@asu.edu](mailto:vandanas@asu.edu)). *Creating a Synchronous Learning environment through Advances in Technology.* Preliminary report.

Distance learning has been around for over 100 years, and it has been defined in many ways in modern literature. The most thorough definition is given by Desmond Keegan (1995), who says that distance education and training result from the technological separation of teacher and learner which frees the student from the necessity of traveling to “a fixed place, at a fixed time, to meet a fixed person, in order to be trained”. Based on this definition, we can see that distance education can take place when the student and teacher are separated by space, but not necessarily by time. In this session, I will explain how we can use technology to create synchronous learning environment to facilitate student - teacher interaction and learning in real time, but in different space. The following video gives a glimpse of this idea: [https://math.la.asu.edu/~sharma/Demo\\_Call1new.mp4](https://math.la.asu.edu/~sharma/Demo_Call1new.mp4) (Received September 20, 2016)

1125-97-2681 **Cody L Patterson\*** ([cody.patterson@utsa.edu](mailto:cody.patterson@utsa.edu)), Department of Mathematics, One UTSA Circle, San Antonio, TX 78249, and **William McCallum** ([wmc@math.arizona.edu](mailto:wmc@math.arizona.edu)), Department of Mathematics, 617 N Santa Rita Ave, Tucson, AZ 85721. *Partnerships to build algebraic thinking for teaching: professional development across geographic and grade-level boundaries.*

In order to help students develop mathematical habits of mind that support success in algebra, teachers must understand how algebraic thinking is woven throughout the K-12 curriculum. The University of Arizona partnered with local school districts on two projects designed in part to strengthen teachers' understanding of how algebraic ideas evolve across grade levels: the NSF-sponsored Arizona Teacher Initiative, which provided graduate coursework for elementary and middle school teachers, and Project CoMPASS, a summer program for middle and high school teachers sponsored by the Arizona Department of Education. We will discuss how partnerships with local school districts enriched these projects, and talk about some of the challenges inherent in conducting content-based professional development with geographically and mathematically diverse groups of teachers. (Received September 20, 2016)

1125-97-2739 **Matthew J Haines** ([haines@augsborg.edu](mailto:haines@augsborg.edu)), 2211 Riverside Ave, Department of Mathematics and Statistics, Minneapolis, MN 55454, and **Nora Helf** and **Lewis Istok\***. *3D Printing in Middle School Mathematics and Preservice Teachers.* Preliminary report.

This is a preliminary report of an Minnesota Council of Teachers of Mathematics funded collaboration that brings together a middle school STEM teacher, a preservice teacher, and an associate professor of mathematics. The report describes the role of the collaborators and one of the activities developed incorporating 3D printing into mathematics standards. (Received September 20, 2016)

1125-97-2862 **Drew C. Youngren\*** ([youngren@nyu.edu](mailto:youngren@nyu.edu)), 251 Mercer St., New York, NY 10012. *The Clickers They Carry: Using smartphones and open platforms for spontaneous student feedback.* Preliminary report.

Many university lecturers make use of 'clickers' (aka audience response systems) to collect student feedback in real-time while presenting material. These systems have been shown effective in a variety of applications but have the drawbacks of proprietary hardware and software. In this session, we present an alternative means of soliciting and analyzing student feedback electronically via QR codes, Google Forms™, and popular Python libraries. Besides the obvious cost advantage, these methods offer more flexibility in the kinds of questions asked and the types of analysis and visualization. We exhibit several examples and neat tricks like client-side data-cleansing via regular expressions. (Received September 20, 2016)

1125-97-2926 **Eun Jung\*** ([eunjung@uga.edu](mailto:eunjung@uga.edu)), 102 College Station Rd. F204, Athens, GA 30605. *Connecting Multiplication, Fractions, and Equations.* Preliminary report.

The present study examines how six future secondary mathematics teachers reason fractions abstractly and quantitatively when they are confused about in confusing the roles of quantities and operators, particularly as they try to create abstract representations of equations. The future teachers were enrolled in mathematics content courses that relied on an explicit, quantitative meaning for multiplication to connect core topics in the multiplicative conceptual field. This report focuses on preservice math teachers' fractional knowledge, analyzing data from task-based clinical interviews designed to investigate participants' figurative and operative reasoning in solving fraction multiplication problems with an unknown multiplier or multiplicand. The main finding is we report is that the future teachers interpreted fractional numbers as operators on magnitude X units in various ways, some of which facilitates the process of representing and producing proportional relationships and linear equations. There are significant gaps for many pre-service teachers around fractions' distinct roles as either quantities or operators. Without grasping this distinction, new teachers will have difficulty implementing standards that require a nuanced approach to teaching fractional concepts. (Received September 20, 2016)

1125-97-3046 **Matt Boelkins** ([boelkinm@gvsu.edu](mailto:boelkinm@gvsu.edu)), **Oscar Levin\*** ([oscar.levin@unco.edu](mailto:oscar.levin@unco.edu)) and **Bruce Yoshiwara** ([byoshiwara@gmail.com](mailto:byoshiwara@gmail.com)). *Interactive textbooks in MathBook XML.*

A growing number of mathematics teachers are creating their own course materials, notes and even textbooks. MathBook XML (<http://mathbook.pugetsound.edu/>) is a free and open source application and markup language which allows these documents to be easily converted to a number of formats, including interactive webpages. These webpages allow for a variety of student centered features, including embedded WeBWorK problems, interactive Sage cells, and a rich and intuitive cross-referencing mechanism. We will demonstrate some of these features by showcasing a number of ongoing open textbook projects from the authors and others. (Received September 20, 2016)

1125-97-3054 **Padmanabhan Seshaiyer\*** ([pseshaiy@gmu.edu](mailto:pseshaiy@gmu.edu)), 4400 University Drive, MS 3F2, Fairfax, VA 22030, and **Jennifer Suh** ([jsuh4@gmu.edu](mailto:jsuh4@gmu.edu)), 4400 University Drive, MS 3F2, Fairfax, VA 22030. *Sustainable Mathematics Partnerships between Public School Districts and Institution of Higher Education: The COMPLETE Center*. Preliminary report.

In this talk, we describe a center for excellence titled COMPLETE (Center for Outreach in Mathematics Professional Learning and Educational Technology) whose mission is to promote excellence in mathematics teaching, learning and collaborative mentoring in Northern Virginia through innovative solution-oriented initiatives and technology integration with emphasis on research and education. The COMPLETE center provides a collaborative network that brings together a consortium of mathematics supervisors and leaders from ten school districts that collaborate with mathematicians and mathematics educators from an institution of higher education in Northern Virginia to develop mathematics based educational programs for students and teachers to solve real-world problems that focus on 21st Century Skills including critical thinking, problem solving, communication, collaboration and creativity, careers, technology and innovation. We will describe both state-wide as well as nationally funded mathematics partnership projects supported through the center that have impacted teachers, undergraduates, graduate students and faculty to become change agents to transform institutional and school district practices. (Received September 20, 2016)

1125-97-3111 **Oswaldo Daniel Soto\*** ([osoto@sandi.net](mailto:osoto@sandi.net)), 9500 Gilman Drive, Mail Code 0036, La Jolla, CA 92093-0036. *Partnering for Success: Developing a high school discrete mathematics curriculum connecting a university course with a local high school course using the standards for mathematical practice*. Preliminary report.

College readiness in mathematics and the subsequent need for remediation have become a concern nation-wide. With few choices available, most college bound students are asked to take pre-calculus, calculus or statistics their senior year in high school. However, discrete mathematics, offers a wide variety of contemporary contexts useful for college preparation if we think about them from the perspective of the standards for mathematical practice. In this session, I present early results of a project bringing pedagogically sensitive mathematicians and math educators together with local high school teachers to redesign the curriculum of a high school course whose goals are stated in terms of BOTH content and standards for mathematical practice with an eye on a sophomore level undergraduate discrete mathematics. (Received September 21, 2016)

1125-97-3138 **Karen D King\*** ([kking@nsf.gov](mailto:kking@nsf.gov)), 4201 Wilson Blvd., Suite 885, Arlington, VA 22230. *Why a mathematics degree? Implications of a Mathematics Major for Secondary Teachers*.

The expectation to be considered a highly qualified secondary mathematics teacher in the US is that a teacher should have a major in mathematics. Yet there has traditionally been weak evidence that having more coursework in mathematics is not associated with improved student achievement. In this talk, I discuss possible reasons that this disconnect between research and our natural instincts about the need for a major by describing the implications of mathematical knowledge for teaching for secondary teachers. While Ball and colleagues have described mathematical knowledge for teaching for elementary teachers and established relationships between this knowledge and student achievement, much less work has been done for secondary teachers. I will describe the ways in which the definition of mathematical knowledge for teaching might be conceptualized for secondary teachers and the implications for the important aspects of the mathematics major that would support such knowledge. (Received September 21, 2016)

1125-97-3160 **Jessica M. Libertini\*** ([libertinijm@vmi.edu](mailto:libertinijm@vmi.edu)), Virginia Military Institute. *The GAIMME Report on Mathematical Modeling in K-16*.

In this session I'll share ideas from the the Early and Middle Grades (K-8) section of the GAIMME report as well as experiences from the IMMERSION program, which is focused on modeling in K-6. If audience members come with questions and experiences to share, we can make this session interactive. (Received October 03, 2016)

1125-97-3161 **Karen Bliss\*** ([kdbliss@ncsu.edu](mailto:kdbliss@ncsu.edu)), North Carolina State University. *Mathematic Modeling at the Undergraduate Level*

Math modeling has long appeared in the undergraduate curriculum as an upper level course (if it appears there at all), typically for math majors who have already had differential equations. In addition to advocating for incorporating modeling earlier and throughout the major, we discuss reasons why departments should develop a course in modeling as an exciting alternative to fulfill non-majors' math general education requirement. Such a course can make math relevant and interesting, and can even support critical non-math skills. (Received October 03, 2016)

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### *MAA Invited Addresses, Presentations by Teaching Award Winners, and SIGMAA Guest Lecturers*

1125-A0-14      **Ingrid Daubechies\***, Duke University, Durham, NC 27708. *Mathematics for Art Investigation.*

Mathematical tools for image analysis increasingly play a role in helping art historians and art conservators assess the state of conservation of paintings, and probe into the secrets of their history. The talk will review several case studies, Van Gogh, Gauguin, Van Eyck among others. (Received May 23, 2016)

1125-A0-16      **Jason Cantarella\***, University of Georgia, Athens, GA 30602. *Random polygons, Grassmannians, and a problem of Lewis Carroll.*

Lewis Carroll once published a set of mathematical “pillow problems”; brainteasers designed to give the reader something to ponder before going to sleep. One of the problems was this: what is the probability that a randomly chosen triangle is obtuse? This talk will explore a surprising connection between the shapes of polygons and the Grassmann manifold of 2-planes in real or complex  $n$ -space. Using this bridge, a number of interesting problems about random polygons and shapes have recently been solved, including explicit computations of some expectations for geometric properties of random polygons and fast algorithms for polygon sampling. The talk will be accessible to a wide mathematical audience, including students, and will also sketch some connections with biology and polymer science. (Received May 23, 2016)

1125-A0-17      **Laura Taalman\***, James Madison University, Harrisonburg, VA 22807. *Math by Design: 3D Printing for the Working Mathematician.*

Mathematicians often spend their days thinking about ideas that exist only in their minds. In this talk we’ll discuss how to use 3D printing to bring models of those ideas into reality, from start to finish. We’ll show how to leverage design software to convert mathematical objects into triangular meshes or voxel representations, and then how those digital representations become code that a 3D printer can understand and implement to create real-world objects. Learn how to get started creating your own mathematical 3D design files, level up your existing design skills, or just enjoy watching the process of turning abstract mathematics into physical plastic. Along the way we’ll explore the essential importance of failure, not only in the design process but also in the study of mathematics itself. (Received May 23, 2016)

1125-A0-18      **Lillian Pierce\***, Duke University, Durham, NC 27708. *From Gauss to today: class numbers and  $p$ -torsion in class groups of number fields.*

Each number field (finite extension of the rationals) has a positive integer associated to it called the class number, defined to be the cardinality of the class group of the field. Class numbers are important objects that arise naturally in many contexts in number theory: for example, Gauss famously investigated class numbers of quadratic fields, in the context of classifying the representation of integers by binary quadratic forms. Today, many deep open questions remain about the structure of class groups and the growth and divisibility properties of class numbers as fields vary over an appropriate infinite family. This talk will focus on the size of the  $p$ -torsion subgroup of the class group: it is conjectured that for any number field and any rational prime  $p$ , the  $p$ -torsion part of the class group of the field should be very small, in a suitable sense, relative to the discriminant of the field. The talk will survey progress on this open problem, from Gauss to today. (Received May 23, 2016)

1125-A0-26      **Susan Holmes\***, Stanford University, Stanford, CA 94305. *Finding meaningful patterns: the decoding of the human microbiome.*

Modern biological data present multiple challenges akin to code breaking. We can learn from the master codebreakers who worked during World War II how to leverage patterns using mathematics and statistics. Turing and his fellow codebreakers used graphs and alignments to process their data. I will show how similar approaches provide rich insight into the complex dynamics of the human microbiome. We still use the Bayes factors and diversity indices developed in the 1940’s to detect changes in the bacterial communities. (Received May 23, 2016)

1125-A0-202 **Glen Van Brummelen\***, Quest University, Squamish, BC. *Trigonometry and the Challenge of the History of Mathematics.*

Writing the history of mathematics is hard. Really hard. Although the apparent purity of mathematics might seem to render its history freer of world view than that of other subjects, in practice it simply makes different ways of thinking harder to recognize. Many monographs these days avoid crossing boundaries between major cultures to avoid imprinting from one to the other. My ongoing project on the history of trigonometry from antiquity to the present goes against this grain. Perhaps quixotically, I portray mathematical ideas as they move through very different peoples and eras, following a complex story while attempting to respect each culture's approach. We shall explore several episodes in trigonometry from ancient Greece, India, Islam, the West, and China where crossing the border is a risky business; we hope to gain some lessons in securing a safe crossing. (Received August 11, 2016)

1125-A0-301 **Martin I Meltzer\*** (qzm4@cdc.gov), Mailstop C-18, 1600 Clifton Rd, Atlanta, GA 30333. *Simple mathematical models for public health decision making during a response.*

The value of simple mathematical models, rapidly built with available data, to aid decision makers during response to catastrophic infectious disease events will be demonstrated by examining the contributions of modeling during 2 public health emergencies: Estimating, in near real-time, the impact of the 2009 H1N1 influenza pandemic; and, Modeling input to aid the decision during the 2014-2016 Ebola epidemic; These examples will be used to illustrate some basic concepts that must guide mathematical modelers as they interact with public health decision makers during responses. (Received August 24, 2016)

1125-A0-602 **Matthew Richey\***, St. Olaf College, Northfield, MN 55057. *Take what you have gathered from coincidence: understanding and using randomness.*

What does it mean to be random? We all encounter randomness every day – it is part of how we talk about the weather, sports, and even love. Despite being so familiar, randomness has proven to be an elusive idea to pin down. Even mathematicians have struggled to define randomness, leading to competing and sometimes conflicting definitions. Whatever it is, randomness is a driving force behind many modern computational algorithms. These algorithms — the Metropolis Algorithm, Markov chain Monte Carlo Methods, and others — use randomness as the secret ingredient that makes it possible to tackle famously difficult problems such as the Traveling Salesperson Problem and image reconstruction. Using many pictures (and even a few Bob Dylan references), this lecture will reveal the historical quest to define randomness and illustrate how randomness allows us to solve many of today's most challenging applied mathematics problems. (Received September 07, 2016)

1125-A0-805 **Brian M. Gurbaxani\***, Office of the Associate Director for Science, Centers for Disease Control and Prevention, Atlanta, GA 30329. *Applied Mathematics and Statistics at the CDC - 2017 and Beyond.*

The Centers for Disease Control and Prevention is one of the world's premier public health agencies, employing about 14,000 people total, and about 300 as mathematical statisticians. Others who use applied mathematics in their day to day life at the CDC include economists, epidemiologists, mathematical disease modelers, and a very small number of engineers and operations researchers.

In this talk I will present just a small portion of the applied math, statistics, and engineering work done at the CDC, much of it from a personal perspective (things I have worked on), in an attempt to showcase the diverse applications of mathematics at the CDC – some even done by mathematicians!. The analytical methods employed range from "routine" statistics, to big data analysis and machine learning, to the more obscure wavelets and fractals. Moreover, I will showcase some of the work done by student groups as part of CDC's push to collaborate more with academic partners, most recently at the Georgia Institute of Technology. Many skills in big data analysis, encryption, high performance computing, smartphone apps, etc. are in need for the future growth of the CDC, skills that the CDC as an institution has largely not developed. (Received September 12, 2016)

1125-A0-1082 **Jeff Buechner\*** (buechner@rci.rutgers.edu). *Computers, mathematical proof, and the nature of the human mind: a surprising connection.* Preliminary report.

Although the use of computers in mathematical proofs antedates the Haken-Appel proof of the four-color theorem in 1976, it was Haken and Appel's proof that created a stir among mathematicians, philosophers, and computer scientists. Was their proof of the four-color theorem a genuine mathematical proof? At that time, Thomas Tymoczko established a conceptual framework for thinking about this issue, and subsequent discussion employed his framework, although some argued that it was deficient. I will argue that a line of thought mentioned (but not developed) by some commentators is necessary for understanding the use of computers in mathematical proofs.

In particular, the consensus view of how computers work (accepted by computer scientists, mathematicians, and philosophers) makes it impossible to understand how computers function in mathematical proofs. I will show why this is so by connecting the consensus view of how computers work with a consensus philosophical view about the nature of the human mind. I will close with a speculation about how we might make progress in understanding how computers work, mathematical proof, and the nature of the human mind. (Received September 14, 2016)

1125-A0-1618     **Janet Heine Barnett\*** ([janet.barnett@csupueblo.edu](mailto:janet.barnett@csupueblo.edu)). *Drinking straight from the source: Learning today's mathematics through its historical roots.*

Over three decades of teaching, I have tried various strategies for engaging students in activities that will help them understand mathematics as a sense-making endeavor used by people as a means to understand the world in which they live. One of the most powerful of these strategies has been the use of primary historical sources. As instructors, we can (naturally) be tempted to give students clear and precise presentations, both in our teaching and in the written materials we provide. But just as a water filtration process intended to remove impurities can also remove healthy minerals and their interesting tastes, efforts to remove potential learning impediments can strip a subject down to a set of facts and formulas lacking in context, motivation and direction. Teaching overly distilled material is also unlikely to help students learn to develop and reason with ideas on their own. Going back to the original source from which a mathematical topic sprang is a means of restoring these vital ingredients. In this talk, I describe one approach to using primary sources with students, and share some of the exciting rewards (and challenges!) that we have experienced together as a result. (Received September 18, 2016)

1125-A0-1675     **Robert A Beezer\*** ([beezer@ups.edu](mailto:beezer@ups.edu)), Department of Mathematics, University of Puget Sound, 1500 N Warner, Tacoma, WA 98416-1043. *Textbooks for the Web from MathBook XML.*

MathBook XML is an open source markup language for describing structured documents, initially mathematics textbooks. By describing structure, along with a careful separation of content from presentation, it is possible to simultaneously create faithful web versions and produce high-quality print and PDF versions.

Web versions of these textbooks make use of various technologies, such as MathJax, knowls, Sage cells, screen readers, SVG graphics, embedded videos, Unicode, while also respecting in a modern format the principles derived from centuries of book design. Students get a dynamic, easily accessible textbook that is superior to printed books in many respects, with few real compromises. Authors are shielded from many technical details, such as the configuration of various technologies, navigating differences in escape characters across formats, and avoiding conflicts between  $\text{\LaTeX}$  packages.

We will demonstrate the aspects of MathBook XML described above, while also discussing the various design decisions and principles involved. This talk will be of interest to textbook authors (current and future), and mathematicians interested in the use of technology in communicating, teaching and learning mathematics. (Received September 18, 2016)

1125-A0-2030     **Caren L. Diefenderfer\*** ([cdiefenderfer@hollins.edu](mailto:cdiefenderfer@hollins.edu)). *Do Your Students Believe that Mathematics is Exciting?* Preliminary report.

If we create a list of adjectives that describes mathematics and ask our students to do the same task, how will the lists compare? This was the introduction to an August 1998 MathFest talk by Dorothy Buerk of Ithaca College. Her words helped me rethink what I do in the classroom. This talk will focus on understanding and supporting our students in order to create a community of learners who believe that mathematics is fun, exciting, beautiful and important. (Received September 19, 2016)

1125-A0-2209     **Tevian Dray\*** ([tevian@math.oregonstate.edu](mailto:tevian@math.oregonstate.edu)), Department of Mathematics, Oregon State University, Corvallis, OR 97331. *The geometry of calculus.*

Geometric reasoning lies at the heart of a coherent view of calculus. Differentiation involves ratios of small quantities. Integration involves chopping, adding—and multiplying. A geometric description of both processes is readily available using infinitesimal reasoning, an approach that transfers well to other disciplines, such as physics and engineering.

Examples of geometric reasoning in calculus include the use of the dot product to derive trigonometric identities, the use of differentials to determine the derivatives of trigonometric functions without the need for limits, and the use of vector differentials to provide a coherent view of vector calculus.

After giving several such examples, this talk describes our efforts to make geometric reasoning, in large part based on differentials, the central theme in calculus, through our very successful 20-year effort to bridge

the gap between lower-division mathematics and upper-division physics, our much less successful attempts to apply what we had learned to first-term calculus, and our recent efforts to introduce a numerical representation of differentiation in terms of experimental data, leading to the concept of “thick derivatives”. (Received September 19, 2016)

1125-A0-2731 **Francis Edward Su\*** ([su@math.hmc.edu](mailto:su@math.hmc.edu)), Department of Mathematics, Harvey Mudd College, 301 Platt Blvd, Claremont, CA 91106. *Mathematics For Human Flourishing*.

Mathematics is valued for its ability to describe the world in beautiful ways. Indeed, beauty is one of many ideals to which we aspire. But why does the teaching and practice of mathematics often fall short of our ideals and hopes? How can the deeply human themes that drive us to do mathematics be channeled to build a more beautiful and just world in which all can truly flourish? (Received September 20, 2016)

1125-A0-3176 **Eva K Lee\***, Georgia Institute of Technology, Atlanta, GA. *Optimization-Based Machine Learning Approach for Predicting Vaccine = Immunity*.

The ability to predict how different individuals will respond to vaccination and to understand what best protects individuals from infection greatly facilitates developing next-generation vaccines. It facilitates the rapid design and evaluation of new and emerging vaccines and identifies individuals unlikely to be protected by vaccine. We describe a general-purpose machine-learning framework, DAMIP, for discovering gene signatures that can predict vaccine immunity and efficacy. Using DAMIP, implemented results for yellow fever demonstrated that a vaccine's ability to immunize a patient could be successfully predicted (with accuracy  $\geq 90$  percent) within one week after vaccination. A gene identified by DAMIP, EIF2AK4, decrypted a seven-decade-old mystery of vaccination. Results for flu vaccine demonstrated DAMIP's applicability to both live-attenuated and inactivated vaccines. Results in a malaria study enabled targeted delivery to individual patients. Our project leads to better vaccines to fight emerging infections, and improve monitoring for poor responses in the elderly, infants, or others with weakened immune systems. In addition, the project's work should help with universal flu-vaccine design. (Received September 20, 2016)

## *The Creation and Implementation of Effective Homework Assignments*

1125-A5-243 **Linda McGuire\*** ([lmcguire@muhlenberg.edu](mailto:lmcguire@muhlenberg.edu)), Department of Mathematics & Computer Science, Muhlenberg College, Allentown, PA 18104. *Putting It Together: An Effective Assignment Model for Upper-division Mathematics Students*. Preliminary report.

The type of assignment that I would like to share in this session is used within the context of studying mathematical history. The 300-level course in question (Landmarks of Mathematics) offers an advanced-level introduction to the development of mathematics from antiquity to the present. An emphasis is placed on the understanding of mathematical developments in broader historical and cultural context.

Small groups of students are given readings from different parts of mathematical history to analyze and are asked to present both cultural context and the underlying mathematics to the rest of the class. Groups share their analyses by leading class sessions using oral presentation and student-authored problem sets.

My presentation will highlight the assignments given, show several samples of student work, discuss assessment techniques used, and share with the audience the positive evidence that these assignments helped students to improve their individual skills with regard to technical reading, mathematical presentation, research methods, and problem-solving. (Received August 18, 2016)

1125-A5-285 **Shelley B. Rohde\*** ([srohde2@msudenver.edu](mailto:srohde2@msudenver.edu)). *Balancing online work and written work in calculus and general studies courses*.

In the age of online homework, we are all challenged to find a balance between efficiency and effective learning. I will discuss various alternatives to the standard written homework implemented for my students in conjunction with the newer online homework systems. Forms include worksheets, short written assignments, and/or quizzes. I will outline the construction of assignments and grading choices, and present results from my classes in calculus and general studies. (Received August 23, 2016)

1125-A5-353 **David McCune\*** ([mccuned@william.jewell.edu](mailto:mccuned@william.jewell.edu)), William Jewell College, Department of Physics and Mathematics, 500 College Hill, Liberty, MO 64068. *Writing effective questions and creating a successful homework system.*

In this talk I will discuss the effectiveness of my individual homework assignments as well as the preconditions I put in place to make those homework assignments successful. The talk focuses on writing effective and interesting questions that push students to think creatively. I will demonstrate some engaging and applicable questions I have created for my Math for the Liberal Arts course that are designed to help my students think abstractly and creatively. Additionally, I will discuss what needs to happen before giving an individual assignment that makes the assignment successful in achieving my learning goals. Being explicit about your expectations and grading, allowing the students to revise their work, ensuring that students know you care about their success, and honoring the struggle they will have with difficult problems creates an environment in which students take the assignments seriously, see the purpose of each individual question, and are less likely to use the internet to cheat. (Received August 29, 2016)

1125-A5-520 **Pangyen Ben Weng\*** ([pangyen.weng@metrostate.edu](mailto:pangyen.weng@metrostate.edu)), 700 E 7th St., Saint Paul, MN 55106. *So Little (Written) Homework, So Much Accomplished.* Preliminary report.

The author presents a homework arrangement as part of a flipped classroom in discrete math. What students are required to do after class include: (1) reviewing the lesson and prepare for quiz, (2) previewing for next lesson by reading an e-textbook and completing (rather simple) online exercises, and (3) keeping notes on the preview reading. He discusses his instructional goals and how this homework arrangement helps meet these goals. He then presents the result of using this design: improved learning outcomes, excellent learning experience and reduced study hours and stress on students. (Received September 05, 2016)

1125-A5-524 **Patricia L Hale\*** ([phale@cpp.edu](mailto:phale@cpp.edu)), Dept of Math and Stat, 3801 W Temple AVE, Pomona, CA 91768. *Learning Math Through Your Arm.*

Educators require students to complete homework to fulfill many objectives they have for their students: mastery of procedures; timely feedback on the students' understanding; practice in critical thinking; practice in problem solving; practice in logical communication of a mathematical solution. However, no one type of homework – online, project, “paper-and-pencil” – meets all the objectives. This talk will describe calculus courses that utilize three types of student work in order to meet these objectives: online, in-class and write-up. Examples of assignments and resources for creating assignments will be presented. How this combination of assignments minimizes time spent grading, and provides opportunity for formative assessment will be discussed. (Received September 05, 2016)

1125-A5-700 **Jacqueline Anderson\*** ([jacqueline.anderson@bridgew.edu](mailto:jacqueline.anderson@bridgew.edu)), 24 Park Avenue, Bridgewater, MA 02325. *Frequent Feedback through Google Forms.* Preliminary report.

I will discuss a teaching practice that I developed in response to a common inequality: The amount of work my students should do outside of class to master the material is greater than the amount of work I have time to grade. After many failed attempts to deal with this reality, I have found a technique that works for me and my students. For day-to-day feedback and homework, instead of collecting assignments, I now have students fill out an online survey to report their experience with assigned problems and readings. I will discuss the content and implementation of these surveys, how they inform my teaching, and their effects on student participation and success in both lower-level and upper-level undergraduate courses. (Received September 09, 2016)

1125-A5-741 **Benjamin Gaines\*** ([bgaines@iona.edu](mailto:bgaines@iona.edu)). *The Practicality of Writing Prompts in Freshman-Level Math Courses.*

In introductory level math classes, writing prompts can be used as part of weekly homework assignments to encourage students to think more deeply about the subject at hand. These writing prompts present a ‘real world’ question in context, and require students to submit a short written response online explaining how the mathematical ideas of the class could be applied in trying to find an answer. These open-ended prompts complement more traditional exercise-based homework, as they allow students to develop their communication and logical reasoning skills while providing some motivation for the concepts being discussed. By creating a detailed rubric and providing it to students ahead of time, these prompts can also be graded without a significant time commitment on the part of the instructor. A sample rubric will be provided, along with examples of some writing prompts and student responses. (Received September 10, 2016)

1125-A5-808 **Susan Jane Colley\*** ([sjcolley@math.oberlin.edu](mailto:sjcolley@math.oberlin.edu)), Department of Mathematics, Oberlin College, 10 North Professor Street, Oberlin, OH 44074. *Turning Problems into Projects.*

While routine homework remains crucial for developing students' skills with computation and basic problem-solving, it is also important that some homework be designed to involve more sophisticated argumentation and writing. I will provide examples of small projects and challenging problems that I have assigned in both single- and multivariable calculus. There is a wealth of material already available; the key is to package the assignments so that they are engaging to students and demonstrate the intellectual heft of mathematics. (Received September 12, 2016)

1125-A5-1141 **Monika Kiss\*** ([monika.kiss@saintleo.edu](mailto:monika.kiss@saintleo.edu)), **Brian Camp**, **Doris Van Kampen-Breit** and **Shawn Weatherford.** *Are graphing skills a thing of the past?*

The influence of technology as part of classroom instruction, homework, and assessment has increased dramatically in the last decade. Students and faculty prefer instantaneous feedback, so that students can immediately correct any misconceptions they have regarding the topic presented, and faculty can quickly adjust content and instruction to address those misconceptions. However, we also want students to be able to think critically when working with mathematical concepts, something which may be interfered with or diminished if the solution is always instantaneously available. It is also important for students to be able to identify and sketch functions graphically as well as understand them algebraically. In this paper, the authors investigate the differences between using online-only homework assignments versus traditional, paper-based homework to assess students' graphing abilities, critical thinking skills, and matching abilities. The eventual goal of this work is to find a hybrid method of engaging students about graphs and their properties that will incorporate the best results from both traditional and online methods of homework delivery. (Received September 15, 2016)

1125-A5-1175 **Przemyslaw Bogacki\*** ([pbogacki@odu.edu](mailto:pbogacki@odu.edu)), Department of Mathematics and Statistics, Norfolk, VA 23529. *Self-Assessment Homework in an Online Linear Algebra Class.*

When developing our online linear algebra class, we faced the challenge of a disconnect between the computer-based components of the course (watching pre-recorded lectures and participating in Blackboard assessments) and the major exams (midterm and final) on which our students are expected to solve linear algebra problems in a paper-and-pencil format at a testing site. Our solution of this issue involves the self-assessment homework problems assigned regularly throughout the course. The problems assigned are chosen to reflect the major learning objectives of the course. For each assigned problem, we ask the student to follow these steps: 1. watch a recording of a similar problem being solved by the instructor using pencil and paper; 2. submit an image (taken with a camera or scanned in) of their paper-and-pencil solution; 3. examine the solution key and self-assess their solution; 4. use the Post-Homework Discussion Forum in case of any questions or comments. Access to subsequent steps in this cycle is controlled using Blackboard's Adaptive Release mechanism. In our course, homework contributes 5% of the course grade and is credited based on completion only. The instructor spot-checks student responses (especially early in the semester, or for underperforming students). (Received September 15, 2016)

1125-A5-1495 **Daniel R. Jordan\*** ([djordan@colum.edu](mailto:djordan@colum.edu)). *Proof-Writing Workshops.*

Workshops—commonly used educationally in the teaching of composition and professionally in the production of artistic works—can be a powerful tool to support students as they develop their proof-writing skills. In a proof-writing workshop, each student independently writes a proof of a statement and then provides feedback on the proofs of their peers and receives comments from them. Workshops can be used in any class where writing proofs, or even less formal mathematical communication, is emphasized. They are highly efficient learning activities, since workshops engage students in multiple roles, namely as author, as critic, and as the recipient of feedback. This presentation will focus on an implementation of proof-writing workshops in an online environment supporting an otherwise face-to-face class, with examples of student work and a discussion of benefits and challenges. (Received September 17, 2016)

1125-A5-1573 **Laura DiMillo Barnes\*** ([laura\\_d\\_barnes@uri.edu](mailto:laura_d_barnes@uri.edu)). *Class Assignments as an Enhancement to Online Homework.*

At the University of Rhode Island, Pre-Calculus is a coordinated course. The course is organized so that all sections cover the same curriculum, take common pre-tests and exams, and work homework through an online system. However, each instructor has approximately 10% of the course grade left to their discretion. Most instructors use that component for quizzes or worksheets that are similar to the online homework, text exercises or test questions. I choose to create Class Assignments that challenge students. One of my goals as a teacher is to produce critical thinkers (even at this level) and I don't think the standard online or text problems provide

that opportunity. My Class Assignments are usually applications of the concepts and often combine concepts. Further, the course does not allow the use of calculators, so these application problems require students to think of alternate ways to get an answer. As an example, here is a right triangle application problem: A person stands 12 feet from a statue. The angle of elevation from eye level to the top of the statue is 30 degrees and the angle of depression to the base of the statue is 15 degrees. How tall is the statue? (Hint: use trig identities to determine values and approximate your answer if necessary). (Received September 18, 2016)

1125-A5-1586 **Shelly Smith\*** ([smithshe@gvsu.edu](mailto:smithshe@gvsu.edu)) and **Robert Talbert** ([talbertr@gvsu.edu](mailto:talbertr@gvsu.edu)). *Making Learning Visible with Student-Generated Video Content*. Preliminary report.

Professors want to assess their students' thought processes and problem solving skills in addition to the correctness of calculations. However, students may have difficulty expressing their reasoning in written work, and class time is often too limited for stage fright-inducing student presentations. Furthermore, traditional assessment techniques can be difficult to use in online courses.

We describe the use of student-generated video content to assess students' engagement with and understanding of problem solving tasks. Using simple and widely available technological tools, students create videos of themselves working through and describing solutions to exercises. The videos are then posted to an unlisted YouTube channel that is only accessible to the students and faculty in the class. We discuss the creation of the assignment, the challenges of logistics and assessment, and the role of student videos in the course. We present results and recommendations based on the use of video assessments in an online asynchronous section of Calculus 1, and in face-to-face sections of Calculus 2. (Received September 18, 2016)

1125-A5-1629 **Matt Zaremsky\*** ([zaremskym@gmail.com](mailto:zaremskym@gmail.com)). *Grading more than just the final answer with an automated grading system: Benefits and pitfalls*.

The new online homework platform *Gradarius* is the first of its kind that grades students not just on their final answer but also on their intermediate steps and "showing their work". It is impressive that technology has reached the point where this is possible, but one can wonder whether this is actually more effective as an instructional tool than other online systems that only grade the final answer. I will discuss the benefits and pitfalls we encountered at Cornell last semester using *Gradarius* for the first time in Calc I, both in terms of the students' experience and our perspective as instructors. (Received September 18, 2016)

1125-A5-1637 **Lakshmi Roychowdhury\*** ([lakshmi.roychowdhury@utrgv.edu](mailto:lakshmi.roychowdhury@utrgv.edu)), School of Mathematical and Stat Sciences, University of Texas Rio Grande Valley, 1201 West University Drive, Edinburg, TX 78539. *How to Implement Effective Homework Assignment in Lower Level Undergraduate Course: Personal Observations*.

In this article we try to show effectiveness of homework assignment. Online homework assignment gives opportunity to students see various questions on topics from text book and have practice on similar type of question where paper-pencil work assignment develop students' logical and critical thinking skill. In our present academic environment, online assignment is useful as long as there are some paper-pencil work involved along with it. Here I will describe my personal observation in an institution where most of the students are first or second generation coming for a college-degree. (Received September 18, 2016)

1125-A5-1691 **Ross Sweet\*** ([rsweet@math.northwestern.edu](mailto:rsweet@math.northwestern.edu)), 2033 Sheridan Road, Evanston, IL 60208. *Peer-Assisted Reflection and Online Homework in a Flipped Calculus Course*. Preliminary report.

A common argument against online homework systems in introductory classes is the lack of practice students have to write and refine solutions. In a first-quarter differential calculus course at Northwestern University, we implemented an additional homework structure to emphasize mathematical writing. Based on the work of Daniel Reinholz, Peer-Assisted Reflection (PAR) problems give students the opportunity to write, reflect on, and revise solutions to challenging problems. We will discuss the complementary roles of online homework systems and PAR problems in a flipped course model, student outcomes, and the use of PAR as a professional development resource for graduate student teaching assistants. (Received September 18, 2016)

1125-A5-1733 **Shahriar Shahriari\*** ([sshahriari@pomona.edu](mailto:sshahriari@pomona.edu)), 610 N. College Ave, Claremont, CA 91711. *Do the homework, then go to the lecture*.

Homework is often thought of as a series of exercises to reinforce what was taught in lectures. In my experience, homework assignments are even more useful for getting students ready for lectures and for encouraging active engagement with the material. By planning ahead and splitting topics into small manageable homework problems, I often try to introduce future topics first through homework assignments. In this presentation, I will give

examples of a calculus class taught entirely through homework problems and other classes where new topics are often introduced first through the homework. (Received September 19, 2016)

1125-A5-1759 **Vicky Williams Klima\*** (klimavw@appstate.edu). *Incorporating Reflection into Calculus Assignments*. Preliminary report.

By developing reflective study habits, students become stronger more independent learners. Using online homework systems allows students to receive instant feedback concerning their mastery of material and also reduces the amount of grading required of instructors. However, encouraging reflection and using online homework seem in opposition to one another. In this talk we present a series of assignments that requires reflection while also employing the use of online homework. Initially, students record what they do understand about a problem and ask pointed questions about what they do not understand. After discussing their observations and questions, students reflect on their initial misconceptions as well the big ideas of calculus that appeared within the problem. Finally, students complete an online homework assignment that asks them to use some of the same skills stressed in the initial problem set. We will present our assessment strategy for the series of assignments, challenges in implementing the series of assignments, and evidence of growth as students complete the series of assignments throughout the semester. (Received September 19, 2016)

1125-A5-1859 **Lori Carmack\*** (lacarmack@salisbury.edu), Salisbury University, Dept. of Mathematics and Computer Science, 1101 Camden Avenue, Salisbury, MD 21804. *Assigning Homework via Mixed Practice*. Preliminary report.

The study of learning and memory is an active area of research among cognitive scientists. In terms of retention and performance, many recent studies favor the concept of mixed practice (working on several related tasks during a single practice session) over blocked practice (working on only one task throughout a single practice session). Results of studies are compelling. Over the past several years, I have experimented with various ways of assigning online and handwritten homework based on the notion of mixed practice. This past semester was the most successful to date in that students actually liked the homework! In end-of-semester course evaluations, several students made comments such as “The mixed practice problems were very helpful.” In this paper, I will discuss various research results of controlled studies on mixed practice, and present in detail what I found to be an effective means of incorporating the idea into homework assignments at the collegiate level. (Received September 19, 2016)

1125-A5-1881 **Judy A. Holdener** and **Brian D. Jones\*** (jonesbd@kenyon.edu). *Mathematics Assignments — a Storied Approach*.

Over millennia, short stories have been used to preserve and pass down human knowledge, ancestry, history, and conventions of morality and behavior. The power of the short story is its ability to concisely and efficiently present information in a framework that is easily retained. In a similar way, perhaps mathematical knowledge is best passed down to future generations of students when mathematics teachers take on the role of storytellers. In this talk, we examine five key elements that are typically found in a successful short story: character, setting, plot, conflict, and theme. Might these elements translate into meaningful elements of a good mathematics homework assignment? Are we effective mathematical storytellers? We consider these questions in relation to assignments that vary in both size and level of difficulty. Additionally, we'll examine the challenges of online homework systems and the internet on effective mathematical storytelling. (Received September 19, 2016)

1125-A5-1884 **Karen B Stanish\*** (kstanish@keene.edu), Keene State College, 229 Main St., Keene, NH 03435-2010, and **Kimberly Schmid-Gagne**. *Developing Critical Thinking Skills in Introductory Statistics*. Preliminary report.

Can we intentionally develop students' critical thinking skills in an introductory statistics course? Many instructors find that some students learn critical thinking skills in their introductory statistics course (and some students do not). But often these skills are not developed purposefully. This talk will share homework assignments from an introductory statistics course that are intentionally designed to improve students' critical thinking skills, the rubrics that make grading these assignments quick and effective, some preliminary results of the assessment of those critical thinking skills, and plans for future implementation of these types of assignments. (Received September 19, 2016)

1125-A5-2064 **Rodica Cazacu\*** (rodica.cazacu@gcsu.edu), 231 W. Hancock St., GC CBX17, Milledgeville, GA 31061. *Online tools for homework assignments in hybrid courses - to use or not to use -*.

Since we started redesigning some of our mathematics courses, the creation of efficient homework assignments became our number one priority. The online tools available on the Pearson's MyMathLab platform that we use

for our hybrid courses could be helpful for one course but could create problems for other. This presentation will show how I changed the content of the homework and how I use those tools since I started designing and teaching such courses. I will also address some challenges I had with these online assignments and show how I try to make them efficient so that they will prepare my students not just for quizzes and tests but for the other courses they will take and for their profession. (Received September 19, 2016)

1125-A5-2076 **Filippo Posta\*** ([filippo.posta@gcu.edu](mailto:filippo.posta@gcu.edu)), 3300 W Camelback Rd, Room 16-323, Phoenix, AZ 85017. *A Blended Approach to Homework Design Promotes Critical Thinking*. The use of online platforms to assess students in Mathematics has provided two fundamental benefits for learners: immediate feedback and content availability. These benefits have generated substantial pedagogical challenges. Immediate feedback can promote a trial and error culture that prevents true understanding. Similarly, content availability can lead learners to the search for the “perfect example” instead than building a solution from some theoretical foundations. In addition, the use of e-assessment technology in mathematics classrooms is promoting isolation and apathy toward group activities. To limit these three consequences of e-assessment, a blended approach was used to create formative assessment that mixes online work with original and personalized classroom/group projects. The presentation outlines this blended approach and includes a data driven discussion of its effects. (Received September 19, 2016)

1125-A5-2218 **Ruthmae Sears\*** ([ruthmaesears@usf.edu](mailto:ruthmaesears@usf.edu)), University of South Florida, College of Education, 4202 E. Fowler Ave., EDU105, Tampa, FL 33620, and **Kenneth Butler** and **Frances Hopf**. *Developing Intermediate Algebra Students Mathematical Communications via Workspace Assignments in MyLabsPlus*. Preliminary report.

In recent times, the use of online learning management software (such as MyLabsPlus and WebAssign) are used more readily in tertiary education courses for assessment purposes. The use of online assessments may not always afford students an opportunity to show their mathematical working, because in many instances only the final answer is required. Hence, there is a need for faculty to focus on the development of students’ ability to communicate mathematically in written format. Therefore, we will showcase how we use Pearson (2016) Workspace assignments in MyLabsPlus for homework assignments in which students are required to show all working as they proceed from the given statement to the final solution. When using Workspace, instructors are able to gain insight into students mathematical reasoning as their students’ progress. At our institution, we assign one Workspace assignment each week, and students are afforded two opportunities to attempt each workspace assignment, with the higher of the two scores being used. Hence, in this presentation we will provide an overview of workspace, discuss how it is used at our institution, and will discuss the frequency of syntactic errors, and various types of mathematical errors observed. (Received September 19, 2016)

1125-A5-2231 **Emma Smith Zbarsky\*** ([smithzbarskye@wit.edu](mailto:smithzbarskye@wit.edu)). *The Three Horsemen of Homework*. I will discuss three different approaches to homework design and implementation: two in calculus and one in partial differential equations. The first example is of online homework I created for a differential calculus course. It was unusual in requiring first a free response, then a multiple choice selection, and finally viewing of the solution to receive credit. The second example, from PDEs, used problems largely taken from our textbook but with the requirement that students repeatedly submit their work until they get it correct. Finally, I will present examples of homework requiring students to use technology and estimation in ways intended to preclude simple internet searches. (Received September 20, 2016)

1125-A5-2233 **Paul E Seeburger\*** ([pseeburger@monroecc.edu](mailto:pseeburger@monroecc.edu)), 1000 E. Henrietta Rd., Rochester, NY 14623. *Creating Effective Online Homework Problems in Algebra, Calculus, and Differential Equations (Using WeBWorK)*. Preliminary report.

Many of us assign our students online homework problems in our courses. These online problems can be valuable learning tools, helping students spend more time practicing the skills and concepts we teach them. But are we assigning pedagogically well-designed problems or are they problems that only ask students to enter a final answer (e.g., limits, definite integrals, differential equations). Unfortunately, even our best students are tempted to use calculators (or Wolfram Alpha) to directly calculate these results, worrying about how to work out the problems later (if they find time). I believe we train students to do math problems in the way we assess them. Problems that only require a numerical (or even symbolic) answer that can be found easily using a calculator or a website train students to use these tools, but may fail to train them to work out the problems in the ways we show them in class and require on exams (showing clear work). Without help, students can also become frustrated in these one-answer problems when their answer is not accepted. I will present some of my attempts

to address these issues in problems I have created or adapted using WeBWorK and propose some best practices for creating online homework problems for mathematics courses. (Received September 20, 2016)

1125-A5-2291 **Andrew J Krause\*** ([krausea3@msu.edu](mailto:krausea3@msu.edu)) and **Ralph Putnam**. *Nuances of online calculus homework: Insights from the student perspective.*

Online homework is propagating rapidly across the nation, especially in large, introductory courses in STEM fields. The literature provides some evidence that the implementation of online homework is correlated with higher exam scores and course grades, but research and theory about how students engage with and experience online homework to support their learning is lacking. This study examines student experiences with online homework by identifying various homework environments, resource use, perceptions, and strategies that characterize diverse student experiences and learning opportunities. Student achievement data is only one of the many forms of data required to understand how online homework supports learning and inform improvements to its implementation; this research draws on interviews and screen recordings and demonstrates the importance of qualitative inquiry about student learning. The findings provide a nuanced portrayal of students' experiences with online calculus homework and illustrate the importance of factors such as individualized homework sets, the types of resources accessed, and the structure of due dates. Interviews with the students provided insights about adjustments that could improve the learning opportunities facilitated by online homework. (Received September 20, 2016)

1125-A5-2303 **Frances Hopf\*** ([fhopf@usf.edu](mailto:fhopf@usf.edu)), University of South Florida, Department of Mathematics and Statistics, 4202 East Fowler Ave, CMC342, Tampa, FL 33620, and **Ruthmae Sears** ([ruthmaesears@usf.edu](mailto:ruthmaesears@usf.edu)), University of South Florida, College of Education, 4202 E. Fowler Ave., EDU105, Tampa, FL 33620. *How does Mastery Learning on Homework Affect Student Success in Precalculus?* Preliminary report.

In the digital era of submitting mathematical assignments electronically, there is a need for students to demonstrate a conceptual understanding of mathematical content while attending to precision in their mathematical communications. Currently, our online PreCalculus course utilizes the MyLabsPlus learning management for all assessment purposes. To facilitate mastery learning of mathematical content, students are required to obtain at least 70% on a guided notes check to be able to access the homework assignments, and at least 70% on the homework assignments to gain access to the related quiz assignments. While completing their homework assignments, students are encouraged to use MyLabsPlus learning resources, which includes: videos, animated slides, and step-by-step examples of how to solve various problems. Students can earn bonus points by submitting their assignments on a weekly basis, instead of waiting until the end of the unit. Therefore, we will describe lessons learned when mastery learning is encouraged on assignments, and the benefits of submitting assignments weekly rather than waiting until the end of the unit. We will also discuss potential implications of this quality improvement venture on cultivating students' success in online mathematics courses. (Received September 20, 2016)

1125-A5-2316 **Matthew J Haines\*** ([haines@augsborg.edu](mailto:haines@augsborg.edu)), 2211 Riverside Ave, Department of Mathematics and Statistics, Minneapolis, MN 55454. *Challenge Investigations in a Sophomore/Junior-Level Geometry Course.* Preliminary report.

In a course where the students meet once a week, small groups are formed to examine weekly challenge investigations. One or two online postings per week are performed to stimulate discussion about the problem. At the start of the weekly class session, the groups meet to discuss their work from the week and report to the class the progress made. Two of the learning objectives are to build students' communication of mathematical ideas and practice problem solving skills in a low-stakes environment. Past implementation has produced mixed but promising success. (Received September 20, 2016)

1125-A5-2404 **Tamas Szabo\*** ([szabot@uw.edu](mailto:szabot@uw.edu)), 800 W Main St, Whitewater, WI 53190. *Grading geometry homework in less than 6 hours a week.* Preliminary report.

Writing intensive, proof oriented, upper level geometry courses for prospective middle and high school teachers are not suitable for online homework systems. Several strategies will be discussed including selecting homework problems, encouraging group homework, and utilizing office hours. While examples may come from geometry courses, many ideas are suitable in other undergraduate math courses. (Received September 20, 2016)

1125-A5-2492 **Leann Ferguson\*** ([leann.ferguson@usafa.edu](mailto:leann.ferguson@usafa.edu)) and **D Scott Dillery** ([dillerys@lindsey.edu](mailto:dillerys@lindsey.edu)). *Recognizing Calculus Outside of Mathematical Settings.*

The authors present three extra credit homework assignments given to students in a Differential Calculus course at an undergraduate military academy. The assignments were given at approximately even intervals during

the semester. The students were asked to model images from outside mathematics with functions, to apply derivatives to real-world contexts utilizing various differentiation rules, and to numerically estimate an inflection point within a chemistry context. The authors will discuss the learning goals of each assignment, as well as present participation rates, student work, and an assessment of the learning goals. (Received September 20, 2016)

1125-A5-2507 **Sara L Malec\*** ([malec@hood.edu](mailto:malec@hood.edu)), 401 Rosemont Ave, Frederick, MD 21702. *Problem Exists Between Keyboard and Chair: Filling in the Gaps in Online Homework*. Preliminary report.

Online homework systems have become ubiquitous in college classes, but sometimes they encourage maladaptive behavior in students that impedes learning. This paper explores a way to assign supplementary assignments that encourage a deeper relationship with mathematical ideas without creating a large amount of grading, and gives statistically significant evidence of the effectiveness of these assignments on final exam performance. (Received September 20, 2016)

1125-A5-2522 **Erika L. Ward\*** ([eward1@ju.edu](mailto:eward1@ju.edu)). *Shifting Feedback and Responsibility: Homework Presentations*.

In math courses, especially at the calculus level and below, homework gives students the opportunity to practice new skills in both familiar and unfamiliar situations, and to develop an understanding of the related concepts. While some of these goals can be easily accomplished in online homework, many systems are less useful for handling open-ended questions. Written homework, on the other hand, takes time to grade and delays feedback. To fill the gap left between these approaches, I supplement online homework with "homework presentations." Students put worked problems up on the board to earn points, and we discuss the problems together to uncover misconceptions and correct misunderstandings. Since preparedness, rather than correctness, earns points, there is less pressure to be perfectly correct, and thus students are more likely to do their own work. In addition to alleviating grading load, homework presentations provide student with feedback at the optimal place in the homework process. (Work partially supported by the Jacksonville University Scholarship of Teaching and Learning Fellows program.) (Received September 20, 2016)

1125-A5-2624 **Kevin Gerstle\*** ([kgerstle@oberlin.edu](mailto:kgerstle@oberlin.edu)), 10 N. Professor St., King 205, Oberlin, OH 44074. *Reflections on Assigning Both Online and Written Homework in Calculus*.

In this talk, I will reflect upon my experience using a combination of both online and written assignments in my Calculus Ia and II classes. In both classes, I assigned online homework sets daily for students to practice computations and receive immediate feedback. In addition, I gave students weekly written "reflection" homeworks to answer thought-provoking questions that required more creativity in their solutions. I will describe my students' and my own perspectives in assigning and completing these joint homework sets, and in doing so, what changes I plan to make in future semesters. (Received September 20, 2016)

1125-A5-2715 **Taylor E Martin\*** ([taylor.martin@shsu.edu](mailto:taylor.martin@shsu.edu)). *Justification and proof-writing in Calculus I through group homework assignments*.

One of the biggest challenges for students in Calculus I is to adapt to the expectations and demands of collegiate level mathematics. Students who succeeded in earlier math classes through memorization and repetition are particularly challenged when faced with the abstract concepts encountered in Calculus I. My goal in crafting homework in Calculus is twofold: to provide scaffolding for students to develop a theoretical understanding of Calculus concepts and to incorporate good mathematical exposition, including justification and proof when appropriate. I achieve these goals by creating weekly homework sets that students work on in groups of 3-4, beginning in class and finishing outside of class. In this talk, I will share examples of questions that promote abstract thought, examples of student work that demonstrates this conceptual understanding, and discuss student attitudes and feedback on this particular method of homework from several semesters of data. I will discuss ways in which group assignments invalidate googled solutions and ease the grading burden for the instructor. Finally, I will discuss pitfalls I have encountered and ways in which to avoid them. (Received September 20, 2016)

1125-A5-2858 **Silvia Saccon\*** ([silvia.saccon@case.edu](mailto:silvia.saccon@case.edu)), Case Western Reserve University, Cleveland, OH 44106. *Promoting students' deep learning in calculus through challenging problem sets*. Preliminary report.

The MAA's Committee on the Undergraduate Program in Mathematics (CUPM) recommends incorporating in every course "activities that will help all students progress in developing analytical, critical reasoning, problem-solving, and communication skills and acquiring mathematical habits of mind." To promote students' learning and development in these areas, I started to include challenging problem sets in calculus courses as part of the

out-of-class assignments. These problem sets challenge students to think deeply about the mathematics discussed in class, emphasize writing as part of the learning process, and provide opportunities for students to share their ideas and collaborate in a team. In this talk, I will describe these out-of-class activities, discuss benefits and challenges of incorporating them in calculus courses, and review feedback from students and impact on their learning. (Received September 20, 2016)

1125-A5-2938 **Michelle L Ghrist\***, HQ USAFA/DFMS, 2354 Fairchild Hall, Suite 6D2, USAF Academy, CO 80840. *Attempting to develop students' communication and critical thinking skills while using an online homework system.* Preliminary report.

Online homework systems can have several benefits: giving students immediate feedback on their understanding of the course material, optimizing instructor time, and utilizing randomizations that can discourage students from copying work from other students or sources. There are also several potential disadvantages to using these systems: a focus on mechanical and lower-level thinking skills, development of bad communication habits, a lack of detailed feedback to students, and student frustration when they cannot find the correct answer.

In this talk, I discuss several iterations of how I have implemented homework in WebWork (and occasionally JiTT, Just in Time Teaching) alongside traditional written homework. In particular, I discuss how I have developed new conceptual questions to focus student efforts on theoretical ideas as well as how I have used and iteratively improved this hybrid approach to achieve desired outcomes while seeking to not unduly burden students or instructors. (Received September 20, 2016)

1125-A5-2988 **Joanna G. Jauchen\*** (jboyett2@gmu.edu). *Intentionally Unstructuring Assignments for future elementary educators.*

In many of our mathematics courses, there is some material where it is difficult to get students to think deeply, carefully or at length about. Vocabulary is one of these. In addition, because we have internalized so many connotations for mathematical vocabulary and understand the language of mathematics deeply, we often underestimate the value of allowing students time to explore vocabulary as an important portion of the curriculum. In this talk, the author describes a single project that she designed for a second semester Math for Elementary Educators course which created time and space for students to explore Geometry vocabulary. The goal of the project was to encourage students to think of the vocabulary not as a laundry list of definitions, but as a vital part of understanding the world of two and three dimensional shapes. She shares the full assignment, her own reflections on the projects, student reflections and samples of student work. (Received September 20, 2016)

1125-A5-3083 **Alana Unfried\*** (aunfried@csumb.edu) and **Roger Woodard** (roger\_woodard@ncsu.edu). *Combining Online Homework and In-Class Writing Prompts for Increased Conceptual Understanding and Critical Thinking in Introductory Statistics.*

Statisticians have noted the importance of teaching conceptual understanding rather than procedural knowledge in introductory statistics. Students should be able to evaluate differing statistical claims, demonstrating critical thinking skills. This particular combination of skills can be developed and assessed through a combination of 1) automatically graded, custom online homework, and 2) comparative free response writing prompts. In online homework, students are given immediate feedback, questions can be randomized so that each student receives a unique assignment, and multiple attempts create a learning experience that moves beyond summative assessment. Banks of conceptual questions, developed by the instructor over time, remove the ability to use search engines to find answers. In the same course, comparative free response questions are given in class. The grading load is minimal, but professors provide immediate feedback while assisting students in-class and then providing a model that would receive full-credit. Questions of the same format are given on exams and graded for accuracy. These questions are specifically aimed at increasing critical thinking skills, and require students to follow a clear four-sentence structure requiring them to support their conclusions. (Received September 21, 2016)

1125-A5-3090 **Robert A Peacock\*** (rpeacock@yhc.edu). *Emphasizing Integral Existentialism in Calculus and beyond.* Preliminary report.

I will present ideas for emphasizing the exercise of establishing the existence of integrals appropriate for Calculus courses. Calculating numerical values of integrals by hand in Calculus is limited, so it is valuable to emphasize existence of integrals in order to move beyond just the numerical calculations. I will present a sampling of a theorems-based and problems-based approach to this exercise featuring an additional (heavy) emphasis on parametrized calculations. Fellow faculty may appreciate the emphasis on parametrized calculations that I will sample since parameters within calculations are not necessarily easily found in commonly used Calculus books.

Finally, fellow faculty may find ideas to take from this talk to enhance, say, undergraduate Real Analysis courses (or other upper-level courses). (Received September 21, 2016)

1125-A5-3128 **A E Francis\*** ([afrancis@carroll.edu](mailto:afrancis@carroll.edu)), Mathematics, Engineering, and Computer Science, Carroll College, 1601 N Benton Ave, Helena, MT 59625. *Are final projects in math classes worth the effort?* Preliminary report.

Semester-long projects can be an effective way to focus students' attention on a problem that has the potential to be richer and more application based than a typical homework problem. In this talk I will discuss some of the benefits and costs of these projects, as well as give some thoughts on how to make them work well. (Received September 21, 2016)

## Current Trends in Mathematical and Computational Biology

1125-AA-901 **Kenneth C Millett\*** ([millett@math.ucsb.edu](mailto:millett@math.ucsb.edu)), Department of Mathematics, UCSB, Santa Barbara, CA 93106. *Entangled Proteins: Knotting and Linking*.

Knots, slipknots, and linked structures are found in quite a few interesting proteins. They exhibit important structural features that correlate with functional features of these proteins. In this lecture, I will describe how one can recognize their presence and measure the extent of their influence on the protein via a historically based description of the development and evolution of the fundamental concepts. I will focus on the study of paradigm examples that inspire important research. Our objective is to shine new light these topics that are the focus of current research effort. (Received September 13, 2016)

1125-AA-1079 **Rachel Walker**, Tampa, FL 33612, **Jaime Mejia**, Cali, Colombia, **Domenico Coppola**, Tampa, FL 33612, and **Heiko Enderling\*** ([heiko.enderling@moffitt.org](mailto:heiko.enderling@moffitt.org)), Tampa, FL 33612. *A mathematical framework to personalize gastric carcinogenesis screening*.

Gastric cancer (GC) is often diagnosed at an advanced stage and consequently remains the third most common cause of cancer-related death worldwide. Early detection and endoscopic surgical therapy has been found to reduce GC-associated mortality, yet an efficient and cost-effective screening program still does not exist. Limiting population-based screening is a high inter-patient heterogeneity in time until progression to cancer, prompting the need for patient-specific screening intervals. We validated the increase in gastric stem cell markers in longitudinal biopsy samples from 70 patients. These data, randomized into training and test cohorts, was used to calibrate and validate a differential equation model to simulate the complex gastric gland dynamics during carcinogenesis. This biology-driven mathematical model was able to reproduce observed dynamics of the stem cell population in a test cohort with high accuracy. Such model can provide a computational tool for the prediction of patient-specific GSC population dynamics, and could guide personalized screening schedules to allow efficient monitoring of disease and the timely detection of malignant transformation. (Received September 14, 2016)

1125-AA-1904 **Doron Levy\*** ([dlevy@math.umd.edu](mailto:dlevy@math.umd.edu)), Department of Mathematics, University of Maryland, College Park, MD 20854. *The role of the autologous immune response in chronic myelogenous leukemia*.

Tyrosine kinase inhibitors (TKIs), such as imatinib (IM), have significantly improved treatment of chronic myelogenous leukemia (CML). However, the majority of patients are not cured for undetermined reasons. It turns out that many patients who otherwise responded well to IM therapy still show variations in their BCR-ABL transcripts. In this talk we will overview mathematical models for CML. Our main focus will be on our recent results concerning mathematical models that integrate CML and an autologous immune response. This is a joint work with G. Clapp, T. Lepoutre, and F. Nicolini. (Received September 19, 2016)

1125-AA-2103 **Christine Heitsch\*** ([heitsch@math.gatech.edu](mailto:heitsch@math.gatech.edu)). *The Combinatorics of RNA Branching*. Understanding the folding of RNA sequences into three-dimensional structures is one of the fundamental challenges in molecular biology. For example, the branching of an RNA secondary structure is an important molecular characteristic yet difficult to predict correctly, especially for sequences on the scale of viral genomes. However, results from enumerative, probabilistic, analytic, and geometric combinatorics yield insights into RNA structure formation, and suggest new directions in viral capsid assembly. (Received September 19, 2016)

1125-AA-2350 **Laurie J Heyer\*** ([lahey@davidson.edu](mailto:lahey@davidson.edu)) and **Jeffrey L Poet**. *Combinatorial and Computational Models in Synthetic Biology*.

Synthetic biology is an interdisciplinary field in which biological machines, designed to do a variety of useful things, are built and tested in the laboratory. Mathematical modeling plays a key role in the design and testing

phases, as we can model and predict the performance of the machine much faster than we can build the living implementation. Our math and biology students have designed, built and tested a system we call Programmed Evolution, a way of forcing cells to produce the desired small molecule (e.g., a drug for treating asthma) at an optimal rate, and not evolve away from this optimal solution. We have applied mathematical and computational tools, including local search algorithms, probability models and agent-based models, to predict and interpret data from the system. (Received September 20, 2016)

1125-AA-2685     **Shelby Nicole Wilson\*** ([shelby.wilson@morehouse.edu](mailto:shelby.wilson@morehouse.edu)), Mathematics Department, 830 Westview Drive, Atlanta, GA 30314. *Determining Near-Optimal Treatment Protocols via Nonlinear Cancer Models.*

This work aims to develop evidence-based treatment protocols designed to optimize the effectiveness of combined cancer therapies. Two mathematical models of cancer growth- each validated with preclinical data are considered. Each model studies the effects of chemotherapy when combined with a non-traditional anti-cancer therapy. The first model considers the dynamics of chemotherapy in the context of the primary immune response, immune vaccines and tumor growth. Second, we study chemotherapy in a context where it is combined with anti-angiogenic drugs (drugs that prevent blood vessel growth). The goal of this work is to determine optimal treatment protocols for combined cancer treatments. A number of heuristic algorithms (genetic algorithms, simulated annealing, Particle swarm algorithms) are used to propose treatment protocols designed to maximize treatment outcomes. (Received September 20, 2016)

## *L-Functions and Other Animals*

1125-AB-640     **Kimball Martin\*** ([kmartin@math.ou.edu](mailto:kmartin@math.ou.edu)), Norman, OK 73019. *Central L-values and functorial transfer.*

Central values of L-functions that appear in number theory encode fundamental arithmetic information. A powerful way to study these values comes via Langlands functoriality. We will first explain these ideas, including old and new results, with explicit examples of elliptic curves and modular forms, and then discuss some mostly conjectural generalizations to higher dimensions. (Received September 08, 2016)

1125-AB-672     **Catherine M Hsu\*** ([cathyh@uoregon.edu](mailto:cathyh@uoregon.edu)), University of Oregon, Fenton Hall, Eugene, OR 97403. *Higher Eisenstein Congruences.*

In this talk, we aim to determine “depths” of congruences between weight 2 newforms and certain Eisenstein series in order to gain information about the structure of the associated Hecke algebra. Indeed, for any squarefree  $N$  with an odd number of prime factors, we consider the Eisenstein series of weight 2 and level  $N$  given by

$$E_{2,N}(z) = \sum_{d|N} \mu(d) d E_2(dz),$$

where  $\mu$  is the Möbius function and  $E_2$  is the weight 2 Eisenstein series for  $\mathrm{SL}(2, \mathbb{Z})$  normalized so that the Fourier coefficient of  $q$  is 1. A recent result of Martin states that if  $p$  divides the numerator of  $\frac{\varphi(N)}{24}$ , then there exists a newform  $f \in S_2(\Gamma_0(N))$  congruent to  $E_{2,N}$  modulo  $p$ ; we wish to refine this result by computing the depths of all congruences between newforms of level  $M$  dividing  $N$  and  $E_{2,N}$ . In particular, because of the close relationship between congruences modulo  $p$  of newforms and completions of the Hecke algebra at certain maximal ideals, our computations, combined with a commutative algebra result of Berger, Klosin, and Kramer, will give us an upperbound for the  $p$ -adic valuation of index of the Eisenstein ideal in the Hecke algebra. This work is currently in progress and partially joint with Krzysztof Klosin. (Received September 20, 2016)

1125-AB-802     **Ian Petrow\*** ([ian.petrow@epfl.ch](mailto:ian.petrow@epfl.ch)), Section des Mathématiques, Bâtiment MA, Station 8, 1004 Lausanne, Switzerland. *L-functions of automorphic forms on non-split tori.*

I will present a concrete description of automorphic forms (characters) on certain non-split tori  $T$ , as well as their associated  $L$ -functions and analytic conductors. E.H. Brooks and I use this description to count automorphic forms on  $T$  when they are ordered by their analytic conductors. (Received September 12, 2016)

1125-AB-1370     **Alexandra M Florea\***, [amusat@stanford.edu](mailto:amusat@stanford.edu). *The mean value of quadratic Dirichlet L-functions over function fields.*

We discuss moments of quadratic Dirichlet  $L$ -functions over function fields, focusing on the mean value of this family of  $L$ -functions. Summing  $L(1/2, \chi_D)$  over monic, square-free polynomials  $D$  of degree  $2g+1$  (with  $q$  fixed and  $g \rightarrow \infty$ ), Andrade and Keating obtained an asymptotic formula with a main term of size  $|D| \log_q |D|$ . We

will describe a different approach which allows us to go beyond the number field square-root cancelation and compute a secondary term of size  $|D|^{1/3} \log_q |D|$ . (Received September 16, 2016)

1125-AB-1713 **Jennifer Beineke\*** ([jbeineke@wne.edu](mailto:jbeineke@wne.edu)), Department of Mathematics, Western New England University, Springfield, MA 01119. *The Porpoise and Relephants of Moments of L-functions and their Asymptotics*. Preliminary report.

In this talk, we will discuss moments of L-functions, and in particular, moments of the Riemann zeta function. We will learn about the importance of studying moments, approaches that have been used to obtain asymptotic results for moments, and some of the challenges of these investigations, “beastly” and otherwise. (Received September 19, 2016)

1125-AB-1802 **Habiba Kadiri\*** ([habiba.kadiri@uleth.ca](mailto:habiba.kadiri@uleth.ca)), Mathematics and Computer Science Department, 4401 University Drive, Lethbridge, Alberta T1K 3M4, Canada, and **Allysa Lumley** and **Nathan Ng**. *New explicit zero density result for the Riemann Zeta Function and consequences for the primes*.

Zero density result for  $\zeta(s)$  consist in estimating the number  $N(\sigma, T)$  of non-trivial zeros of the Riemann zeta function with real part greater than  $\sigma$  and imaginary part between 0 and  $T$ . In 1940, Ingham showed the following asymptotic result

$$N(\sigma, T) \ll T^{\frac{3(1-\sigma)}{2-\sigma}} (\log T)^5.$$

Many other asymptotic bounds of different form have been proven since then, but very few provide an explicit bound. Ramaré recently proved

$$N(\sigma, T) \leq 4.9(3T)^{\frac{8}{3}(1-\sigma)} (\log T)^{5-2\sigma} + 51.5(\log T)^2,$$

for  $\sigma \geq 0.52$  and  $T \geq 3.061 \cdot 10^{10}$ . We will present here an improvement of this result, together with applications to explicit estimates for the prime counting function  $\psi(x)$ . (Received September 19, 2016)

1125-AB-2272 **Micah B. Milinovich\*** ([mbmilino@olemiss.edu](mailto:mbmilino@olemiss.edu)), Department of Mathematics, Hume Hall, University of Mississippi, University, MS 38677. *Simple zeros of L-functions and related problems*. Preliminary report.

Many problems in number theory are related to the question of whether or not there are any  $\mathbb{Q}$ -linear relations among the ordinates of the nontrivial zeros of automorphic  $L$ -functions. In general, this question is wide open. I will discuss recent progress on some special cases of this question. For instance, are the (non-real) zeros of an  $L$ -function simple? Given two  $L$ -functions, are their zeros distinct? Can an  $L$ -function simultaneously vanish at height  $t$  and height  $2t$ ? (Received September 20, 2016)

1125-AB-2319 **Min Lee\*** ([min.lee@bristol.ac.uk](mailto:min.lee@bristol.ac.uk)), Howard House, Queen’s Avenue, Bristol, BS8 1SD, United Kingdom, and **Andrew R Booker** and **Andreas Strömbergsson**. *Numerical Computations with the Selberg trace formula*.

Ten years ago A. Strömbergsson and A. Booker worked out many explicit examples of the Selberg trace formula for square-free levels and have improved the evidence in favour of Selberg’s eigenvalue conjecture. This approach should be expanded to the general levels. Based on the detailed notes on the Selberg trace formula by Strömbergsson, Booker, Strömbergsson and I are investigating the Selberg trace formula for general levels numerically. We will also discuss a couple of applications. (Received September 20, 2016)

1125-AB-2602 **Edva Roditty-Gershon\*** ([er14265@bristol.ac.uk](mailto:er14265@bristol.ac.uk)), United Kingdom. *Arithmetic statics over function fields*.

In my talk I will explain how to express the variance of sums of certain arithmetic functions in both arithmetic progressions and short intervals in terms of zeros of certain associate L-functions. This is in the context of the ring of polynomials over a finite field of  $q$  elements, in the limit  $q$  goes to infinity. This study uses several recent equidistribution results due to N. Katz to express the zeros of the L-functions in terms of matrix integrals over the unitary group. (Received September 20, 2016)

1125-AB-2943 **Djordje Milićević\*** ([dmilicevic@brynmawr.edu](mailto:dmilicevic@brynmawr.edu)), Bryn Mawr College, Department of Mathematics, 101 North Merion Avenue, Bryn Mawr, PA 19010. *How many L-functions are there?*

$L$ -functions and their cousins known as automorphic forms are borne out of, and play a central role in our understanding of, arithmetic problems ranging from the distribution of prime numbers and elliptic curves to partition numbers, quadratic forms, equidistribution of geodesics on arithmetic surfaces, and many more. This affectionately nicknamed “zoo” is a rich landscape that is often best studied in large naturally occurring families of related objects. The size of such a family in turn acts as its universal essential characteristic in analytic

approaches to  $L$ -functions such as moment evaluations. In this talk, which will emphasize the underlying intuition, we will address the problem of counting automorphic forms and  $L$ -functions, surveying both the classical asymptotic results and our recent joint work with Farrell Brumley. (Received September 20, 2016)

## *Modeling & Understanding Environmental Risks*

1125-AC-498 **Martin E. Walter\*** (Department of Mathematics, Campus Box 395, University of Colorado, Boulder, CO 80309). *Current Civilization Plus Climate Change Equals Collapse*. Preliminary report.

We need to change the fundamental structure of our so-called “modern” civilization if it is to survive. For example, industrial agriculture is said to convert “fossil fuels into food” via its extensive reliance on natural gas, oil and coal for fertilizers, pesticides, transportation, refrigeration. But burning fossil carbon needs to be phased out as quickly as possible. Growing populations concentrated in regions susceptible to extreme weather events imply more frequent and devastating losses. Mathematics plays a basic role in understanding the forces of change and in optimizing our response. (Received September 04, 2016)

1125-AC-617 **James Case\*** (jcase66777@aol.com). *Modeling the Energy Future*.

The collapse of fossil fuel markets is likely to prove extremely traumatic. Among other things, the high-nitrogen fertilizers necessary to sustain the so-called “green revolution” are a by-product of the fossil fuel industry. When that starts shutting down, crop yields will return to 1950 levels, severely restricting food availability, worldwide. A desperate refugee crisis is likely to result. Predicting the collapse of fossil fuel markets is a lot like predicting earthquakes. You know pretty well WHAT is going to happen, but you can’t say WHEN. A new method of making such predictions will be demonstrated. (Received September 07, 2016)

1125-AC-785 **Michael Olinick\*** (molinick@middlebury.edu), Department of Mathematics, Warner Hall, Middlebury, VT 05753. *The Interaction Term in Population Models*.

Many situations arising in the social and life sciences involve interaction between two populations. Examples include two species competing for the same resource, predators and prey in biology, susceptibles and infectives in epidemiology, hawks and doves in evolutionary game theory, etc. Mathematical models of the resulting dynamics, borrowing a law of mass action from chemistry, frequently containing a term proportional to the product of the two groups in a system of differential equations. Alternative models using the product of their square roots often exhibits similar qualitative behavior and yield systems with closed form solutions. We investigate several such models and their relevance to real world phenomena, especially overshoot and collapse situations. (Received September 11, 2016)

1125-AC-1091 **Shandelle M Henson\*** (henson@andrews.edu). *Climate Change and Tipping Points in Seabird Colonies*.

Changes in sea surface temperature (SST) are associated with changes in reproductive and feeding strategies in colonial seabirds at Protection Island National Wildlife Refuge, Washington, USA, located in the Strait of Juan de Fuca. Years of high SST are associated with high egg cannibalism and ovulation synchrony in the colony. We hypothesize that decreased fish availability associated with high SST is correlated with the rise in cannibalism, and that reproductive synchrony is a response to cannibalism. Proof-of-concept models illustrate the conditions under which these hypotheses are supported and show that prolonged rises in SST can create tipping points that allow colony collapse. This talk is coauthored by James L Hayward and J. M. Cushing. (Received September 14, 2016)

1125-AC-1171 **Charles R. Hadlock\*** (chadlock@bentley.edu). *Commonalities and differences among environmental calamities*.

Common themes arise in studying environmental calamities, but so too do important differences. Using a wide range of examples, this talk will describe an organizational framework for analyzing such events and for studying their dynamics through mathematical concepts and models. Some of the examples may be useful in connection with various courses as well as in guiding managers, policy makers, and operators. (Received September 15, 2016)

1125-AC-1521 **William Dean Stone\*** (william.stone@nmt.edu). *A hysteresis effect in a simple sea-ice model*. Preliminary report.

A simple, phenomenological model of planetary ice-coverage, insolation, and albedo is considered. Changes in heat-loss, due to atmospheric changes, can lead to rapid changes in ice cover, and a hysteresis effect. The melting

of ice decreases the albedo leading to a sharp fall-off, with a hysteresis effect so that just returning to the previous atmospheric CO<sub>2</sub> does not return to the previous climate - an example of non-linearity leading to a collapse. (Received September 17, 2016)

1125-AC-3050 **Ben Fusaro\*** ([fusaro@math.fsu.edu](mailto:fusaro@math.fsu.edu)). *Humanistic Conceit, Unintended Consequences and Collapse*. Preliminary report.

This element of conceit is suggested by the Challenger disaster, or reading an article such as “Averting a post-antibiotic apocalypse” (Science News, 10 JUN’16), or the collapse of a 150 year research effort by linguists to explain why human language is so different from (i.e., superior to) the communication systems of the rest of the animal kingdom. This conceit distorts modeling by underestimating the probabilities of dangerous events, overestimating the benefits and minimizing the costs in trade-offs, etc. We are not immune to this. In 2013, the AMS Notices printed several letters in which E.O. Wilson, an outstanding biologist/ecologist, is stigmatized as someone who doesn’t understand mathematics or its uses. He might not be able to solve a PDE, but anyone who understands complex system at his level must be doing some modeling and clear thinking. This issue of conceit is a challenge to practitioners/teachers of modeling. More examples, and some predictions of conceit-driven collapses, will be provided. (Received September 20, 2016)

## *New Directions in Quantitative Literacy for General Education, in honor of Lynn Steen*

1125-AD-613 **Victor I Piercey\*** ([piercev1@ferris.edu](mailto:piercev1@ferris.edu)). *A Quantitative Reasoning Approach to Algebra for Business Students: Analysis and Preliminary Results*.

Students in certain professional programs, such as business, often fall through the cracks when it comes to mathematics pathways. They need some algebra, but not necessarily the full sequence that is designed for Calculus-bound students (even when those students have to take business calculus). While quantitative reasoning courses often include some algebra in the form of linear and exponential modeling, that may be insufficient for their needs in their courses on finance and accounting, where algebraic manipulations are necessary.

In this talk I will describe how I included traditional algebraic manipulations in a quantitative reasoning course for business students. The approach taken is consistent with the perspective of quantitative reasoning in terms of constructing meaning and contextualized mathematical experiences. A naïve understanding of APOS theory is used as a theoretical framework. The talk will include preliminary results from research whose goals include defining “grade-level performance” in algebraic manipulations and measuring that grade level for students who complete this course. (Received September 07, 2016)

1125-AD-781 **Gizem Karaali\*** ([gizem.karaali@pomona.edu](mailto:gizem.karaali@pomona.edu)), Claremont, CA 91711. *On Utilitarian and Aesthetic Goals of Mathematics Education: Quantitative Literacy and Humanistic Mathematics*.

Mathematicians, educators, policy makers, parents, and students submit a variety of reasons for the teaching of mathematics in our school system. The question becomes somewhat more nuanced at the college level. From basic quantitative literacy skills essential for a healthy civic life to a keen understanding of the power and limitations of mathematical tools, college courses in mathematics offer students a wide range of justifications. Mathematics courses developed to fulfill a general education requirement often fall into one of two categories: courses in quantitative literacy or courses in mathematical appreciation. In this presentation I explore the history of this duality of offerings and focus on the tension that underlies it. Finally I claim that our stance on this issue sends specific messages to students about the roles of mathematics in their lives and what mathematics might or should mean for them. (Received September 11, 2016)

1125-AD-1853 **Eric Gaze\*** ([egaze@bowdoin.edu](mailto:egaze@bowdoin.edu)). *Thinking Quantitatively: Teaching and Assessing a Quantitative Reasoning Course*.

The text *Mathematics and Democracy* carefully lays out the rationale for a robust Quantitative Reasoning (QR) curriculum: “Quantitatively literate citizens need to know more than formulas and equations. They need a predisposition to look at the world through mathematical eyes, and to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of careful reasoning.” This talk will explore how to create assignments for a QR course that address these challenges by providing a firm foundation in proportional reasoning and modeling with spreadsheets embedded in meaningful, real contexts. We will discuss the use of articles and reading assignments alongside worksheets and

problem based learning. Activities for group work in class, and strategies for creating assignments that scaffold QR skill development while deepening students' reasoning capabilities will be discussed. To truly empower our students to actively participate in today's data driven society, we must engage them with well thought out QR curriculum and active learning strategies. (Received September 19, 2016)

1125-AD-2252 **Lily S. Khadjavi\*** (lkhadjavi@lmu.edu). *Quantitative Literacy and Social Justice: From Basic Examples to Transformative Experiences.*

In *Mathematics and Democracy*, Steen and his collaborators set an inspiring goal for their work: "Quantitative literacy empowers people by giving them the tools to think for themselves, to ask intelligent questions of experts, and to confront authority confidently." This talk will feature several examples of applications of mathematics and statistics to issues of social justice, such as racial profiling and environmental justice. Some applications simply involve single class meetings while others build over the course of a term, but all invite the students to raise questions and test hypotheses of their own. (Received September 20, 2016)

1125-AD-3029 **Cinnamon Hillyard\*** (ch7@uw.edu). *Carnegie's Quantway Pathway: Using a Network Improvement Approach to Improve Quantitative Literacy Pedagogies.*

The Carnegie Foundation for the Advancement of Teaching developed two pathways to get student through developmental math and on to college careers. These pathways have seen unsurpassed success at achieving this goal year after year. We will discuss how one of these pathways, Quantway, used a quantitative literacy curriculum to achieve these results, moving students through developmental math into college algebra and pre-calculus courses. Beyond the curriculum, we will discuss how the integration of non-cognitive strategies focused on mindset and social belonging supported students' success. We will also discuss how a faculty network contributed to the sustained and continued growth of the curriculum, improvement of the pedagogical strategies, and curricular variations for diverse student populations. We will discuss how this networked improvement approach is critical to the success of Quantway. (Received September 20, 2016)

## Office Hours with a Geometric Group Theorist

1125-AE-81 **Eric M Freden\***, freden@suu.edu. *Growth in Groups.*

Given a finitely presented group, let  $\sigma_r$  denote the number of group elements with word metric length  $r \geq 0$ . The growth series for the presentation is defined as the formal power series  $S(z) = \sum \sigma_r z^r$ . Computing  $\sigma_r$  has often involved ad-hoc methods based on counting normal geodesic forms. A more mechanized approach appeared in 1984 when Cannon showed that any Fuchsian group has normal geodesic forms that constitute a regular language and consequently  $S(z)$  is a rational function. By 1992, Cannon, Thurston, et al generalized this result to all word hyperbolic groups as well as some other classes.

This presentation is based on the book chapter of the same name. It considers counting problems in groups where the underlying objects (group elements, geodesic words, words representing the identity, etc) are in correspondence with a context-free (or indexed) language rather than a regular language. In many cases the counting method of Deleste-Viennot-Schützenberger is then applicable and can be used to compute growth series. (Received July 13, 2016)

1125-AE-424 **Adam Piggott\*** (adam.piggott@bucknell.edu), Department of Mathematics, Bucknell University, Lewisburg, PA 17837. *Pitching Coxeter Groups to a Curious Undergraduate.*

Suppose an interested undergraduate arrives in your office hours and expresses an interest in undergraduate research. How can we pitch a topic? We discuss our approach to introducing Coxeter groups in such a situation. (Received September 01, 2016)

1125-AE-491 **John Meier\*** (meierj@lafayette.edu), Department of Mathematics, Lafayette College, Easton, PA 18042. *Finite generation and subgroups of infinite index.*

If  $G$  is a finitely generated group and  $H$  is a subgroup of finite index, then  $H$  is also finitely generated. This claim is not always true when  $H$  is of infinite index. Quite interesting geometry arises even in seemingly elementary cases, such as when  $G$  is a direct product of free groups and  $H$  is the kernel of a map from  $G$  to  $\mathbb{Z}$ . In this talk we will consider this and other examples, leading toward an instinct for how one might establish that a given infinite index subgroup is or is not finitely generated. (Received September 03, 2016)

1125-AE-689 **Leah Childers\*** (lchilders@pittstate.edu), and **Tara Brendle** and **Dan Margalit**.  
*Mapping class groups: a pictorial introduction.*

A key example in geometric group theory is the mapping class group of a surface, a group that arises from the geometry and topology of surfaces. In this talk we give an elementary, pictorial introduction to the mapping class group of a surface. We will explore standard examples and discuss basic tools for studying the group. Our focus will be on the interplay between the geometry of the surface and the algebraic properties of this group. (Received September 09, 2016)

1125-AE-831 **Greg Bell\*** (gcbell@uncg.edu), 144 Petty Building, 317 College Avenue, Greensboro, NC 27412. *Asymptotic dimension of groups.*

Dimension is a fundamental concept that can be applied in many areas of mathematics. We describe a way to assign dimension to countable groups due to Gromov. This so-called asymptotic dimension is the large-scale equivalent of the topological (covering) dimension and so it has properties that are analogous to the properties of covering dimension. We will survey the basic results before describing some deep consequences and open problems. (Received September 12, 2016)

1125-AE-991 **Jennifer Taback\*** (jtaback@bowdoin.edu), 8600 College Station, Bowdoin College, Brunswick, ME 04011. *An Introduction to Lamplighter Groups.*

We will define the family of lamplighter groups, which are examples of wreath products in group theory. Lamplighter groups can be understood purely algebraically, using mathematical trees, using decks of (very special) cards, or strings of light bulbs! We will meet the group elements themselves, the group as a whole, and end with a reference to other wreath products and the Traveling Salesman Problem. This talk will be accessible to undergraduates. (Received September 13, 2016)

1125-AE-1165 **Nic Koban\*** (nicholas.koban@maine.edu) and **John Meier**. *The Ends of a Group.*

In a typical pre-calculus course where real-valued functions are studied, a natural question to ask about these functions is, “What is happening with the function at infinity?” This leads to studying limits of the function as the independent variable “approaches infinity”. A similar question can be asked about a finitely generated group. The Cayley graph gives us a picture of the group, and we can wonder, “What is happening with this group at infinity?” As we approach infinity, are we approaching a space with a certain structure? In this talk, we examine this idea. (Received September 15, 2016)

1125-AE-2099 **Johanna Mangahas\*** (mangahas@buffalo.edu). *Ping-Pong for Free Groups.*

This talk is for anyone curious to understand and appreciate Ping-Pong. The Ping-Pong Lemma gives a method for detecting a free group. Simple yet useful, it neatly illustrates the geometric group theory tenet that one can get to know a group by observing its action on a space. (Received September 19, 2016)

1125-AE-2346 **Robert W Bell\*** (rbell@math.msu.edu), Department of Mathematics, 619 Red Cedar Road, Room C305, East Lansing, MI 48824. *An exploration of right-angled Artin groups.*

Suppose that we are given elements  $a, b, c, \dots, z$  and their inverses. Suppose that some of these letters commute; for instance this can be recorded by a graph with vertices  $a, b, c, \dots, z$  and edges  $xy$  whenever  $x$  and  $y$  commute. Such a graph defines a graph group; this group is more commonly referred to as a right-angled Artin group because of connections to reflections in mirrors which meet at right angles and because of their similarity to the braid groups studied by Emil Artin. These groups play a surprisingly important role in the study of geometric group theory. This talk will serve as an introduction and an invitation to research problems on graph groups. (Received September 20, 2016)

## *Random Polygons and Knots*

1125-AF-354 **Veronika Irvine\*** (veronikairvine@gmail.com). *TesseLace: An interesting family of doubly-periodic alternating braids.*

Bobbin lace is a 500-year-old art form in which threads are braided in an alternating manner to produce a lace fabric. A key component in its construction is a small pattern, called a bobbin lace ground, which is repeated periodically in two dimensions to fill a region of any size. I have developed a mathematical model for bobbin lace grounds representing the structure as the pair  $(\Delta(G), \zeta(v))$  where  $\Delta(G)$  is a 2-regular digraph  $G$  embedded on the torus and  $\zeta(v)$  is a mapping from the vertices of  $G$  to a set of braid words. The properties  $\Delta(G)$  must possess in order to produce workable lace along with an equivalence relation will be presented. These criteria are used to prove that an infinite number of prime bobbin lace grounds exist and to exhaustively enumerate and

generate results for increasing numbers of vertices in  $G$ . Over 5 million workable patterns have been identified, well in excess of the roughly 1,000 found in lace ground catalogs. To draw out of these results some of the more aesthetically interesting examples for lacemakers, the combinatorial search was tailored to select patterns with a high degree of symmetry. Lace ground representatives from each of the 17 planar periodic symmetry groups have been found. (Received August 29, 2016)

1125-AF-423      **Harrison Chapman\*** ([hchapman@math.uga.edu](mailto:hchapman@math.uga.edu)). *Slipknotting in the Knot Diagram Model*.

The presence of slipknots in configurations of proteins and DNA has been shown to affect their functionality, or alter it entirely. Historically, polymers are modeled as polygonal chains in space. As an alternative to space curves, we provide a framework for working with subknots inside of knot diagrams. We prove using a pattern theorem for knot diagrams that not only are almost all knot diagrams slipknotted, almost all *unknot* diagrams are slipknotted. This proves in the random diagram model a conjecture yet unproven in random space curve models. (Received September 01, 2016)

1125-AF-616      **Allison Henrich\*** ([henricha@seattleu.edu](mailto:henricha@seattleu.edu)). *Knot Fertility and Heredity*. Preliminary report.

In this talk, we will introduce and explore the following new knot theoretical notions. A knot  $D$  is said to be a *descendant* of another knot  $P$  if there is a minimal crossing diagram of  $P$  on which some subset of crossings can be changed to produce a diagram of  $D$ . In this case,  $P$  is said to be a *parent* of  $D$ . A knot  $P$  is *fertile* if all knots with smaller crossing number are descendants of  $P$ . More generally, a knot  $K$  is *n-fertile* if all knots with crossing number less than or equal to  $n$  are descendants of  $K$ . We will discuss families of related (and insular) knots as well as knots that have many parents and knots that are particularly fertile.

This is joint work with Jason Cantarella, Elsa Magness, Kayla Perez, Eric Rawdon, and Briana Zimmer. (Received September 07, 2016)

1125-AF-902      **Kenneth C Millett\*** ([millett@math.ucsb.edu](mailto:millett@math.ucsb.edu)), Department of Mathematics, UCSB, Santa Barbara, CA 93106. *Entanglement of Confined Random Polygonal Chains*.

Consider a collection of long pipes of varying diameter and containing a flowing fluid. One can ask about the spatial nature of the flow vortices as they relate to the character of the fluid's motion. If this fluid contains linear molecular chains, varying in size and character from macromolecules to linguini, is their structure knotted or linked or otherwise entangled. In order to understand the physical or biological properties of the system, one must understand the consequences of their presence. A coarse grained model of such systems consists of linear polygons confined to tubes with a circular cross-section of varying diameter and subjected to an axial alignment bias such as a physical force. We will describe how one might model such systems and as well as how one might quantify the extent to which the presence of knotting and linking can be measured. In addition, we will see how the diameter and alignment influence the presence of such forms of entanglement. (Received September 13, 2016)

1125-AF-1405      **Joel Hass\*** ([hass@math.ucdavis.edu](mailto:hass@math.ucdavis.edu)). *Generating random knots and links from random permutations*.

We study random knots and links using the Petaluma model, which is based on the petal projections developed by Adams. In this model we obtain a formula for the distribution of the linking number of a random two-component link. We also obtain formulas for the expectations and the higher moments of the Casson invariant and the order-3 knot invariant  $v_3$ . These are the first precise formulas given for the distributions and higher moments of invariants in any model for random knots or links. We also use numerical computation to compare these to other random knot and link models, such as those based on grid diagrams. This is joint work with C. Even-Zohar, N. Linial and T. Nowik. (Received September 16, 2016)

1125-AF-2068      **Christine Soteris\*** ([soteris@math.usask.ca](mailto:soteris@math.usask.ca)). *Entanglement complexity in lattice polygon models of polymers under confinement*. Preliminary report.

We use self-avoiding polygons in a tubular sublattice of the simple cubic lattice (the 3-dimensional integer lattice  $\mathbb{Z}^3$ ) to model polymers under confinement. Such lattice tube models have potential applications for modelling single DNA molecules in nanochannels, DNA under tight confinement, or protein configurations. In all three of these cases, there are interesting open questions about the effects of confinement on the nature and frequency of "entanglements" (knotting and linking) in the relevant biopolymer. To address these questions, we have used lattice tube models to study knotting probabilities and knot-distributions as a function of tube dimensions, as well as questions about knot-localization. In this talk, I will give an introduction to lattice models of polymers

and then present recent theoretical and numerical results, obtained in collaboration with Nick Beaton and Jeremy Eng, about knotting in tubes. The numerical results are for small tube sizes and involve exact and Monte Carlo generation of polygons using transfer matrix methods. (Received September 19, 2016)

1125-AF-2072 **Laura K. Plunkett\*** (plunkett@hnu.edu) and **Kyle Chapman**. *Knotting and Size in Ergodically Generated Off-Lattice Walks with Excluded Volume*.

We describe a new algorithm, the reflection method, to generate off-lattice random walks of specified, though arbitrarily large, thickness in  $\mathbb{R}^3$  and present the data resulting from our implementation of this method. We use this new data to describe the complex relationship between the presence and nature of knotting and size, thickness and shape of the random walk. We extend the current understanding of excluded volume by expanding the range of analysis of how the squared radius of gyration scales with length and thickness, including an analysis of very thin walks. We also examine the profound effect of thickness on the probability of knotting in open chains, and how the distribution of knot types varies with thickness. (Received September 19, 2016)

## *Research in Improving Undergraduate Mathematical Sciences Education: Examples Supported by the National Science Foundation's IUSE: EHR Program*

1125-AG-566 **Nicholas A Scoville\*** (nscoville@ursinus.edu), 601 E. Main Street, Math and CS, Collegeville, PA 19426, and **Jerry Lodder, Diana White, Dominic Klyve, Danny Otero, Janet Barnett and Kathy Clark**. *Collaborative Research: Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources (TRIUMPHS)*. Preliminary report.

The NSF funded TRIUMPHS (Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources) grant is a five year (year one just completed) seven institution collaborative project devoted to designing, implementing, and assessing primary source projects (PSPs) to be used in the teaching of undergraduate mathematics courses. The grant was developed based on the growing interest in the mathematics community to use primary historical source material in the classroom, the lack of such resources, and the lack of any kind of assessment of such methods. After sharing some reasons why there is a growing interest in primary sources, we will take a look back at the accomplishments of the first year including project development, conference presentations, and initial assessment. We will then discuss the future plans for the next four years of the grant. (Received September 06, 2016)

1125-AG-1250 **Jennifer J Kaplan\*** (jkaplan@uga.edu) and **Kristen E Roland** (kristen.roland25@uga.edu). *Fostering Active Learning in Statistics: Research on Students and Graduate Teaching Assistant*. Preliminary report.

Statistical literacy is crucial to an educated citizenry and statistics has one of the fastest growing course enrollments of the undergraduate STEM disciplines. Many statistics courses are taught by Teaching Assistants (TAs). Little is known, however, about the training needs of TAs to foster active learning, a critical recommendation to improve undergraduate STEM education. Active-learning experiences were developed using underlying principles and lessons learned from similar work at NC State University. The TAs associated with the project attend weekly training sessions to prepare to facilitate the experiences in labs. The training sessions and some of the labs are observed by a researcher. In addition, the TAs provide feedback through weekly surveys and periodic focus groups. We describe the lessons learned from the first year of piloting and discuss how the lessons are being translated into revisions of the GTA training. We found that TAs appreciate time spent doing the activities as students and reviewing the content of the activities. They have difficulty connecting the activities to the content and in providing closure for the activities. We will discuss modifications in our training plans, such as sharing sample student responses to the assessment items with the TAs. (Received September 15, 2016)

1125-AG-1561 **Michael S Jacobson\*** (michael.jacobson@ucdenver.edu), Department of Mathematical & Statistical Sci, Denver, CO 80217, and **Michael Ferrara, RaKissa Manzanares and Gary Olson**. *Promoting Success In Early College Mathematics Through Graduate Teacher Training*.

Mathematics is a central part of all STEM disciplines, and undergraduate success in mathematics courses is an increasingly critical piece of the growing national need to train the next generation of U.S. STEM professionals. The Promoting Success in Early College Mathematics through Graduate Teacher Training project (supported

by NSF Award #1539602) at the University of Colorado Denver (CU Denver) addresses this critical national issue through improved teaching preparation for Graduate Teaching Assistants (GTAs) in the Mathematical Sciences. In particular, the project focuses on how a more comprehensive approach to training GTAs can impact their classroom practice and benefit their students, ultimately increasing success, particularly in early undergraduate mathematics courses. Further, a large proportion of PhD graduates, with appointments ranging from community colleges to large research universities, have a significant part of their professional responsibilities focused on teaching undergraduates. By providing an effective model of broad graduate student training that encompasses a wide variety of components, the project will study the impact on the overall undergraduate teaching and student learning effectiveness of the GTAs. (Received September 18, 2016)

1125-AG-1716 **Paul E Seeburger\*** (pseeburger@monroecc.edu), 1000 E. Henrietta Rd., Rochester, NY 14623, and **Monica VanDieren** and **Deborah Moore-Russo**. *Collaborative Research: Improving Conceptual Understanding of Multivariable Calculus Through Visualization Using CalcPlot3D*. Preliminary report.

Our project seeks to improve students' geometric intuition (both 2D and 3D) about surfaces, vectors, vector fields, and curves, thereby better preparing students to more fully understand engineering and physics problems in their STEM coursework. At the core of our project is CalcPlot3D, an interactive online 3D JavaScript app designed to enhance the teaching and learning of multivariable calculus. This app brings the concepts of multivariable calculus to life and makes it easy to visually explore the concepts and relationships between them. It facilitates the graphical representation of many multivariable concepts including contour plots, velocity and acceleration, directional derivatives, and gradients.

In addition to the creation of this app, our project seeks to create a series of new visual concept explorations and applications to improve student understanding of multivariable calculus, differential equations and linear algebra and to use the app to conduct research investigating how student understanding of multivariable calculus concepts changes through the use of visualization and dynamic concept explorations.

See <http://web.monroecc.edu/calcnSF/>. This project is funded by NSF-IUSE #1524968, NSF-IUSE #1523786, and NSF-IUSE #1525216. (Received September 19, 2016)

1125-AG-1813 **Heather A Moon\*** (hamoon@lsc.edu), 500 8th Ave, Lewiston, ID 83501, **Thomas J Asaki** (tasaki@wsu.edu), PO BOX 641067, Neill Hall, Pullman, WA 99164-1067, **Marie Snipes** (msnipes@kenyon.edu), 103 College Rd, Gambier, OH 43022, and **Chris Camfield** (camfield@hendrix.edu), 1600 Washington Ave, Conway, AR 72032. *Collaborative Research: Data-Driven Applications Inspiring Upper-Division Mathematics*.

Our project provides undergraduate math students with a deep appreciation for Analysis tools and their real-life uses. Inspired by current data problems, we are creating adaptable and transportable application-based modules that are being tested and improved in an iterative fashion. Our module based approach differs markedly from the standard method of definition-theorem-proof-example. It also differs from other application-integrating approaches which illustrate the use of learned tools on real problems. Each module begins by introducing a cutting edge research problem in data, image, or video analysis. Solution paths inspire the development of mathematical concepts. Because the level and frequency of courses vary from institution to institution, we are creating these modules so that they are easily adaptable at both introductory and advanced levels. Our modules will be intended as supplements to the standard curriculum or as meldable into one or more full-length, problem-driven courses. In this talk, we present details about each module and the associated data-driven problems. We give examples of the topics that are inspired in each module. (Received September 19, 2016)

1125-AG-1899 **Estrella Johnson\***, strej@vt.edu, **Christine Andrews-Larson**, cjlarson@fsu.edu, and **Karen Keene**, kakeene@ncsu.edu. *Teaching Inquiry-oriented mathematics: Establishing Support*.

The Teaching Inquiry-oriented Mathematics: Establishing Supports (TIMES) project is using a designed-based research to develop and investigate a three-part instructional support model. The TIMES model consists of three instructional supports: curricular support materials, a three-day summer workshop, and weekly online workgroups. Within the context of this model, the research team is investigating the relationships among instructional supports, instructors, and instruction that are important for informing effective instructional change. In this presentation, we offer preliminary findings related to the facilitation of meaningful discussions around instructors' pedagogical reasoning, the use of videos in online faculty workgroups, and departmental contexts and norms that support instructors in adopting research-based instructional materials and strategies. We end with a discussion of our next steps, including an investigation into student outcomes (disaggregated by gender

and race/ethnicity) to understand how instructional practices influence students': performance on content assessments, perceptions of instruction, and performance and perseverance in subsequent mathematics courses. (Received September 19, 2016)

## *Technical Tools for Mathematical 3D Printing*

1125-AH-67 **Elizabeth Denne\***, Department of Mathematics, Washington & Lee University, Lexington, VA 24450. *Using Cinema 4D to create Calculus models.*

3D printing can be a powerful tool to help everyone visualize mathematical models. In this talk, I'll discuss how it can be used in the classroom in Calculus II and Multivariable Calculus. There are a variety of different ways to do this; from using models to illustrate ideas, to having the students design, print and share their own models. I'll also show how Cinema 4D can be used to create quadratic surfaces. (Received July 06, 2016)

1125-AH-70 **Ergun Akleman\*** ([ergun.akleman@gmail.com](mailto:ergun.akleman@gmail.com)), 5126 Bellerive Bend Dr., College Station, TX 77845. *Orientable Mesh Modeling.*

In this work, I describe a solid foundation for orientable 2-manifold mesh modeling using graph rotation systems. Based on this theory, we have developed TopMod, which is an orientable 2-manifold mesh modeling system. TopMod provides a wide variety of High Genus Modeling tools, Remeshings & Subdivisions, and Extrusions & Replacements. Using TopMod, one can find a wide variety of ways to create high genus shapes; almost all subdivision algorithms, wide variety of ways to remeshing shapes and new extrusions. These tools are also useful for Architectural applications, Design and Sculpting and Sketch Based Modeling. (Received July 08, 2016)

1125-AH-72 **Ergun Akleman\*** ([ergun.akleman@gmail.com](mailto:ergun.akleman@gmail.com)), 5126 Bellerive Bend Dr., College Station, TX 77845. *Cyclic Woven Object Modeling and Topological Constructions.*

Cyclic Woven Object Modeling: We have developed provided a solid foundation for knot, link and cyclic woven object modeling using extended graph rotation systems. If we twist an arbitrary subset of edges of a mesh on an orientable surface, we can obtain non-orientable surfaces. The resulting extended graph rotation system can be used to induce a cyclic weaving on the original surface, that corresponds a 3-space embedding of a non-orientable surface.

Topological Constructions: Discrete Gaussian-Bonnet theorem and Gaussian curvatures related mesh topologic concepts to geometry. Using this relationship, we have developed methods to physically construct shapes. (Received July 08, 2016)

1125-AH-73 **Henry Segerman\*** ([segerman@math.okstate.edu](mailto:segerman@math.okstate.edu)). *Design by transformation.*

Many mathematical models can be constructed via the following general procedure: Take a relatively simple shape in three-dimensional space, and transform it in some way to produce the desired model. An example would be to take a rectangular grid, and twist it into a Möbius strip grid. This technique is particularly useful when one designs the input geometry by-hand in a graphical CAD program, then transforms that geometry using a script. This allows for rapid iteration of a design, in a far more convenient and expressive form than if the geometry of the final model were entirely generated by programming. (Received July 10, 2016)

1125-AH-365 **Jonathan M Gerhard\*** ([gerha2jm@dukes.jmu.edu](mailto:gerha2jm@dukes.jmu.edu)) and **Laura Taalman**. *Visualizing Homotopies with 3D Printing.* Preliminary report.

A key interest in algebraic topology is determining whether two spaces are homotopy equivalent, i.e. does there exist a continuous deformation from one space to the other? The classic example of two objects which are homotopy equivalent is the donut and the coffee mug, which might not be so easy to see. We can determine that two spaces are homotopy equivalent by defining the homotopy between them, but this is often incredibly difficult to visualize. For example, if we remove a torus  $T^2$  from the 3-sphere  $S^3$ , then the space we are left with is homotopy equivalent to the space of two disjoint solid tori! In order to visualize some of these difficult homotopies, we use 3D-printing to create a series of models which allow us to get an intuitive grasp on the deformation involved. (Received August 29, 2016)

1125-AH-849 **David Bachman\*** ([bachman@pitzer.edu](mailto:bachman@pitzer.edu)), Pitzer College, 1050 N Mills Ave, Claremont, CA 91711. *Using Grasshopper to design 3D-printable models.*

Grasshopper is a graphical scripting platform for the popular CAD program, Rhinoceros 3D. By dragging and dropping component boxes onto a blank canvas, and connecting them with virtual wires, the user creates complex scripts which generate any kind of mathematical design. By using sliders and control knobs these scripts become interactive, allowing the user to make aesthetic design choices. Furthermore, there are many plug-ins for

Grasshopper that expand its functionality by adding physics-based modeling tools, voxel-based modeling, etc. However, as with any modeling program, particular care is required to guarantee the resulting designs are 3D printable. (Received September 12, 2016)

## *Cryptology for Undergraduates*

1125-B1-39 **Joseph H Silverman\*** ([jhs@math.brown.edu](mailto:jhs@math.brown.edu)), Mathematics Department - Box 1917, Brown University, Providence, RI 02912. *Cryptology as a Post-Linear Algebra Gateway to Advanced Mathematics.*

In this talk I will describe the Brown University course in Mathematical Cryptology created jointly with Jeff Hoffstein and Jill Pipher. The course uses public key cryptology as the framework to introduce science and math majors to a variety of mathematical topics, including group theory, number theory, probability, information theory, and analysis of algorithms, as well as more advanced topics such as elliptic curves and lattices. The mathematics is tied together through the study and comparison of public key cryptosystems and digital signature schemes based on factorization (RSA), discrete logarithms (ElGamal, ECC), and lattice problems (NTRU). This course, which attracts students from across the sciences, is one of our most popular upper level courses. (Received June 14, 2016)

1125-B1-96 **David A Brown\*** ([dabrown@ithaca.edu](mailto:dabrown@ithaca.edu)), Ithaca College, Dept. of Mathematics, 953 Danby Road, Ithaca, NY 14850. *Using Declassified Intelligence Documents in a Cryptology Course.*

Often, the stories behind cryptology are as compelling as the underlying mathematical concepts, and sometimes more so. The people and events throughout the history of cryptology easily motivate students (majors and non-majors) to dig deeply into the material. In this presentation, I will highlight my use of primary cryptologic documents, available via the Freedom of Information Act, from the FBI, NSA, and CIA as a basis for student research. I will provide several specific examples, including cryptology used in organized crime, serial killer cases, and personal communication between John Nash and the NSA. I will also provide examples of projects that students worked on and the cryptology connections that they found. (Received July 22, 2016)

1125-B1-213 **Scott C. Batson\*** ([scott.batson@navy.mil](mailto:scott.batson@navy.mil)), **Hemant Pendharkar**, **Tuwaner Hudson Lamar**, **Georgianna L.T. Campbell** and **Kayla A. Capitan**. *The Suitability of Lattices for Project-based Introductions to Cryptology.*

Through the completion of common prerequisite courses, the majority of undergraduate students have been exposed to the mathematical concepts necessary to characterize lattices, which may be conceptualized as an arrangement of points in geometric space. Yet, there are also perspectives of lattices that may only be expressed with knowledge of advanced Algebraic Number Theory and Abstract Algebra. While lattice-based cryptography is studied as a strong candidate for post-quantum cryptography, the various aspects of lattices, from simple to complex, are capable of supporting introductions to cryptology at different levels. We first introduce lattices as a topic for cryptologic study. We then identify problems and mathematical aspects of lattice-based cryptography that are interesting and accessible to undergraduate students, graduate students, and/or faculty. Finally, results and outcomes of a lattice-based cryptography research project, with contributions ranging from those of a high school student to terminal degree holders, are discussed. This research experience is presented as an innovative model for project-based introductions to cryptology. (Received August 12, 2016)

1125-B1-245 **N. Paul Schembari\*** ([schembariesu@gmail.com](mailto:schembariesu@gmail.com)), Mathematics Department, 200 Prospect Street, East Stroudsburg, PA 18301. *The Simulation and Cryptanalysis of Rotor Ciphers.*

Ciphers based on rotor machines were the state-of-the-art in the mid-1900s, with arguably the most famous being the German Enigma. We have found that students have great interest in the Enigma and its cryptanalysis, so we created our own rotor cipher which is simulated with shifting tables and can be cryptanalyzed. Ours and the historic rotor ciphers are based on rotating wheels which change the encryption with each character encrypted. Our cipher is easily programmed or can be simulated in Excel for students with less experience, and its cryptanalysis leads to the re-creation of the rotor wheels. The cryptanalysis of our cipher is executed on simulated machines with one or two wheels and is based on a known-plaintext attack.

Our classroom experiences include discussion of Enigma encryption and the analysis of our rotor cipher. Also as part of our instruction in cryptology, a National Security Agency historian visited campus to display an Enigma, and we display another rotor machine (US Navy CSP-1500 Six Rotor Portable Mechanical Cipher Machine) on loan from the National Cryptologic Museum, Fort Meade, MD. In this paper we discuss the

simulation of our rotor cipher, its cryptanalysis, external cryptologic educational experiences, and experiences with our exercises. (Received August 18, 2016)

1125-B1-417 **Rick Klima\*** ([klimare@appstate.edu](mailto:klimare@appstate.edu)), Department of Mathematical Sciences, 342 Walker Hall, Appalachian State University, Boone, NC 28608, and **Neil Sigmon** ([npsigmon@radford.edu](mailto:npsigmon@radford.edu)), Department of Mathematics and Statistics, P.O. Box 6942, Radford University, Radford, VA 24142. *Enigma: A Combinatorial Analysis and Maple Simulator.*

The German Enigma machine was the most widely used mechanical field cipher during World War II, and is arguably the most famous military cipher ever. The various possibilities for rotors, ring settings, notch positions, reflectors, and plugboard connections combined to give a theoretical number of potential initial configurations of the machine that was astronomical, each resulting in a unique way in which the machine functioned. However, rotors and reflectors with only a very small number of different wirings were never produced, and for most of the war a fixed number of plugboard cables was used. This dramatically reduced the actual number of potential initial configurations of the machine, and was an important contribution to the Allied codebreaking effort at Bletchley Park that resulted in its cryptanalysis. In this talk we will show how combinatorics can be used to determine the number of theoretical and actual potential initial configurations of the Enigma, and demonstrate a Maple simulator of its operation. (Received September 01, 2016)

1125-B1-431 **Neil P. Sigmon\*** ([npsigmon@radford.edu](mailto:npsigmon@radford.edu)), Department of Mathematics and Statistics, P.O. Box 6942, Radford University, Radford, VA 24073, and **Rick E. Klima** ([klimare@appstate.edu](mailto:klimare@appstate.edu)), Department of Mathematical Sciences, Appalachian State University, Boone, NC 28608. *Teaching Cryptology to Increase Interest in Mathematics for Students Majoring in Non-Technical Disciplines and High School Students.*

Cryptology can be taught at a variety of levels, including college students from non-technical backgrounds and high school students. Currently, a general education course for Radford University's Honors Academy involving cryptography is being offered and has been taught for many years. This course has been a very popular course with students. An integral part for teaching a course in cryptology at this level is the integration of easy to use technology that allows students to see realistic examples of cryptographical algorithms, while not requiring a detailed knowledge of a programming language.

The purpose of this presentation will be to give an overview of the course being taught at Radford and how Maplets have been used to provide students with an easy to use tool for encrypting, decrypting, and cryptanalyzing messages. In addition, efforts with this approach to use cryptology to increase student interest in mathematics at the high school level will be described. (Received September 01, 2016)

1125-B1-436 **Nathan M Dasenbrock-Gammon\*** ([dasenbrocn1@nku.edu](mailto:dasenbrocn1@nku.edu)). *Attacking Even False Addition.*

The primary World War II Japanese naval cipher JN-25 was enciphered code. Initially random additives were combined with clear code by the operation of false addition – addition of digits modulo 10. Near the end of the war, the operation was changed by the use of conversion tables, which describe operations that are not group operations. This presentation will explore the recovery of additives from JN-25 when the operation is not false addition. (Received September 01, 2016)

1125-B1-944 **Darren B Glass\*** ([dglass@gettysburg.edu](mailto:dglass@gettysburg.edu)), 300 N. Washington Street, Gettysburg, PA 17325. *Teaching Information Security to First-Year Students.*

For the last decade we have taught a First-Year Seminar at Gettysburg College on Cryptography. In this talk, we will discuss a couple of activities that we do in the class to introduce students to some of the ethical and philosophical issues raised by cryptography in the modern world, and how these may differ from classical and historical cryptography. (Received September 13, 2016)

1125-B1-1252 **Paul Jenkins\*** ([jenkins@math.byu.edu](mailto:jenkins@math.byu.edu)). *Broken one-time pads and other projects.*

The Brigham Young University mathematics department teaches both a senior-level mathematical cryptography course, for science and engineering students who have had linear algebra, and a freshman cryptography seminar open to all majors. This talk describes classroom activities and projects used in these courses, including cipher-solving competitions, personalized final exams, and software which helps solve Vigenère ciphers and incorrectly used one-time pads. (Received September 15, 2016)

1125-B1-1401 **Eric J Landquist\*** (elandqui@kutztown.edu), Dept. of Mathematics, Kutztown University, Kutztown, PA 19530. *Secure Hands-on Cryptosystems in an Undergraduate Cryptology Course*. Preliminary report.

Could something as commonplace as a 52-card deck of playing cards hold the potential for encryption stronger than even the most rigorously scrutinized ciphers in use today? Large files that are transmitted over the internet are often encrypted using a symmetric cipher (such as AES) with a random 128-bit key. Students in an upper-division undergraduate cryptology course were surprised to learn, however, that a well-shuffled deck of cards holds over 225 bits of entropy (randomness). Utilized to its maximum potential, this deck of cards could provide better security than even 128-bit AES. Throughout the course, students were challenged to design cryptosystems in which both encryption and decryption could be carried out efficiently by hand, but would be able to withstand modern cryptanalysis. In this talk, we discuss a few of the ideas for secure hands-on cryptosystems that arose, the inspiration for these ideas, and the overall structure of the course that fostered curiosity and exploration. Moreover, these hands-on cryptosystems would be accessible to any undergraduate audience and would make for a fun, albeit somewhat time-consuming, in-class activity. (Received September 16, 2016)

1125-B1-1416 **Manmohan Kaur\*** (mkaur@ben.edu), Department of Math and CS, 5700 Colleg Road, Lisle, IL 60532. *Computer Implementations of Certain Cryptographic Methods*. Preliminary report.

Cryptology, the science of sending and receiving secret messages, is at the intersection of mathematics and computer science, and encompasses every aspect of modern life. Cryptographic methods are easily accessible to undergraduates, and can help kindle their long term interest in mathematics and its applications. Computer simulations of various cryptographic methods add to the excitement in the classroom, as the students encrypt and decrypt in real time. While implementations of the more traditional cryptosystems are easily available online, the newer ones are not. As a part of our undergraduate course on cryptology, the students work on a project, relating cryptography with another topic of their interest. Some students choose to implement cryptosystems like Enigma, RSA, Diffie-Hellman Key Exchange, El Gamal, etc., in a computer algebra system. Working through their implementation allows them not only to better understand the method, but also introduces them to various nuances and gaps between theoretical cryptography, whose main concern is security, and practical cryptography, which is guided by efficiency. We will describe some of these implementations, how they are used in the classroom, and student outcomes. (Received September 16, 2016)

1125-B1-1523 **Eleanor Farrington\*** (efarrington@maritime.edu). *Using Cryptology to Motivate the Study of Functions*. Preliminary report.

Satisfactorily answering the dreaded question “When will I ever use this in the real world?” is always a challenge. In the case of conceptual algebra topics such as the introduction of the idea of a function it gets even harder. Luckily the basic ideas of cryptology provide an excellent motivation for both the study of compositions of functions and inverse functions. This talk will look at how these topics have been used in College Algebra classes for non-majors. (Received September 17, 2016)

1125-B1-1700 **Anne M. Ho\*** (aho@coastal.edu), Conway, SC. *Unlocking Ideas: Using escape room puzzles in a cryptology classroom*. Preliminary report.

Escape rooms have become an increasingly popular game and team building exercise in which participants are locked in a room and asked to use puzzles to get out within a set time limit. According to a recent article entitled “The Rise of Educational Escape Rooms” (*The Atlantic*, 2016), educators are starting to use the ideas of escape rooms in classrooms. A natural setting for such puzzles is an undergraduate cryptology course. In this talk, I will discuss the experience of running an interactive class session where my students were asked to use their knowledge of classical ciphers to decrypt ciphertext, unlock padlocks, and determine passwords. Some topics covered included the shift, affine, Vigenère, substitution, Playfair, and Hill ciphers. In addition, I will address the logistics of implementing these puzzles with the use of the open-source math software system, SageMath. Lastly, I will briefly mention how I have utilized these ideas in other classes such as Calculus. (Received September 18, 2016)

1125-B1-1785 **Chris Christensen\*** (christensen@nku.edu). *Don't Forget (Enciphered) Codes*. Preliminary report.

Codes are often a forgotten topic in cryptology courses, but they have a distinguished history. In particular, the creation and attack of enciphered code involves interesting mathematical ideas. This presentation will explore the construction of an enciphered code that mimics the World War II Japanese naval cipher JN-11 and the technique of differencing, which is the primary attack on enciphered code. (Received September 19, 2016)

1125-B1-2225 **Joy L. Becker\*** ([joy.becker@wartburg.edu](mailto:joy.becker@wartburg.edu)). *Cryptology Examples for a Variety of Course Levels*. Preliminary report.

Topics related to cryptology can be introduced in a variety of mathematics courses, such as abstract algebra, linear algebra, and first-year general education courses. For example, the applications of matrices in methods such as the Hill cipher can give students a taste of the mathematics behind cryptology. This talk will include ways to introduce cryptology examples in various mathematics courses for majors and non-majors. The impact on student motivation will also be discussed. (Received September 20, 2016)

1125-B1-2376 **Jennifer M. Magee\*** ([jmagee@umw.edu](mailto:jmagee@umw.edu)). *Cryptology for first year students*.

The University of Mary Washington offers a cryptology class for freshmen as part of the larger first-year experience. This seminar class is designed to incorporate active learning and uses cryptology as a vehicle to develop the speaking, writing, and research skills needed to succeed in college. This course has no prerequisites and requires only basic algebra skills and an interest in cryptology. In this talk, we will discuss the cryptologic topics covered in the course, as well as how the course meets the student learning outcomes associated with the first year seminar program. (Received September 20, 2016)

1125-B1-2584 **Tamara B Veenstra\*** ([tamara\\_veenstra@redlands.edu](mailto:tamara_veenstra@redlands.edu)). *Alan Turing and his Contributions to Cryptology*. Preliminary report.

The movie “Imitation Game” attempted to give audiences some historical information about the life of Alan Turing and the work he conducted in cryptology during World War II. However, the depiction of his work in this field was lacking in depth and technical details. This talk will present a more technical description of some his mathematical ideas related to breaking the enigma machine as well as a broader introduction to the different areas of his work during this time. (Received September 20, 2016)

## *The Development and Adoption of Open Educational Resources for Teaching and Learning*

1125-B5-215 **Jim Fowler\*** ([fowler@math.osu.edu](mailto:fowler@math.osu.edu)), 100 Math Tower, 231 West 18th Avenue, Columbus, OH 43212, and **David Farmer**. *Find, Review, Promote: CuratedCourses aligns OER to the course syllabus*.

Finding online math curricular resources that are readily available, field-tested in a variety of settings, and known to be high quality is a challenge. CuratedCourses is an NSF-funded IUSE project to address that challenge.

CuratedCourses is an open platform where metadata is associated with educational content on the web, where experts review the educational content, and where instructors can then find the vetted content aligned with popular textbooks and course calendars. Challenges are locating high-quality content, developing rubrics for evaluating content, and creating standard formats for tagging topics and learning outcomes. GitHub and other open technologies are used to address these challenges. (Received August 15, 2016)

1125-B5-216 **Petra Bonfert-Taylor\*** ([petra.b.taylor@dartmouth.edu](mailto:petra.b.taylor@dartmouth.edu)), Thayer School of Engineering, Dartmouth College, 14 Engineering Drive, Hanover, NH 03755, **Sarah E. Eichhorn** ([s.eichhorn@uci.edu](mailto:s.eichhorn@uci.edu)), 110 Theory Dr., University of California, Irvine, Irvine, CA 92617, and **Tom Roby** ([tom.robby@uconn.edu](mailto:tom.robby@uconn.edu)). *Curated Courses in Mathematics: Resources for Creation of Online Mathematics Content*. Preliminary report.

Curated Courses in Mathematics (CCIM) is a project to create, gather, curate, tag, review, organize and make available high quality online open educational mathematics resources. The CCIM project aims to coordinate work being done at multiple institutions on similar courses, enabling faculty to share resources they create or curate from other sources.

In this session, we focus on sharing resources we have created for faculty about how to design and produce online mathematics content. This includes video and written guides for creating a variety of types of mathematics videos such as writing over slides on a tablet.

By creating a system for curating and tagging resources our hope is that faculty can more easily find high quality materials to utilize in their classes and more broadly disseminate good resources they create. (Received August 12, 2016)

1125-B5-236 **Matt Boelkins\*** (boelkinm@gvsu.edu), Department of Mathematics, Grand Valley State University, Allendale, MI 49401. *Active Calculus: Recent Developments*. Preliminary report.

The single variable version of *Active Calculus* was made publicly available (<http://gvsu.edu/s/Ym>) in August 2012. With a growing base of individual users and departments who have formally adopted it, the text has become popular in both PDF and print-on-demand formats, while remaining in need of some improved features. Through attending the American Institute of Mathematics' (AIM) Mathbook XML Workshop in April 2016, I learned of several new technological developments that present opportunities to strengthen the quality and utility of *Active Calculus* and other similar books. In this talk, I'll give an overview of ongoing work to convert the text to Mathbook XML and the resulting now-possible HTML format, with particular emphasis on the new addition of live WeBWorK exercises that provide immediate and anonymous feedback to the reader. Along the way, I'll briefly reflect on some key experiences and lessons learned in writing a free and open text. (Received August 16, 2016)

1125-B5-510 **Anne Quinn\***, quinna@edinboro.edu. *Using Instructional Apps to Visualize Graph Theory*.

I will discuss several free and inexpensive graph theory smartphone apps that I have used in the classroom and have written about in recent articles (Mathematics Teacher, 2015; College Mathematics Journal, 2016). Using these apps in lecture and homework assignments for the past few years has helped me produce more professional lectures and has helped my students visualize vertex-edge graphs, investigate the interplay between matrix and graphical representations, develop insight into writing proofs, and explore transformations that led to isomorphic graphs that were bipartite and/or planar. In addition, I will use the apps to demonstrate Kuratowski's Theorem and other classic problems. Moreover, I will discuss how students became more proficient at solving problems and creating proofs without technology during this unit. Supplemental storage apps such as Dropbox will also be discussed. (Received September 04, 2016)

1125-B5-562 **Brad Bailey\*** (bbailey@ung.edu) and **Thomas Cooper** (tom.cooper@ung.edu). *Do you get what you pay for?: Students' opinions of textbooks' formats*.

With the increasing costs of printed textbooks along with the increasing availability of open-educational resources (OER), faculty may be tempted to consider adopting open-access materials. But how do our students feel about these options? Does the format of the text influence how and how often the students use their textbooks? Are students using free or online texts just as satisfied with the quality of these resources as other students? The speakers administered a survey to students in introductory courses with required texts in a variety of formats, including OER's. This survey addresses the above research questions, and in this presentation, the speakers will reveal and discuss the results of this survey. (Received September 06, 2016)

1125-B5-959 **Thomas W. Judson\*** (judsonw@sfasu.edu), Department of Mathematics and Statistics, P.O. Box 13040-3040 SFA Station, Nacogdoches, TX 75962. *Sage Cells: Making Sage Accessible to Students, Teachers, and Authors*.

Sage cells are self-contained Sage calculations that can be embedded in any web page. No software needs to be installed, and no prior knowledge of Sage is required because the commands are pre-loaded. Students can learn Sage by modifying existing commands, lowering the barrier to learning a computer algebra system. In an online text (HTML) a Sage cell is evaluated with a single click. There are no browser plugins to install, nor additional software to download or install. The Sage code is present in the text and requires no programming. By making modifications to the code students and faculty can learn Sage as needed. They can work without fear of making mistakes, because when anything goes wrong they can reload the page to restore the original. For authors it is a straightforward process to add Sage cells to an existing book in MathBook XML, an XML application for authors. The print, pdf, and ePub versions of a document simply display the Sage code as static, but no special handling is required by the author. Project UTMOST is creating a repository of Sage cells organized by mathematical topic to help authors work more efficiently and to allow authors with minimal Sage knowledge to incorporate Sage into their documents. (Received September 13, 2016)

1125-B5-1166 **Mitchel T. Keller\*** (kellermt@wlu.edu), Department of Mathematics, Washington & Lee University, 204 W Washington St, Lexington, VA 24450. *The Journey of one Open Source Applied Combinatorics Text*.

Like many of today's open source texts, *Applied Combinatorics*, written jointly with William T. Trotter, began as a set of course notes to replace a commercial textbook. It gradually evolved while the presenter was a doctoral student, and in 2010 a free PDF was released that contained a full-developed set of exercises for almost all

chapters. In the intervening years, a clear statement of a Creative Commons license was added, and the text has been adopted at a number of institutions throughout North America. After a 2016 workshop at the American Institute of Mathematics, we decided to convert the text from L<sup>A</sup>T<sub>E</sub>X to MathBook XML because of the ease of creating a reader-friendly HTML version (with interactive SageMath components) in addition to PDF and inexpensive print on demand versions. This new version was released in time for Fall 2016 classes. This talk will discuss the evolution of the text, experiences using it in the classroom, and the challenges and benefits of the recent conversion to MathBook XML. To view *Applied Combinatorics*, visit <http://rellek.net/appcomb/>. (Received September 15, 2016)

1125-B5-1305     **Mark C McClure\*** ([mcmcc1ur@unca.edu](mailto:mcmcc1ur@unca.edu)), Department of Mathematics CPO 2350, University of North Carolina Asheville, 1 University Heights, Asheville, NC 28804.  
*Classroom Discourse - your class's own private Stack-like forum.* Preliminary report.

Discourse is open source software designed to make it easy to host your own online discussion forum. It is similar in many ways to [math.stackexchange.com](http://math.stackexchange.com) - the mostly awesome, online discussion forum that many tech savvy mathematicians will be familiar with. Discourse has many of the same features, including real time interaction, the ability to upload images, and mathematical typesetting via MathJax - but you can make it your own and have complete control. This allows a teacher to harness the benefits of encouraging students to communicate mathematically using modern technological tools but without forcing undergraduates into the online wilderness. The author discusses his experiences and results, educational and technical, of three years of using Discourse and similar tools in the classroom. (Received September 16, 2016)

1125-B5-1703     **Eric D Bancroft\*** ([edbancroft@gcc.edu](mailto:edbancroft@gcc.edu)), Eric Bancroft, Campus Box 3121, Grove City College, 100 Campus Dr, Grove City, PA 16127. *Building a Better Business Calculus (for Free!)*.

In this talk I will share my adaptation of an open source business calculus textbook to an electronic, web-based format with integrated video lectures and interactive applets, suitable for use in a traditional lecture-based course, a flipped classroom, or an online environment. I will also discuss the free online homework system I use as well as changes I have made to the course to increase the effectiveness of the textbook. (Received September 18, 2016)

1125-B5-2108     **Jeff Zeager\*** ([jzeager@lorainccc.edu](mailto:jzeager@lorainccc.edu)), Division of Science and Mathematics, Lorain County Community College, 1005 Abbe Road North, Elyria, OH 44035. *How the Ohio Mathematics Initiative Influenced our OER Precalculus book.*

The Stitz-Zeager OER Precalculus book debuted in August 2009 and has been modestly revised by the authors over the past several years. During those years, the authors were heavily involved in the Ohio Mathematics Initiative and the work of that group has led the authors to undertake a major restructuring of their Precalculus text. Specifically, the text is being remodeled to support a co-requisite remediation strategy. Also, the decision to reserve College Algebra exclusively for Calculus-bound students (and provide alternative pathways to other students) has provided an opportunity for the authors to delve more deeply into the language and concepts required for Calculus. In this talk, a brief overview of the Ohio Mathematics Initiative will be given followed by a discussion of the textbook revisions completed during the authors' sabbaticals in Fall 2016. (Received September 19, 2016)

1125-B5-2391     **John Watson\*** ([jwwatson@atu.edu](mailto:jwwatson@atu.edu)), Tomlinson 126, Arkansas Tech University, Russellville, AR 72801. *Using open resources in a freshman general education course for non-STEM majors to promote learning and improve attitudes towards mathematics.*

For the past three semesters I have been using open resources a freshman level general education mathematics course for non-STEM majors. This presentation is a report on the resources used in teaching the class, in lieu of a textbook, their effect on student attitudes, and their effectiveness in promoting student learning. (Received September 20, 2016)

1125-B5-2701     **Jonathan Dahl\***, [dahlj@lafayette.edu](mailto:dahlj@lafayette.edu). *Algorithmic generation of calculus problems: beyond random coefficients.* Preliminary report.

Typical open homework systems currently provide randomization only in problem coefficients, requiring separate code for structurally distinct problems. An assignment for differentiation practice, for example, will require the instructor to hand select each problem and rely on enough (correctly coded) examples to be available. This talk will discuss an algorithmic approach to providing problems with a randomized mathematical structure. In the case of differentiation, every "Find  $f'(x)$  where  $f(x) = \underline{\quad}$ " problem is based on the same code, which roughly estimates the complexity of  $f$  and the difficulty of the problem during problem generation. Implementations for

a calculus course will be discussed, as well as potential applications to algorithmic production of test banks with difficulty and quality estimates for each problem. (Received September 20, 2016)

1125-B5-2747 **Chester I Ismay\*** ([cismay@reed.edu](mailto:cismay@reed.edu)), 2043 College Way, UC4703B, Forest Grove, OR 97116. *Using the bookdown R package to create a free modern introductory statistics textbook focused on data visualization, reproducibility, and resampling techniques.*

The bookdown R package has allowed for resources to be freely available while also encouraging collaboration and reproducible analyses. It allows one to write just one source document and export to HTML, PDF, and ebook formats. As examples, this package can be used to create textbooks that include GIFs for students to follow along with while using RStudio, to show R code and its results, and to easily update your resource as you need. In this talk, I'll discuss how I created an open source textbook for my Social Statistics course (available at <http://ismayc.github.io/moderndiver-book>) and feedback I received from my students this past semester on using it.

I'll also discuss why I believe everyone should be using modern R packages in their introductory statistics course and how this textbook can be used as a guide for doing just that. Instructor resources are in development and will be available to further support open access to a modern introductory statistics curriculum. I'm hoping this provides the mathematics and statistics community with a guide to how you could incorporate modern R packages into your classes and why it is so important to teach your students introductory statistics as an introduction to the practice of reproducible statistical analysis. (Received September 20, 2016)

1125-B5-2838 **Zephyrinus C Okonkwo\*** ([zephyrinus.okonkwo@asurams.edu](mailto:zephyrinus.okonkwo@asurams.edu)), Department of Mathematics and CS, 504 College Dr, Albany, GA 31705, and **Anilkumar Devarapu** ([anilkumar.devarapu@asurams.edu](mailto:anilkumar.devarapu@asurams.edu)), Department of Mathematics and CS, 504 College Dr, Albany, GA 31705. *Design and Implementation of Affordable Learning Georgia Basic Statistics Project.*

Over the years, the cost of college textbooks have become prohibitive, making it difficult for students to enroll in certain courses. The overall cost of mathematics and statistics books are exceedingly high, and in some classes, student participation in active learning is very difficult. The University System of Georgia is working to reduce textbook cost burden on students by awarding the Affordable Learning Georgia Textbook grant to instructional faculty at various colleges and universities. In fall 2015, we received one of such grants to develop a no cost textbook MATH 2411-Basic Statistics course. In this paper, we present some measurable positive results from the design and implementation of Affordable Learning Basic Statistics project at Albany State University. (Received September 20, 2016)

1125-B5-3010 **Lindsay Orlando\*** ([lrandlo@austincc.edu](mailto:lrandlo@austincc.edu)), Austin Community College, 6101 Airport Blvd, Austin, TX 78752, and **Mary Parker**. *Open Source Materials for QL: Modeling & Personal Financial Mathematics.*

The OER materials for our quantitative literacy course take a different direction than most of the QL texts currently available. We focus on modeling, making significant use of spreadsheets. In our personal finance section, students explore the mathematics of savings and debt situations by using spreadsheets to set up savings plan and amortization schedules, and then to make modifications to them, such as adding an extra payment annually. Our open source materials include text, homework problems, Excel activities, and instructor resources. (Received September 20, 2016)

1125-B5-3025 **Roger Wolbert\*** ([rwolbert@edinboro.edu](mailto:rwolbert@edinboro.edu)). *Using OERs Extensively in a Flipped Geometry Classroom.*

Open Educational Resources (OERs) are widely available and can be used in flipped classrooms to introduce students to new concepts outside of class before engaging in class discussions. The presenter has used specific OERs for a college geometry course geared toward preservice middle school teachers. OERs, including videos, applets, dynamic geometry software, and worksheet generators, helped the students to learn geometric constructions, vocabulary, and theorems outside of class. This left classroom time for guided inquiry-based activities. In this session, a review of how students reported on the effectiveness of OERs for the flipped geometry course will be covered. In addition, the presenter will include suggestions on how to build and organize modules in any learning management system for any flipped classroom that uses OERs. (Received September 20, 2016)

## *Discrete Mathematics in the Undergraduate Curriculum - Ideas and Innovations for Teaching*

1125-C1-611      **Susanna S. Epp\*** ([sepp@depaul.edu](mailto:sepp@depaul.edu)). *Introductory Combinatorics: Language, Visual Representations, and Models.*

Many beginning combinatorics students believe that the way to answer a given counting question is to quickly choose and apply a formula, but they often make mistakes because they have little insight into the concrete nature of the objects they are supposed to count. This talk will describe techniques to encourage students to analyze counting situations more deeply, through the use of visual representations, careful and unambiguous use of language, and emphasis on thinking in terms of models. (Received September 07, 2016)

1125-C1-668      **Christopher J. Catone\*** ([catonec@gmail.com](mailto:catonec@gmail.com)), 1621 N. 13th Street, Reading, PA 19604. *Bridging Calculus and Discrete Math via the Discrete Derivative.*

Most mathematics majors take Calculus and Discrete Mathematics early in their college careers and often consider them disjoint. Although well known among mathematicians, the discrete derivative often does not appear in either course. By making this exclusion we miss an opportunity to emphasize the connections among mathematical disciplines. In this talk we present the discrete derivative as it could appear in a Discrete Math class and illustrate the parallels with the derivative studied in Calculus. Lastly, we give examples where this tool can be used to solve discrete problems and discuss the benefits of adding this topic to your discrete math course. (Received September 08, 2016)

1125-C1-680      **Katie V. Johnson\*** ([kjohnson@fgcu.edu](mailto:kjohnson@fgcu.edu)), 10501 FGCU Blvd. S., Fort Myers, FL 33965. *Team-Based Learning in Discrete Math.* Preliminary report.

Team-Based Learning is a pedagogical method that uses diverse groups of 6-7 students and focuses on applications and decision making. Research has shown that teams in TBL classes can be very powerful, due primarily to the structure of the course that holds both teams and individuals accountable for learning. In this talk, I will explain how I have transformed my Discrete Math course for TBL, including: 1) how I have organized the course into five modules, 2) the assessment process for each module, and 3) examples of application activities. (Received September 09, 2016)

1125-C1-760      **Jeffrey W. Clark\*** ([clarkj@elon.edu](mailto:clarkj@elon.edu)). *Teaching Combinatorics to Diverse Student Interests.*

Students taking Combinatorics often come from a mix of pure and applied backgrounds. This talk will address strategies for developing pedagogies and assessments that engage such a heterogeneous student demand, synthesizing the stories approach of Benjamin and Quinn, the computer science foundation of Graham, Knuth, and Patashnik, as well as the use of calculus techniques with generating functions popularized by Wilf. (Received September 11, 2016)

1125-C1-910      **Aviva Halani\*** ([ahalani@exeter.edu](mailto:ahalani@exeter.edu)), 20 Main Street, Exeter, NH 03833. *The Evolution of Problem Posing Approaches for Counting Problems.*

For many counting problems, it can be beneficial for students to construct and answer related combinatorics problems by modifying one of the criteria involved in the problem. This presentation focuses on such *Problem Posing* approaches to counting problems and describes a model of how an epistemic student's approaches could evolve as she progresses through a sequence of counting tasks. Data for this study come from two teaching experiments. Open coding was used to identify the students' approaches to conceptualizing the set of elements being counted, called the *solution set*, as they engaged in counting problems. Four Problem Posing approaches emerged from the data analysis. Once these approaches were identified, perturbation experienced by students and the resulting accommodation of their thinking were analyzed. It was found that such perturbation and its resolution was often the result of an instructional intervention. (Received September 13, 2016)

1125-C1-1159      **Robert Talbert\*** ([talbertr@gvsu.edu](mailto:talbertr@gvsu.edu)), 1 Campus Drive, Mackinac Hall A-2-168, Grand Valley State University, Allendale, MI 49401. *Computational thinking in undergraduate discrete mathematics using Python and Jupyter notebooks.* Preliminary report.

Undergraduate discrete mathematics courses offer an excellent opportunity to link the world of abstract mathematics with the world of computation through the use of activities that focus on problem analysis, abstraction, algorithm development, and communication. When coupled with activities that empower learners to use computation to think in this way, this process is often called *computational thinking*. Originally envisioned by mathematician and educator Seymour Papert, the notion of computational thinking is a powerful paradigm for

learning in which the learner uses computation to understand and frame solutions to problems and to construct new knowledge.

A recent technology that promotes computational thinking in any learning environment is the *Jupyter notebook* platform (<http://jupyter.org/>), a free and open-source application for creating documents that combine mathematical notation using LaTeX, formatted text using Markdown, and executable code using the Python programming language. In this talk, we will demonstrate use cases of Jupyter notebooks in set theory, combinatorics, and graph theory and discuss general frameworks for using Python and Jupyter in undergraduate discrete mathematics. (Received September 15, 2016)

1125-C1-1449 **Sandy Irani** ([irani@ics.uci.edu](mailto:irani@ics.uci.edu)), Dept of ICS, Univ. of California, Irvine, CA 92697, **Frank Vahid\*** ([vahid@cs.ucr.edu](mailto:vahid@cs.ucr.edu)), Dept. of CSE, Univ. of California, Riverside, CA 92521, and **Alex Edgcomb** ([aedgcomb@cs.ucr.edu](mailto:aedgcomb@cs.ucr.edu)), Dept. of CSE, Univ. of California, Riverside, CA 92521. *New web-native animated interactive learning material for discrete math.*

An undergraduate course in discrete mathematics presents students with concepts that are often very different from the kind of math they have been exposed to in high school. Furthermore, in topics such as counting, there is no well-defined algorithm to solve problems. Instead students are given a set of mathematical tools and need to figure out by intuition and practice which tool to apply in a given situation. We have thus created new web-native interactive learning material for discrete mathematics, to replace existing textbooks. The material makes extensive use of interactive activities like animations and learning questions. Students are led through sample problems in which they have to reason about each concept to answer questions. Common misconceptions lead to incorrect solutions which are then explained. The animations help students understand visual and dynamic concepts such as bijections for counting or the execution of a finite state machine. At our university, the material has been used by several thousand students for the past three years. Instructors indicate positive results, and student ratings of the material is much higher than for the previous standard discrete math textbook. The material has been used at over 50 universities over the past two years. (Received September 16, 2016)

1125-C1-1659 **Gregory V. Bard\*** ([bardg@uwstout.edu](mailto:bardg@uwstout.edu)), Dept. of Math., Stat., and Comp. Sci., Jarvis Hall, Science Wing, The University of Wisconsin—Stout, Menomonie, WI 54751. *Four Problems from Computer Engineering to Enhance Student Enthusiasm in the Discrete Mathematics Classroom.*

Discrete mathematics is of central importance to both computer science and computer engineering. Some of the topics are easy to motivate. The applications of truth tables, binary/hexadecimal numbers, and graph theory to computing are easy to teach, and can enhance student enthusiasm—resulting in a more positive experience for both the student and the instructor. Similarly, modular arithmetic can be motivated by the RSA cipher of Rivest, Shamir, and Adleman.

In stark contrast to this, combinatorics is a major topic, yet most of the textbooks are very light on applications, focusing on problems that can only be described as bizarre. The speaker did his Bachelor's and first Master's Degree in Computer Engineering, and worked in industry prior to getting a second Master's and PhD in Applied Mathematics. The speaker will present four particular sample problems (for two minutes each) that are highly realistic and applied, that can bring a breath of fresh air, as well as a sense of purpose, to the combinatorics chapter. (Received September 18, 2016)

1125-C1-1688 **Joshua Hallam\*** ([hallamjw@wfu.edu](mailto:hallamjw@wfu.edu)). *Definitions and Asimov's Three Laws of Robotics.* Understanding the importance of definitions in mathematics is an important skill for undergraduates to develop. Last spring, during a discrete mathematics course, my students did a project intended to help them understand this importance. The project was based on Asimov's three laws of robotics:

- (1) A robot may not harm a human, or through inaction allow a human to come to harm.
- (2) A robot must obey orders given to it by humans unless it conflicts with the first law.
- (3) A robot must protect its own existence unless doing so conflicts with the first and second law.

The students created a short story in which one of the laws is “violated” because of a faulty definition. The students also had to discuss how their faulty definition could be fixed and whether this new definition led to unintended consequences. In my talk, I will discuss the outcomes of the project. (Received September 18, 2016)

1125-C1-1799 **Debra L. Hydorn\*** ([dhydorn@umw.edu](mailto:dhydorn@umw.edu)), 1301 College Avenue, Fredericksburg, VA 22401. *“Small Teaching” in Introduction to Discrete Mathematics.* Preliminary report.

In his book “Small Teaching: Everyday Lessons from the Science of Learning,” James M. Lang provides examples of assignments and activities instructors can use to help students build their knowledge and understanding of

course concepts as well as their inspiration for learning. Lang defines “small teaching” as brief activities, one-time interventions, or small modifications to a course that have their foundations in the learning sciences, have a positive impact on real-world learning environments, and that he has used in his own teaching. While I was pleased to see that I was already doing some of the “small teaching” Lang recommends, for the Fall 2016 semester I incorporated more of Lang’s ideas into my introduction to discrete mathematics course. Examples include providing students with more opportunities to retrieve learned information, interleaving concepts and activities to space out learning throughout the course, and making more connections with previously learned material. This presentation will provide summaries of the “small teaching” activities I used and student outcomes, as well as student responses to mid- and end-of-course surveys. (Received September 19, 2016)

1125-C1-1976     **Oscar Levin\*** ([oscar.levin@unco.edu](mailto:oscar.levin@unco.edu)), School of Mathematical Sciences, Campus Box 122, 501 20th Street, Greeley, CO 80639. *Tricks to make counting harder for students.*

One of the most challenging topics for students in discrete mathematics courses is counting. Perhaps even more challenging is figuring out how to teach the topic. One approach is to distill commonalities of problems into simple rules for students to follow. This attempt to make counting easier does not seem to help much. In this talk, we will consider the opposite approach. We will explore some ideas about how to make counting harder, and look at how this might benefit students. (Received September 19, 2016)

1125-C1-2192     **Yun Lu\*** ([lu@kutztown.edu](mailto:lu@kutztown.edu)), Mathematics Department, Kutztown University, Kutztown, PA 19530. *Projects for Graph Theory Course.*

In this talk, I will share my experience of using projects for graph theory course to improve students’ learning effectiveness. I will discuss some of my successful and unsuccessful approaches, as well as students’ feedback and performance if time allows. (Received September 19, 2016)

1125-C1-2363     **Jean Marie Linhart\*** ([jeanmarie.linhart@cwu.edu](mailto:jeanmarie.linhart@cwu.edu)). *Success with Standards-Based Grading in Discrete Mathematics.*

Examinations told me that students often were unable to solve combinatorics problems and solve recurrence relations in my discrete mathematics course. Many students were earning just enough partial credit to pass, often without really understanding these subject areas that are central reasons that they are required to take the course. To move students towards mastery learning and full proficiency, I implemented standards-based grading. Grades are based on the number learning objectives students master, and in return there are multiple opportunities to demonstrate mastery and to learn from mistakes. (Received September 20, 2016)

1125-C1-2505     **Ali S Shaqlaih\*** ([ali.shaqlaih@untDallas.edu](mailto:ali.shaqlaih@untDallas.edu)), 7400 University Hills Blvd, University of North Texas at Dallas, Department of Mathematics, Dallas, TX 75241. *Teaching Approaches in Discrete Mathematics for Pre-service Teachers.*

Discrete Mathematics is a fundamental course in teachers’ preparation programs for both Math 4-8 and Math 7-12 tracks. However, topics introduced in such course vary by institution. In this talk, I will present how this course was taught, in different semesters; by different approaches including the hybrid inquiry based learning approach. Course content and class activities along with the assessment methods in each approach will be discussed. A quantitative analysis of the students’ assessment and a qualitative analysis of Students’ perceptions will be presented. (Received September 20, 2016)

1125-C1-2606     **Hyman Bass\*** ([hybass@umich.edu](mailto:hybass@umich.edu)), 610 E. University, 4124 School of Education, Ann Arbor, MI 48109-1259. *Many Incarnations of Pascal.* Preliminary report.

I present examples of two novel ways that combinatorial ideas might productively enter the undergraduate, or even high school curriculum. The first is based on a more general idea, intended to simulate an aspect of theory-building practices in mathematics. To this end, students are presented with a variety of apparently quite different problems. They are asked to not only solve the problems, but to identify and articulate some mathematical structure that they can demonstrate is fundamentally involved in each of the problems. I provide a particular, combinatorially based problem set, which was produced for use in such a designed activity. The second, quite different design, is a treatment of discrete calculus based on the binomial polynomials, “ $x$ -choose- $d$ ”  $= \frac{x(x-1)\cdots(x-d+1)}{d!}$ . This provides a compact and coherent mathematical unit that is not only a discrete analogue of calculus, but it also incorporates some substantial doses of algebra, and has several interesting applications. I will present the developmental sequence for this unit (Received September 20, 2016)

1125-C1-2710 **Feryal Alayont\***, Grand Valley State University, Department of Mathematics, 1 Campus Dr., Allendale, MI 49401. *Guided Discovery Based Learning in Discrete Mathematics via Pre&In-class Activities.*

The topics in a discrete mathematics course directly tie to a wealth of real-life applications that can be used to motivate student interest in these topics. Asking students questions related to these applications in a pre-class activity before a topic is discussed in class provides a great hook for the topic. In the classroom, students then build on their initial work by pondering further related questions in the in-class activity. These questions are designed to guide them to discover the ideas of the topic through individual and/or group work. In this presentation, I will describe the teaching structure and provide examples of the activities from the course along with student feedback on the course. (Received September 20, 2016)

1125-C1-2804 **Elise Lockwood\***, 064 Kidder Hall, Department of Mathematics, Corvallis, OR 97331. *How to Help Your Students Prove Combinatorial Identities.*

Proving combinatorial identities is an important mathematical skill that requires nuance and can be a mysterious undertaking for students. To learn how students reason about combinatorial proof, I gave a pair of undergraduate students a series of tasks designed to help them prove combinatorial identities. In this talk, I share results from these interviews, including a) beneficial interventions that helped students prove combinatorial identities, and b) students' productive and unproductive ways of thinking about combinatorial proof. I draw upon a model of students' combinatorial thinking (Lockwood, 2013) in order to frame this discussion. I conclude with tips for how to effectively incorporate combinatorial proof in your classroom. (Received September 20, 2016)

1125-C1-2887 **Cayla D. McBee\*** (cmcbec@providence.edu). *A Candy Exchange, Legos and a Brand New Car!*

Incorporating active learning activities into lectures help hold student interest and engage students in the mathematical material, which in turn can lead to a deeper level of understanding. In this talk I will discuss some active learning activities that can be used when teaching discrete mathematics. Examples include exchanging candy to illustrate one-to-one and onto functions, using Legos to appreciate precise mathematical language and involving students in a game of Let's Make a Deal. (Received September 20, 2016)

1125-C1-3100 **Zackery Kevin Reed\*** (reedzac@oregonstate.edu). *The Password Activity: An Instructional Tool for the Combinatorics Classroom.*

Elementary combinatorics provides students with a unique experience in mathematics where problems can be accessible and yet difficult to solve. When engaging with the material, students have the opportunity to develop rich understandings of mathematical structures and engage in fundamental mathematical practices like generalizing and proving. In this presentation, I present results from interviews in which we gave students a series of scaffolded combinatorial tasks that culminate in a proof of the Binomial Theorem. Through these tasks, students had the opportunity to build a robust understanding of the multiplication principle and binomial coefficients and to hone their generalization and proving skills. I discuss the mathematics and the implementation of the activity, and I offer episodes of specific examples of student interaction with the activity that demonstrate its affordances. The aim is to present the activity as a powerful instructional tool to implement in a variety of classroom settings. (Received September 21, 2016)

1125-C1-3110 **Osvaldo D. Soto\*** (osoto@sandi.net), 9500 Gilman Drive, Mail Code 0036, La Jolla, CA 92093-0036. *Partnering for Success: Developing a high school discrete mathematics curriculum Connecting a university course with a local high school course using the standards for mathematical practice.* Preliminary report.

College readiness in mathematics and the subsequent need for remediation have become a concern nation-wide. With few choices available, most college bound students are asked to take pre-calculus, calculus or statistics their senior year in high school. However, discrete mathematics, offers a wide variety of contemporary contexts useful for college preparation if we think about them from the perspective of the standards for mathematical practice. In this session, I present early results of a project bringing pedagogically sensitive mathematicians and math educators together with local high school teachers to redesign the curriculum of a high school course whose goals are stated in terms of BOTH content and standards for mathematical practice with an eye on a sophomore level undergraduate discrete mathematics. (Received September 21, 2016)

1125-C1-3131 **John S Caughman\*** (caughman@pdx.edu), PO Box 751, Portland, OR 97207. *Avoiding minimal elements in the poset of ways to introduce posets.*

Partially ordered sets (posets) and their properties are a common feature of many undergraduate courses in Discrete Mathematics. Since they are easy to define using simple logical expressions and familiar mathematical

concepts, posets offer students opportunities to sharpen their proof skills and reinforce their knowledge of sets and relations. Motivating or contextualizing posets, however, can be challenging, and a rushed treatment can result in a misrepresentation of posets as little more than an academic exercise in abstract formalism.

In this talk, I will share ideas and activities that have remedied this issue in my own classroom. With accessible, open-ended tasks involving little more than systems of linear equations, students at many levels can engage and develop an intellectual need for posets through a number of surprising but natural entry points. Generating discourse and activity rich with conjecture and proof, these tasks also offer launch points for other combinatorial properties of posets, such as maximal chains, rank functions, and the notorious problem of counting linear extensions. (Received September 21, 2016)

## *Do Mathematicians Really Need Philosophy?*

1125-C5-248 **James R Henderson\*** (jrh66@psu.edu). *Otavio Bueno's Mathematical Fictionalism*. Mathematical Platonists claim that mathematical objects actually exist (though not spatio-temporally) and that, therefore, there is but one mathematical Truth corresponding to those objects and the relations that hold between them. Nominalists, on the other hand, claim mathematical objects do not exist and, therefore, almost all of mathematics is false. (This includes any proposition involving an existence claim; universal propositions are taken to be vacuously true.) Not surprisingly, both Platonism and Nominalism have their own particular strengths and weaknesses. The question for the practicing mathematician or philosopher of mathematics is this: Does one have to "pick a side"? Fictionalism, at least as advocated by Otavio Bueno, takes its lead from Bas van Fraassen's Constructive Empiricism, a version of scientific anti-realism. Bueno's Fictionalism is a middle position where the sticky problem of the existence of mathematical entities is simply not addressed (or not completely, at any rate), and it gives a non-standard version of mathematical truth. Whether this brand of Fictionalism can provide a sound basis for mathematics as practiced by professional mathematicians will be discussed. (Received August 18, 2016)

1125-C5-507 **Bonnie Gold\*** (bgold@monmouth.edu), Long Branch, NJ 07740. *Melding realism and social constructivism*. Preliminary report.

My own philosophical viewpoint has always been something of a blend of realism (platonism) and social constructivism: realism about mathematical objects, and social constructivism (Reuben Hersh's version, not Paul Ernest's) about our knowledge of those objects. More recently, while reading José Ferreirós's *Mathematical Knowledge and the Interplay of Practices*, I have been working on how to integrate his approach, which seems to me a more sophisticated version of social constructivism, with my viewpoint. I will discuss this version of pluralism, and briefly comment on the main topic, whether mathematicians need philosophy. (Received September 04, 2016)

1125-C5-1670 **Katalin Bimbó\*** (bimbo@ualberta.ca), 2-40 Assiniboia Hall, Department of Philosophy, University of Alberta, Edmonton, Alberta T6G2E7, Canada. *The unexpected usefulness of epistemological skepticism*.

David Hilbert believed that mathematical problems have definite answers. Some philosophers of mathematics concentrate on metaphysical questions such as "Do numbers (or sets, triangles, etc.) exist?" However, epistemological problems are probably more important for mathematical practice than taking a stance in an ontological debate. I will illustrate that moderate skepticism can help us to produce a definite answer to a precisely formulated mathematical problem. The example comes from theoretical computer science, which I take here to be a (relatively) new branch of mathematics. Objects in theoretical computer science are often more structured and complicated than an equilateral triangle, but at the same time, they are more abstract than an app or an OS. Occasionally, our intuitions come up short in reasoning about these kinds of objects. I will conclude that a certain skepticism together with insistence on more formal definitions and proofs can be fruitful. (Received September 18, 2016)

1125-C5-2521 **Thomas Drucker\*** (druckert@uwv.edu), Department of Mathematics, University of Wisconsin-Whitewater, 800 West Main Street, Whitewater, WI 53190. *Why Can't Those With Conflicting Views on the Foundations of Mathematics Just Get Along?* Preliminary report.

There has been ongoing strife over the issue of whether set theory or category theory is the appropriate foundation for mathematics. Claims have been made as to the relative merits of one or the other with regard to certain branches of mathematics. For many mathematicians the issue of foundations is irrelevant, but that has not

stopped the arguments. Can mathematicians do some, most, or even all mathematics without worrying about the choice of foundation? One can do arithmetic in different bases, but somehow different logics affect the content of mathematics rather more profoundly. The content of this talk will continue earlier investigations along the lines of Henle's 'The Happy Formalist'. (Received September 20, 2016)

## Humanistic Mathematics

1125-D1-86 **Carol E. Gibbons\*** ([gibbonsc@salve.edu](mailto:gibbonsc@salve.edu)), Salve Regina University, 100 Ochre Point Ave, Newport, RI 02840, and **Jayne Hennessy** ([hennessj@salve.edu](mailto:hennessj@salve.edu)), Salve Regina University, 100 Ochre Point Ave, Newport, RI 02840. *The Geometry and Spirituality of Islamic Tiling*. Preliminary report.

We will discuss our current interest in the geometry of Islamic tiling patterns. The tiling patterns are useful models when studying symmetry, infinity, and groups. This ancient and beautiful art, while based on simple geometric shapes, produces patterns that are amazingly complex. Although the works are centuries old, some have properties of recently developed mathematics such as fractals and aperiodicity. The link between the geometry and the spiritual effect of the tiling is a major focus. (Received July 16, 2016)

1125-D1-461 **Mike Pinter\*** ([mike.pinter@belmont.edu](mailto:mike.pinter@belmont.edu)), Department of Mathematics and Computer Sci, Nashville, TN 37212. *Mathematics Intersecting with Other Modern World Ideas: 1850-1950*. Preliminary report.

For this presentation, I will describe ideas, people and topics from mathematics in the mid 19th century to the mid 20th century that I incorporate into a general education mathematics course that students typically take during their first two years. One version of the course is offered in the Honors Program. The courses do not have any particular prerequisite; thus, the courses do not directly rely on ideas from calculus or algebra. Topics addressed from that 100-year time period include symbolic logic, basics of set theory, and social choice theory, with particular emphasis on logic. Logicmix is a very helpful resource for framing a large discussion that draws in the arts, some social sciences, philosophy, and the beginnings of computer science. In addition to names from mathematics such as Bertrand Russell, Kurt Godel and Alan Turing, students also encounter Kenneth Arrow and Thomas Kuhn. I will include some associated course assignments and student feedback. (Received September 02, 2016)

1125-D1-1088 **William Wood\***, [bill.wood@uni.edu](mailto:bill.wood@uni.edu). *Very Special Functions: Perspectives of Generalized Trigonometry*.

The classical trigonometric functions can be generalized to parameterize circles in other geometries. One example is the "squigonometric" functions for the planar  $p$ -norm, an idea that has been rediscovered and repackaged in a number of ways. We will look at some of the different approaches mathematicians have taken to these functions and offer them as an illustration of mathematical discovery that starts with basic calculus. (Received September 14, 2016)

1125-D1-1172 **Sam Cowling** and **May Mei\*** ([meim@denison.edu](mailto:meim@denison.edu)). *Big, Small, and Nowhere at All: The Nature of Numbers - A Denison Seminar*.

Mathematics and philosophy intersect historically, conceptually, and methodologically. Among other things, the notions of proof, argument, and deductive inference at the heart of mathematics are the same that drive much of contemporary analytic philosophy. At the same time, there are results in mathematics that illuminate perennial philosophical questions and even rule out historically influential answers to these questions. In this talk, we outline and assess the aims of an interdisciplinary second- and third-year seminar on mathematics and philosophy and describe some strategies for integrating mathematical results within philosophical debates. Some challenges for contextualizing the humanistic value of philosophy within a mathematics-driven course will be discussed. Ideas for topics of discussion, historical anecdotes, and in-class activities are actively solicited. (Received September 15, 2016)

1125-D1-1320 **Julie C. Beier\*** ([beierju@earlham.edu](mailto:beierju@earlham.edu)), Earlham College, 801 National Rd W, #138, Richmond, IN 47374. *Using Math to Improve Cultural Understanding*.

Culture is often used to motivate student learning in mathematics, but here I flip the paradigm and ask: How can mathematics be used to learn about culture? We will explore this idea through a case study implemented during a study abroad experience that is focused on Tibet and its culture. We will discuss not only what was done in this setting, but why do it, how to think about these ideas more generally and thoughts about teaching such courses when not abroad. (Received September 16, 2016)

1125-D1-1351 **Albert W Schueller\*** ([schuelaw@whitman.edu](mailto:schuelaw@whitman.edu)), Dept of Math & CS, 345 Boyer Ave, Walla Walla, WA 99362. *Mathematical Computer Doodles.*

Browser based graphical programming libraries like Processing, Geogebra and even just HTML5 make it relatively easy for a mathematician with a small amount of programming skill to “sketch” and share beautiful, thought-provoking, and maybe even educational visual art. We will review the many examples of this kind of computer doodling that are out on the web—including some of my own. We will also review some resources available to help you do your own computer doodling. (Received September 16, 2016)

1125-D1-1456 **Christine von Renesse\*** ([cvonrenesse@westfield.ma.edu](mailto:cvonrenesse@westfield.ma.edu)) and **Jennifer DiGrazia.** *Mathematics, Writing and Rhetoric: Deep Thinking in First-Year Learning Communities.*

In this talk, we will present the idea of combining a composition and a mathematics for liberal arts course in a fully integrated learning community. Professors Christine von Renesse (Mathematics) and Jennifer DiGrazia (English) have been teaching four iterations of such a learning community, each with a different focus based on a campus-wide first year read. The two seemingly disconnected courses work toward the same meta-goals and help the students in becoming confident and critical thinkers and writers. To this effort, both instructors use inquiry-based techniques from their subject areas, as well as the mathematical curriculum from the “Discovering the Art of Mathematics” project ([www.artofmathematics.org](http://www.artofmathematics.org)). We will present student work and survey results measuring students’ beliefs and attitudes to show the deep impact such a course can have on students (and instructors). (Received September 16, 2016)

1125-D1-1483 **Diana B. McGinnis\*** ([dmcginnis@gsu.edu](mailto:dmcginnis@gsu.edu)). *Facilitating Student Self-Direction in Learning Mathematics.*

This paper will address some of the ways in which humanistic approaches can inform pedagogy and lead students toward becoming more self-directed learners. Although I have just begun the research process into one of these approaches, I have learned that looking at my profession through a humanistic lens has improved my own sense of fulfillment as well as convinced me that my students are capable of much more than I anticipated. The process whereby students acquire those skills and modes of reasoning that increase their understanding, confidence, and long-term learning is a journey with many turns and struggles, but one which they ultimately take alone. How can humanistic attitudes and methodologies play a role in improving the student journey? How does the teacher encourage self-direction and sense of ownership? How do we structure our classes so that students become more reflective learners? These are some of the challenges that have motivated me to implement change and to investigate the results. (Received September 17, 2016)

1125-D1-1677 **Aaron Trocki\*** ([atrocki@elon.edu](mailto:atrocki@elon.edu)), Duke 209E, 2320 Campus Box, Elon University, Elon, NC 27244. *Using History as a Vehicle for Humanizing Mathematics.*

Completing a first course in Calculus is typically accomplished in the first two years of a liberal arts education. A challenge facing educators is that of humanizing topics of Calculus, a content heavy subject with little room for supplemental activities. This presentation delineates one project implementation that requires Calculus learners to incorporate other pillars of a liberal arts education such as history, writing, and communication while simultaneously meeting course content goals.

The project was implemented in two Calculus I courses in a liberal arts university. Giving a Historical Context to Calculus had the purpose of providing students an opportunity to familiarize themselves with the origins, archetypal problems, and historical characters that led to our current conception of Calculus. The project took the form of short papers followed by structured presentations. Project details and implementation are shared along with student work samples and reactions to the experience. For example, students investigated the human struggle with conceptualizing limits and codifying methods of Calculus. Instances of how the assignment drew out connections among Calculus content and the humanities are highlighted. (Received September 18, 2016)

1125-D1-2394 **John Kellermeier\*** ([kellermeierjh@gmail.com](mailto:kellermeierjh@gmail.com)), 9850 Tanglewood Dr, Vancouver, WA 98664. *From Menstruation to Triathlons: Ethnomathematics for the College Classroom.*

The field of ethnomathematics views quantitative reasoning as arising from the human struggle to survive and thrive.

This presentation will showcase ethnomathematical material appropriate for a liberal arts quantitative reasoning course such as those commonly required as part of college general education programs for non-math and science majors. The topics covered will include “How Menstruation Created Mathematics,” “Divination and Synchronicity” and “Triathlon Ethnomathematics.” Emphasis will be placed on exposition of these topics for this audience and how they can be presented to enhance students’ understanding of the role of mathematical thinking in all human activities. (Received September 20, 2016)

1125-D1-2581 **Debra K. Borkovitz\*** (dborkovitz@wheelock.edu). *Math Teaching Stories from the Kingdom of the Sick*.

Susan Sontag famously wrote, “Everyone who is born holds dual citizenship, in the kingdom of the well and in the kingdom of the sick. Although we all prefer to use the good passport, sooner or later each of us is obliged, at least for a spell, to identify ourselves as citizens of that other place.” In the past fifteen years, chronic illness and medical emergencies have obliged me to frequently use the other passport. These “travels” have profoundly affected my teaching, quite often for the better. In this talk I’ll share some stories and some thoughts about communication, compassion, building community, and promoting responsibility in the mathematics classroom. I’ll also touch on some topics in quantitative reasoning and liberal arts mathematics. (Received September 20, 2016)

1125-D1-2839 **Kira Hylton Hamman\*** (kira@psu.edu). *Mathematics beyond Mathematics: Uses and Abuses*. Preliminary report.

Mathematicians get a bit bristly when non-mathematicians use mathematics for non-mathematical purposes. Uses that are obviously figurative are usually okay: love > hate, silence = death, that kind of thing. And the fine arts, for some reason, get a pass. You rarely hear mathematicians complain about, say, Escher. But when the non-mathematician attempts to use an abstract mathematical concept (applied math is a different story, of course) outside of its mathematical context in a serious academic way, all hell breaks loose. In fairness, some of this work is at best problematic, and mathematicians are right to call it out. But sometimes – too often, I will argue – work that has merit is dismissed by mathematicians in their enthusiasm to point out its mathematical flaws. This talk will focus on the work of Piaget and Lacan, in a preliminary attempt to characterize work that advances knowledge and to sort it from work that uses mathematics for intellectual posturing, or worse. (Received September 20, 2016)

1125-D1-2908 **Maura Chhun** (maura.chhun@metrostate.edu) and **Matthew Moynihan\*** (mmoynihan@wooster.edu). *Team Teaching the Math and History of Global Pandemics*. Preliminary report.

Incorporating equal measures of mathematics and history, we describe a course that engages students with varying levels of experience in each subject. Through the study of the Black Death, cholera, the 1918 flu, and HIV/AIDS, students are introduced to mathematical modeling. Class discussions will focus on race, gender, and class as well as the historical outcomes of the pandemics. These class discussions will in turn lead to student-designed compartmental models in epidemiology. While the models use differential equations, we focus on quantitative exploration with historical grounding as opposed to mathematical rigor. This course will provide students with an understanding of how diseases are spread and the ways in which governments can respond effectively to public health crises. (Received September 20, 2016)

1125-D1-3071 **Cindy Traub\*** (ct@wustl.edu). *Playing Nice in the Math Sandbox: Mathematics in Support of Digital Humanities*.

How can a mathematician contribute to the understanding of the developing federal government in the late 1700s? Can network analysis contribute to a more cohesive computation of genre classification? How can one compare the rarity of words in early modern English printed text? Pedagogically, what is our duty as mathematicians in collaboration with humanists, in this age of open-source software and rapidly expanding black-box solutions that leave us all a few keystrokes away from performing complex computations? I will examine these questions in light of my experience as a mathematician turned embedded librarian in Washington University’s Humanities Digital Workshop Summer program. I will give examples of successful collaborations with scholars in multiple humanities disciplines, as well as useful technological tools. These experiences may be of interest to faculty or students interested in building collaborations across disciplinary boundaries, or those seeking to design just-in-time workshop-based instruction to support a diverse set of learners. (Received September 20, 2016)

1125-D1-3104 **David L Neel\*** (neeld@seattleu.edu), PO Box 222000, 901 12th Ave, Seattle, WA 98122. *Mathematics beyond Mathematics: Uses and Abuses (Part II)*. Preliminary report.

Mathematicians can become surly when non-mathematicians use mathematics for non-mathematical purposes. Uses that are obviously figurative are usually okay, as are art/geometry connections. But when the non-mathematician attempts to use an abstract mathematical concept outside of its mathematical context in a serious academic way? Mayhem and consternation from the mathematicians. True, sometimes the use is abuse. But sometimes work that has merit (perhaps measured using unfamiliar metrics) might be unduly dismissed by mathematicians in their enthusiasm to point out its mathematical flaws. This talk will focus on the work of

Krauss and Badiou, in a preliminary attempt to characterize work that advances knowledge and to sort it from work that uses mathematics for intellectual posturing, or worse. (Received September 21, 2016)

1125-D1-3124 **James M Henle\*** ([jhenle@smith.edu](mailto:jhenle@smith.edu)). *The Mathematical Art: A course for beginning artists*. Preliminary report.

By “the mathematical art” I do not mean mathematics in the service of art, or mathematics inspiring or enabling art. I do not mean, for example, the sort of art exhibited at this conference. I mean mathematical structures, ideas, as the works of art.

This is a course I have taught at Smith College for students with no special background in math. I introduce students to some simple genres of the art (games, dances, numeration systems, puzzles, etc.) and they create their own works. The students serve alternately as artists and as critics. We discuss aesthetic principles of the art.

The course concludes with a curated campus exhibit of the students’ mathematical artworks. (Received September 21, 2016)

## *Humor and Mathematics*

1125-D5-1336 **Elizabeth A Peitz\*** ([elizabeth\\_peitz@knights.ucf.edu](mailto:elizabeth_peitz@knights.ucf.edu)), 12613 Victoria Place Circle, Apt. 3308, Orlando, FL 32828. *The pun of introducing students to Calculus: Why was the parent function upset with its child?*

As said by Michael Fox “I think the scariest person in the world is the person with no sense of humor.” Second to this, many people may believe that the scariest person in the world would have been their Algebra teacher. The majority of individuals have a very serious approach to mathematics, especially when we are talking about those who do not plan to use it in their futures. I believe that the best way to get students who would not normally be interested in mathematics involved is to make them laugh. In this presentation I plan to give an example of a mathematical comedy skit that I use to help my students understand and enjoy Calculus I. The comedy skit hits the main aspects of the lesson to grab the attention of the students and then I use the opportunity while I have their attention to better explain the topic in depth. I also plan to display the relationship between comprehension of the material after a comedic lesson versus comprehension after a standard lecturing lesson. I believe that this approach to Calculus I helps show students how much fun advanced math can be and hope that it ultimately leads to more students choosing to major in mathematics. (Received September 16, 2016)

1125-D5-1805 **Andrea Young\*** ([younga@ripon.edu](mailto:younga@ripon.edu)). *Improvisation in a Senior Capstone Course*.

Improvisational comedy has long been within the purview of intimate theaters or TV shows like “Whose Line is it Anyway?” However, I have found that exercises and activities from improv comedy can be useful in teaching mathematics, as they help students become less fearful of doing mathematics and more trusting of each other. Improv games can also help students develop communication skills. In this talk, I will discuss and demonstrate the ways I used improv in a senior capstone course to strengthen group dynamics and to improve students’ ability to think on their feet. (Received September 19, 2016)

1125-D5-2014 **Janine E. Janoski\*** ([janinejanoski@kings.edu](mailto:janinejanoski@kings.edu)). *Brace Yourself, Calculus Memes are Coming*. Preliminary report.

Webster’s defines a meme as “an amusing or interesting picture, video, etc., that is spread widely through the Internet.” Modern day memes tend to be captioned photos that are intended to be funny and often ridicule human behavior. In this talk, we will discuss using calculus memes as a way to engage students, learn concepts, and review material. Examples of memes created by calculus students will be given as well as student feedback on effectiveness of memes as a teaching aid. (Received September 19, 2016)

1125-D5-2027 **Keith Weber\*** ([keith.weber@gse.rutgers.edu](mailto:keith.weber@gse.rutgers.edu)), Graduate School of Education, 10 Seminary Place, Rutgers University, New Brunswick, NJ 08901. *Using recent advanced in humor theory to understand how we do mathematics and why we enjoy it*.

I use Hurley, Dennett, and Adams’ theory of humor to analyze jokes about mathematics and jokes regarding mathematical work. I argue that doing so can provide insights into how we think when we do mathematics. In particular, Hurley and colleagues’ theory show how jokes about mathematical errors can illustrate the fallible intuitive inferences that are common in our reasoning and jokes about semantic ambiguity can illuminate why we enjoy visual proofs. Perhaps most enjoyably, this talk will provide a collection of amusing mathematical jokes. (Received September 19, 2016)

1125-D5-2534 **Thomas Drucker\*** ([druckert@uwv.edu](mailto:druckert@uwv.edu)), Department of Mathematics, University of Wisconsin–Whitewater, 800 West Main Street, Whitewater, WI 53190. *A Funny Thing Happened on the Way to Foundations*. Preliminary report.

Bertrand Russell was one of the leading advocates of the need for a satisfactory foundation for mathematics in the early part of the twentieth century. He was also an entertaining writer and was careful to work humour into his presentations of logic and foundations for a general audience. Those who were inclined to take issue with some of Russell's efforts, e.g., Henri Poincare, sometimes resorted to satire in response. Perhaps the most extensive example of this was Philip E.B. Jourdain's 'The Philosophy of Mr. B\*rti\*nd R\*ss\*ll', which appeared in 1918. This talk looks at the way in which foundations has lent itself especially to humour among branches of research in mathematics. (Received September 20, 2016)

1125-D5-2844 **James P. Howard, II\***, [jh@jameshoward.us](mailto:jh@jameshoward.us), [jphoward@faculty.umuc.edu](mailto:jphoward@faculty.umuc.edu). *And Behind Door #3...*

In 2016, the web comic Saturday Morning Breakfast Cereal by Zach Weinersmith presented a problem as a comic, naming this "The Monty Hall Trolley Problem:"

Imagine you're in an out-of-control trolley. You're headed toward three buildings, and you control which you slam into. Two buildings contain five people one building contains online one person. You randomly select a building to slam into. Then, one of the other buildings is revealed to contain five people. Should you switch tracks?

Combing the elements of the ethical staple, "The Trolley Problem," with the historical statistical trauma of "The Monty Hall Problem," the Monty Hall Trolley Problem presents a relatively simple analytical challenge. However, the applications of this problem and its solutions transcend comic-inspired classroom problems. The fundamental question will be faced by autonomous machines, including the rising class of self-driving vehicles.

This presentation will investigate the problem presented by Weinersmith, generalize it, and show the connections to modern systems design. (Received September 20, 2016)

1125-D5-3014 **John J Weber\*** ([jweber13@gsu.edu](mailto:jweber13@gsu.edu)), 555 North Indian Creek Drive, Clarkston, GA 30021, and **Larry Lesser** ([lesser@utep.edu](mailto:lesser@utep.edu)) and **Dennis K Pearl** ([dkp13@psu.edu](mailto:dkp13@psu.edu)). *Using Interactive Songs to Engage Students in Learning Introductory Statistics: Overview of NSF-Funded Project*.

We discuss the benefits of the educational uses of song (e.g., Lesser, An, & Tillman, 2016) combined with a MadLibs-style activity (Trumpower, 2010) to engage statistics students. Our NSF-funded Project SMILES (Student-Made Interactive Learning with Educational Songs for introductory statistics) has created 21 songs containing lyrics focusing on vetted content objectives (e.g., from GAISE). Students complete MadLibs-style prompts to supply examples/context or conceptual information and receive immediate feedback on their entries. The results are high quality songs, with lyrics partially generated by students. Students can play the songs containing their inserted phrases and sing along with the displayed lyrics. A randomized experiment will assess how well this web-based, machine-run, auto-graded resource enhances learning and reduces anxiety. (Received September 20, 2016)

1125-D5-3052 **Cesar Martínez-Garza\*** ([cxm58@psu.edu](mailto:cxm58@psu.edu)), Penn State Berks, Tulpehocken Road, P.O. Box 7009, Reading, PA 19610. *Through the Looking Glass: a collage of images that adds character to mathematical concepts in Calculus courses*.

This presentation will expose images that the author has concocted (generally on the spot) to exemplify or add some color and dimension to mathematical concepts while teaching undergraduate Calculus courses. A little bit of humor can help students focus on concepts that they might find intimidating upon initial exposure. As such, a chimpanzee with a one-button calculator can easily replace a summation operator, and the process of using the limit definition to find a derivative can be analogous to the "Hunger Games", provided District 13 does not exist. Then, there is the Blue Hell of the people who divide by zero... That one is not funny, but it is as intimidating as it sounds. (Received September 20, 2016)

## *Incorporating Big Data Ideas in the Mathematics and Statistics Classroom*

1125-E1-867 **Robin L Angotti\*** ([riderr@uw.edu](mailto:riderr@uw.edu)), School of STEM, Campus Box 358538, 18115 Bothell Way NE, Bothell, WA 98011. *Using Big Data in the Sciences: Integrating Mathematics and Plant Ecology.*

In a collaborative partnership, faculty from mathematics and biology co-taught a course for undeclared freshmen students using a big data set of 35 years of plant succession data on Mt. St. Helens in which data was analyzed and visualized within the scientific discipline of plant ecology. Mathematical and scientific literacy were developed in conjunction with each other as they arose in context of analyzing large-scale biological patterns in the data.

By working with the data set, freshmen students learned key mathematical, statistical, and biological concepts which were integrated into hands-on tasks assigned in the course while utilizing cutting edge software tools, specifically Tableau software. Instruction started from the application of trying to visualize the data and worked at a level appropriate for student analysis. First year students were exposed to authentic research data and were asked to investigate and explain the data in terms that anyone could understand. Students and faculty created online interactive visualizations to allow access to the information found in the data. This session will also describe insights on cross-disciplinary collaborative teaching to provide students access to rich, data applications of scientific research. (Received September 12, 2016)

1125-E1-2757 **Patti Frazer Lock\*** ([plock@stlawu.edu](mailto:plock@stlawu.edu)), Dept of Math, CS, and Stats, St. Lawrence University, Canton, NY 13617, and **Robin H. Lock, Kari Lock Morgan, Eric F. Lock and Dennis F. Lock.** *Big Data Visualization in Intro Stats (in 15 minutes!).*

It would be wonderful if we could help our Intro Stats students become aware of some of the new and creative and amazing ways to tell stories with data visualization. But how can we find the time in an already packed Intro Stats course to go beyond histograms, boxplots, and scatterplots? This talk will discuss how to, in just 15 minutes of class time, help students get a sense of what is possible in data visualization, and hopefully to pique their interest to explore further. (Received September 20, 2016)

## *Innovative and Effective Ways to Teach Linear Algebra*

1125-E5-247 **Nicholas J. Higham\*** ([nick.higham@manchester.ac.uk](mailto:nick.higham@manchester.ac.uk)), School of Mathematics, The University of Manchester, Manchester, M13 9PL, United Kingdom. *Exploiting Recent Developments in MATLAB.*

The last few releases of MATLAB have contained some significant changes to the core linear algebra functionality and have also included new features that can be used to illustrate linear algebra in applications. I will describe some of these features and their impact on linear algebra courses that make use of MATLAB. (Received August 18, 2016)

1125-E5-729 **Emily J Evans\*** ([ejevans@mathematics.byu.edu](mailto:ejevans@mathematics.byu.edu)), 275 TMCB, Brigham Young University, Provo, UT 84602. *Undergraduate Spectral Theory with Computer Labs.*

In this talk I will discuss our success in teaching advanced spectral theory to junior students as part of Brigham Young University's applied and computational mathematics emphasis. In particular I will address how we introduce, motivate, and teach topics typically not seen until graduate school including the Perron-Frobenius theorem, the spectral mapping theorem, Krylov subspace methods and the pseudospectrum. Special care will be taken to discuss the computer labs that accompany the material taught. These labs provide motivation, practice, and interesting problems for the students to solve. (Received September 10, 2016)

1125-E5-1098 **Martha Lee H Kilpack\*** ([mlhkilpack@mathematics.byu.edu](mailto:mlhkilpack@mathematics.byu.edu)). *Helping non-math majors see the power in linear algebra theory through proofs.*

At the beginning of a linear algebra course I envision my students ending the semester with an understanding and respect for the powerful theory of linear algebra. How do we help students from a broad spectrum of fields understand this theory? Could proofs be part of the answer? In this talk, I will discuss an approach to helping students dive into the proofs and thus learn the theory needed to complete them. (Received September 14, 2016)

1125-E5-1396 **Spencer Payton\*** ([spayton@math.wsu.edu](mailto:spayton@math.wsu.edu)). *Student Mathematical Connections in an Introductory Linear Algebra Course Employing Both Inquiry-Oriented Teaching and Traditional Lecture.*

Inquiry-oriented teaching is becoming increasingly popular in linear algebra classes, but certain institutional considerations may provide obstacles to incorporating inquiry in an undergraduate mathematics course. These obstacles may include large class sizes, class time constraints, amount of material to cover, and a requirement to coordinate with other sections of the class. This session will describe an action research study that was conducted in an attempt to explore how inquiry-oriented teaching could be implemented in an introductory linear algebra that faced the aforementioned challenges. Throughout the study, an approach to inquiry-oriented teaching was developed that involved teaching through both traditional lecture and inquiry-oriented teaching. Additionally, this study involved an investigation into what mathematical connections students appear to form within this classroom context. The data suggests that focusing inquiry-oriented teaching on particular concepts such as span and linear independence can be useful in creating opportunities for students to develop certain mathematical connections. (Received September 16, 2016)

1125-E5-2338 **Michael E. Gage\*** ([gage@math.rochester.edu](mailto:gage@math.rochester.edu)), Department of Mathematics, Hylan Building, University of Rochester, Rochester, NY 14627. *WeBWorK, linear algebra and the simplex method.* Preliminary report.

This talk will showcase the WeBWorK "scaffold" question type which enables the construction of worksheet like on-line homework, delivered in segments. Later segments are not available to students until the first segments have been completed correctly.

The principal example will be a worksheet from operations research that guides students to a better understanding of the connection between the Gauss elimination procedure from linear algebra and the simplex method.

Students often memorize the simplex procedure without coming to grips with its connection to Gauss elimination and to geometric principals from calculus. Two dimensional examples help make the geometric connection but are sufficiently obvious visually that students ignore the algebra. Using the sectioned worksheet to force careful step-by-step analysis and explanation of two and then three dimensional examples of the simplex method leads students to a deeper understanding both of the simplex method and of procedures from their matrix algebra course.

Automatic grading allows examples like this to be used in large classes or assigned as optional independent study. The interoperability of WeBWorK with Geogebra and Sage also plays a role in constructing these worksheet problems. (Received September 20, 2016)

1125-E5-2345 **Steven Schlicker\*** ([schlicks@gvsu.edu](mailto:schlicks@gvsu.edu)), Department of Mathematics, Grand Valley State University, 1 Campus Drive, Allendale, MI 49401, and **Feryal Alayont.** *Active Learning in Linear Algebra.*

Linear algebra provides significant learning obstacles for many students. Students need to master algorithms, understand a beautiful yet complicated web of mathematical concepts, and often also wrestle with writing mathematical proofs for the first time. We will describe how we use a collection of preview and in-class activities throughout the semester to help improve student success in these course goals. With each preview activity, students are exposed to the day's material through concrete examples of newly defined concepts or algorithms, or through questions asking them to reflect on previous concepts. In-class activities are built on students' preview work and strengthen their conceptual understanding and connections between the different topics. The text we are writing, which contains all of the preview and in-class activities, will be available as a free open-source text. (Received September 20, 2016)

1125-E5-2561 **Gilbert Strang\*** ([gilstrang@gmail.com](mailto:gilstrang@gmail.com)), MIT, 2-245, Cambridge, MA 02139. *The Rank of a Circle of 1's in a Matrix.*

The rank and the singular values tell us essential information about compressing a matrix. For a 1-0 matrix with a disk or a triangle full of 1's, we estimate those numbers in joint work with Alex Townsend. (Received September 20, 2016)

1125-E5-2635 **Marie A. Snipes\*** ([snipesm@kenyon.edu](mailto:snipesm@kenyon.edu)) and **Amanda Harsy Ramsay** ([harsyram@lewisu.edu](mailto:harsyram@lewisu.edu)). *Inspiring Linear Algebra with Problems in Image Analysis.* Preliminary report.

There are many rich and beautiful connections between linear algebra and problems in image processing. Two examples are radiography/tomography, the process of recording radiographic images and then recreating a 3d

picture from the images, and image manipulation using the heat diffusion operator. In this talk we describe an approach that brings these two applications to the foreground of a first-semester linear algebra course. In particular, we use Matlab/Octave to introduce the applications to students early in the course, and use the applications to inspire the development of central concepts including span, linear transformations, eigenvectors and eigenvalues, and the invertible matrix theorem. This is joint work with Tom Asaki, Chris Camfield, and Heather Moon. (Received September 20, 2016)

1125-E5-2770 **Risto Atanasov\*** ([ratanasov@email.wcu.edu](mailto:ratanasov@email.wcu.edu)). *WeBWorK, Reading Quizzes, and Proof Portfolio in Linear Algebra Course*. Preliminary report.

In the Fall semester of 2015 I used a modification of the flipped classroom learning technique in my Linear Algebra course. The students had to complete a reading assignment before each class. In these assignments students read selected topics from the textbook, completed WeBWorK assignments focusing on the computational part of the readings, took short in-class quizzes assessing the understanding of the concepts covered on these assignments, and worked on more complex suggested problems that were discussed in class. To develop and demonstrate students' mathematical writing abilities, students completed a proof portfolio. Each week I assigned two to four proof-oriented problems from the textbook as portfolio problems. Since writing mathematics requires practice and feedback, the students submitted each portfolio proof for one critique, made corrections, and resubmitted the proofs again. At the end of this presentation I will talk about the effectiveness of the teaching approach described above compared to more traditional approaches that I used when I taught the class in previous years. (Received September 20, 2016)

1125-E5-2977 **Timothy O Trujillo** and **Kelley Tatangelo\*** ([kbtatangelo@mines.edu](mailto:kbtatangelo@mines.edu)), 1500 Illinois St., Golden, CO 80401. *Implementing a partially flipped team-based approach to linear algebra*.

We discuss a coordinated effort over the past three years among instructors at the Colorado School of Mines to implement a partially flipped linear algebra classroom. The model followed by the instructors, described in [1], was developed by three professors from two universities.

Students are asked to watch short 10-15 minute videos before each class which prepare them for daily activities in the classroom. In turn, daily group activities are used to introduce students to the concepts and ideas covered in the lecture at the end of each class. Students are assessed in a variety of ways including: online video response questions through Google Forms, daily group activities, mathematical reasoning days, weekly worksheets and exams. We will discuss the technology used to implement the courses and the impressions of the students and instructors using the model.

#### REFERENCES

- [1] DEBRA CARNEY, NICHOLAS ORMES & REBECCA SWANSON, *Partially Flipped Linear Algebra: A Team-Based Approach*, *PRIMUS*, vol. 25 (2015), no. 8, pp. 641–654.

(Received September 20, 2016)

1125-E5-3020 **Gulden Karakok\*** ([gulden.karakok@unco.edu](mailto:gulden.karakok@unco.edu)) and **Emilie Hancock** ([emilie.hancock@unco.edu](mailto:emilie.hancock@unco.edu)). *Implementation of Various Teaching Practices to Address an Identity Crisis in Elementary Linear Algebra*.

Teaching practices are chosen and implemented in service of accomplishing learning objectives. At our university, one semester Elementary Linear Algebra course serves many different populations, including math, physics, computer science, applied statistics, and pre-service secondary education majors. As a result, learning objectives of this course regularly face tensions between finding balance between applications and formal proofs. Thus, the course feels like two courses being forced into just one and faces an identity crisis. To overcome this issue, we chose to implement various teaching practices and pedagogical tools. In this presentation, we share our implementation of these practices and tools, as well as the benefits of such choices. For example, we implemented Inquiry Based Learning (IBL) inspired activities to encourage formal connections between multiple representations and a partially-flipped classroom design to aid in strategically balancing content delivery with inquiry. The flipped classroom model also inspired a final project in which students selected different topics related to their majors and, in addition to written reports, created videos for other students to watch out of class. Videos were followed by a question-and-answer session during face-to-face time. (Received September 20, 2016)

1125-E5-3037 **Sepideh Stewart\*** ([sstewart@math.ou.edu](mailto:ssstewart@math.ou.edu)), 601 Elm Ave, Norman, OK 73019. *Examining linear algebra students' endeavors in moving between the embodied, symbolic and formal worlds of mathematical thinking*. Preliminary report.

Linear algebra is made out of many languages and representations. Hillel (1997, p. 233) suggests that "knowing when a particular language is used metaphorically, how the different levels of description are related, and when

one is more appropriate than the others is a major source of difficulty for students". As Dreyfus (1991, p. 32) declares "One needs the possibility to switch from one representation to another one, whenever the other one is more efficient for the next step one wants to take. Teaching and learning this process of switching is not easy because the structure is a very complex one." We hypothesize that most students do not have the cognitive framework to perform the switch that is available to the expert. In this talk, employing Tall's three-world model, we present specific linear algebra tasks that are designed to encourage students to move between the embodied, symbolic and formal worlds of mathematical thinking. These tasks will be used to examine students' challenges with certain directions, for example, embodied to formal. (Received September 20, 2016)

1125-E5-3065 **Maria Trigueros\*** ([mtrigueros@gmail.com](mailto:mtrigueros@gmail.com)), Depto. Matematicas, ITAM, Rio Hondo 1, 01000 Mexico, CdMx, Mexico, **Edgar Possani**, Departamento de Matematicas, ITAM, Rio Hondo 1, 01000 Mexico, CdMx, Mexico, and **Ana Paulina Figueroa** ([apaulinafg@gmail.com](mailto:apaulinafg@gmail.com)), Departamento Matematicas, ITAM, Rio Hondo 1, 01000 Mexico, CdMx, Mexico. *Multiplying Matrices: an activity based approach.*

In this talk we will present an analysis of the results of the use of a set of activities we have designed to teach matrix multiplication and its links with matrix functions. The analysis uses APOS Theory as theoretical framework and covers results obtained by three groups of students taking an introductory course in Linear Algebra. Students modeled a problem, dealing with pesticide consumption by plants and animals, which motivates and gives meaning to matrix multiplication. Additional activities were designed based on a genetic decomposition to help students to construct the link of matrix multiplication with functions. We later evaluate the effectiveness of this approach, and explain the main difficulties encountered by students in terms of the theoretical framework. (Received September 20, 2016)

## *Innovative Strategies to Inspire and Prepare Potential STEM Majors Who Are Not Yet Ready for Calculus*

1125-F1-94 **Harry F Hoke\*** ([hhoke@richmond.edu](mailto:hhoke@richmond.edu)), Mathematics and Computer Science Department, 28 Westhampton Way, Univ. of Richmond, VA 23173, and **Kathy W Hoke** ([khoke@richmond.edu](mailto:khoke@richmond.edu)). *Science Math and Research Training (SMART) Calculus at University of Richmond.*

University of Richmond Integrated Science Experience (URISE) aims to remove barriers that impede the persistence, retention, and success of underrepresented students in STEM disciplines in part through interdisciplinary STEM education. Students selected for URISE complete a community building summer science experience prior to their first semester and then enroll in one of two courses: Science, Math and Research Training (SMART) for those with limited math background or Integrated Quantitative Science (IQS) for those who have had calculus. This talk focuses on SMART, a two semester integrated Biology and Chemistry course coordinated with two semesters of SMART calculus. We present aspects of SMART calculus that encourage persistence in STEM. There is no assumption of prior calculus, algebra and pre-calculus are incorporated just-in-time rather than in a separate course before calculus, and a workshop day each week relates the analysis of data the students collected in Bio-Chem lab to calculus. We will also present outcome data from three years as the first SMART class approaches graduation. (Received July 21, 2016)

1125-F1-385 **Sharon Lanaghan\*** ([slanaghan@csudh.edu](mailto:slanaghan@csudh.edu)) and **Matthew G Jones** ([mjones@csudh.edu](mailto:mjones@csudh.edu)). *Building and Sustaining Success in Pre-calculus Through a Multi-Pronged Approach.*

Nationwide, many students are not getting through key Mathematics courses that are required for most STEM majors. At CSUDH, pass rates in Pre-calculus were modest (79.2% of all students, calendar years 2009-2013). In addition, only 74.3% of CSUDH freshmen STEM majors were persisting into their second year. To address these concerns, faculty began an overhaul of Pre-calculus in the fall of 2014. A wide range of changes were made, including standards-based grading and an electronic PDF course pack, and classroom strategies such as active learning, problem solving, frequent assessment and digital tools and resources for students. To date, 86% of 272 students have passed the redesigned course. In this session, we will describe the changes, the outcomes including the impact on Calculus 1, and the current work to sustain the changes and expand the use of the redesigned course to more faculty. (Received August 30, 2016)

1125-F1-453 **Paul N Runnion\*** (prunnion@mst.edu) and **Barbara Wilkins** (bwilkins@mst.edu). *Hit the Ground Running: A Summer Bridge to Success at Missouri S&T*.

The majority of incoming freshman at Missouri University of Science and Technology enter in a STEM field (primarily engineering), yet many are unprepared for the demands of a STEM program. During the placement and registration process, all incoming freshmen are invited (and underprepared students are strongly encouraged) to participate in Hit the Ground Running (HGR), a 3 week summer bridge program. HGR, which began as the Minority Engineering Program and has grown to encompass a broader range of students, provides students with intensive instruction in Math, English, and Chemistry and culminates in an opportunity to improve their course placement for the fall. We will discuss the structure of HGR, show how it fits into our overall math placement process, and present data on the impact of the workshop on student success. (Received September 02, 2016)

1125-F1-932 **Ellen J Goldstein\*** (ellen.goldstein@bc.edu), Boston College, Department of Mathematics, Chestnut Hill, MA 02467. *Calculus with Integrated Precalculus for Underprepared STEM Majors*. Preliminary report.

For the past two years, I have been teaching a full-year differential calculus sequence that includes comprehensive pre-calculus instruction for underprepared students whose major requires a course in calculus. Following the structure of Laura Taalman's *Calculus with Integrated Precalculus*, the course focuses on calculus concepts, which are used as motivation to learn precalculus and hone algebra skills. After transitioning from a lecture format to a flipped format, and then to a combination of the two, I have found a balance that meets the diverse needs of this particular student group and adequately prepares them to move on to a standard integral calculus course. (Received September 13, 2016)

1125-F1-1232 **Martha J. Siegel\*** (msiegel@towson.edu), Mathematics Department, Towson University, 8000 York Road, Towson, MD 21252. *Helping Students "Function in the Real World"*.

We discuss the goals and outcomes of a pre-calculus course that emphasizes active learning for mastery of basic pre-calculus mathematical concepts. STEM students are asked to work in teams on real world problems requiring graphical, numerical, and symbolic analysis. They tackle problems in biology, chemistry, and physics that require their understanding of the fundamental mathematical ideas in pre-calculus. In a natural setting, the concepts are shown to be essential tools for solving typical problems in STEM areas. (Received September 15, 2016)

1125-F1-1601 **Stuart Boersma\*** (stuart.boersma@cwu.edu). *Reasoning with Functions: A STEM prep pathway*. Preliminary report.

The Charles A. Dana Center has created an accelerated mathematics pathway to prepare STEM majors for calculus. This new and innovative curriculum, *Reasoning with Functions*, has a firm foundation in research-based methods for teaching precalculus concepts, fosters an active learning environment, and integrates student support throughout. This paper will outline the development of the curriculum, detail the innovative features of the curriculum, and report on the current state of the pilot project. (Received September 18, 2016)

1125-F1-1767 **Allan D. Mills\*** (amills@tntech.edu), **Michael Allen** and **Anna Litchford**. *Math Success for STEM Majors at Tennessee Tech University*.

This talk describes the efforts of Tennessee Tech's "Math Success for STEM Majors" NSF-STEP grant team to increase the number of STEM graduates at TTU. In particular, the redesign of a 5 credit hour Precalculus course, the creation of a STEM-flavored "Introduction to University Life" course, and the introduction of Supplemental Instruction in Calculus I will be discussed and project findings and data from these initiatives will be shared. (Received September 19, 2016)

1125-F1-1786 **Khushikumari Patel-Desai\*** (khuship@clemson.edu), 104 Holtzendorff Hall, Dept of Engineering and Science Education, Clemson University, Clemson, SC 29634, **Eliza Dargan Gallagher** (egallag@clemson.edu), 104 Holtzendorff Hall, Dept. of Engineering and Science Education, Clemson University, Clemson, SC 29634, **Claire L.A. Dancz** (cdancz@clemson.edu), 104 Holtzendorff Hall, Dept. of Engineering and Science Education, Clemson University, Clemson, SC 29630, **Jeffery M. Plumblee II** (jplumb1@clemson.edu), 104 Holtzendorff Hall, Dept. of Engineering and Science Education, Clemson University, Clemson, SC 29630, and **Charity N. Watson** (cwatson@clemson.edu), O-110 Martin Hall, Department of Mathematical Sciences, Clemson University, Clemson, SC 29634. *To Work or Not To Work: Understanding How Natural Work Habits Can Help or Hurt Students in Self-Paced Courses.*

At Clemson University, all STEM majors who are not yet calculus ready take precalculus under a self-paced hybrid course model that includes an asynchronous online component using ALEKS® and a face-to-face component with targeted direct instruction in small groups. The model works very well for the majority of the students, but not at all for others.

The KOLBE-A Index is a measure of how an individual works most effectively when given the flexibility to choose how to approach a task. We looked at the correlation between KOLBE-A Index categories, work patterns, and student performance among students in precalculus to determine if certain indexes were particularly well-suited or poorly-suited to this course model. We present our results and recommendations for using KOLBE-A Index data to help at-risk students successfully complete courses with significant self-paced components. (Received September 20, 2016)

1125-F1-1933 **Jenna P. Carpenter\*** (carpenter@campbell.edu), PO Box 115, Buies Creek, NC 27506. *Engaging and Retaining Underprepared Engineering Majors With Math-Heavy Applications.*

Campbell University has launched a new School of Engineering in Fall 2016. We are interested in creating a pipeline (instead of a filter) for students interested in engineering who enter college without the appropriate mathematical background. To accomplish this, we have created an Introduction to Engineering Applications Course that takes students through mathematics content from beginning algebra through integral calculus using a variety of engineering applications. Emphasis is placed on hands-on exposure to engineering concepts, teaching the early mathematics content along side the related applications, teaching students MatLab to handle the mathematics content beyond their reach, and incorporating professional development and service requirements for students in the course, in order to more fully engage them in School of Engineering activities. We will report on our first year success, as well as identify areas for further improvement. Student feedback on the course will be included. This course is based on work from NSF grants by Dr. Nathan Klingbeil, NSF Grant Numbers EEC-0343214, DUE-0618571, DUE-0622466, DUE-0817332. (Received September 19, 2016)

1125-F1-1967 **Margaret H. Dean** (mdean@bmcc.cuny.edu), 199 Chambers Street, N599, New York, NY 10007, and **Yi Han** and **Fatima Prioleau\*** (fprioleau@bmcc.cuny.edu). *Prepare Potential STEM Majors Who Are Not Yet Ready for Calculus Sequence.* Preliminary report.

Many high school students intend to major in a STEM field in college, but most of them won't be able to handle the rigorous courses they'll need to earn a degree in science, technology, engineering and math. The ACT report for the year 2014 stated that only 5% of African American and 13% of Hispanic high school graduates met the STEM readiness benchmark. At BMCC-CUNY, the vast majority of its student population is comprised of minority groups, many of whom are academically underprepared. Most students come from urban high schools with weak science and math programs and must be provided remedial instruction to address these deficiencies. A BMCC report showed that there is at least one extra year delay for a math major who starts at the remedial mathematics level. In helping prepare potential STEM majors who are not yet ready for the Calculus sequence, we use WeBWork (An Educational Open-source platform). We have also rebuilt our calculus sequence starting from the intermediate algebra/trigonometry course. In this presentation we will share our preliminary report. (Received September 20, 2016)

1125-F1-2012 **Jessica L Williams\*** (jessica.williams@converse.edu), 580 East Main Street, Converse College, Spartanburg, SC 29302. *Effects of Active Learning Techniques on Precalculus Students' Beliefs.* Preliminary report.

The enrollment of a precalculus course often consists of a mix of students completing precalculus as a terminal math course and potential STEM majors completing the course as preparation for calculus. Encouraged by research supporting the effectiveness of active learning and inquiry-based learning techniques on both student

learning and attitudes, a study was conducted in a precalculus course at a small college. A comparison is made between a lecture-based section and a section incorporating a variety of active learning techniques, with simultaneous evaluation of potential STEM majors and non-STEM majors. Structure of the overall course, breakdowns of lecture time versus active time, and individual activities will be discussed. Evidence presented will include data and commentary regarding changes in beliefs and attitudes surrounding the study of mathematics. This data is gathered from pre- and post- course surveys in addition to surveys regarding specific activities. (Received September 19, 2016)

1125-F1-2078 **Branden Stone\*** (bstone@adelphi.edu) and **Salvatore J. Petrilli, Jr.** (petrilli@adelphi.edu). *A year long Calculus course versus the traditional Pre-calculus/Calculus I sequence.* Preliminary report.

Since the Fall of 2000 Adelphi University has been offering a year long calculus course for those students who would normally place into pre-calculus. To determine the impact this course had on our students, Dr. Petrilli conducted a statistical analysis using data from Fall 2000 to Fall 2011. The sample consisted of  $n = 332$  mathematics majors and did not include any transfer students. In this talk we will discuss enrollment rates, the influence a year long sequence had on the upper level calculus courses, as well as the quantitative and qualitative results. (Received September 19, 2016)

1125-F1-2136 **Marie P. Sheckels\*** (msheckel@umw.edu), Department of Mathematics, University of Mary Washington, 1301 College Avenue, Fredericksburg, VA 22401. *Enhancing quantitative reasoning and skills through exploring scientific applications.*

Many college students enter college considering one of the STEM majors. However, some students are underprepared for the mathematics and reasoning skills needed for success in sciences courses. MATH 120: Quantitative Reasoning for the Sciences at the University of Mary Washington is designed to help refresh and enhance students' mathematics understanding and skills while at the same time introducing them to various topics in the STEM fields so to both interest them and prepare them to be successful in those areas. There are numerous science-related applications of mathematics that are accessible to students in pre-calculus level courses. In this general education mathematics course students become familiar with how mathematics is used to answer scientific questions and in so doing strengthen their quantitative literacy. This talk will explain the structure of the course and the alignment between the science and mathematics topics. It will describe how students engage in problem-based learning and provide examples of the various projects and activities that students work on involving relevant applications in areas of environmental science, astronomy, biology, genetics, physical science and medicine. Results from a classroom research study will be shared. (Received September 19, 2016)

1125-F1-2204 **Xiangming Wu\*** (xwu1@mix.wvu.edu), West Virginia University, Department of Mathematics, P.O. Box 6310, Morgantown, WV 26505, and **Jessica Deshler** and **Edgar Fuller.** *A Peer-Mentoring Program for STEM-Intending Developmental Mathematics Students.* Preliminary report.

Students enrolling in developmental mathematics courses face significant obstacles while pursuing a science, technology, engineering or mathematics degree that requires even basic levels of mathematics mastery. The gap between their self-perceived abilities and the expectation of most university level mathematics courses can strongly impact their performance, in turn impacting retention of STEM-intending developmental mathematics students in STEM fields. At West Virginia University, a non-credit-bearing program had been offered to students in past thirty years attempts to provide a bridge to college level mathematics for students whose backgrounds are significantly deficient. In the fall of 2015, this program became a credit-bearing university course; and in 2016 we began a peer-mentoring program for students who enrolled in this course. In this presentation, we will present some preliminary results from the peer-mentoring program with respect to retention of STEM-intending developmental mathematics students. This peer-mentoring program is part of a larger project that also looks at students' anxiety and personality traits and we will also present correlations between the success of students in the peer-mentoring program and these affective traits. (Received September 20, 2016)

1125-F1-2582 **Joel Kilty\*** (joel.kilty@centre.edu), 600 W. Walnut Street, Centre College, Danville, KY 40422, and **Alex M McAllister** (alex.mcallister@centre.edu), 600 W. Walnut Street, Centre College, Danville, KY 40422. *Mathematical Modeling and Applied Calculus: An Integrated Approach for Less Prepared Students.*

In our modern world, we are inundated and grapple with data on a daily basis. As mathematicians, we are often more comfortable discussing the behavior of functions presented analytically, in contrast with the data-driven or tabular presentations of functions ubiquitous in our culture. In this talk, we present an entry level Mathematical

Modeling and Applied Calculus course for students who will (most likely) only take one mathematics course in college and most of whom have a weak algebraic background that (most likely) indicates they will not be successful in a traditional calculus course. Our course is designed to develop a student's ability to model data with elementary functions and then improve their models using the Method of Least Squares, which is also fully developed in the course. The tools of Calculus are used to analyze these models in both the discrete and continuous contexts. Throughout the course, students review pre-calculus ideas while learning about mathematical modeling and the central ideas of calculus. (Received September 20, 2016)

1125-F1-2684 **Padmanabhan Seshaiyer\*** ([pseshaiy@gmu.edu](mailto:pseshaiy@gmu.edu)), 4400 University Drive, MS 3F2, George Mason University, Fairfax, VA 22030, and **Mary Nelson** ([mnelso15@gmu.edu](mailto:mnelso15@gmu.edu)), 4400 University Drive, MS 3F2, George Mason University, Fairfax, VA 22030. *Engaging and retaining pre-college STEM students in Calculus through innovative pedagogical practices.* Preliminary report.

In this talk, we will describe the STEM Accelerator Program at George Mason University which was initiated to increase the number of STEM majors, improve retention rates of STEM students, reduce their time to graduation, and help them join the STEM workforce or continue their education upon completion of their Bachelor's degree in STEM disciplines. Specifically, we will describe some novel initiatives and pedagogical approaches in engaging students that plan to take Calculus in their freshmen year. These include (a) an active-learning approach to teaching calculus; (b) oral reviews co-facilitated by faculty and advanced undergraduates and; (c) a three day mathematics readiness with a one-week pre-college program for incoming college students. The students that were impacted through these initiatives not only completed Calculus successfully, but also went on to pursue a STEM major. (Received September 20, 2016)

1125-F1-2786 **Brian J Lindaman\*** ([blindaman@csuchico.edu](mailto:blindaman@csuchico.edu)), Dept. of Math and Statistics, CSU Chico, 400 W. 1st St., Chico, CA 95929. *Planets, Earthquakes, and Airbags: The Challenge of Incorporating Significant Mathematics Content in STEM Activities.*

The mathematics in a STEM lesson can be underwhelming at times; it can be of minimal importance, too advanced to be applicable, or too shallow to inspire curiosity about the math itself. A grant-funded project in Northern California designed a series of STEM-integrated activities which foster the learning and use of rich mathematics. We found STEM to be an enticing entry point for students; by focusing on real-world STEM scenarios, such as planetary exploration, earthquake activity, and auto collisions, students were motivated to investigate the math deeper. In addition, they were better able to understand the science, engineering, and technology aspects, and their overall curiosity about STEM-integrated problems increased. We are excited to share our activities with others and engage in the important conversation on seeking more ways to promote robust mathematics learning within a larger STEM context. Session participants will receive an overview of how the activities were developed, the activities themselves, and results of their use in precalculus courses. The session will end by sharing the challenges encountered when developing math-intensive STEM classroom activities, and open the conversation to identify ways to collect and share this type of knowledge. (Received September 20, 2016)

1125-F1-2806 **Cathy W Grilli\*** ([cgrilli@cbu.edu](mailto:cgrilli@cbu.edu)), Christian Brothers University, 650 East Parkway South, Memphis, TN 38104, and **Andrew Diener**. *Two Approaches to Precalculus.* Preliminary report.

Christian Brothers University split our precalculus population by ACT scores. Those with higher Math ACT scores take the traditional Precalculus, those with lower scores take a combination of a Functions course and a Trig course. The courses are coordinated and run concurrently which is different from most schools that divide the subject into an algebra and a trig course. Topics are aligned so that circles in the Functions course lead to the unit circle trig definitions. Solving quadratic equations leads to quadratic form equations involving trig functions. Inverse functions in the Functions course lead to the inverse trig functions. This immediate reinforcement helps students see how mathematics builds on itself and promotes student success. CBU is a small school where the majority of students taking precalculus are engineering majors. While half of those that declare engineering as their major begin in precalculus the engineering paradigms begin with Calculus I. The increased need for immediate success in precalculus driven by our structured engineering curriculum led us to explore new options. We will share our experiences in the combination of courses as well as the impact on students in the traditional Precalculus course. (Received September 20, 2016)

1125-F1-2863 **Jeff Pullen\***, Mathematics Department, 1501 Mercer University Drive, Macon, GA 31207, and **Katherine Northcutt, Jarred Jenkins** and **Chamaree de Silva**. *Four Faculty, Twenty Students, and the University's Squirrel Population: Reconceptualizing Undergraduate Research for Non-Calculus Ready Science Majors*. Preliminary report.

Program in Integrative Science and Mathematics (PRISM) is a pilot project in its second year at Mercer University, designed to guide a selected cohort of our at-risk, non-calculus ready, incoming student population. In this program, students study Statistics, Precalculus, Biology, Physics, and Psychology in an integrated manner over the course of an entire academic year as they learn how to conduct an authentic research project using the squirrel population on campus. We will share our experiences, assessment results, and planned changes for future installments. (Received September 20, 2016)

1125-F1-2989 **James S Rolf\*** (jim.rolf@yale.edu) and **John Hall**. *Impact of an Online Bridge Program for Preparedness for Quantitative Reasoning*. Preliminary report.

Online Experiences for Yale Scholars (ONEXYS) is a summer online bridge program to help incoming students who are bright, but underprepared for quantitative reasoning courses. The past three summers we have taught pre-calculus material through online videos, adaptive practice, and challenging applications, and have employed upperclassmen as coaches and mentors to foster online learning communities. A large proportion of the students in ONEXYS come from underrepresented groups, and we purposefully use strategies that we believe will ease their transition to campus life and increase STEM retention.

We will discuss the evidence that led us to adopt this structure as well as the results of pre/post surveys and focus groups designed to assess the impact of the program on student perception of preparedness for quantitative reasoning courses. (Received September 20, 2016)

1125-F1-3057 **Rachid Ait Maalem Lahcen\*** (rachid@ucf.edu), Department of Mathematics, University of Central Florida, Orlando, FL 32816, and **R. N. Mohapatra** (ram.mohapatra@ucf.edu), Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Preparing students to succeed in Calculus through adaptive instructional approach*.

Experience and consideration of data shows that there is a trend of students changing from STEM majors due to low success rate in Calculus. In spite of funds allocated to universities for Calculus reform, there is no significant success in increasing pass rate in Calculus in major metropolitan public universities. In this talk we shall mention how we plan to approach improvement of instructional method in pre-calculus courses with a view to providing students with a platform to improve their confidence and better problem solving skills through inquiry based instruction. (Received September 20, 2016)

1125-F1-3082 **Stacy M Musgrave\*** (smmusgrave@cpp.edu) and **Marilyn P Carlson**. *Improving Student Success through Deepening GTAs' Meanings*.

At many universities, courses leading up to calculus are taught by graduate teaching assistants (GTAs). These typically novice instructors face the challenge of teaching ideas foundational to calculus, yet our data suggests GTAs often hold impoverished meanings for those ideas themselves. We share data revealing the varied meanings held by GTAs regarding average rate of change, angle measure and the sine function, before and after being involved in an intervention designed to support them in developing richer meanings for these and other ideas found in algebra and pre-calculus courses. We highlight aspects of the intervention and corresponding materials focused on deepening the content knowledge of these instructors which have contributed to increased student success. (Received September 21, 2016)

1125-F1-3086 **Martha Shott\*** (shott@sonoma.edu), 956 Hawthorne Cir., Rohnert Park, CA 94928. *A Watershed Year: Modeling and Data Interpretation as Pathways to Building Mathematical Confidence in First-Year Students*. Preliminary report.

The School of Science and Technology at Sonoma State University, one of the 23 campuses in the California State System, has developed an interdisciplinary first-year experience for prospective STEM majors that integrates biology, mathematics, and critical thinking to study issues surrounding the regional Russian River watershed. Each year's cohort is comprised of students of varying mathematical background, ranging from students with developmental needs to students ready to enroll in second-semester calculus. In order to support the students with weaker preparation to pursue technical majors, the course's instructional team has implemented a variety of strategies including targeted small-group tutoring, interactive classroom activities to foster peer discussion and mentoring, and assignments that enhance students' mathematical intuition and confidence. Students are able to develop mathematical models to mimic natural processes, understand the limitations of those models, and interpret numerical results in a critical and meaningful way. Preliminary data indicate that students completing

our program are successful in subsequent mathematics courses and are able to obtain a STEM degree in a timely fashion. (Received September 21, 2016)

1125-F1-3113 **Junalyn P Navarra-Madsen\*** ([jnavarramadsen@mail.twu.edu](mailto:jnavarramadsen@mail.twu.edu)), Texas Woman's University, P. O. Box 425886, Denton, TX 76204. *STEM Women Majors: A Path to Success.*

Texas Woman's University (TWU) is one of the largest US university for women by population with record-breaking enrollment this fall of more than 15,000 students attending the university's three campuses in Denton, Dallas and Houston. TWU recently received NSF S-STEM grant, "Quantitative and Analytical Sciences for Academic Reinforcement and Success (QuASARS)" - NSF Grant 1154394. The presenter is a co-PI of this grant and shares some information and insights on how to lower the attrition rate of female STEM majors in courses such as Calculus, Linear Algebra and Differential Equations using early intervention approaches, tutoring, mentoring which ultimately leads to building a learning community of successful career women in the STEM fields. (Received September 21, 2016)

## *Innovative Teaching through Recreational Mathematics*

1125-F5-139 **Maura A. Murray\*** ([mmurray@salemstate.edu](mailto:mmurray@salemstate.edu)), Mathematics Department, 352 Lafayette Street, Salem, MA 01970. *Using the Mathematics in the Simpsons in a First Year Seminar.* Preliminary report.

A first year seminar (FYS) course is designed to introduce freshman to the experience of academic exploration that is at the heart of a liberal arts education. The mathematical ideas infused in the long running Simpsons TV show offer a fun way to draw students into exploring new, unfamiliar concepts. I will share how, "*The Simpsons and Their Mathematical Secrets*", by Simon Singh, framed our class discoveries involving logic puzzles, graph theory, number theory and geometry. Pertinent short video clips will be mentioned and activities, geared for non-mathematics majors, will be shared. (Received August 03, 2016)

1125-F5-995 **Robert W. Vallin\*** ([robert.vallin@lamar.edu](mailto:robert.vallin@lamar.edu)), Department of Mathematics, Lamar University, P.O. Box 10047, Beaumont, TX 77710. *A Magic Trick That is Full of Induction.*

Mathematical induction is an extremely important device in every mathematician's toolkit. It is mostly taught via a succession of problems involving formulas or inequalities that seem to students either abstract or forced. This talk begins with a magic trick whose secret is shown via induction. We then exhibit some mathematical fallout from the trick and examine certain properties using mathematical induction, strong induction, and, finally, the proof of a summation formula done in the manner of Benjamin & Quinn. (Received September 13, 2016)

1125-F5-1064 **Jay A Malmstrom\*** ([jmalmstrom@okcc.edu](mailto:jmalmstrom@okcc.edu)), 7777 S May Ave, Oklahoma City, OK 73159. *Using a Kiowa Game to Increase Student Understanding of Expected Value.*

The Kiowa roamed the Southern Plains in the late 19th century. They played a game involving marked peach pits to pass the time in camp. A version of the game can be used to help non-Math majors develop a feel for the concepts of expected value and expected time of play. (Received September 14, 2016)

1125-F5-1329 **Stephen M. Adams\*** ([sa3236@cabrini.edu](mailto:sa3236@cabrini.edu)), Cabrini University, 610 King of Prussia Road, Radnor, PA 19087. *Instant Insanity: Using Colored Blocks to Teach Graph Theory.* Preliminary report.

Instant Insanity is a puzzle comprised of four blocks whose sides are painted with one of four colors. The goal is to stack the four blocks in a column such that all four colors appear on each of the four sides of the column. In this talk we will demonstrate a hands-on way to introduce this puzzle in a sophomore-level discrete mathematics course to help motivate the study of graph theory. (Received September 16, 2016)

1125-F5-1346 **Jennifer L Sinclair\***, 1000 University Center Lane, Lawrenceville, GA 30043. *Music composition utilizing probabilistic methods as an applied project in an upper level mathematical statistics course.*

A Markov chain (discrete time version of Markov process) is a mathematical model depending on randomness and passage of time. A fundamental property of Markov processes is the memory-less property, which implies the conditional probability of the process taking on a particular value at time  $n$  only depends on the location at time  $n-1$ , disregarding the previous observations at times  $0, 1, \dots, n-2$ . This work addresses a creative and potentially artistic classroom activity related to music. Historically, probabilistic music composers used rolled dice to choose the next note, which typically results in a scattered array of disorganized notes. Markovian music

composition appeared in the twentieth century and takes into consideration the current location of a note to choose the next note and can be designed to consider note location, scale, timing, and more. During Fall 2015, junior and senior level students engaged in a research approach to Markovian music composition at a public regional college. This presentation addresses a description of the project and best practices in the classroom in addition to a presentation of the results of pre and post assessments and attitudinal surveys. (Received September 16, 2016)

1125-F5-1420 **Edmund A Lamagna\*** (eal@cs.uri.edu), Department of Computer Science and Statistics, Kingston, RI 02881. *Frogs + Puzzles = Algorithmic Thinking.*

Recreational mathematics provides a rich source of fun, interesting problems that can be used to develop mathematical thinking and problem-solving skills. Importantly, recreational mathematics provides a way to “level the playing field” among students with vastly different mathematical backgrounds. Students also enjoy working on puzzles, and will devote more effort to them than routine exercises.

The presenter teaches a liberal arts math course based on puzzles and games. The course is taught entirely through active learning, with no lectures. Students spend each class working in small groups solving puzzles that revolve around a common theme.

One of the activities is a set of sequential movement puzzles in which frogs are required to perform various feats (e.g., reversing positions). These puzzles are a vehicle for learning about algorithms in an “unplugged” way, with no computer programming. The problems conceal much deep mathematical content and provide a springboard into such topics as algorithm analysis, lower bounds, parity, and the generalization of solutions. The problems also encourage the use of problem-solving paradigms such as starting with small cases and observing patterns. (Received September 16, 2016)

1125-F5-1658 **Tanya Cofer\*** (tcofer@ccga.edu). *The Surprising Mathematics Hidden Inside the Trihexaflexagon: Using hinged polygons to teach group theory.* Preliminary report.

Flexagons are easily constructed paper objects that can be manipulated as an amusement or game. But these clever little objects also hold subtle and intriguing algebraic structure. In this talk, will report on the design and implementation of a project I gave in a course on abstract algebra. In the project, students were challenged to get creative and decorate trihexaflexagons with designs and colors in order to realize the different subgroups of a dihedral group. I’ll share some of the cool designs students came up with as well as some surprising and enlightening outcomes of this engaging class project. (Received September 18, 2016)

1125-F5-1830 **Hossein Behforooz\*** (hbehforooz@utica.edu), Mathematics Department, Utica College, Utica, NY 13502. *Linear Algebra Properties of Magic Squares.*

We know that, over all, every magic square is a very especial square matrix and in this talk, we will present some interesting linear algebra properties of these magic square matrices. There are some published short articles or notes related to this subject but they are not very complete papers with all details in one place. Since the time of the lecture is short and the title of the session is “Innovative Teaching through Recreational Mathematics”, I promise to follow the title of the session and state the theorems without proofs. That will change my presentation from theoretical approach lecture to a fun and amusement type of lecture. Yes, MATH is FUN and we must remember that we had Magic Squares, may be more than four thousand years, before new born entertainment with Sudoku Squares.

Reference: Hossein Behforooz: Linear Algebra Properties of Magic Squares, Topics in Recreational Mathematics, No. 1, 2016, 18-23. (Received September 19, 2016)

1125-F5-1912 **Michael A. Brilleslyper\*** (mike.brilleslyper@usafa.edu), Department of Mathematical Sciences, U. S. Air Force Academy, USAF Academy, CO 80840. *How many push-ups did they do?* Preliminary report.

You happened to turn on the TV towards the end of the Air Force versus Navy football game in Oct 2014, just in time to see the Air Force cadets rush from the stands into the end zone and quickly do 30 push-ups. You notice that the game score is AF 30, Navy 21. It is tradition that after each scoring play by their respective teams, first-year cadets or midshipmen rush onto the field and do the number of push-ups equal to their team’s new score. Over the course of a high scoring game, they can do a lot of push-ups. How many push-ups they might do is the subject of this talk. Along the way we will see that for a given game score there is a minimum and a maximum number of push-ups that could have been done. We characterize the set of possible “push-up numbers” that fall between the minimum and maximum. We also discuss a “push-up counter vector” and provide a geometric interpretation of a scoring vector. Finally, we use a result known to Euler to help count distinct ways

in which certain game scores can be obtained. This talk is suitable for undergraduates. (Received September 19, 2016)

1125-F5-1956 **Ricardo V Teixeira\*** ([teixeirar@uhv.edu](mailto:teixeirar@uhv.edu)), 615 newhaven, Victoria, TX 77904.  
*Exemplifying Mathematical Concepts through Magic Tricks.*

World-famous magicians perform acts heavily based on mathematical concepts, but rarely do the explanations get proper attention. By learning the reasons why certain tricks work, students (and educators) will see fun aspects of mathematics. There might be several areas of mathematics involved in magic. In this talk, we will cover tricks whose explanations involve: error detection methods, binary and ternary number systems, cyclic groups, divisibility rules, and more. In information theory, error detection is a technique that enables reliable delivery of digital data over unreliable communication channels. A magic trick will exemplify this technique with redundant information on “magical cards”. Binary and ternary number systems are positional notation systems with base 2 and 3. An easy trick with binary cards and a more elaborated trick with cards from a normal deck will show how they can relate to magic. In algebra, a cyclic group is a group that is generated by a single element. We use that each suit has 13 cards, and 13 is a prime number, to create a cyclic group with cards that will allow us to perform a great act. Finally, if time permits, we will also relate divisibility rules, arithmetic operations, probability and more, in our presentation. (Received September 19, 2016)

1125-F5-2001 **Anthony Rizzie\*** ([anthony.rizzie@uconn.edu](mailto:anthony.rizzie@uconn.edu)), 341 Mansfield Road, U1009, Storrs, CT 06269, **Myron Min-Thu-Aye** ([myron.minn-thu-aye@uconn.edu](mailto:myron.minn-thu-aye@uconn.edu)), 341 Mansfield Road, U1009, Storrs, CT 06269, and **Amit A Savkar** ([amit.savkar@uconn.edu](mailto:amit.savkar@uconn.edu)), 341 Mansfield Road, U 1009, Storrs, CT 06269. *Multivariable calculus: A Play-Doh adventure.*  
Preliminary report.

Over the last several years there has been a push to create alternative tools to help students visualize concepts in Calculus 3. Some have been successful using online graphing calculators while others have been more focused on animations provided by publishing companies. This year at the University of Connecticut, there has been a conscious effort to encourage students in large lecture Calculus 3 to take a hands-on approach to understanding the various visual aspects of the course.

We have developed several lesson plans that involve students using Play-Doh in the classroom to explore and interact with surfaces and solids in three dimensions. We will discuss the success of these learning activities as it relates to the learning objectives of the lessons that were taught using them. Student reflections and their perceptions of using Play-Doh will be shared with the audience. (Received September 19, 2016)

1125-F5-2224 **M. Reba\*** ([mreba@clermson.edu](mailto:mreba@clermson.edu)), Department of Mathematical Sciences, Martin Hall, Clemson University, Clemson, SC 29631, and **D. Shier**. *Using Games and Puzzles to Motivate and Introduce Students to Mathematical Concepts and Strategies Underlying Complex Societal Applications.*

In both a developmental mathematics course and in an honors course, we have used recreational math problems to motivate and introduce students to recurring mathematical representations and associated solution strategies. Solving the Thai 21 challenge or a change-making problem illustrates the essential features of the dynamic programming approach used in solving large optimization problems. Solving a maze, jug, or egg-timer problem highlights the concept of a breadth-first search and leads to a discussion of shortest-path algorithms used in modern GPS systems. Conceptualizing a domino arrangement problem or the Instant Insanity cube problem introduces the notion of Hamiltonian circuits in graphs and motivates the need for heuristics when seeking minimum-cost Hamiltonian circuits for practical routing/scheduling problems. Evaluating a gambler’s wager or a solving a Car-Talk Puzzler can be illuminated by using decision trees, which in turn aid in calculations of conditional probability. Group problem-solving sessions and projects have also been used in these courses. (Received September 20, 2016)

1125-F5-2464 **Michael A. Jones, Brittany Shelton\*** ([bshelton@albright.edu](mailto:bshelton@albright.edu)) and **Jennifer M. Wilson**. *Deal or No Deal in the classroom.*

Deal or No Deal was a prime time game show on the National Broadcasting Corporation network, in which dollar amounts ranging from \$.01 to \$1 million were concealed in 26 suitcases. In a series of rounds, the Contestant was asked to accept a monetary offer to stop playing (deal) or open a specified number of suitcases revealing dollar amounts (no deal). The game ends when the Contestant accepts an offer or gets the amount in the selected suitcase.

Through a series of activities, we use Deal or No Deal as a framework for thinking about decision making under conditions of uncertainty, which can be applied to any real-life situation. These activities are designed to reinforce students' familiarity with function notation, graphical representation of functions, and inverse functions. They also allow students to apply concepts from probability in the context of making decisions involving risk. These activities have been used at the high school, undergraduate, and masters levels. In each case they were tailored to the appropriate level and the emphasis was tailored to the course learning objectives. (Received September 20, 2016)

1125-F5-2557     **Jennifer F. Vasquez\***, Department of Mathematics, The University of Scranton, Scranton, PA 18510, and **Michael P. Allocca** and **Steven T. Dougherty**. *Permutation games with signed and circular permutations.*

Permutations are of central importance in abstract algebra and can be applied to help model and understand biological processes. In this talk we will present games based on signed and circular permutations. Time permitting we will discuss connections of these games to codes and the evolution of genomes. The ideas here may be used in discrete mathematics and abstract algebra courses. (Received September 20, 2016)

1125-F5-2638     **Lee G. Windsperger\*** ([lwindsperger@winona.edu](mailto:lwindsperger@winona.edu)). *Using a Mathematical Excursion in Calculus to Challenge and Expand Student Understanding of Continuous Functions.* Preliminary report.

In this talk, I will discuss a nonstandard, mathematical excursion which I have used in my calculus courses to uncover the limitations of our basic visualization and understanding of continuous functions. Through this recreational retreat from the standard calculus content, I hope to reveal the definition of generalized functions and the concept of completeness. Furthermore, my goal is to temporarily level the field between students with calculus experience and those without, solidify current and past content knowledge for students, open doors to new opportunities for undergraduate research, and expand the students' view of mathematics. (Received September 20, 2016)

1125-F5-2876     **Tyler J Markkanen\*** ([tmarkkanen@springfieldcollege.edu](mailto:tmarkkanen@springfieldcollege.edu)), Department of Mathematics, Physics, and CS, Springfield College, 263 Alden Street, Springfield, MA 01109-3797. *The "mathemagical" classroom.*

Math-based magic tricks provide a way to incorporate rich and engaging activities in the classroom. In this talk, we will demonstrate a variety of these "mathemagic" tricks. These include tricks in which the magician miraculously predicts a secret number chosen in advance by an audience member. Such guess-my-number tricks use the sums of digits and special cards that exploit the properties of combinations and binary numbers. Other tricks involve predicting a card at a seemingly random location in a deck of cards. Here, high school algebra can be used to prove why the magic tricks work, giving an opportunity for students to discover the proofs for themselves. Finally, we will briefly discuss how some students have responded to classroom activities that combine math and magic. (Received September 20, 2016)

1125-F5-2981     **Paul R Coe\*** ([coepaul@dom.edu](mailto:coepaul@dom.edu)), Dominican University, 7900 W. Division Street, River Forest, IL 60305. *Does Monte Hall know Bayes' Rule?*

You are a contestant on the game show "Let's Make a Deal". The host, Monte Hall, shows you three curtains. He tells you that behind one of these curtains is a car and behind the other curtains is a goat and asks you to choose a curtain. You choose curtain number two. Oddly, the host now asks you what the probability is that you chose the curtain with the car behind it. A bit bewildered, you answer  $1/3$ . Monte Hall now reminds you that at least one of the two curtains that you did not choose must have a goat behind it. To prove this, he opens curtain number one. He claims that he has not given you any information about what is behind your curtain, but you now see only two curtains on the stage, one of which has a car behind it. Being Monte Hall, he now offers you the chance to keep your curtain or trade for what's behind the other curtain. Should you keep your curtain because you don't trust Monte Hall or trade it for the other curtain? I use this problem in class as an example of Bayes' Rule. Depending on what you believe about Monte Hall's behavior, the posterior probability that the car is behind your curtain can be anything between zero and one half. In my presentation I will describe how I use this problem and the importance of observing Monte Hall very carefully. (Received September 20, 2016)

## *Inquiry-Based Teaching and Learning*

1125-G1-53 **Vilma Mesa, Mollee Huisinga** (mollee@umich.edu) and **Ashley N Jackson\*** (jacksash@umich.edu). *Managing tensions within a coordinated inquiry-based learning algebra course: The role of worksheets.*

We present a description of the ways in which nine instructors teaching a linear algebra course at a research university managed tensions that emerged because of the requirement of using Inquiry-Based Learning (IBL) in a coordinated way. Through interviews and analysis of documents (pacing chart, textbook, and worksheets), classroom observations, and focus groups, we identified features of the course organization that created tensions that were resolved by creating worksheets that were used to support the teaching and learning of the course. The tensions were related to content to cover, the pacing, and what counts as IBL. There were differences in how instructors used the textbook (e.g., some reorganized sections in the textbook and others used the textbook as given) and how they designed their worksheets (some tried to create worksheets that emphasized applications of concepts while others sought to increase the level of abstraction and development of mathematical theory). We propose that differences were observed in managing tensions might be related to the research orientation the instructors brought and to their status in the institution. (Received September 18, 2016)

1125-G1-256 **Zoë Misiewicz\*** (zoe.misiewicz@oneonta.edu). *Inquiry-Based Learning and the History of Mathematics: Discovering the Geometric Procedure for Completing the Square through an Ancient Mesopotamian Text.* Preliminary report.

Ancient Mesopotamian numerical tablets provide an accessible approach to the earliest history of mathematics; students can look at a drawing of a cuneiform text and immediately begin to discover for themselves certain characteristics of the Babylonian number system, such as its sexagesimal nature. Mesopotamian texts that describe mathematical procedures in words are less familiar and less accessible, but are ultimately even more rewarding when approached from an inquiry-based perspective.

An excerpt from one such text, BM 13901, is presented in literal translation and initially appears incomprehensible. Yet with some guidance, pre-service mathematics teachers are ultimately able to work step-by-step through the instructions provided in this text to discover the geometric procedure for completing the square. Once they have discovered the process for themselves using this ancient text, students often report that they understand it much better than the familiar algebraic formulation and are eager to apply it in their own teaching. They have simultaneously gained insight into the mathematics of ancient Mesopotamia and attained a deeper understanding of a standard modern procedure that they will be teaching regularly in secondary schools. (Received August 19, 2016)

1125-G1-379 **Dywayne A Nicely\***, Ohio University-Chillicothe, 101 University Drive, Chillicothe, OH. *Flipping Precalculus through Guided Notes.*

After discovering that other instructors had success in implementing the flipped classroom model in their mathematics courses, we decided to analyze the effects of the flipped classroom model in our Precalculus course during the past academic year at Ohio University-Chillicothe (OUC). In this final report, we compare final grade data of the control and experimental groups by performing a t-test on the overall averages and calculating the effect size (Cohen's d). Students from the fall semester of 2015 populated the control group while students from the spring semester of 2016 populated the experimental group. Along with the grade data, we will detail the methods and procedures that were conducted during the intervention. Specifically, how we flipped the classroom through the use guided notes and providing the students with annotated PowerPoint slides. Lastly, we will provide some student comments about the intervention that were collected from a survey. (Received August 30, 2016)

1125-G1-512 **Khairul Islam\*** (kislam@emich.edu), Mathematics Department, Ypsilanti, MI 48197. *Examples of Inquiry-Based Teaching and Learning: Applications with Public-use Cancer Data.*

Does cancer discriminate? How to justify any existence of discrimination? Can the discrimination, if exists, be attributable to gender or ethnicity? Such questions were asked while teaching fitting models in a Calculus - I or Introductory Statistics class, where optimization (maximum or minimum) criteria are being introduced. Other questions were asked as well for similar inquiries. Objectives were to assess the level of understanding of forms of models (linear and/or non-linear) being attempted in problem solving, and to see how students approach answering some pre-assigned questions. In this presentation, some examples and applications of model-based teaching and learning will be addressed using public-use cancer data, where students can be active learners guided by the instructor. Such approaches help students understand the underlying mathematical tools effectively, with positive attitudes towards further learning. (Received September 04, 2016)

- 1125-G1-625 **Amanda Sutherland\*** ([asuther1@su.edu](mailto:asuther1@su.edu)) and **Beth Dodson** ([bdodson94@su.edu](mailto:bdodson94@su.edu)). *Using Guided-Inquiry Activities to Promote Stronger Foundations in Introductory Statistics*. Preliminary report.

Inquiry-based learning (IBL) is a popular method for encouraging autonomous thinking. By combining IBL methods with group work founded in immersive content discovery through constructivist activities, we convert the instructor's role from lecturer to coordinator. This allows the educator to provide unobtrusive guidance, manage class time without manipulating a student thought process, and build a foundation for future in-depth analyses. These guided-inquiry activities provide students the opportunity to help each other through problem manipulation and actively contributing to a group, not only to create a sense of teamwork but also to build a strong knowledge base through the proven art of "learning by teaching". By testing these activities in classrooms, we are able to observe students' journey through the learning cycle. Using low-risk assessments and student testimonials, we begin to determine the effectiveness of these activities in helping students retain information and develop a broader understanding of the material. (Received September 08, 2016)

- 1125-G1-645 **Patricia Baggett\*** ([baggett@nmsu.edu](mailto:baggett@nmsu.edu)), Dept. of Math Sciences, MSC 3MB PO Box 30001, New Mexico State University, Las Cruces, NM 88003-8001, and **Andrzej Ehrenfeucht** ([andrzej@cs.colorado.edu](mailto:andrzej@cs.colorado.edu)), Computer Science Dept, University of Colorado, Engineering Center, Boulder, CO 80309. *A Graduate IBL Course in the History of Mathematics Education*.

We offer a one-semester graduate mathematics course at New Mexico State University, History and theories of mathematics education. It is a history course that differs from others typically offered in that students produce most of the research themselves. The course has three components. Students first choose an antiquarian mathematics text (from a private collection, Google Books, etc.) and give a thorough 30-minute report on it to the class. They read articles dealing with history and theories of math education and turn in answers to questions provided by the instructor. Finally, with help from the instructor, they come up with a research question in the history of mathematics education that they attempt to answer based on original sources, and present their research in a 15-20 minute talk at a mini-conference open to all faculty and students. We will describe the structure of the course, the students' opinions of it, and some of the research topics that have been developed into single-author presentations at national mathematics conferences (7) or published articles (2). (Received September 08, 2016)

- 1125-G1-655 **Timothy Redl\*** ([redlt@uhd.edu](mailto:redlt@uhd.edu)), University of Houston-Downtown, Department of Mathematics and Statistics, One Main Street, Suite S705, Houston, TX 77002. *Teaching Honors College Algebra with Inquiry-Based Instruction at the University of Houston-Downtown*. Preliminary report.

During the Fall 2015 semester, I had the pleasure and privilege of teaching College Algebra to 17 students in the Honors Program at UH-Downtown. This was the first-ever offering of an Honors-designated section of College Algebra (or any Honors-designated mathematics course, for that matter) at UHD. As an Honors section, it focused more on using algebra and functions in the context of inquiry-based learning to model and solve real-world problems within the context of natural science, advanced mathematics, social science, business, statistics and engineering, than a traditional College Algebra course. We used technology such as graphing calculators, Wolfram Alpha, and Microsoft Excel to model, analyze, and predict solutions, and engaged in meaningful conversations and made decisions about the appropriateness of different modeling strategies and solutions to these problems. In lieu of a final exam, students completed end-of-semester group projects to gain exposure to and acquire knowledge of mathematical topics that were beyond the scope of the traditional College Algebra course – topics usually encountered in pre-calculus or calculus. In this talk, I will describe and demonstrate some of the technologies, class activities, and projects from the course. (Received September 08, 2016)

- 1125-G1-704 **Nicholas Long\*** ([longne@sfasu.edu](mailto:longne@sfasu.edu)). *Creating Independent Readers: Ideas for Effective Student Reading of Textbooks*.

"Students will only go as far in mathematics as their reading takes them." This statement is more than a good truism for how non-mathematical ideas can inhibit our students' mathematical progress. This statement is a call for us, as educators, to ensure that our students learn how to effectively read their textbooks. We will present some common and uncommon ideas on how to help students read more effectively with an emphasis on implementation, especially as this applies to Inquiry Based Learning activities. (Received September 09, 2016)

1125-G1-706 **Randall E. Cone\*** ([recone@salisbury.edu](mailto:recone@salisbury.edu)), 128 Henson Hall, Salisbury University, Salisbury, MD 21801. *Pushing Symbols: IBL in Mathematics and Computer Science.*

At some universities and colleges, mathematics and computer science (CS) programs are co-located within the same department. One benefit of such co-location is the possibility that mathematics faculty sometimes (regularly) teach computer science courses, especially if those faculty have a background in the subject. Does IBL naturally cross the disciplinary boundary from mathematics to computer science? Our experience in this indicates a resounding 'yes!' which, in this presentation, we support with a discussion of using IBL in a variety of CS courses, including: Scientific Programming (in Python), Theory of Computation, and High-Performance Computing. (Received September 09, 2016)

1125-G1-826 **Miriam Harris-Botzum\*** ([mharrisbotzum@lccc.edu](mailto:mharrisbotzum@lccc.edu)), 4525 Education Park Drive, Schnecksville, PA 18069. *Implementing POGIL Activities in a Community College First-Semester Calculus Course.* Preliminary report.

After attending a POGIL (Project Oriented Guided Inquiry Learning) workshop at a previous JMM, I began introducing POGIL activities into my Calculus I and Calculus II classes.

Using a combination of objective assessments, personal observations, and surveys of student attitudes and perceptions, this paper explores the effects of using POGIL in the Calculus I class, in terms of student learning and attitudes, and also addresses some of the challenges in instituting POGIL activities to students who are more accustomed to passive learning. (Received September 12, 2016)

1125-G1-848 **Milos Savic, OK** , and **Emily L Curtis\***, 601 Elm St., Norman, OK 73019. *Productive Failure in Proving – Perspectives of a Student and Instructor.* Preliminary report.

One aspect of inquiry-based learning that has been emphasized is the concept of a productive failure, in which a student fails in their problem-solving process, attempts to correct their solution, and reflects meta-cognitively about why that failure was productive for future problem-solving attempts. However, in our search, there has been little investigation into what a productive failure looks like from the student and teacher perspective. We aim to present a case study in which both the instructor and the student share their perspectives prior to the productive failure. The productive failure, on the second exam of an abstract algebra course, was presented to the class, and a reflection of that presentation by the instructor and student will also be presented. This dissection of a productive failure might be helpful to future instructors who want to demonstrate a "typical" productive failure. (Received September 20, 2016)

1125-G1-952 **Cassie Williams\*** ([w111i5c1@jmu.edu](mailto:w111i5c1@jmu.edu)), James Madison University, Harrisonburg, VA. *Unintended Consequences: How IBL experiences influence future teachers.* Preliminary report.

There is a large and growing body of research pointing to the effectiveness of IBL (and active learning in general) on learning content and developing productive mathematical habits for students at all levels. What other consequences exist for students who experience these non-lecture classroom structures? In this talk I will present some preliminary data about one potential unintended (or at least unexpected) byproduct of IBL: Are future secondary teachers more likely to consider employing IBL (or any active learning strategy) in their future classrooms after experiencing such a classroom structure as a student? (Received September 13, 2016)

1125-G1-962 **James R Eby\*** ([jeby@blinn.edu](mailto:jeby@blinn.edu)). *In a traditional Calculus class, students explored several topics using Excel with data. This helped connect the topics with their Engineering classes and introduced integration early in the course.*

This presentation will focus on several IBL activities used in my traditional, first semester Calculus class will be discussed. All of the activities use data points to explore Calculus topics in a discrete setting using Excel. Some of the activities introduce the idea of a definite integral early in the class, to allow for a deeper understanding of the Fundamental Theorem of Calculus before it is formally introduced. In addition, some data will be presented that indicates students tend to think of more examples than simply continuous functions after working through these activities. Copies of all activities will be provided in electronic form. Topics included in the activities include: Limits, ARC compared with Instantaneous Rate of Change, directional limits, Mean Value Theorem, linearization of linear, exponential, and logistic functions, local extrema, concavity, Fundamental Theorem of Calculus, and the idea of accumulated compared with net change. (Received September 13, 2016)

- 1125-G1-984      **Suzanne Ingrid Dorée\*** ([doree@augsborg.edu](mailto:doree@augsborg.edu)), Augsburg College Campus Box 61, Minneapolis, MN 55454. *Making Discrete Inquiries: Effective IBL Structures for a Multi-Audience Discrete Mathematics Course*. Preliminary report.

Mathematical inquiry is the core of what mathematicians do. Teaching inquiry helps students grow their inner mathematician. For example, teaching students to pose/test conjectures motivates writing proofs, improves reasoning skills, and builds confidence. The MAA CUPM Curriculum Guide 2015 agrees: “programs should include activities designed to promote students’ progress in learning to . . . assess the correctness of solutions, create and explore examples, carry out mathematical experiments, and devise and test conjectures . . . Students should develop mathematical independence and experience open-ended inquiry.” (Cog. Rec. 1,4) How do we teach inquiry? Discrete mathematics and inquiry-based learning is an ideal content-pedagogy pair. For 17 years I have taught a sophomore level discrete mathematics course using IBL. This talk will describe the practical elements of the IBL course structure and highlight a few activities that have been particularly effective in teaching conjecturing. Materials are available for class testing. Our results couldn’t be better: students love the course, spend a lot of time working in and out of class, are prepared for advanced proof courses, and more students are continuing in mathematics. Work supported in part by AIBL Small Grant. (Received September 13, 2016)

- 1125-G1-1205      **Karl-Dieter Crisman\*** ([karl.crisman@gordon.edu](mailto:karl.crisman@gordon.edu)). *When IBL drops in to Calculus: A cautionary tale*. Preliminary report.

Having experimented with a few mini-IBL activities in our one-semester applied calculus course, I received an Academy of Inquiry-Based Learning grant to develop and test more such activities to ‘drop in’ to an otherwise non-IBL course at strategic points. After several semesters of using them, I will present some conclusions - both encouraging and cautionary – for practitioners with constraints keeping them from doing a completely student inquiry driven course.

Briefly, it is possible to successfully use quite inquiry-based activities in such a limited context, but (in line with the MAA National Study of College Calculus regarding ‘ambitious teaching’) the relationship with student achievement is mixed. Naturally, there will also be plenty of demonstrations of the activities themselves. (Received September 15, 2016)

- 1125-G1-1400      **Celil Ekici\*** ([celil.ekici@uvi.edu](mailto:celil.ekici@uvi.edu)), RR01 Box 10,000, University of the Virgin Islands, Department of Mathematical Sciences, Kingshill, VI 00850, and **Christopher Plyley**. *Inquiry based Calculus with Difference: Continuous and Discrete Modeling of Mathematics in Population Growth*.

Building towards the Common Vision for Mathematical Sciences in 2025, this work contributes to the reform of college math teaching by developing inquiry based learning activities which allow undergraduate students to experience deeper connections between continuous and discrete approaches to mathematical modeling. We use population growth involving locally relevant data (fly and lionfish) as a context to develop both calculus and discrete mathematical concepts, such as recurrence and rates of change. We will share the lessons learned from three cycles of implementation of these novel inquiry based learning activities across Calculus I and II and Discrete Mathematics courses. These novel activities provide a thread across these courses which develop connections between difference and differential equations in the context of population dynamics. The developed activities engage students to use descriptive, numerical, algebraic, and graphical representations while incorporating relevant instructional technologies to facilitate student inquiry and discussion. We will share student learning outcomes as well as their engagement and assessments. (Received September 16, 2016)

- 1125-G1-1505      **Thomas J Clark\*** ([tom.clark@dordt.edu](mailto:tom.clark@dordt.edu)), 498 4th Ave. NE, Sioux Center, IA 51250. *Weighing Fog: Hands on Modeling for Day 1 of Differential Equations*.

The first day of many mathematics classes contains formalities and very little mathematics. Here an alternative is presented where modeling is placed as the centerpiece to orient students to the real work of differential equations. Namely, to capture as beautifully and compactly as possible through the process of conjecture and investigation, the deep and interesting aspects of the physical world. A demonstration of the sublimation of dry ice sits at the center of the lesson. Students collaborate in groups to design an experiment that could measure the change in mass of a piece of dry ice that is dropped into water, the experiment is then carried out, and finally the students (with some guidance) build and solve a model for the phenomenon. I will also present my reflections on the lesson as well as guidance and resources for use in the differential equations classroom. (Received September 17, 2016)

1125-G1-1724 **Susan B Crook\*** ([susan.crook@loras.edu](mailto:susan.crook@loras.edu)). *IBL Calculus I Successes and Failures.*

This talk will detail assignments and activities given in two sections of Calculus I in Fall 2016, totaling more than 50 students. Some activities have been used previously, but tweaked due to feedback and others were newly developed for this semester. The presentation will provide the prompts, worksheets, or assignments and samples of student responses. Anecdotal evidence of success or failure will be given along with discussion of how the materials will be changed in the future. (Received September 19, 2016)

1125-G1-1781 **Meghan De Witt\*** ([mdevitt@stac.edu](mailto:mdevitt@stac.edu)). *First steps in IBL with students who have never proved a mathematical result before.*

The author recently taught her first IBL course—Complex Analysis—at an institution where students receive little to no training in proofs or logical argument. We describe the lessons learned and modifications made to the teaching method in order to reach the students and teach them not only the required topic but also how to formulate, structure, and defend their results. This includes a self-grading scheme whereby the students recognized their own level and consistently self-evaluated their progress. (Received September 19, 2016)

1125-G1-1811 **Anneliese H. Spaeth\*** ([aspaeth@hawks.huntingdon.edu](mailto:aspaeth@hawks.huntingdon.edu)). *Using a Problem Sequence to Teach Mathematics Majors Basic Programming Skills.* Preliminary report.

Many effective problem sequences and materials exist to assist educators in using Inquiry-Based Teaching and Learning techniques over a variety of mathematical topics. The author notes a lack of such materials covering basic programming skills, oriented specifically for mathematics majors. The author developed one such problem sequence addressing the language PHP, during the Fall 2016 semester in a one credit seminar for mathematics majors. Topics discussed will include: the developed problem sequence, the author's experience in teaching this course, and a rubric created by the author for grading student code solutions. (Received September 19, 2016)

1125-G1-1821 **Audrey Malagon\*** ([amalagon@vvc.edu](mailto:amalagon@vvc.edu)), **Lydia Kennedy** and **Kristin Burney**. *Don't Drink the Kool-Aid!*

This hands-on inquiry based differential equations project gives students the opportunity to experiment and explore mixing problems in a lab setting. Using powdered drink mix, and two stacked beverage dispensers, students explore flow rates and how the amount of drink powder in a tank changes when varying concentrations of powder and water mixture are used. (Received September 19, 2016)

1125-G1-1829 **Andrew-David Bjork\*** ([abjork@sienahights.edu](mailto:abjork@sienahights.edu)), **John Duvall** and **Jesse Stires**. *Experiences in an IBL Numerical Analysis course.*

A couple of years ago I had the privilege of attending an MAA workshop at Kenyon College put on by AIBLM (the Academy for Inquiry-Based Learning in Mathematics). While there I worked specifically on course notes for a numerical analysis course. This past Fall I have used these notes for a second time. Along with some of my students, I propose to give a quick overview of the course, and live feedback from some of those who survived these trials (that would include me), and some of those who actually enjoyed the class (hopefully my students). Two of the co-presenters are students who will share their perspectives on the course. (Received September 19, 2016)

1125-G1-1944 **James B Collins\*** ([jbcollie2@gmail.com](mailto:jbcollie2@gmail.com)). *Effect of Classroom Setup on Student Learning.* Preliminary report.

The physical setup of the classroom can be an important tool in making a classroom more active and student centered. This semester I was able to move the desks around in my classroom to accommodate group work and discussion. In this talk I will discuss how I was able to make this change within my department. I will also discuss the benefits I have seen in student interaction and learning with this new classroom setup, both in a strict IBL classroom, as well as a more lecture based classroom. (Received September 19, 2016)

1125-G1-2009 **Jessica L Williams\*** ([jessica.williams@converse.edu](mailto:jessica.williams@converse.edu)), 580 East Main Street, Converse College, Spartanburg, SC 29302. *A Novice Attempt at Teaching IBL Real Analysis.* Preliminary report.

Real Analysis is an upper level course often taken as an elective in the later stages of a student's undergraduate career. At least one semester of a Real Analysis course is typically offered as part of the undergraduate mathematics curriculum at four-year institutions. Encouraged by research supporting the effectiveness of inquiry-based learning in undergraduate proof-based courses, a first attempt at employing IBL was made in an undergraduate real analysis course at a small college. Structure of the overall course, breakdowns of typical class periods, resources utilized, and modifications made will be discussed. Challenges and successes of teaching in an IBL

fashion for the first time will be the focus, along with anecdotal evidence of changes in student's attitudes and beliefs gathered from surveys and written assignments. (Received September 19, 2016)

1125-G1-2041 **Heather A Lewis\*** (hlewis5@naz.edu). *Writing IBL Notes for a Textbook-Free Class.* Preliminary report.

The switch from teaching a class lecture-based to activity-based is often accompanied by finding or writing a suitable collection of problems to use in place of a textbook. In this talk I will share my own experiences with both non-majors and majors courses: where I looked for inspiring source material, how I went about writing my own problems, and the most effective ways I've found for polishing those notes. (Received September 19, 2016)

1125-G1-2049 **Matthew A Morena\*** (mamorena@yhc.edu). *Students Teaching Students Through Video Presentations.* Preliminary report.

Teaching students to conceptual rather than procedural levels is often a goal of mathematics instructors. Strategies for doing so include interdisciplinary projects, flipped classrooms, and class presentations. One barometer for success is an affirmative to the question: can a student effectively teach a concept to other students? In a recent Multivariable Calculus class, I sought to combine all three teaching strategies into one technique: student-made video presentations. First, students select challenging, interdisciplinary problems and then, outside of class, the students video-record themselves demonstrating their solutions. Using the media platform VoiceThread, the video presentations are uploaded to a class website whereupon the students are also encouraged to study and learn from their classmates' videos. In this talk, I will share my observations and experiences using this technique, as well as those of my students. (Received September 19, 2016)

1125-G1-2256 **Nermin Bayazit\*** (nbayazit@fitchburgstate.edu) and **Florian Enescu.** *How Can We Foster Collaboration and Inquiry in an Online Mathematics Course?* Preliminary report.

Fully online courses are getting more attention and demand among students at every level, arguably more among professional graduate students like teachers pursuing master's degrees. Conference Board of the Mathematical Sciences (CBMS) recently called for action to incorporate active learning into post-secondary mathematics classrooms. In response to this call, we will share an assignment that was carefully crafted to provide opportunities to "engage in mathematical investigation, communication, and group problem solving, while also receiving feedback on their work from both experts and peers" (CBMS, 2016). This assignment was designed to be used in a fully online graduate mathematics course offered to in-service secondary mathematics teachers. We will share our perspective in regard to the challenges and affordances as the instructors of such course and some feedback that we received from the students of this course. (Received September 20, 2016)

1125-G1-2306 **Jonathan P Keiter\*** (jkeiter@esu.edu), East Stroudsburg University, Mathematics Department, 200 Prospect St, East Stroudsburg, PA 18301. *Using Inquiry-Based Learning to Explore Applications of Integration.*

The content and concepts in a second semester Calculus course challenges many students. The course is a math major course, but it serves all science majors. The majority of the students in each section are not majoring in mathematics. We developed inquiry-based in-class projects to help the students explore together the techniques and applications of integration. We will discuss the methods used to get the class engaged. We will describe and share the projects that we use. (Received September 20, 2016)

1125-G1-2313 **Elizabeth W Schott\*** (eschott@fsw.edu), Department of Mathematical Sciences, 8099 College Parkway, Fort Myers, FL 33919, and **Laurice Garrett** (laurice.garrett@fsw.edu), Department of Mathematical Sciences, 8099 College Parkway, Fort Myers, FL 33919. *Incorporating Inquiry Based Learning into a Mathematics Foundation Course at Florida SouthWestern State College.*

In recent years, inquiry based learning has risen in the literature as an innovative method to teach college mathematics. Originally founded in the Moore Method and applied to theory and proof courses, the method has more recently been applied to a vast number of courses and topics in many disciplines. Personal faculty teaching experience led us to be interested in this teaching methodology. We believed that this method of teaching had the potential to greatly increase student engagement and thus increase student success not only in upper level mathematics courses but also in mathematics foundation courses. Thus, we incorporated inquiry based learning into the curriculum for MAT 1100 - Mathematical Literacy for College Students at Florida SouthWestern State College. We will share the results of a one semester pilot program that included two different textbooks, four professors and multiple sections. Measured outcomes focused on the assessment of student engagement in the course. (Received September 20, 2016)

1125-G1-2317 **Jessie Hamm\*** ([hammj@winthrop.edu](mailto:hammj@winthrop.edu)). *Lessons Learned from a First Attempt at IBL*. Teaching a new course is always challenging but teaching a new course using a new technique can be overwhelming. In this talk I will discuss my first attempt at teaching an Introduction to Proofs course using a Modified Moore Method. I will share lessons learned, plans for future improvement, and advice to other novice IBL instructors. (Received September 20, 2016)

1125-G1-2347 **Carol Gwosdz Gee\***, St. Edward's University, 3001 S. Congress Ave, Austin, TX 78704. *Cut-Apart Proofs: a hands-on activity in varied contexts*. Preliminary report. In this hands-on session, we present several examples of "cut-apart proofs" – jigsaw puzzle-like small group activities that introduce and emphasize the importance of logical structure in a mathematical proof. A selection of proof examples reinforcing essential definitions will be drawn from calculus, discrete math, abstract algebra, and topology. We will also include ideas to provide differentiated levels of challenge within the same activity and ways to use these activities at various stages in the learning process. (Received September 20, 2016)

1125-G1-2434 **Xiao Xiao\*** ([xixiao@utica.edu](mailto:xixiao@utica.edu)). *IBL in very small classes*. In very small IBL classes (between 3 and 10 students), ideas are often harder to generate compared to larger classes when students get stuck. In this talk, we discuss two practices that address this challenge.

The first practice is an online draft-feedback process which we learn from Patrick Rault. To students, the feedback from the instructor will give them an additional chance to improve their work and sometimes even a jump start to a problem before the formal discussion of the problems in class. To the instructors, this will give them a much better idea of how much can be achieved and what needs to be prepared for the next class, which makes the class much more effective. If the extra feedback is still not enough to generate ideas and discussions, we will use the second practice, which is team-solving the problem during the class. I have been using both of these practices for at least six courses and students were able to solve nearly all the problems in the task-sequence (including the case when the class only had 3 students). (Received September 20, 2016)

1125-G1-2497 **Robert L Sachs\*** ([rsachs@gmu.edu](mailto:rsachs@gmu.edu)), MSN 3F2, George Mason University, Fairfax, VA 22030-4444. *Towards guided reinvention of Riemann sums and the Fundamental Theorem of Integral Calculus*.

Student reinvention of the Riemann sum of products and the Fundamental Theorem of Calculus makes for an inspiring climax to first-semester calculus. Presented with distance/velocity, cost and earnings tasks requiring use of average values on subintervals, students create Riemann sums. Some then go on to formulate the Fundamental Theorem of Calculus in its global (fixed endpoint integral) and then local (moving endpoint and derivative thereof) versions. Ongoing refinements of tasks aims to increase the number of successes and productive misses. (Received September 20, 2016)

1125-G1-2541 **Jonathan K. Hodge\*** ([hodgejo@gvsu.edu](mailto:hodgejo@gvsu.edu)). *Implementing inquiry-based learning via online polls*. Preliminary report.

In this talk, we'll explore ways that online polling software can be used to promote inquiry-based learning in courses throughout the mathematics curriculum. The talk will include a demonstration, so please be sure to bring a phone, laptop, or other mobile device. (Received September 20, 2016)

1125-G1-2564 **Sandra L. Laursen\***, 580 UCB, Boulder, CO 80309-0580, and **Zachary Haberler** and **Charles N. Hayward**. *Messaging for a movement: Names, ideas, and inclusion in the movement for inquiry-based learning in mathematics*.

A robust body of education research literature supports the use of active learning approaches to strengthen student learning and persistence in undergraduate STEM courses. Within mathematics, inquiry-based learning (IBL) is one approach that has gained currency in recent years. The number of IBL practitioners and the legitimacy of IBL as an educational change movement have grown through conferences, workshops, a journal, and new support organizations, led by volunteer leaders with help from funders and professional societies. These changes offer evidence that the movement is having some success, yet the challenge is still large: many students do not yet have access to these enriching mathematical experiences. To further inform and assist instructors to implement and succeed with IBL methods, the community must not only continue its outreach and professional development work, but consider how its messages, whether explicit or unintended, are understood by others. We present recent research findings that demonstrate the importance of terminology such as 'IBL' and 'Moore Method' for explaining the movement and defining its membership. These findings have implications for how the IBL community acknowledges its history and for how it communicates and enacts welcome to all. (Received September 20, 2016)

1125-G1-2568 **Brian Katz\*** ([briankatz@augustana.edu](mailto:briankatz@augustana.edu)), 639 38th St, Rock Island, IL 61201, and **Victor Piercey**. *SIGMAA IBL: Making our Future Proactively Inclusive*.

Teachers and researchers come to the IBL community for diverse reasons, and this SIGMAA is committed to a broad and inclusive definition of IBL in terms of both classroom implementation and the populations it can serve, including those populations historically underserved by our discipline. The previous sentence is taken from the mission statement of the SIGMAA, and it represents a powerful commitment to inclusivity. However, as SIGMAA IBL is born, we must go further, from a statement of values to a plan to make our organization proactively inclusive. This session will be highly interactive, with discussion of next steps and barriers to living up to our commitment. (Received September 20, 2016)

1125-G1-2586 **Debra K. Borkovitz\*** ([dborkovitz@wheelock.edu](mailto:dborkovitz@wheelock.edu)). *Clock Buddies: An Engaging, Open-Ended Scheduling Activity with Mathematical Depth and Pedagogical Flexibility*.

Clock Buddies is my favorite first day of class activity. It starts as a non-threatening icebreaker activity that helps students learn each other's names, but it soon asks students to find their own strategies for solving a real-world scheduling problem. Even highly math phobic students work with others and succeed. Over the years students have devised numerous strategies for approaching the problem, with connections to graph theory and other topics in discrete mathematics, as well as to number theory, mathematical modeling, algorithms, and probability. Students gain insight from creating different visual representations of the problem and then comparing and contrasting them. I have used the activity mostly for the first few days in my Math for Elementary Teachers classes, but it can be adapted for both shorter and longer lengths and for different courses and levels. The talk will include discussion of the mathematics and the pedagogy of Clock Buddies. (Received September 20, 2016)

1125-G1-2590 **Kassie Archer\*** ([karcher@uttyler.edu](mailto:karcher@uttyler.edu)), 3900 University Blvd., Tyler, TX 75799. *Discovering Geometry*.

The presenter will discuss her experience with teaching undergraduate-level Euclidean and Non-Euclidean Geometry. This includes daily homework, presentations, projects, hands-on learning, making conjectures, and a discovery-based final exam. (Received September 20, 2016)

1125-G1-2599 **Tim Whittemore\*** ([twhittemore@exeter.edu](mailto:twhittemore@exeter.edu)) and **Aviva Halani** ([ahalani@exeter.edu](mailto:ahalani@exeter.edu)). *Shared Presentations: Encouraging Clear Communication through Divided Roles*. Preliminary report.

Some inquiry-based classes focus on students working through tasks selected from a problem-based curriculum for homework and then presenting their solutions to the class. These solutions are sometimes incomplete, imprecise, and perhaps impractical. Mathematical discussions can lead to more complete, more precise, and more practical solutions that are understood by all members of the class. Clear communication is essential for this to happen, but might fall out of focus as more attention is given to students mastering content. In this presentation, we discuss an alternative approach to the structure of math classes described above. In Shared Presentations, one student submits a solution to a given task and another student presents that solution to the class. The purpose of this alternate structure is to push students to interpret, communicate, and critique the arguments shared by a peer. For these actions to be possible, students must pay special attention to the clarity of their written work so that it accurately conveys the intended reasoning to an informed audience. The presenter must be able to step outside of his or her own solution method in order to understand the first student's solution. We report on an implementation of this instructional approach in this presentation. (Received September 20, 2016)

1125-G1-2613 **Mariah Birgen\*** ([mariah.birgen@wartburg.edu](mailto:mariah.birgen@wartburg.edu)), 100 Wartburg Blvd., Waverly, IA 50677. *Liberal Arts Mathematics and Guided Learning Worksheets – IBL for non-majors*. Preliminary report.

After teaching Real Analysis and Introduction to Proofs using IBL techniques I was sold on the significantly improved learning that happens when a professor puts some, or most, of the burden of learning mathematics on the students. However, it seems to be unfair to only give my best effort to the students who are already convinced of the value of mathematics. I have been teaching Mathematics for Democracy for over five years and decided that these students deserved my attention as well. After all, they are the ones who are really paying my salary.

In Summer 2013, I was able to get ahold of notes for a Liberal Arts math course – IBL style by Anders O.F. Hendrickson. Using a significant portion of his notes and adding my own as necessary, I created a set of notes for a 14 week semester course focused on the mathematics involved in the election of the U.S. President. Unlike my more advanced courses, this classroom is run in more of a guided-lecture style. Students work discovery

problems in the coursepack and then I take the time to help them craft definitions of terms that are used to discuss the concepts more generally. Thus, the students are able to internalize the definitions because they are written in their own language. (Received September 20, 2016)

1125-G1-2619 **James Carter\*** (jmcarter@butler.edu). *Autonomous Learning in College Algebra*. Preliminary report.

By providing students with a large list of problems on which they are allowed multiple attempts, assigning them to groups rotating weekly, and using a binary grading system this method of presenting the material in a College Algebra course will enhance several aspects of student learning. Since students are allowed to submit problems in any order, they are able to focus on those with which they already have experience boosting their confidence early in the term while allowing multiple attempts on each problem alleviates some pressure for immediate understanding, however, the binary grading system moves the focus to ultimately understanding the material. This talk will summarize the results of applying this presentation method to a college algebra course during the fall term of the 2016 school year. (Received September 20, 2016)

1125-G1-2645 **Marie Snipes\*** (snipesm@kenyon.edu), **Tom Asaki** (tasaki@wsu.edu), **Chris Camfield** (camfield@hendrix.edu) and **Heather Moon** (hamoon@lcmail.lcsc.edu). *Using Image Processing to Inspire Inquiry in Real Analysis Courses*. Preliminary report.

Real analysis techniques play an integral role in modern techniques for examining and manipulating data, but all too often these applications are left out of undergraduate classes on the subject. One particularly nice application is the problem of removing the noise from a corrupted image. In this talk we describe how this application has been incorporated into inquiry-based undergraduate real analysis classes. Students start by examining images in Matlab/Octave and deciding what makes an image noisy. They then use their observations to brainstorm methods for removing noise, culminating in the development of a calculus of variations technique for image denoising. Their explorations provide the motivation for understanding fundamental real analysis concepts such as metric spaces, Cauchy sequences, convergence, and derivatives. We will describe results of using this material with our classes and briefly talk about how the material can be modified for use in either a one-semester introductory analysis course or a two-semester sequence. This work is part of the NSF-funded IMAGE Math project. (Received September 20, 2016)

1125-G1-2659 **Kayla Bradley Dwelle\*** (dwellek@obu.edu), 410 Ouachita Street, Box 3739, Arkadelphia, AR 71998. *Do Math Long and Prosper: An Experiment in "Gamifying" an Active Learning Classroom*. Preliminary report.

Whether a short time or long time practitioner of teaching in an active learning classroom, most of us find that the heart of our success depends greatly on students taking personal responsibility for their learning and, consequently, their performance in an active learning course. Though ultimately that responsibility falls on them, I believe we can be powerful influences to and motivators for this personal responsibility.

I now have several years of practicing teaching in active learning classrooms. I have discussed active learning with other practitioners at workshops, conferences, and during campus visits. I have tried many strategies and ideas to promote personal responsibility. Some have been successful, others not so much.

Aspiring to try something new in my fall courses, I launched a synthesis of several moderately successful to very successful strategies, culminating in an overall theme, rules, and class currency. In other words, I "gamified" my courses. In this talk, I will discuss the details of this implementation, what worked, what did not, and how I attempted to assess the change in student's personal responsibility for their learning as a result of the game. (Received September 20, 2016)

1125-G1-2676 **Stephen F Bismarck\*** (sbismarck@uscupstate.edu), 800 University Way, Spartanburg, SC 29303. *Inquiry-Based Teaching and Learning in the Mathematics Classroom*.

The session will define inquiry-based teaching and learning, relate this practice to the mathematical process standards (more specifically the standards for mathematical practice), and provide examples of tasks showing ways students can use mathematics (and in some cases technology) to make sense of situations. (Received September 20, 2016)

1125-G1-2726 **Galit Eizman\*** (galiteiz7@gmail.com), po box 1034, Brookline, MA 02446. *"Rethink, Revise, Research" Encouraging Critical and Scientific Thinking*.

What could encourage, promote and establish research skills? The existing literature reflects the importance of critical thinking in developing research abilities for young students and prospective research scholars. Glaser (1941) was one of the first to point out that critical thinking is a crucial tool for questioning, inquiring and rethinking, hence for creating the future generation of researchers. Moreover, as early as the exposure to critical

thinking is, the better it would be for further research purposes, as it maintains the natural curiosity of young students and teaches to question concepts and theories without taking them as obvious or for granted. The case of international students with lack of critical thinking culture or background might be challenging in particular. This paper presents an experiment of applying a sample of interactive and original teaching methods, some are surprisingly simple to implement, in pre-college summer courses for Chinese students in Beijing, in mathematical courses of macro-economics and international economics, taught by American professors. The results reflect the tendency of students to choose a current and future research path. As a following project, an academic initiative was created for promoting young researchers in sciences. (Received September 20, 2016)

1125-G1-2820 **Joel T. Patterson\*** ([jpatterson@cpsd.us](mailto:jpatterson@cpsd.us)), 459 Broadway, Cambridge, MA 02138.  
*Constructing Inquiry Lessons in High School Geometry.*

Take the instructions out of constructions so the compass and ruler become tools students choose to answer problems. The presenter will offer a few simple examples of activities that high school geometry teachers can use to begin to explore incorporating inquiry and active learning into their curriculum on constructions. The first of these is an activity that leaves students asking and figuring out “What happened here?”: what transformations could have been applied to cause one figure to be changed into another, and how can we learn as much as possible about those transformations? The second is an activity asking “Is this polygon cyclic... or not?”: can a circle intersect all vertices? Based on work to study how and why other high school geometry teachers begin to experiment with inquiry and activity-based learning in their classrooms, the presenter will describe his experience with the activities, and offer a few guidelines to encourage other instructors to consider incorporating inquiry and activity-based learning. (Received September 20, 2016)

1125-G1-2834 **Theron J Hitchman\*** ([theron.hitchman@uni.edu](mailto:theron.hitchman@uni.edu)), Department of Mathematics 0506, University of Northern Iowa, Cedar Falls, IA 50613. *The IBL SIGMAA: Chair’s farewell and Business Meeting.*

This will be a proxy for a shortened business meeting for the IBL SIGMAA. (Received September 20, 2016)

1125-G1-2843 **Gail Tang, El Turkey Houssein, Cilli-Turner Emily\*** ([emilyct@uw.edu](mailto:emilyct@uw.edu)), **Milos Savic, Gulden Karakok and David Plaxco.** *Inquiry as an Access Point to Equity.*

Many policy documents and statements from several organizations (e.g., NCTM, CBMS, MAA, PCAST) have issued calls to improve STEM education through the use of active learning practices to provide learning opportunities for all students in STEM fields. Existing research indicates that such teaching practices support greater learning gains for historically underrepresented students in STEM, while not harming others (e.g., Kogan & Laursen, 2014). In this presentation, we will explore the theoretical ways in which inquiry teaching practices (e.g., Inquiry-Based Learning (IBL) or Inquiry-Oriented Learning (IOL)) could potentially support equity in the classroom. Specifically, we will merge characteristics of inquiry (Student Ownership, Knowledge Building, Peer-Involvement, Doing Mathematics, Student-Instructor Relationship, and Student Success) put forth by Cook, Murphy, and Fukawa-Connelly (2016) with the Four Dimensions of Equity (Access, Achievement, Identity, and Power) proposed by Gutiérrez (2009). Finally, we will discuss how engaging in practices of IBL/IOL could provide opportunities for instructors to implement more equitable practices. (Received September 20, 2016)

1125-G1-2885 **Aviva Halani\*** ([ahalani@exeter.edu](mailto:ahalani@exeter.edu)) and **Tim Whitemore** ([twhitemore@exeter.edu](mailto:twhitemore@exeter.edu)). *Practicing Peer Review: Making Sense of Other Peoples’ Mathematical Perspectives.*

Inquiry-based classes often revolve around students presenting solutions to their peers. Students are typically more comfortable with solutions with which they are more familiar and, because of this, alternate solutions are often injected into the conversation, disrupting the flow of the discussion. Although exploring multiple approaches is commendable and educative, these solutions are frequently proposed because classmates have trouble interpreting and processing the presented solution. In this presentation, we discuss an attempt to increase students’ willingness and ability to entertain solution methods different than their own. Students were asked to analyze and critique solutions previously produced by others. We report on our implementation and the perceived impact of this instructional exercise. (Received September 20, 2016)

1125-G1-2898 **Gary A. Olson\*** ([gary.olson@ucdenver.edu](mailto:gary.olson@ucdenver.edu)), University of Colorado Denver, Denver, CO. *College Algebra TACTivities and the TA Coach Experiment.*

In this session, I will share sample TACTivities that were developed to promote active learning in the college algebra classroom. These TACTivities were designed to be engaging, tactile in nature and foster discussion among students regarding the algebra content. The TACTivities were utilized along with a TA Coach in an effort to redesign College Algebra recitations at our university. The impact of this project on DFW rates and final

exam performance will also be discussed. This work is sponsored by an NSF DUE grant #1539602. (Received September 20, 2016)

1125-G1-2953 **Joshua P. Bowman\*** (joshua.bowman@pepperdine.edu), Natural Science Division, Pepperdine University, 24255 Pacific Coast Highway, Malibu, CA 90263-4321. *Learning real analysis through discussion and presentation*. Preliminary report.

I will report on the effectiveness of a first-semester real analysis course structured around alternating discussion based on reading assignments and presentation of solutions to assigned exercises. Time permitting, I will also discuss the use of specifications grading to support the goals of these class activities. (Received September 20, 2016)

1125-G1-2971 **Joanna G. Jauchen\*** (jbozett2@gmu.edu). *Reflective Journaling in Quantitative Reasoning*.

Reflective practice in education has a long history, including the foundational tenets proposed by Dewey in the 1950s. In contemporary life, including our math curriculum, we rarely create time or space for reflection and the learning that accompanies it. In this talk, the author describes her experience piloting a Quantitative Reasoning course designed around mini-lectures, small-to-medium-sized problems and reflective journaling. Students in the course were given time to explore mathematical concepts in contexts of their own choosing. Emphasis was placed on concrete application of quantitative reasoning principles to subject matter that was individually meaningful to each student. Within the talk, I will describe the course, a typical class day and describe assignment structures. (Received September 20, 2016)

1125-G1-3074 **Carl Toews\***, toewsc@gmail.com. *IBL with Jupyter notebooks*.

This talk reports on a recent series of upper level IBL-style classes featuring guided computational inquiry (CGI) activities with Jupyter notebooks. One of the barriers to providing substantive computational inquiry activities in the undergraduate classroom is the overhead in teaching students how to code. Jupyter notebooks breach this barrier by allowing executable code and typeset mathematical exposition to be embedded in the same document, tremendously streamlining the passage from exposition to code. Unlike other notebook-type platforms, the Jupyter kernel is configurable, allowing the same basic notebook structure to be used with Python, R, Octave and many other languages. This talk summarizes the results of using Jupyter notebooks with a Python kernel to construct IBL flavored optimization and numerical analysis classes. Student feedback from these courses suggests that the Jupyter notebook can be a powerful element around which to design classroom activities that appeal to students with divergent coding backgrounds. (Received September 20, 2016)

1125-G1-3089 **Elizabeth Thoren\***, ethoren@pepperdine.edu. *Leveraging Context to Make Old Ideas New Again*.

Most Precalculus and many Calculus students have been exposed to the concepts from those courses in high school, and often come away from the experience with significant holes in their procedural fluency and/or conceptual understanding. This situation leaves educators with a dilemma: How do we re-introduce these concepts to a varied audience that includes the “experienced” as well as the “uninitiated”. One way to address this problem is to leverage context to open problems up and foster deeper conceptual understanding. In this talk I share some carefully crafted mathematical situations that promote inquiry to level the playing field. (Received September 21, 2016)

1125-G1-3129 **Jeffrey J King\*** (jeffrey.king@unco.edu), **Gulden Karakok** (gulden.karakok@unco.edu) and **Nathaniel Miller** (nathaniel.miller@unco.edu). *Students’ Social Adaptation to Mathematical Tasks*. Preliminary report.

In this study, an advanced undergraduate geometry class taught in an inquiry-based learning setting was observed for social and socio-mathematical norms. Three pairs of students engaged in three task-based, semi-structured interviews: paired, individually, then paired again, solving the Seven Bridges of Königsberg and related tasks. A fourth stimulated-recall interview was performed using episodes from the last paired interview. Classroom observations and interview discourses were open coded for themes, structure, and function to analyze the norms developed within the classroom and by each pair as shaped by their social interactions. Tentative findings include: 1) norms of consensus, autonomy, and argumentation produced within the classroom, 2) varying metaphors across interview contexts, and 3) reliance on empirical strategies rather than structural reasoning. In this preliminary report, evidence from collected data is shared and a brief discussion how these results could help inform IBL teaching methods is included. (Received September 21, 2016)

## *Integrating Research into the Undergraduate Classroom*

1125-G5-522     **Tanweer J Shapla\*** (tshapla@emich.edu), Mathematics Department, Eastern Michigan University, Ypsilanti, MI 48197, and **Khairul Islam** (kislam@emich.edu), Mathematics Department, Eastern Michigan University. *Using classroom as a venue for undergraduate research.*

A classroom is the place where students find problems firsthand and learn strategy to solve them. Using real-life applications in the lesson plan, giving activities in the class, incorporating adequate technology in problem solving, assigning projects as assessment tools, discussing benefits of undergraduate research with students while teaching, etc. help students motivated and engaged in undergraduate research. Due to the early exposure to research activities, we have found that students with an undergraduate research get into internship, job or the desired graduate program by making them competitive among others. In this presentation, we provide examples and applications that we have found useful to engage students in undergraduate research while teaching, and share some impact on students in achieving their goals. (Received September 20, 2016)

1125-G5-1603     **Caroline Haddad\*** (haddad@geneseo.edu), 1 College Circle, SUNY Geneseo, Geneseo, NY 14454. *Final Projects that Give a Taste of Research.* Preliminary report.

Final group projects in lieu of final examinations can be a rewarding end to an undergraduate course, culminating in a project and presentation that brings together everything they learned in the course, and more. Final projects work well in some courses, and are now a permanent feature in my Numerical Analysis, Intro to Wavelets, and Linear Programming and Operations Research courses. Student groups of 2-3 must research a relevant topic, investigate, solve or recreate a solution, disseminate a summary of the results in a short paper, and orally present the entire projects. Students handouts describing my expectations, a timeline, as well as some project ideas will be presented. In addition, sample final project presentations will be shown, along with student opinions of the course and their projects. Student challenges and instructor difficulties may be touched upon, including those where I believe the projects didn't work as well as a final exam. (Received September 18, 2016)

1125-G5-1980     **Abe Edwards\*** (aedwards@msu.edu). *Great Pedagogical Gains from Mentoring Undergraduate Research in Calculus I.*

“What are some limitations of common images and metaphors from freshman calculus?” This question is illustrative of the independent research questions that students have pursued in my Calculus I courses. This talk describes the start-to-finish experiences of students who elected to pursue undergraduate research as part of an honors option for Calculus I. In particular, the talk explains how students work through the research process—from choosing a question, to collecting and analyzing data, to drawing conclusions and reporting the research to a broader audience. The talk emphasizes how mentoring undergraduates in research has improved my own teaching of calculus, and how I learned that the Unit Circle may occasionally fail us. (Received September 19, 2016)

1125-G5-2120     **Carolyn A Otto\*** (ottoa@uwec.edu) and **Manda R Riehl** (riehlar@uwec.edu). *Development and Implementation of a Research Methods Course.* Preliminary report.

In this talk, we will discuss the development and implementation of our new math research methods course. The main premise of this course is to instruct future mathematicians on the art and procedures of mathematics research. We will discuss the structure and content of the course, which includes an original student research project that utilizes all the skills they had learned over the course of the semester. A description on how this course and research project help prepare students for future undergraduate and graduate research will be addressed. In addition, we will show how this course and its course work fits into our new comprehensive math research major. (Received September 19, 2016)

1125-G5-2352     **Malgorzata Aneta Marciniak\*** (mmarciniak@lagcc.cuny.edu), 4266 Phlox Place, Apt C7, Flushing, NY 11355, and **Marina Nechayeva** and **Vladimir Przhhebelskiy**. *Aspects of Calculus 3 in flexible solar panels and other renewable sources of energy.*

Renewable sources of energy are not only trendy but can also serve as a source of topics for math projects in all levels of Calculus classes. In the past we experimented with aerodynamics and recently our offer got extended to wind turbines and solar panels with a strong emphasis on creativity. In the project that will be presented during the talk students design new shapes of solar panels and evaluate the energy collected through a sunbeam with a modified formula of the flux. The project uses Calculus 3 topics: surfaces, space curves, vector fields, and multiple integrals. (Received September 20, 2016)

1125-G5-2377 **Jeffrey S. Powell\*** ([jspowell@samford.edu](mailto:jspowell@samford.edu)), Dept. of Mathematics & C.S., Samford University, 800 Lakeshore Dr, Birmingham, AL 35229. *Mickey Mouse, Kevin Bacon, and How Undergraduate Research Opened a Whole New World For Me.*

Our Senior Project course pairs faculty with our graduating seniors to produce a paper and presentation on a topic that is new to the student and typically in an area of the student's interest. For some students, there's an opportunity to discover new mathematics. In this talk, I'll share a particularly interesting case where I worked with a Disney-obsessed student to build and to analyze the previously unstudied Disney short film network. I'll discuss how the project allowed the student to learn about social network analysis while it simultaneously expanded my own research horizons into this fruitful area for undergraduate research. In this talk, I'll also answer the following important question: Are Mickey Mouse and Kevin Bacon equivalent? (Received September 20, 2016)

1125-G5-2628 **George E Cazacu\*** ([george.cazacu@gcsu.edu](mailto:george.cazacu@gcsu.edu)), 3023 McComb CT NE, Milledgeville, GA 31061. *The irresistible attraction of big mathematical ideas - Creating an interest in undergraduate research.*

This work is exploring ideas about creating and perfecting classroom environments and activities that facilitates undergraduate research. It presents a collection of mathematical problems and examples that are meant to attract and challenge students' mathematical thinking. With topics ranging from simple geometry to paradoxes, from arithmetic to million dollar problems, undergraduate research will open the way for new fermats. (Received September 20, 2016)

1125-G5-2643 **Tom Asaki\*** ([tasaki@wsu.edu](mailto:tasaki@wsu.edu)), Department of Mathematics and Statistics, PO Box 643113, Washington State University, Pullman, WA 99164, and **Marie Snipes, Heather Moon** and **Chris Camfield**. *Image and Data in the Classroom: Research and Research-like Experiences.*

A student's ability to engage in scientific research is becoming the norm for undergraduates. The classroom can be a significant stepping stone toward achieving necessary skills and relevance through research practices. We present students with real-world questions as initial motivation for mathematical concepts, rather than as a justifying application. Activities designed as open-ended explorations provide research-like experiences, training students to grapple with undefined concepts and the unknown. Current research topics for classroom use include limited view tomography scenario development, improved noise removal from images, and visually-pleasing image blending. While students gain new appreciation for the utility of mathematics in everyday life, they also gain confidence in their own skills and ability to address research questions. Research questions presented near the end of the semester/quarter can now be viewed through a mathematically enlightened and critical lens. Modules for classroom use are being developed and tested as part of the NSF-funded IMAGE Math project. (Received September 20, 2016)

1125-G5-2828 **Shannon R Lockard\*** ([slockard@bridgew.edu](mailto:slockard@bridgew.edu)). *Creating and Investigating Classes of Graphs*. Preliminary report.

Due to its visual nature, Graph Theory is an excellent class for students to study classic problems as well as discover their own ideas. In this presentation I will describe a research project assigned in an upper level Graph Theory course. After creating their own infinite graph classes, students further describe their graphs by investigating different parameters discussed in class. Finally, students choose what they feel is interesting about the graph class to present to the class. Over the course of the project, students have the opportunity to create their own unique mathematical objects, strengthen their understanding of course topics, have their own say in what makes interesting mathematics, and usually find out that not all questions have nice, neat answers. This talk will include several examples of student work. (Received September 20, 2016)

## *Intertwining Mathematics with Social Justice in the Classroom*

1125-H1-148 **Samuel R Kaplan\*** ([skaplan@unca.edu](mailto:skaplan@unca.edu)), One University Heights, CPO#2350, Asheville, NC 28804. *Enriching Student Experiences Through Service Learning*. Preliminary report.

The University of North Carolina Asheville has offered several versions of a special topics course on Math and Social Justice. Discussion about social justice, math education, access to resources, power relationships and structural inequality take on palpable dimension for undergraduates when service learning is added to the equation. Students in Math and Social Justice are required to tutor math in local after-school programs

provided for low-income families. By the end of the semester, students are expected to demonstrate a working knowledge of key issues of numeracy and society based on synthesizing their reading and experience. The course is designed to lay the basic groundwork of intellectual, personal and bibliographic resources for students to continue investigation and participation in math and social justice long past the end of the semester. (Received August 04, 2016)

1125-H1-212 **Mikael Vejdemo-Johansson\*** ([mvj@math.csi.cuny.edu](mailto:mvj@math.csi.cuny.edu)), 2800 Victory Boulevard, 1S-215, Staten Island, NY 10314. *Mapping Police Violence in Introduction to Statistics*. Preliminary report.

Mapping Police Violence is a data collection and presentation project emerging from the Black Lives Matter movement, aiming to collect and present reliable data on instances of police violence. Their data set is publicly available and the organization encourages derivative analyses and extensions.

For our Introduction to Statistics this year, we invited the data scientist working on the Mapping Police Violence project to talk to the class, and built one of the data analysis labs around this data set.

In this talk, we will discuss our experiences from the invited speaker and the data source, its impact on the course, and ideas for extensions and developments of this approach. (Received August 12, 2016)

1125-H1-232 **Samuel Luke Tunstall\*** ([tunstall1@msu.edu](mailto:tunstall1@msu.edu)). *Athletes, Education, and Welfare: Problems that Promote Quantitative Literacy and Social Justice*. Preliminary report.

In this talk, I describe a module on social justice we've created at Michigan State University (MSU) for our new course in quantitative literacy. Given the terminal nature of the course and its increasing enrollment, we find it paramount to discuss topics related to social justice. In our initial offerings of the course, topics have included NCAA student athletes, the U.S. and world wealth distributions, and incarceration rates, among other things. In the class, we talk about how we can use basic mathematics to develop informed opinions on these issues—precisely what students will need when they graduate. I will discuss samples of these assignments and concerns we have in moving forward with the module in future semesters. (Received August 15, 2016)

1125-H1-294 **Charles Peter Funkhouser\*** ([cfunkhouser@fullerton.edu](mailto:cfunkhouser@fullerton.edu)), 231 Coursey Lane, Port Ludlow, WA 98465, and **Miles R Pfahl** and **Harriet C Edwards**. *Discovering Undergraduate Mathematics in American Indian Culture*.

This project has developed and researched undergraduate mathematics materials based in the culture and mathematics of Native American Peoples for integration into undergraduate courses to foster social justice and equity in mathematics education. While mathematics topics include probability and statistics, number theory, transformational geometry, and preservice elementary and secondary education-related content, social concerns are reflected in cultural topics such as public health (diabetes) and Tribal membership (blood quantum). The presenters discuss how standard undergraduate mathematics can impact daily life among Tribes when cultural awareness is used as a basis for lesson development. These materials—both paper and electronic—are classroom ready, and are developed, piloted and assessed in consultation with Tribes in the the Rocky Mountains, the Plains, the Pacific Northwest, and the Southwest. This work is an NSF DUE TUES funded project. (Received August 24, 2016)

1125-H1-778 **J D Berg\*** ([jberg5@fitchburgstate.edu](mailto:jberg5@fitchburgstate.edu)), **Catherine Buell**, **Danette Day** and **Rhonda Evans**. *Social justice general education statistics course*. Preliminary report.

This talk will give an overview of the authors' work to add social justice topics to an applied statistics course (a general education course) as well as review the initial results of pre-and post-course surveys of students beliefs on issues related to social justice. The authors took an existing course and adjusted course materials so that the topics addressed in both in-class examples, and out of class projects were all socially justice themed. Examples of both in-class materials and class projects will be discussed as well as suggestions for others considering similar course development. (Received September 11, 2016)

1125-H1-987 **Erin R Moss\*** ([erin.moss@millersville.edu](mailto:erin.moss@millersville.edu)). *Progress and Resistance in Exploring Social Justice Mathematics with Graduate Students*.

"Equity Issues in Mathematics Education," a course I developed for my university's M.Ed. in Mathematics students, primarily serves practicing secondary mathematics teachers. Topics include: the achievement gap; the "receivment gap"; culturally-relevant pedagogy; tracking; issues of identity, power, and privilege in math education; and mathematics for social justice. In addition to reading both scholarly and practitioner-oriented articles about math for social justice, I gave students two related written assignments. The first assignment was to select a task from the book *Rethinking Mathematics*, design a full lesson plan incorporating the task, and teach it to classmates. The second assignment required each student to develop his or her own math lesson involving

a real-life social justice issue of their choosing. As expected, I encountered resistance to challenging the status quo. However, in several instances, exploring social issues in the context of mathematics helped students come to a deeper understanding of the ways people are marginalized in American society. In this presentation, I will provide participants with copies of the assignment descriptions, explain the implementation of each assignment, and discuss the content of the lessons designed by students. (Received September 13, 2016)

1125-H1-1058 **Priya V. Prasad\*** ([priya.prasad@utsa.edu](mailto:priya.prasad@utsa.edu)). *Revolutions in Flatland: Questioning Social Hierarchies with Geometry*.

A long-standing tradition in our Fundamentals of Geometry course has been to assign students to read Flatland and write a story that takes place in that universe. But at a minority-serving institute with a large number of non-traditional and first-in-family college students, we have noticed that students tend to interpret the (now almost impenetrable) satire of Edwin A. Abbott's classic romance literally – and they question the social hierarchies represented in the book through the lens of their own experiences. This presentation will discuss some of the clever ways our students have used geometric concepts to create “revolutions” in Flatland and some of our thoughts on how to use this eminently mathematical book to encourage students to reflect on social justice concepts. (Received September 14, 2016)

1125-H1-1789 **Kathy W Hoke** and **Lauren Keough\*** ([keoulaur@gvsu.edu](mailto:keoulaur@gvsu.edu)). *Raising Awareness of Social Justice Issues in Calculus I: How to Get Started*. Preliminary report.

We will present our first-time experiences at Grand Valley State (a public regional comprehensive university) and University of Richmond (a private liberal arts college) including data related to a topic in social justice in a standard Calculus I course. At University of Richmond, we chose eviction data in order to tie in with the One Book, One Campus choice for the year. At Grand Valley State we used data on the US correctional population. In both cases, we incorporated real data to reinforce calculus concepts of average rate of change, tangent line approximation, relationship between the graphs of a function and its derivative, and numerical integration, while simultaneously raising awareness of the social justice issues. We will share materials as well as discuss pedagogical issues that arose in our first attempt to bring a relevant societal issue related to equity and fairness into a calculus classroom. (Received September 19, 2016)

1125-H1-1855 **Phong Le\*** ([phong.le@goucher.edu](mailto:phong.le@goucher.edu)), Center for Data, Math and Computer Science, Goucher College, 1021 Dulaney Valley Road, Baltimore, MD 21204. *Authentic Messiness: Using data sourced from community-based partner organizations in an introductory level statistics course*. Preliminary report.

In data and statistics courses, we are always in search of interesting, accessible sources of data. Especially at the general education level, students also demand relevance as they develop comfort with abstraction. Many students are also motivated by social justice issues. In this talk we describe our partnership with a small nonprofit organization as a co-educator in a general education statistics course. The partner provides a level of authenticity, messiness, and engagement not typically possible in a typical classroom. The class acts as a data consultant. Working with partner provided datasets and gathering publicly available information to address issues and needs identified by the partner. Preparations for this project are underway. This partnership goes live Spring of 2017. (Received September 19, 2016)

1125-H1-2010 **Tricia Muldoon Brown\*** ([patricia.brown@armstrong.edu](mailto:patricia.brown@armstrong.edu)). *A Quantitative Literacy Project on Poverty in the United States*.

We present a three-part module that combines mathematical goals of data literacy and critical thinking, and social justice goals of addressing stereotypes on poverty and social programs and promoting self-reflection about societal and personal responsibility towards those in poverty. This module is designed for use across an entire semester of an introductory mathematics class, specifically one focused on quantitative reasoning. The presentation will introduce the the module as well as share classroom notes and experiences. (Received September 19, 2016)

1125-H1-2082 **Victor I Piercey\*** ([piercevi@ferris.edu](mailto:piercevi@ferris.edu)). *Quantitative Ethics – the Other Side of Mathematics and Social Justice*. Preliminary report.

Teaching mathematics for social justice empowers citizens to take action and speak truth to power. But there is another side: teaching those in power to act responsibly with mathematics. In this talk, I will share how I have included something that I call “quantitative ethics” in a class for business students. I will include issues discussed in class as well as ideas for the future. After all, with great mathematics comes great responsibility. (Received September 19, 2016)

1125-H1-2118 **Jason Quinley\*** ([jquinley@brookstoneschool.org](mailto:jquinley@brookstoneschool.org)), 9 Spruce St, Columbus, GA 31904. *Social Science and Servant Leadership Reflections on Game Theory at the Secondary Level*. Preliminary report.

In this talk we give several highlights from teaching a *Game Theory* seminar to advanced high-schoolers at a Southern college-prep school with a focus on *Servant Leadership*. Notable assignments include using NetLogo for simulating Schelling’s segregation model in the week of the Martin Luther King Jr. holiday, white paper assignments exploring Arrow’s Impossibility Theorem using L<sup>A</sup>T<sub>E</sub>X, and classroom experiments on the repeated Prisoner’s Dilemma.

Several themes emerge: connections between tolerance, population density, and economic status; the difficulty of selecting third party candidates and more generally new voting mechanisms; and mathematical techniques for modeling cooperation at both a psychological and social level. Samples of student work on altruism, evolution, and bargaining will be included. A secondary theme is the application of mathematical approaches to issues of character education.

	<i>C</i>	<i>D</i>
<i>C</i>	3,3	0,5
<i>D</i>	5,0	1,1

FIGURE 1. **The Prisoner’s Dilemma:** Why do some situations lend themselves to cooperative behavior?

**Example Game Tables.** (Received September 19, 2016)

1125-H1-2137 **Kyle Evans\*** ([kyle.evans@uconn.edu](mailto:kyle.evans@uconn.edu)) and **Fabiana Cardetti.** *Lessons Learned from School Mathematics and Global Citizenship*. Preliminary report.

The teaching, learning, and applications of mathematics are often discussed on the P-20 spectrum and in this talk, the preliminary findings of a study involving K-12 teachers and their understanding and implementation of global citizenship into their mathematics lessons and classrooms will be presented. Encompassing social justice, the framework of global citizenship includes the knowledge of local, national, and global issues, particular attitudes towards the issues, and behaviors and actions to improve communities, countries, and the world. K-12 teachers provide unique perspectives and materials that illustrate ways global citizenship and mathematics can be used together. Implications for the undergraduate mathematics classroom will also be discussed as an extension of the lessons learned from school mathematics. (Received September 19, 2016)

1125-H1-2430 **Nathan Napoleon Alexander\*** ([nalexander@usfca.edu](mailto:nalexander@usfca.edu)), 2130 Fulton Street, School of Education, ED 304B, San Francisco, CA 94117, and **Aditya Adiredja** ([adiredja@math.arizona.edu](mailto:adiredja@math.arizona.edu)), 617 N. Santa Rita Ave., P.O. Box 210089, Department of Mathematics, Tuscon, AZ 85721. *Historical Perspectives on Social Justice in Mathematics*.

Both as a profession and academic discipline, the field of mathematics lives within a much broader history which has privileged some and marginalized others. This talk presents historical examples and conceptual models that relate to the role and need of critical historical perspectives in contemporary mathematical spaces. By uncovering the ways in which these critical histories have and continue to inform counter-narratives that questions the traditions of teaching and of mathematics that we tend to inherit, this session invites faculty to reflect on and understand the sociology of mathematics. Implications for inclusion and teaching mathematics are discussed. Discussion from this session will be closely tied to the "Hard Conversations on Social Justice in Mathematical Spaces" talk. (Received September 20, 2016)

1125-H1-2454 **John Kellermeier\***, [kellermeierjh@gmail.com](mailto:kellermeierjh@gmail.com). *Using Context to Address Social Justice Issues in the Statway Classroom*.

Statway is a student success program developed by the Carnegie Foundation that is designed to take developmental math students to and through completion of a college level statistics course. Over the past six years, this program has proved to be highly successful, resulting in triple the college credit completion rate of students in half the time.

A key aspect of the Statway curriculum is the use of contexts relevant to student’s lives. Effort has been made to make these contexts both relevant and culturally appropriate, and we are investigating the use of contexts to specifically address issues of social justice in the Statway classroom.

Any lesson in teaching mathematics in an applied fashion always has two curricula, an explicit curriculum and an implicit curriculum. The explicit curriculum includes the mathematical content of the lesson while the

implicit curriculum reflects that which is learned through and about the lesson context. This presentation will give examples of the implicit curriculum in a statistics course that can be leveraged to promote understandings of social justice. In particular, we explore how lesson contexts can be used to (1) increase awareness and cultural competence and (2) address issues of equity, diversity and social justice. (Received September 20, 2016)

1125-H1-2501 **Nathan Napoleon Alexander, Zeynep Teymuroglu and Carl R Yerger\*** (cayerger@davidson.edu). *Hard Conversations on Social Justice in Mathematical Spaces.*

Many faculty and students struggle to see how mathematics can inspire social change. Given the ever-changing global contexts of mathematics, it is important that faculty understand how mathematics is culturally and socially relevant to today's society. The purpose of this presentation is to engage and reflect on hard conversations on social justice in mathematical spaces. The conversations in this presentation are based on the Associated Colleges of the South's (ACS) 2016 Faculty Workshop on Mathematics for Social Justice. The central question asked by participants of the ACS workshop that will be engaged is discussing issues of social justice both within the classroom and in one's department. Lessons from the workshop will be shared. Reflections to important common questions asked by participants will help to frame questions and comments from the audience. This presentation is an extension of Nathan Alexander's presentation. (Received September 20, 2016)

1125-H1-2566 **Jared Warner\*** (jared.warner@guttman.cuny.edu). *Bias in the Courts? A Student-led Study of New York City's Arraignment Courts.* Preliminary report.

According to the 2015 Annual Report of the Criminal Court of New York City, last year in New York City there were on average over 1,000 arraignments per day in which criminal defendants appeared before a judge to be notified of charges filed against them. Since arraignment sessions are open to public observation, such sessions provide a wealth of accessible and topical data for statistics and quantitative literacy instructors who want to integrate issues of justice into their courses. This talk will describe a study of New York City's arraignment courts completed by an introductory statistics class at Guttman Community College in Midtown, Manhattan in the fall of 2016. The project was implemented in partnership with the Police Reform Organizing Project (PROP), and it required students to attend arraignment sessions, record data on cases observed, and write reports performing analyses of this data. In their reports, students were asked to determine if there is statistical evidence supporting claims of the existence of policing and prosecuting practices that target people of certain ages, genders, and races. These reports were subsequently submitted to PROP to assist in its community development initiatives. (Received September 20, 2016)

1125-H1-2670 **John Ross\*** (rossjo@southwestern.edu), 7371, Southwestern University, 1001 E. University Avenue, Georgetown, TX 78626, and **Therese Shelton** (shelton@southwestern.edu). *Supermarkets, Highways, and Oil Production: Statistics and Social Justice.*

We present several modules that address social justice issues in an introductory statistics course. We look at inequalities of housing location, including access to healthy foods, air pollution via proximity to traffic, and health concerns via proximity to fracking sites. Statistical content includes survey design, contingency tables, various quantiles, and hypothesis testing. We present the assignments and report on their implementation in the classroom. (Received September 20, 2016)

## *Mathematical Technology in the Calculus Classroom*

1125-H5-112 **Kristen Mazur\*** (kmazur@elon.edu). *A Basic Approach to Creating Interactive Calculus Lessons in Mathematica.*

Mathematica is a powerful computational tool and has the potential to also be a powerful teaching tool. However, effective integration of technology in the classroom is challenging, and bringing Mathematica into a calculus lesson can be daunting. In this talk I will discuss Mathematica-based lesson templates that allow students to explore concepts, perform calculations and take notes in an easy-to-use and easy-to-create format. These templates not only reduce technology anxiety for both students and instructors, they also become cohesive notes that students can revisit after class. Moreover, the templates support discovery-based and active learning. I will demonstrate the templates themselves and how to create one from a blank Mathematica file. I will also discuss some advantages and disadvantages for both students and instructors. (Received August 01, 2016)

1125-H5-167     **Alexander Y Vaninsky\*** ([avaninsky@hostos.cuny.edu](mailto:avaninsky@hostos.cuny.edu)), 475 Grand Concourse, Room A-242a, Bronx, NY 10451. *Teaching and learning mathematics in the AR/VR environment.*  
 This presentation discusses teaching and learning mathematics in augmented (AR) or virtual (VR) reality created by a combination of goggles and earphones. It claims that interactive learning in such an environment is more attractive and efficient. It increases motivation and interest in the subject matter. The approach is underlain by the findings of educational neuroscience considering the learning process as the formation of domains in the brain forming mathematics knowledge centers. The teaching process provides sensory excitation and establishes connections among these and other domains. Hardware and software are available in the market. The suggested approach allows for practical implementation of different teaching techniques properly featuring the particularities of an individual student. It may be adopted to meet the needs of visually or hearing-impaired students, multiple intelligence, different learning styles, group learning, research-based teaching, etc. The choice may be made by a student himself. For example, virtual group participants and their style of behavior may be adjusted to provide a comfortable learning environment. Different aspects of the AR/VR approach are discussed, including implementation, curriculum, possible problems, and the ways to overcome them. (Received August 06, 2016)

1125-H5-173     **Christine A. Herrera\*** ([caherrera@csuchico.edu](mailto:caherrera@csuchico.edu)) and **Alexander White** ([aw22@txstate.edu](mailto:aw22@txstate.edu)). *Exploring Sequences through Technology to Expand Students' Example Space.*

Instructors who teach calculus often struggle to find a meaningful way to build students' conceptual understanding of limits of sequences. One way to deepen students' understanding is to enrich their example space. However, a student's example space may be dominated by either the number line model or the Cartesian plane and there is the potential for cognitive conflict when an example of a sequence is presented in a visual model that varies from the model that dominates their example space. The talk will address the difficult transition between the two different visual models and show how to reduce this potential cognitive conflict using the Wolfram Demonstration, Examples of Limits of Real Sequences. We will share tasks that help students visualize and understand the formal mathematical language of limits. By using the dynamic graphs which incorporate both models, explore different general examples, and demonstrate how their properties relate to the formal definition, students are able to build a robust example space. (Received August 08, 2016)

1125-H5-181     **Mike May\*** ([maymk@slu.edu](mailto:maymk@slu.edu)). *Building and Using GeoGebra Books in Calculus.* Preliminary report.

GeoGebra is a free, cross platform application for exploring and visualizing mathematics that can be used through most lower division courses. There is a large online repository of approximately a half million free activities and materials. A GeoGebra Book is a collection of materials, either created by the book author or collected from the archive. This talk looks at building an using such a book in single variable calculus for in class visualizations, for student explorations, and for out of class student explorations. (Received August 09, 2016)

1125-H5-339     **Lina Wu\*** ([lwu@bmcc.cuny.edu](mailto:lwu@bmcc.cuny.edu)), 529 West 42nd Street, Apt. 5K, New York, NY 10036. *Maple Software Technology as a Stimulant Tool for Dynamic Interactive Calculus Teaching and Learning.* Preliminary report.

The presenter is interested in sharing her research experiences on how to use the Maple Software technology as a valuable and powerful stimulant tool that provides dynamic and interactive methods in teaching and learning. During a sequence of pilot summer Calculus courses from 2013 to 2016 at Borough of Manhattan Community College, the presenter has designed many Calculus projects using Maple Software as her teaching strategies to assess students' learning. Maple projects of Polar Art in 2013, Funny Face in 2014, Geometric Abstract Art in 2015, and Culture Art in 2016 will be presented. Visualizing the beauty of math-related artwork created by computer-generated diagrams in Maple has ignited students' passion to learn Calculus. Demonstrating animated graphic images and simplified symbolic calculation in Maple has enhanced students' understanding abilities on abstract math concepts. Practicing on Maple projects has increased students' problem-solving skills in math. These pilot Calculus sequence courses were supported by MSEIP Grant (Minority Science Engineering Improvement Program) and ELIC (Enhanced Learning in Classroom Teaching Fund). (Received August 28, 2016)

1125-H5-404     **Laura R Lynch\*** ([llynch@ccga.edu](mailto:llynch@ccga.edu)), College of Coastal Georgia, One College Drive, Brunswick, GA 31520. *Empowering Calculus Students through Mathematica.*

This talk will illustrate how I have incorporated the technologies of *Mathematica*, the Wolfram CDF player and the Wolfram Demonstrations Project into the (single and multi-variable) Calculus classroom via lecture

and projects. *Mathematica*, a software program developed by Wolfram Research, allows instructors to create presentations using on-the-fly calculations and visualizations. Students attain a deeper level of understanding when they can see and explore concepts of the infinitesimal and infinite using interactive graphs, both in class through *Mathematica* but also at home through free Wolfram products. (Received August 31, 2016)

1125-H5-1185 **Grace McClurkin\*** ([mcclurkin@math.utk.edu](mailto:mcclurkin@math.utk.edu)) and **Joshua Mike** ([mike@math.utk.edu](mailto:mike@math.utk.edu)). *Utilizing Mathematica for Higher Level Thinking in Multivariable Calculus*. Preliminary report.

We will describe our design and implementation of an inverted multivariable calculus classroom, which utilizes computer-based computation via *Mathematica*. In our approach, we delegate the lower cognitive thinking to outside class using notes and short practice. Conversely, higher-level thinking is developed in-class through *Mathematica*-based conceptual activities and other explorations. Our goal is to use *Mathematica* to simultaneously alleviate computational burden and reveal mathematical concepts. (Received September 15, 2016)

1125-H5-1390 **Nicole R Juersivich\*** ([njuersi9@naz.edu](mailto:njuersi9@naz.edu)), 7 Morgan Chase, Honeoye Falls, NY 14472. *Promoting Mathematical Proficiency with Technology and Structured Inquiry in Calculus I*. Preliminary report.

Technology is a natural cognitive support for inquiry because it provides students with a tool to guide their experimentation, conjecturing, and communication. Using the National Research Council's components of mathematical proficiency, I created three structured- inquiry workshops employing three different types of mathematical action technology that addresses common misconceptions in differential calculus. This presentation will focus on the design, implementation, and informal assessment of the technology workshops with respect to student understanding and engagement. (Received September 16, 2016)

1125-H5-1692 **Ross Sweet\*** ([rsweet@math.northwestern.edu](mailto:rsweet@math.northwestern.edu)), 2033 Sheridan Road, Evanston, IL 60208. *Analyzing Student Usage of Online Video Lectures in a Flipped Calculus Course*. Preliminary report.

When teaching a course using a “flipped” or “inverted” model, it is a delicate balancing act between students' outside preparation and the difficulty and comprehensiveness of in-class assignments. Following a first-quarter calculus course at Northwestern University, we will focus on Panopto, a web-based video hosting platform. Panopto was used to host video lectures to improve students' self-guided learning outside of class. We will discuss the analytic tools Panopto provides as a resource to measure student engagement and the effectiveness of video lectures. (Received September 18, 2016)

1125-H5-1990 **Darrin Weber\*** ([dweber3@vols.utk.edu](mailto:dweber3@vols.utk.edu)). *Using Videos to Augment In-Class Instruction*. Preliminary report.

In our hybrid Calculus III classroom (a partly inverted classroom that still has some in-class lectures), we implement instructional videos to be viewed outside the classroom that supports in-class discussion and lectures, as well as frees up classroom time to dedicate to working problems. These videos also serve as a reinforcement and guide into the use of Wolfram *Mathematica*, which we use as a platform for all work done in the course. We talk about how we created the videos, as well as what material we choose to cover in them and why. (Received September 20, 2016)

1125-H5-2111 **Mel Henriksen\*** ([henriksenm@wit.edu](mailto:henriksenm@wit.edu)). *Learning Calculus Concepts with Desmos – In and Out of the Classroom*.

Today's technology makes the ‘rule of four’ all the more implementable. But not all implementations are equal. Graphing technology can serve as an excellent way to demonstrate concepts and relationships, but can be used in an even more powerful way by allowing students to build their own graphical models. In this way instructor demonstrations give way to student creations (or, at least, re-creations.) But this requires a graphing technology that is easily learned, is interactive and whose solutions can be stored. The graphing application *Desmos* meets these criteria and allows students in Calculus to build their own models, exploring, for example, the transition of a secant line into a tangent line. What was once shown by a sketch on the white board, a graphic in a text or even a static plot now can become an interactive tool. Using a combination of an instructor-made video tutorial, pre-designed *Desmos* files, wireless video presentation and *Desmos* files created by students in real time in the classroom, *Desmos* becomes a terrific tool for teaching Calculus concepts. With the addition of web-based conferencing it can be used in virtual office hours as well. (Received September 19, 2016)

1125-H5-2139 **D Scott Dillery\*** ([dillerys@lindsey.edu](mailto:dillerys@lindsey.edu)). *Desmos Calculator and SageMath Cell Server in Calculus.*

Desmos Calculator and SageMath Cell Server are two web-based free technologies. Both technologies have APIs which allow for them to be incorporated into individual web pages. Their features are compared using examples of classroom use and assignments given in calculus 1 and calculus 2 courses. (Received September 19, 2016)

1125-H5-2314 **Neal Coleman, Christopher Judge, Deniz Kutluay, Ranjan Rohatgi\*** ([rrohathgi@saintmarys.edu](mailto:rrohathgi@saintmarys.edu)), **David Sprunger, Zhixu Su and Tristan Tager.** *Flip-mastery learning in applied calculus.* Preliminary report.

In the fall of 2014, over 700 students participated in an experiment testing the efficacy of the flip-mastery model in teaching calculus. Four instructors taught two at least two classes each — the first a traditional lecture-style class and the second a flipped class. Graduate students created an online content-delivery and homework system giving the students a single website on which they could learn new material and answer homework problems. In this talk I will show how we combined a flipped classroom with mastery learning techniques and discuss some of our results. (Received September 20, 2016)

1125-H5-2330 **Monica M VanDieren\*** ([vandieren@rmu.edu](mailto:vandieren@rmu.edu)), Dept of Mathematics, RMU, 6000 University Blvd, Moon Township, PA 15237. *Three Ways of Using CalcPlot3D in the Multivariable Calculus Classroom.* Preliminary report.

CalcPlot3D is an on-line, freely available, 3D graphing applet developed by Paul Seeburger (See <http://web.monroec.edu/calcNSF/>). This applet was originally developed to help students explore and visualize multivariable calculus concepts, but it has the potential to be used in other mathematics, engineering, and science courses in which three-dimensional visualization may aid in student understanding. Through concrete examples, I will describe three different ways in which I employ CalcPlot3D in my classroom: demonstrations, labs, and homework assignments. I will summarize some on-going research which identifies some of the pros and cons of this implementation and how it may improve student understanding. This work is funded by NSF-IUSE #1524968, NSF-IUSE #1523786, and NSF-IUSE #1525216. (Received September 20, 2016)

1125-H5-2372 **D Brian Walton\*** ([walttondb@jmu.edu](mailto:walttondb@jmu.edu)), 60 Bluestone Drive, MSC 1911, Harrisonburg, VA 22807. *Web-based apps for practice, scaffolding and conceptualization in calculus.*

Over the last few years, the author has developed a collection of web-based apps to facilitate student learning in a variety of ways. Some apps provide instant, randomized practice in basic skills (e.g., differentiation and antidifferentiation). Other apps illustrate core concepts (e.g., the definition of a limit and the definition of the definite integral). Most recently, the author has worked to create apps that scaffold mathematical reasoning (e.g., constrain algebra manipulations to follow rules). The author will describe the basic framework on which these tools have been built and demonstrate how they might be used. (Received September 20, 2016)

1125-H5-2405 **Nikolay Brodskiy\*** ([nbrodski@utk.edu](mailto:nbrodski@utk.edu)), 227 Ayres Hall, 1403 Circle Drive, Knoxville, TN 37996. *Computation and cloud collaboration in a Calculus class.* Preliminary report.

Computational capability of current technology is becoming readily available and user friendly at a rapid pace. Cloud sharing and collaboration is getting seamless and intuitive. I have been using various technologies to enhance my calculus classes over several years: WolframAlpha and Wolfram Mathematica for computations, Google documents and Mathematica Online for collaboration and sharing, Google Hangouts and Zoom for online meetings. I will describe the advantages and disadvantages of using these technologies in a regular class, an online synchronous class, and an online asynchronous class. Proctoring an online test is particularly challenging, and a successful method of proctoring online will be described. I will share my perspectives on how technology choices depend on the instructor's pedagogical preferences. (Received September 20, 2016)

1125-H5-2623 **Bart Snapp\*** ([snapp.14@osu.edu](mailto:snapp.14@osu.edu)). *Teaching Calculus with Ximera.*

Since the fall of 2015, The Ohio State University has been running experimental sections of Calculus 1, 2, and 3 where students forgo the expenditure of a traditional text and use a free, open source text instead. This text was developed by faculty at OSU using the Ximera platform. In this presentation we will discuss the effects of this technology on student outcomes, explain how these resources can be reused by instructors at other institutions, and demonstrate how one can author such materials themselves. (Received September 20, 2016)

1125-H5-2669 **Philip B. Yasskin\*** ([yasskin@math.tamu.edu](mailto:yasskin@math.tamu.edu)), Dept of Mathematics, Texas A&M University, 3368 TAMU, College Station, TX 77843-3368, and **Douglas B. Meade** and **Andrew Crenwelge**. *MYMathApps Calculus - Building on Maplets for Calculus*. Preliminary report.

MYMathApps Calculus is a calculus text, under development, which is totally online. It includes:

- randomly generated tutorials and exercises in addition to the standard text, examples and exercises
- user-controllable animations as well as standard static graphics
- hyperlinks both internally and externally. In particular, exercise pages link back to the pages where the material is discussed.
- pop-up notes, for things like proofs, which do not obstruct the flow of the text.

This text has grown out of the WebCalc project by the author, Don Allen and Mike Stecher at Texas A&M University and the Maplets for Calculus project by the author and Doug Meade at the University of South Carolina. The text is being written in HTML5, Javascript and MathJax so it is available on all devices. The work is supported in part by NSF DUE TUES-2 grant 1123255. (Received September 20, 2016)

1125-H5-2727 **Jody Sorensen\*** ([sorensj1@augsborg.edu](mailto:sorensj1@augsborg.edu)), Mathematics and Statistics, 2211 Riverside Ave., Augsburg College, Minneapolis, MN 55454. *A Surprising Use of Technology to Find Leaf Area*.

When getting students to think about how to calculate area in Calculus II, one unusual piece of technology I use is a LI-COR portable leaf area meter. This machine is designed for scientists to find the areas of leaves when studying photosynthesis and carbon uptake. Students calculate the area a few different ways and compare results. After discussing this example, I'll also talk about how we are reducing the use of graphing calculators in our Calculus labs, and using more online tools like Desmos. (Received September 20, 2016)

1125-H5-2791 **Amit A Savkar\*** ([amit.savkar@uconn.edu](mailto:amit.savkar@uconn.edu)), 341 Mansfield Road, U1009, Storrs, CT 06269, and **David Nichols**, 341 Mansfield Road, U 1009, Storrs, CT 06269. *3D visualizations in multivariable calculus: A pedagogy through technology*. Preliminary report.

In the fall of 2016 multivariable calculus switched its offering from a small lecture format to a large lecture format at the University of Connecticut. The issue of visualizing surfaces, planes, intersections, normals etc in multivariable calculus was considered to be most difficult for the students by the instructors who taught this course. Thus a project was undertaken to create visualization tools that could not only help students with visualization but also allow them to interact with the figures while grasping the mathematics around it.

We have developed several different visualizations for multivariable calculus using open source libraries GNU and MIT. Using basic JAVA scripts we are able to create these tools that interact with students online. We will discuss these tools as a meaningful way of using technology within the classroom and s (Received September 20, 2016)

## Mathematics and the Arts

1125-II-97 **David A Reimann\*** ([dreimann@albion.edu](mailto:dreimann@albion.edu)), Math/CS Department, Albion College, 611 E. Porter, Albion, MI 49224, and **Liliya Chernysheva** ([lc11@albion.edu](mailto:lc11@albion.edu)), Math/CS Department, Albion College, 611 E. Porter St., Albion, MI 49224. *Sheets, tubes, and capsules constructed from corner connected rectangles*.

We have previously demonstrated that visually interesting open lattice structures can be created by replacing edges with flexible rectangular plates in regular polyhedra. The plates are connected at corners resulting in an open lattice structure where vertices and faces are transformed into open space and edges are rectangular plates. In this work, we show constructions based on uniform tessellations of the plane and prove that exactly four (3-3-3-3-3-3, 4-4-4-4, 6-6-6, and 6-3-6-3) will remain planar after this edge expansion procedure. These sheets can be rolled into cylindrical tubes of arbitrary length along either of the two primary symmetry axes of the corresponding tessellation. Capsules can be constructed by capping the tubes with sections of regular polyhedra, specifically pyramids, cupolas, and rotundas. Example constructions for a variety of capsules will be shown. (Received July 22, 2016)

1125-I1-198 **Maura Twillman\*** (mtwill12@students.towson.edu), MD, and **Diana Cheng** (dcheng@towson.edu). *Linear momentum in pairs figure skating: Mathematics behind the art of lifts.*

Lifts are an aesthetically pleasing and athletic part of a pairs skating program, and certain aspects of these lifts can be described mathematically. Pairs skaters' combined velocity in the lift is a function of their individual masses & initial velocities. We report on undergraduate mathematics education research analyzing how secondary students reasoned quantitatively about conservation of linear momentum in the context of pairs lifts. (Received August 11, 2016)

1125-I1-264 **James Walker** (walkerjs@uwec.edu), **Emily Gullerud\*** and **Claire Arneson**. *Creating Symmetric Designs and Animations.*

This talk will describe our work on developing an open-source GUI for creating symmetric designs, based on extending the work of Frank Farris. One new aspect of our work is a method for producing animations of symmetric designs, including designs that perform multiple independent rotations in time. (Received August 21, 2016)

1125-I1-327 **Susan A. McBurney\*** (smcburney@prodigy.net). *Bitwise Artwork.*

Bitwise curves are by definition, smooth curves (without cusps) formed from arcs of circles with a variety of radii. What is intriguing is that any curve can be constructed by smoothly joining arcs from appropriately-sized circles. This presentation will highlight a variety of these curves as well as their cousins, piece-wise curves. It will explain methods of construction, provide a proof of one method in particular, and then go on to examine the generation of new families of curves and their application to decorative ornamentation. (Received August 26, 2016)

1125-I1-368 **Lina Wu\*** (lwu@bmcc.cuny.edu), 529 West 42nd Street, Apt. 5K, New York, NY 10036. *Digital Graphic Calculus Art Design in Maple Software.* Preliminary report.

The presenter is interested in sharing her experience on how to make design of digital graphic art possible in Calculus teaching thru the use of Maple Software technology. A sequence of projects of "Culture Art" in 2016, "Geometric Abstract Art" in 2015, "Funny Face" in 2014, and "Polar Art" in 2013 created at the Borough of Manhattan Community College (BMCC) will be presented at the conference. In order to complete these projects, students were required to apply their Calculus knowledge regarding different curve equations or various surface equations as well as transformation and composition. Furthermore, students were expected to use Maple Software technology to make their designs in computer-generated images. Combining Calculus knowledge and the use of Maple Software technology enables students to create their artistic artwork according with their mathematical thinking. The combination of math knowledge and mastering the use of Maple Software has created a beautiful and interesting way of math learning. (Received August 30, 2016)

1125-I1-387 **Beth Malmskog\*** (beth.malmskog@villanova.edu) and **Katie Haymaker** (kathryn.haymaker@villanova.edu). *Quilting Squares.*

This talk starts with a real-world question about when it is possible to form a special type of quilting circle configuration. This simple premise leads to the study of latin squares, directed graphs, number theory, and group theory, and opens the door to unsolved problems. (Received August 30, 2016)

1125-I1-421 **Simei Tong\*** (tongs@uwec.edu), Department of Mathematics, University of Wisconsin-Eau Claire, 105 Garfield Ave, Eau Claire, WI 54702. *Rotation and Symmetry in Mathematical Quilt Design.* Preliminary report.

Squares, triangles, and circles are building blocks for mathematical quilts. With basic knowledge of rotation, symmetry, and color combinations, quilts could present some mathematical concepts in a simple and colorful way. This presentation will explain some standard rotations used in quilting design, such as log cabin method or triangular and circular paper piecing method. Some projects will be presented during the presentation. The ideas of projects could be used in a math service course or faculty/student research projects for undergraduates in math majors and/or art & design majors. (Received September 01, 2016)

1125-I1-447 **Saul Schleimer** and **Henry Segerman\*** (segerman@math.okstate.edu). *Squares that Look Round: Transforming Spherical Images.*

We propose Möbius transformations as the natural rotation and scaling tools for editing spherical images. As an application we produce spherical Droste images. We obtain other self-similar visual effects using rational functions, elliptic functions, and Schwarz-Christoffel maps. (Received September 02, 2016)

1125-II-519 **Annalisa Crannell\*** ([annalisa.crannell@fandm.edu](mailto:annalisa.crannell@fandm.edu)), F&M Mathematics, Box 3003, Lancaster, PA 17604-3003. *Curve constraints in ruler-and-compass perspective drawings*. Preliminary report.

“You can draw a spiral staircase, but you can’t draw the handrail.” This statement describes a limitation that arises in ruler-and-compass perspective drawings. Projective geometry allows us to draw certain curves either as a collection of points or as an envelope of tangent lines (think of string art which creates hyperbolas from a collection of lines). Perspective art applies the tools of projective geometry to create realistic images of curves lying in three space. But ruler and compass constructions have their limitations, and those limitations translate into un-drawable curves in perspective art, including (as we will explain) the spiral handrail of a drawable staircase. (Received September 05, 2016)

1125-II-664 **Betty Love\*** ([blove@unomaha.edu](mailto:blove@unomaha.edu)), Mathematics Department, University of Nebraska - Omaha, Omaha, NE 68182, and **Victor Winter** ([vwinter@unomaha.edu](mailto:vwinter@unomaha.edu)), Computer Science Department, University of Nebraska - Omaha, Omaha, NE 68182. *The Mathematics and Art of the Wunderlich Cube*.

A Wunderlich curve is constructed using rotations and reflections of an initial seed shape to create patterns which can be connected to form a space-filling curve. A Wunderlich cube is a cube whose faces contain raised stamp shapes corresponding to reflections of a seed shape and reflections of its 90-degree rotation. Wunderlich cubes embody numerous spatial reasoning puzzles. We will discuss how an understanding of the properties of the Wunderlich cube can be developed through manual construction using LEGO<sup>®</sup> mathematical analysis, computational thinking, coding, and 3D printing. (Received September 08, 2016)

1125-II-881 **Douglas Dunham\*** ([ddunham@d.umn.edu](mailto:ddunham@d.umn.edu)), Department of Computer Science, University of Minnesota, Duluth, Duluth, MN 55812-3036, and **John Shier** ([johnpf99@frontiernet.net](mailto:johnpf99@frontiernet.net)), 6935 133rd Court, Apple Valley, MN 55124. *Creating Wallpaper Patterns that are Locally Random Fractals*. Preliminary report.

We show an algorithm for wallpaper patterns whose fundamental regions are filled with fractal patterns composed of progressively smaller copies of a motif (the basic subpattern). The motifs can be very simple or complicated. The local part of the algorithm begins by placing the largest copy of the motif at a random location in the fundamental region. After placing  $i$  motifs, the algorithm keeps trying random locations within the region at which to place the next motif until a location is found for which the new motif does not overlap any previously placed motif. Then  $i$  is incremented and this process is repeated. The sizes of the motifs obey an inverse power law which guarantees they will fill the fundamental region in the limit, though we stop after a finite number of successful placements.

The global part of the algorithm copies the contents of the fundamental about the Euclidean plane by using the transformations that generate the entire wallpaper pattern from the fundamental region. We have implemented this algorithm for the wallpaper groups  $p1$ ,  $p2mm$ ,  $p4mm$ ,  $p3m1$ ,  $p6mm$ ,  $p4$ , and  $p6$ . We will show sample patterns. (Received September 12, 2016)

1125-II-931 **Vincent J. Matsko\***, University of San Francisco, Department of Mathematics and Statistics, 2130 Fulton St., San Francisco, CA 94117. *Teaching a Mathematics and Digital Art Course*.

I taught Mathematics and Digital Art for the first time in Fall 2016. I’ll present an overview of the course, as well as give examples of student work. Here is the course description: What is digital art? It is easy to make a digital image, but what gives it artistic value? This question will be explored in a practical, hands-on way by having students learn how to create their own digital images and movies in a laboratory-style classroom. We will focus on the Sage/Python environment, and learn to use Processing as well. There will be an emphasis on using the computer to create various types of fractal images. No previous programming experience is necessary. (Received September 13, 2016)

1125-II-956 **Nick C Mendler\*** ([nickmendler101@gmail.com](mailto:nickmendler101@gmail.com)). *Self-Similar Polygon Spirals*. Subsets of fractal trees similar to those generated by L-systems are artistically selected as structures for generative art. The geometry of the trees reflects the study of nested chains of polygons whose centers lie on nested logarithmic spirals. (Received September 13, 2016)

1125-II-1270 **Luke Wolcott\*** ([luke.wolcott@gmail.com](mailto:luke.wolcott@gmail.com)). *The effects of altitude sickness on mathematical cognition*.

The Berlin gallery HORSEANDPONY ran an October 2016 exhibition “Altitude Sickness” of artifacts derived from artistic actions conducted by artist Elizabeth McTernan and mathematician Luke Wolcott during a summer

2015 trip to the Indian Himalayas. There they investigated the effects of high altitude on mathematical cognition, informed by The Great Trigonometrical Survey, weightlessness and the genius myth. Luke will talk about the piece and show images. (Received September 15, 2016)

1125-I1-1444 **Samaneh Gholizadeh Hamidi\*** ([s.hamidi@mathematics.byu.edu](mailto:s.hamidi@mathematics.byu.edu)), Department of Mathematics, Brigham Young University, Provo, UT 84602. *Math and Persian Art*. Preliminary report.

Persia has left numerous marks on the civilizations and cultures of human beings, dating back to ancient times. From the first scientific notions of astronomy and the measurement of the celestial sphere and determination of the new year according to the vernal equinox, to the mathematics of shapes, solids, and numbers. Also, Persian arts have played important roles. Persian workers have combined skills in theoretical mathematics and practical techniques such as ceramics, along with artistic ideas from their own history and around the world. Putting these together, they have expressed several appealing forms, which bridge mathematics and the arts. The purpose of this talk is to get a glimpse of Mathematics in Persian arts in different eras and also to present a more detailed visual analysis of a ceramic design. (Received September 16, 2016)

1125-I1-1731 **Karl H. Schaffer\*** ([karl\\_schaffer@yahoo.com](mailto:karl_schaffer@yahoo.com)), 325 Lucinda St., Scotts Valley, CA 95066. *Dichromatic Dances*.

Dancers and choreographers employ symmetric patterns of bodies in space fluidly and in fleeting ways. Ballet, folk, and contemporary dances often arrange dancers in two categories: men and women, two colors of costumes, antagonistic groups, etc. In this talk we will look at how two-colored symmetry patterns are employed in a variety of dance forms, and how the dichromatic symmetry patterns may be seamlessly morphed from one symmetry type to another, in a manner similar to visual parquet deformations. These explorations may make for engaging activities for liberal arts math classes. (Received September 19, 2016)

1125-I1-1740 **Rosanna Iembo\*** ([rosannaiembo@libero.it](mailto:rosannaiembo@libero.it)), via Federico Cozzolino 18, 84018 Scafati, Salerno, Italy, and **Irene Iaccarino** ([irene.iaccarino@hotmail.it](mailto:irene.iaccarino@hotmail.it)), via Interna Marina 19, 88900 Crotona, Italy. *Myia married Milo. And mathematics, music and athletic melted in beautiful harmony in Crotona's Pythagorean School*.

When Myia, the daughter of Pythagoras, married Milo no one was surprised. Nevertheless she was a musician and a mathematician in Pythagoras and his wife Theano's School. Who was this man, capable of capturing the accomplished musician and mathematician's love? Milo, from Crotona, Italy, a famous athlete, a true legend. As a child, Milo had shown extraordinary strength, so it was rumoured in Crotona that he was the son of Heracles. When Milo was thirteen he won the Olympic Games of the sixtieth Olympiad. When Pythagoras founded the School, twenty year old Milo, was among the first to be admitted because he took care of his body and mind trying to achieve harmony, the fundamental principle of the Pythagoreans. In his life he won 7 times the Olympics, 7 times the Pythian Games of Delphi, 9 times the Nemean Games and 10 times the Isthmian Games in Corinth. During his 28 years career, he won 33 times. The Pythagorean School was basically a school of life so ethics was the leitmotiv of science, knowledge and human relationships. Milo lead an army bringing Crotona to victory when it was attacked by the nearby city of Sybaris. He also saved many Pythagoreans during an earthquake by substituting himself in place of a collapsed column of the School. Milo's myth is still alive. (Received September 19, 2016)

1125-I1-1849 **Margaret Kepner\*** ([renpek1010@gmail.com](mailto:renpek1010@gmail.com)). *The Art of Geometric Dissections*. Preliminary report.

A geometric dissection is a subdivision of a shape into pieces that can be reassembled to create a different shape. For example, a square can be cut up into seven pieces that will form a regular five-pointed star when they are rearranged. I will present examples of some two-dimensional geometric dissections, and will show how I have used the associated imagery in my artistic work. (Received September 19, 2016)

1125-I1-1856 **Douglas G Burkholder\*** ([doug.burkholder@lr.edu](mailto:doug.burkholder@lr.edu)). *Hidden Beauty in Penrose Tiling: Weavings and Lace*. Preliminary report.

We explore Penrose's subdivision of kites and darts to search for hidden beauty. By dividing the kites and darts in half and selectively coloring them based only on their relative position, we create 15 unexpected and distinctive patterns hidden within Penrose tiling. These patterns tend to have the appearance of a weaving. Alternately, by selectively removing tiles as we subdivide, we can obtain fractal patterns that appear to be lace. (Received September 19, 2016)

1125-I1-1858 **Elizabeth Whiteley\*** ([contact@elizabethwhiteley.com](mailto:contact@elizabethwhiteley.com)). *Turning the Corner: Symmetry, Botanical Art, and Metalpoint Drawing.*

Line groups, or frieze patterns, can be used to create aesthetically pleasing borders in graphic designs, carpets, quilts and other planar art forms. Special consideration must be given to the corners if one also wants the border to have a planar symmetry. This talk presents ways for linear repeat patterns to turn the corner for 2-D applications. I utilize this methodology in my original metalpoint drawings. This allows me to combine linear and planar symmetries, Renaissance metalpoint drawing techniques, and botanical subject matter to create aesthetically pleasing drawings. Examples of my botanical art will demonstrate all seven line groups. (Received September 19, 2016)

1125-I1-1860 **Sarah Katherine Stengle\*** ([stenglesarah@centurylink.net](mailto:stenglesarah@centurylink.net)), 586 Smith Avenue South, St. Paul, MN 55107, and **Genevieve Gaiser Tremblay** ([gtremblay@cornish.edu](mailto:gtremblay@cornish.edu)). *Criterion of Yielding is a group of drawings with elements from the mathematics of plasticity superimposed on vintage stereoscopic images exploring paths of stress and strain visually and emotionally.*

Criterion of Yielding is a drawing and virtual reality collaboration inspired by mathematical figures used to describe the elasticity or resilience of a material under many possible combinations of stresses and strains. Using historic stereoscopic landscape images from the Robert N. Dennis collection of the New York Public Library as the foundation, these collaborative drawings and animations are layered landscapes exploring how the mathematics of plasticity might apply to our human experience. This talk will present a group of finished drawings and several preliminary animation prototypes. A virtual reality art project will be developed using three-dimensional models extracted from the stereoscopic images animated together with elements taken from the drawings. (Received September 19, 2016)

1125-I1-2046 **Gareth E Roberts\*** ([groberts@holycross.edu](mailto:groberts@holycross.edu)), 1 College Street, Worcester, MA 01610. *We Got The Beat: Using Rhythm to Teach and Motivate Mathematics.*

Rhythmic structure in music is inherently mathematical. For instance, the lengths of standard notes (whole, half, quarter, etc.) form a decreasing geometric sequence with ratio  $r = 1/2$ . Adding more and more dots to increase the length of a note is equivalent to constructing a geometric series. The mathematical principle underlying polyrhythmic music is the least common multiple. These and other examples will be offered as creative ways to teach and motivate mathematical study. Musical examples presented will range from rock (e.g., The National) to classical (e.g., Verdi, Ligeti). (Received September 19, 2016)

1125-I1-2065 **Madhuri Bapat\*** ([durga1950@hotmail.com](mailto:durga1950@hotmail.com)), 1666 S. Cactus Wren Lane, Thatcher, AZ 85552. *Mathematics and Science in Rangolee Art from India.* Preliminary report.

Rangolee is an ancient folk art from India. This author has categorized the rangolee designs into the ones drawn with dots and the ones without dots. Some designs with dots are drawn by connecting the dots and others by drawing a line around the dots. These designs are also categorized as traditional and contemporary. She has investigated mathematical models and principles in those designs. Four principles that are found in the designs are 1. Eulerian Graphs, 2. Iterations or recursions, 3. Algorithm and 4. Symmetry. Some designs are drawn using Laws of Reflection from physics. Some designs follow cyclic order. Iterations of some designs are drawn more than one way. In her 10 min talk the author will do the case study of selected designs. (Received September 19, 2016)

1125-I1-2100 **Karl M Kattchee\*** ([kkattchee@uwlax.edu](mailto:kkattchee@uwlax.edu)), University of Wisconsin-La Crosse, and **Craig S Kaplan** ([csk@uwaterloo.ca](mailto:csk@uwaterloo.ca)), University of Waterloo, Cheriton School of Computer Science. *Combinatorial Poppies.*

We consider orthogonal closed paths that dwell on the edges of square grids. A certain subset of these paths are called “poppies.” We derive a formula for the number of asymmetrical poppies on an  $n$  by  $n$  grid, and we explore the aesthetics of the poppies. (Received September 19, 2016)

1125-I1-2221 **Mark Kozek\*** ([mkozek@whittier.edu](mailto:mkozek@whittier.edu)), Department of Mathematics, Whittier College, Whittier, CA 90608-0634. *Mathematics in Literature and Cinema.*

We describe our team-taught, interdisciplinary course *Numb3rs in Lett3rs & Films: Mathematics in Literature and Cinema*, which explores mathematics in the context of modern story-telling. To date, we have taught three iterations of this course, most recently in Spring 2015.

In this iteration, we examine the representation of mathematical characters – of female mathematical characters in particular – in literature and cinema. We highlight several pieces that have female main characters and pieces that were created by women, and we use these pieces as our entryway into mathematics. We define

“mathematical characters” loosely to include programmers, hackers and game designers, and we define “literature and cinema” broadly to include all forms of creative/theatrical story-telling, such as web-based content, documentaries and video games.

In this talk, we reflect on this course’s evolution over its three iterations. We describe several of the pieces we used, the mathematical topics they led us to, some of our more successful assignments and some of our challenges. We discuss what we learned from each iteration and what we hope to accomplish next (Spring 2017). Lastly, we mention other courses that have followed *Numb3rs in Lett3rs & F1ms*. (Received September 19, 2016)

1125-I1-2295      **Anil Venkatesh\*** ([anilvenkatesh@ferris.edu](mailto:anilvenkatesh@ferris.edu)). *Pythagoras to Secor: Improving the Miracle Temperament*.

In music theory, a temperament is a system of tuning. The concept of temperament has been studied since antiquity; it arises as a consequence of the unique factorization property of integers, applied to the pitches of musical notes.

In this talk, we briefly review historically significant temperaments such as Pythagorean tuning and 12-tone equal temperament. We then introduce George Secor’s “miracle temperament.” Discovered in 1974, the miracle temperament produces highly harmonious intervals compared to traditional tunings. In fact, it is considered by music theorists to be the best approximation of the first eleven overtones. Last year, we presented a new temperament with better approximations of these intervals than the miracle temperament. In this talk, we extend last year’s result with an analysis of the space of all temperaments. Our main result is an algorithm that fully categorizes all miracle-like temperaments in this space. (Received September 20, 2016)

1125-I1-2523      **Michael Sebastian Gagliardo\*** ([mgagliar@callutheran.edu](mailto:mgagliar@callutheran.edu)) and **Rafaella Fiore-Urizar**. *Barcelona Through the Looking Glass: A travel seminar on Mathematics, Architecture, and Detective Fiction*.

In Spring/Summer 2015 we ran a semester long course and two week travel seminar to Barcelona, Spain investigating the connections between Mathematics, Architecture, and Detective Fiction. The main mathematical content focused on the Art and Architecture of Antoni Gaudi. This course was for non-majors and included many hands on activities designed to illuminate some of Gaudi’s creation techniques. In this talk, I will discuss some of these activities and some connections between Mathematics, Architecture, and Detective fiction. (Received September 20, 2016)

1125-I1-2594      **Debra K. Borkovitz\*** ([dborkovitz@wheelock.edu](mailto:dborkovitz@wheelock.edu)). *Math Through Crochet, Quilts, and Temari: A Liberal Arts Math Course*.

Math Through Crochet, Quilts, and Temari is a course designed to appeal both to students who like crafts and are filling a math requirement and as an elective for Math for (Elementary) Teaching majors. I’ve taught it once and plan to teach it again this spring. The course meets MWF, and Fridays are craft days that aim for a quilting bee vibe, where students can bring their friends, and other members of the college community help teach the crafts. Some math topics included are alternative renderings of traditional elementary and high school math topics, such as the relationship between the circumference and volume of a sphere; pi; latitude/longitude and spherical coordinates; and exponential and trigonometric functions. Other topics are geometric topics often connected with the crafts, such as tessellations, polyhedra, symmetry, and some spherical and hyperbolic geometry. The craft projects include several temari balls (embroidered spheres), amigurumi (little crocheted stuffed animals and other figures), and hyperbolic plane models. In the talk I’ll focus especially on aspects of the course that are different from some of the published accounts of using crafts in math classes. (Received September 20, 2016)

1125-I1-2596      **Craig M. Johnson\*** ([johnsonc@marywood.edu](mailto:johnsonc@marywood.edu)), 18 Fieldstone Dr., Scott Township, PA 18411. *Identifying Dihedral Groups of Inversions in Music*.

Certain concepts in music theory can be used to expose students to interesting and straightforward problems that involve pattern recognition to identify subgroups of the dihedral group of order 12. These groups appear as sets generated by inversions and transpositions in the works of many classical composers such as Bach and Beethoven. (Received September 20, 2016)

1125-I1-2720      **John H Wilson\*** ([john.wilson@centre.edu](mailto:john.wilson@centre.edu)), Centre College, 600 West Walnut Street, Danville, KY 40422. *Mathematics in a Dramatic Warm-up Exercise*.

Many drama warm-up exercises involve actors standing around in a circle responding to cues given by other actors in the circle. In this talk, I devise a mathematical model for one such exercise and present some mathematical questions that arise. In the warm-up, which I will call Swing, an actor calls out a cue that passes the cue-calling

responsibility clockwise to another actor in the circle who in turn calls a cue and so forth around the circle of actors. The “Swing” cue passes the action to the person immediately adjacent to the caller. The “Ramp” and “Tunnel” cues skip one and two people respectively. By using modular arithmetic and some basic number theory we will identify sequences of calls that are fair and balanced. (Received September 20, 2016)

1125-I1-2745 **Kurt E. Ludwick\*** ([keludwick@salisbury.edu](mailto:keludwick@salisbury.edu)), Department of Mathematics & Computer Science, Salisbury University, 1101 Camden Avenue, Salisbury, MD 21801. *Recurrence Relations for Melodies and Tilings*. Preliminary report.

We consider the number of distinct melodies that can be written subject to specified restrictions, such as the number of pitch classes and allowed lengths of notes and/or rests. If  $x_n$  denotes the number of allowable melodies of length  $n$  beats, it turns out that the sequence  $\{x_n\}$  is generated by a recurrence relation, the equation for which depends on said restrictions.

Certain versions of this problem are isomorphic to well-known combinatorial “tiling” problems. In particular, there is an obvious one-to-one correspondence between the following:

- Melodies of length  $n$  beats using specified note lengths and pitch classes
- Tilings of length  $n$  cells using tiles of specified lengths and colors

On the other hand, if we allow our melodies to include *rests*, then the tiling analogy breaks down in a subtle but significant way, resulting in a different family of recurrence relations which (to the best of my knowledge!) has no well-known combinatorial analog. We will discuss recurrence relations for counting melodies which include rests, along with an analogous variation on the tiling problem. (Received September 20, 2016)

1125-I1-2918 **Jennifer Wilson\*** ([wilsonj@newschool.edu](mailto:wilsonj@newschool.edu)). *The mathematical problems of Sol LeWitt*. The work of the Conceptual artist Sol LeWitt is often compared to mathematics. There is his “subject matter”: the geometric forms and the exploration of dimension, scale, perspective, shape, and permutation. There is also the formal language: the apparent order, the economy of gesture, the reliance on notation and precise syntax. Most crucially, there is the sense that LeWitt is working out the solution to an explicit problem. In this talk we look at several problems LeWitt “posed” in his artwork and his proposed solutions. We compare works arising from “well-posed” and “ill-posed” problems and contrast the rational and irrational (his words) aspects of his artistic practice. Finally, we explore how LeWitt’s ideas can generate new art for us and for our students. (Received September 20, 2016)

1125-I1-2945 **Samuel Jinglian Li\*** ([samuel.wgx@gmail.com](mailto:samuel.wgx@gmail.com)), 204 Krider Rd., Sanford, FL 32773. *Polyphonic Piano Transcription with an Infinite Training Dataset*. Preliminary report.

We suggest the use of a procedurally generated dataset to train a supervised learning system for polyphonic piano transcription. Data can be generated quickly, on-demand, and in large quantities, reducing the chance of overfitting and avoiding the time-consuming process of curating large training datasets. The data generation algorithm is based on the linearity of sound, and creates training data for arbitrary note combinations by linearly combining the spectra of individual notes. Additional transformations and distortions are added to improve generalization. The generated data are used to train a standard feedforward neural network by using the backpropagation supervised learning algorithm. The network input is a high-dimensional vector representing the frequency spectrum of the input sound, calculated through a constant Q transform; the network output is an 88-dimensional binary vector representing note activations. The resulting networks are run on the MAPS piano dataset and evaluated through note- and frame-based metrics. We show that even very simple neural network architectures achieve reasonable transcription performance and excellent generalization qualities when trained using this procedurally generated data. (Received September 20, 2016)

1125-I1-3041 **Bill Linderman\*** ([wclinder@king.edu](mailto:wclinder@king.edu)), 1350 King College Rd, Bristol, TN 37620. *A Novel Idea: Teaching Mathematics using Apostolos Doxiadis’s Uncle Petros and Goldbach’s Conjecture*. Preliminary report.

We discuss how Apostolos Doxiadis’s gem of a novel, *Uncle Petros and Goldbach’s Conjecture*, can be used as required reading in an upper-level mathematics course as a way to explore several important themes in higher mathematics - the nature of proof, creativity, truth, and obsession. A work of fiction about a mathematical genius, the novel is rich with historical mathematical references and includes cameos by Ramanujan, Gödel, and Turing. (Received September 20, 2016)

1125-I1-3043 **Felicia Y. Tabing\*** ([tabing@rose-hulman.edu](mailto:tabing@rose-hulman.edu)), 5500 Wabash Ave, Terre Haute, IN 47805. *Incorporating the Arts in a Mathematics Classroom*.

Bringing art and design in a mathematics class can be a valuable way for students to experience mathematics. It gives students a different point of view, and shows the value of creative thinking that they may not get in their typical math course. I will present the different ways that I incorporate the arts in my math classes, even in the most unlikely classes, such as calculus and linear algebra aimed at engineering students. I will talk about how I infuse everything in my classes with art, from the design of notes, quizzes, and exams, to the assignments I give my students. (Received September 20, 2016)

1125-I1-3056 **Cheri L Boyd\*** ([cboyd4@naz.edu](mailto:cboyd4@naz.edu)), 4245 East Avenue, Rochester, NY 14618. *Bragdon and Trautmann's Math at the MAG*. Preliminary report.

The Memorial Art Gallery in Rochester, NY created an exhibit to highlight the friendship and collaboration between architect Claude Bragdon and artist Fritz Trautmann. Concurrent with the exhibit, our math education majors offered a workshop for art teachers exploring Bragdon's use of magic squares and higher dimensional geometry in *The Frozen Fountain: Being Essays on the Art of Design in Space* (1932). This talk will introduce Bragdon's development of projective ornament which appears in several of his notable Rochester buildings from the early 20th century. We'll segue to Bragdon's influence on Trautmann whose *Galaxy* (1942) illustrates a scientific, uniform use of color. The fourth dimension reminds us to assign *Flatland* reading and viewing. Inviting students to see beyond their usual dimension engages everyone, and naturally allows art students in our history of math course to think and write mathematically about their study of art. (Received September 20, 2016)

1125-I1-3125 **James M Henle\*** ([jhenle@smith.edu](mailto:jhenle@smith.edu)). *The Art that is Mathematics*. Preliminary report. I argue that one aspect of mathematics constitutes an art, an art that can take its place among the fine arts—painting, music, literature, etc. I propose that the objects of this art are mathematical structures.

Let me be clear about what I am not doing. I am not discussing mathematics in the service of art, or mathematics inspiring or enabling art. The sort of art exhibited at this conference, for example, while it is indeed art, is not what I call the art of mathematics. I am arguing for mathematics itself as art.

I am also not simply arguing that mathematics is beautiful. I am setting up a framework and identifying specific objects as the works of art. There are issues here of the philosophy of art (What is art? What is an artist?), the philosophy of mathematics (Are mathematical structures created or discovered?), and the history of mathematics (Who is or was a mathematical artist? Which mathematicians have been motivated primarily by aesthetic considerations?). (Received September 21, 2016)

1125-I1-3143 **Elizabeth C Rogers\*** ([brogers@piedmont.edu](mailto:brogers@piedmont.edu)), Piedmont College, 165 Central Ave., Demorest, GA 30535. *Art as a Pedagogical Innovation That Can Provide a Multicultural Dimension to the K-12 Classroom*.

Although mathematics has been described as a “universal language,” it is difficult for teachers to make it meaningful for today's diverse student populations. The link between mathematics and the arts is multicultural with a deep historical tradition that spans the globe. Creativity that allows the artist to take a mental vision and bring it forth as art is similar to the creativity of a mathematician looking at a problem from a totally new perspective and finding a solution. Throughout history, art and mathematics have contributed to the function and beauty of civilization. This presentation will focus on mathematics and how it is distinctly evident in art. Examples of art forms from global cultures will include architecture, weaving, painting, sculpture. These will be tied to multiple levels of mathematics as a pedagogical innovation that is inclusive of all students and cultures (Received September 21, 2016)

## Mathematics and Sports

1125-I5-48 **Paul R. Bouthellier\*** ([pbouthe@pitt.edu](mailto:pbouthe@pitt.edu)), 504 East Main Street, Titusville, PA 16354. *Tennis Anyone? Mathematical Modeling and Markov Processes*.

In this talk we shall create a mathematical model for the game of tennis. This model will then be used to create computer simulations for a singles tennis match. Our model is based on factors such as: first and second serve percentages for both players, percentages of points won on first and second serves, percentage of points won in rallies lasting so many shots (duration), and win-loss records in tie-breaks. Other factors such as court surfaces, best of three or five set games and endurance can also be built into the model. Estimations of such parameters can be obtained from recent head-to-head matches between players on various surfaces. Simulations are then run

to predict the outcome of matches based on the parameters described above. These will allow us to determine the “keys of the match.” The outcomes of the simulations will be compared to the probabilities generated from Markov processes. (Received June 22, 2016)

1125-I5-84

**Stanley Rothman\***, stanley.rothman@quinnipiac.edu, and **Alexander Everett**. *A theoretical approach for generating linear theorems to predict winning percentages for the teams in the mlb, nfl, nba and nhl at any point in a season.*

In a previous paper published in the Fall Edition, 2014 of The Baseball Research Journal, I presented an alternate formula to Bill James’ Pythagorean Theorem of Baseball. His formula states a team’s expected winning percentage  $EXP\ W\% = \frac{[(Runs\ Scored)^2]}{[(Runs\ Scored)^2 + (Runs\ Allowed)^2]}$ . My original alternate formula called The Linear Formula for Baseball states a team’s expected winning percentage  $EXP\ W\% = 0.000673 * (Runs\ Scored - Runs\ Allowed) + 1/2$ . I derived this linear formula using empirical data. Both these formulas were shown to accurately predict a team’s actual winning percentage for a season. In my latest paper, recently submitted for publication in the above journal, I have developed a new theoretical approach for deriving four new linear formulas for the MLB, NBA, NFL and NHL. This new theorem allows me to compare the four formulas for the four leagues. In my presentation, I will prove this new theorem and then show how easy it is to produce these four new linear formulas. I will explain why these four formulas differ and demonstrate that each of these formulas accurately predict a team’s actual season winning percent in all four leagues. I will introduce a new metric to determine which teams are over performing or under performing at any point in a season. (Received July 16, 2016)

1125-I5-88

**Gerry Myerson\*** (gerry.myerson@mq.edu.au), Mathematics, Macquarie University, NSW 2109, Australia. *The convex hull of a ballplayer.*

Athletes have their ups and downs. We form expectations of their level of performance based on their best years, and may consider they have not lived up to their potential in their other years. We propose a way to quantify an athlete’s potential, and the extent to which the athlete has fallen short of this potential, based on the familiar concept of the convex hull of a finite point set. We present our method in the context of yearly home run production in American major league baseball. (Received July 17, 2016)

1125-I5-249

**James R Henderson\*** (jrh66@psu.edu). *Maximizing Utility of Challenges in Professional Tennis.*

It is well documented that most incorrect line calls in tennis are made on balls landing on or inside the lines but which are nevertheless called “out.” This means that of the calls players want to challenge, most are made at the far end of the court. Unfortunately, there are several reasons that it is much more difficult to see the impact points of balls landing tens of yards away than those bouncing nearby. Thus, given that players want to challenge all and only (disadvantageously) incorrect calls, they are pulled in two directions: the potential for lineperson error leads players to challenge calls made on the far side of the court, and the potential for player error (with respect to linesperson error) leads players to challenge calls on the near side of the court. Further, speed of shot, direction of shot (with respect to the line it approaches), lighting conditions, and a variety of other factors make shots easier or more difficult to call correctly. How might players maximize the utility of their challenges? IBM and tournaments own (and do not generally release) Hawkeye challenge data, so figures have been independently collected from recorded matches to study this question. (Received August 18, 2016)

1125-I5-309

**Glenn D Sidle\*** (gdsidle@ncsu.edu) and **Hien Tran**. *Using Machine Learning to Predict the Next Major League Pitch*. Preliminary report.

Since the introduction of PITCHf/x in 2006, there has been a plethora of data available for anyone who wants to access the minute details of every baseball pitch thrown over the past decade. Everything from the initial velocity and release point to the break angle and strike zone placement is tracked, recorded, and used to classify the pitch according to an algorithm developed by MLB Advanced Media. Given these classifications, we developed a model that would predict the next type of pitch thrown by a given pitcher, using only data that would be available before he even stepped to the mound. We used data from three recent MLB seasons (2013-2015) to compare individual pitcher predictions based on linear discriminant analysis, multi-class support vector machines, and classification trees. Using a committee approach to reduce the variability in each prediction, we achieved results that beat the best guess for almost every pitcher examined while improving on the best guess by an average of close to 10%. (Received August 25, 2016)

1125-I5-405 **Reza O Abbasian\*** (rabbasian@tlu.edu), TLU Dept. of Math-CS-IS, 1000 W. Court St., Seguin, TX 78155, and **Kareem K Christmas** and **John T Sieben**. *Statistics of a Proposed Mercy Rule in College Football to Reduce Major Injuries*. Preliminary report.

Over the last thirty years, eleven sports have accounted for about 2000 catastrophic injuries of high school and college players. More than half of those injuries are attributed to athletes playing football. In our work, we have utilized twelve years of data from over 10,000 college football games to develop statistical models which can be used to approximate the probability of winning as a function of score difference at various stages of the game. Using our model we can then determine the score difference, at the end of first, second and third quarter, at which there is a near-zero probability of a comeback. Given the probability distribution of major injuries in college football we will then attempt to estimate the optimal stoppage criteria, in terms of score difference and time, which produces the maximum reduction in the number of catastrophic injuries. (Received August 31, 2016)

1125-I5-427 **Sheldon H Jacobson\*** (shj@illinois.edu), **Arash Khatibi** and **Douglas King**. *Seeds of Victory: Big Ideas with Small Data in March Madness*.

Every March, sports fans meticulously scrutinize the teams participating in the NCAA Division I Mens Basketball Championship Tournament (aka March Madness), seeking to design a perfect bracket that correctly predicts the outcomes of all tournament games. With several quintillion possible brackets, such perfection is not achieved by chance. This work proposes probability models that describe seed and bracket position performance in the tournament using a small data set, containing the results of past NCAA tournament games, and hence, these models do not rely on expert knowledge of the participating teams. Using these data, the proposed probability models estimate the likelihoods of every potential combination of seeds that can appear in each round of the tournament; statistical analysis shows that these estimates provide a close fit to the historical seed distributions in past tournaments. These models can provide a set of tools for building brackets; users wishing to rely on the models alone can generate bundles of random brackets according to the estimated distributions. Comparison of the brackets generated using these models with the winners of the ESPN Bracket Challenge tournaments for the past several years is reported. (Received September 01, 2016)

1125-I5-940 **John Kersch Mayberry\*** (jmayberry@pacific.edu), 3601 Pacific Ave, Stockton, CA 95211. *Tracking Athlete Wellness*. Preliminary report.

Developing simple, repeatable, and reliable methods for monitoring athlete wellness is an important goal for trainers, athletes, and coaches. Such measures not only have the potential to predict fatigue related declines in athlete performance, but also the potential to assess risk of injury. In this talk, we will discuss two efforts currently underway in the athletic department at our university aimed at addressing this issue: tracking self-reported athlete health statistics through qualitative surveys and tracking quantitative force-time jump statistics through SpartTrac<sup>TM</sup>'s patented software. We will present results of one case study with the men's water polo team in which self-reported health statistics were highly correlated with team effort, but not necessarily with team execution, and conclude by discussing some of the promising features of working with the more quantitative SpartTrac<sup>TM</sup> measurements. (Received September 13, 2016)

1125-I5-961 **Jacqueline Brannon Giles\*** (jbgiles@yahoo.com), 13103 Balarama Drive, Houston, TX 77099. *The Existence and Uniqueness of Metrics in Sports*.

The presenter will identify historical sports figures such as Simone Manuel, Paul Warfield, Warren Wells, Jesse Owens and others. The uncovering of metrics that establish the uniqueness of their accomplishments during certain timeframes will be discussed. Outliers will be identified with comments on the methods that suppress the identification of accomplishments by truncating data. Questions of validity and reliability of data manipulation will be posed. (Received September 13, 2016)

1125-I5-969 **Megan Olivia Powell\*** (mpowell@stfrancis.edu), University of St. Francis, 500 Wilcox St., Joliet, IL 60435. *Markov Chain Models of NFL Overtime Rules*.

In recent years, the National Football League's (NFL) rules for overtime have changed from a sudden death format to a modified sudden death format. Now, the team receiving the ball first during overtime can only win on their first possession with a touchdown. In this talk, I will discuss a Markov Chain model of the two overtime formats and use NFL overtime data to analyze the fairness of the two models. The modified sudden death is still criticized, especially for playoff games, so I will propose a new model where each team is allowed at least one possession of the ball. (Received September 13, 2016)

1125-I5-980 **Roland Minton\*** ([minton@roanoke.edu](mailto:minton@roanoke.edu)), 620 Beech Road, Salem, VA 24153. *Skill and Randomness on the PGA Tour.*

A standard test of whether or not a statistic is measuring an athletic skill is the correlation of players' statistics in one year to their statistics in the next year. This correlation is computed for Strokes Gained from ShotLink data for numerous types of golf shots (e.g., putting, fairway shots from 100-150 yards, drives on par 4's). With the exception of driving statistics, the correlations are all considerably lower than typical sports statistics, and on the order of dubious measures such as clutch play in baseball or basketball. This is not evidence that Strokes Gained is a flawed statistic or that putting is not a real skill, but rather indicates the small margins of error for golfers on the PGA Tour. (Received September 13, 2016)

1125-I5-983 **David Moreau\*** ([dbmoreau@mail.roanoke.edu](mailto:dbmoreau@mail.roanoke.edu)). *A Student Stat Crew at Roanoke College.* The Maroon Stat Crew is a group of students at Roanoke College who provide data collection and statistical analysis for the school's athletic teams and coaches. As a Division III school with a small budget, the data collection problem is substantial. The Crew has created manual collection and laptop collection strategies that can be handled by two students at the games. Data analysis has focused on visual representations that are meaningful for coaches and players. Examples will be given from soccer, basketball, and lacrosse of data collection procedures, statistical analysis, and feedback from the coaches and players. (Received September 13, 2016)

1125-I5-1068 **Tetyana Berezovski\*** ([tberezov@sju.edu](mailto:tberezov@sju.edu)), Saint Joseph's University, 5600 City Avenue, Philadelphia, PA 19131. *Mathematics with Apparatus: explorations into rhythmic gymnastics.* Preliminary report.

Mathematics of Sports began to be recognized as an attractive platform for innovative instruction of mathematics. In this presentation participants will learn about mathematical modeling tasks with rhythmic gymnastics as a lens for application of mathematical knowledge. These tasks were designed for prospective secondary mathematics teachers, but could be appropriate for usage in college level courses. Evaluation of apparatus position during elements performance with Euclidean Geometry, Trigonometry and Calculus models is discussed. (Received September 14, 2016)

1125-I5-1384 **Adam F Childers\*** ([childers@roanoke.edu](mailto:childers@roanoke.edu)), 2521 Robin Hood Rd., Roanoke, VA 24014. *Thinking Outside the Box-Score in Lacrosse.*

The data available for men's college lacrosse is very limited and therefore making a predictive model for goals scored is primitive at best. Through the use of a custom application, we recorded spatial data for the Roanoke College men's lacrosse team for the 2016 season. This more comprehensive data describes the spatial build up to shots (passing and running), shot location (field and goal), and any penalties awarded. With this more detailed game data we are able to build a more accurate model for goals scored, describe the quality of shots, and get a better understanding of where the defense is vulnerable. We will compare this custom data to box-score data to quantify the value of the more detailed spatial data. (Received September 16, 2016)

1125-I5-1402 **John David\*** ([davidja@vmi.edu](mailto:davidja@vmi.edu)), 512 Letcher Ave., 430 Mallory Hall, Lexington, VA 24450. *Poor Man's Total Quarterback Rating.*

ESPN's Total Quarterback Rating (QBR) has been one of the most fascinating developments in NFL analytics in the last few years. It does however have a few shortcomings, specifically its data and analyst intensive requirements, lack of a publicly available algorithm and its availability only for QBs going back to 2006. The Poor Man's QBR (PQBR) captures much of the variation in QBR with readily available and commonly used statistics. Additionally, it can be applied to all QBs back to 1969. We will discuss the development of this rating, how it compares to other common QB metrics, and how it can be applied to understanding various aspects of a QBs career. Finally we will use this metric to make our contribution to the perennial discussion of who are some of the greatest NFL QBs of all time. (Received September 16, 2016)

1125-I5-1436 **Eric Alan Eager\***, 1725 State St., La Crosse, WI 54601. *A Data Science Approach to Picking National Football League Games.* Preliminary report.

In this talk I discuss a statistical model for picking National Football League (NFL) games, both against the spread and straight up, using a collection of openly-available and proprietary data. Features of the model include historical Vegas betting lines and spreads, as well as Pro Football Focus' individual player grades. Ranking algorithms including the Keener method are also used. The performance of this model, on test data from 2007-2015, as well as the 2016 NFL regular season, are compared with that of media experts. Future possible features of the model are also discussed. (Received September 16, 2016)

1125-I5-1565 **Andrew B Perry\*** ([aperry@springfieldcollege.edu](mailto:aperry@springfieldcollege.edu)), Department of Mathematics (MPCS), Springfield College, 263 Alden St., Springfield, MA 01106. *The Newest Football Statistics and Football Analytics Research.*

Organizations such as Sports Info Solutions and Football Outsiders have been recording extremely detailed football play-by-play data in the past decade. Some examples of this data are: (a) players participating in each snap, (b) types of runs or passes (draw play, screen pass, quick hitch, etc.), (c) yards after catch, passes thrown away (by QB), and (d) location of passes caught (chest, high, low). We discuss the scope of currently available data. Some of this data has been exploited in published research and we also consider the potential of this data both in player and team evaluation and in rethinking football strategies. (Received September 18, 2016)

1125-I5-1612 **Tien Chih\*** ([tien.chih@newberry.edu](mailto:tien.chih@newberry.edu)), 517 Turkey Pointe Lane, Chapin, SC 29036, and **Demitri Plessas**. *A Search for Champion Boxers.*

In the world of boxing, fighters are judged by the quality of their opponents. This way of determining value is reminiscent of Google's PageRank algorithm, which assigns values to websites based on the value of sites linked to it. We describe the mathematics behind PageRank and give a scheme to adapt it to boxing. We illustrate how this algorithm would behave in several scenarios. Then 20 top heavyweight boxers from the 1990's are selected and ranked using this method. We also demonstrate show how Sage code may be easily implemented to execute this algorithm and finally, describe some possible extensions of this method. (Received September 18, 2016)

1125-I5-1685 **Howard H Hamilton\*** ([hhamilton@soccermetrics.net](mailto:hhamilton@soccermetrics.net)), Atlanta, GA 30339. *A Bayesian Analysis of Draft Pick Value in Major League Soccer.*

Major League Soccer (MLS), like other North American professional sports leagues, uses multiple draft schemes to distribute talent among its member clubs. A club's draft selection can be viewed as an asset that can be exercised or traded, so it is useful to understand a draft selection's value and its evolution over time. In this talk I will present valuation models that result from a Bayesian local regression of the expected career value of a draft pick. The models are valid for a specific draft year and are trained by the career values of draftees in previous years. The models are differentiated by the use of a time horizon to filter players in the training set and restricting career values to those earned at the drafting team. Results indicate that valuation models trained with no time horizon exhibited smoothed curves but significantly overvalued draft slots, particularly in the middle of the draft. Further analysis indicates that the expected value of later selections in the draft has collapsed in the last five years, which might explain club behaviors at recent MLS SuperDrafts. (Received September 18, 2016)

1125-I5-2025 **Franklin H. J. Kenter\*** ([kenter@usna.edu](mailto:kenter@usna.edu)). *How To Win Your March Madness Pool with Jensen's Inequality and The Law of Large Numbers.*

Every year during March Madness, articles appear informing people they are silly by selecting unlikely upsets in their brackets thereby decreasing their expected score.

In this talk, we show that, in fact, these articles are incorrect. Indeed, for a winner-take-all NCAA March Madness pool, participants should not select a bracket by only choosing the favored teams to win. Rather, participants should "mimic" the appearance of an actual bracket by selecting a reasonable number of upsets.

More generally, we determine the optimal strategy for a winners-take-all betting pool with sufficiently many participants using only elementary tools. (Received September 19, 2016)

1125-I5-2151 **Diana S Cheng\*** ([dcheng@towson.edu](mailto:dcheng@towson.edu)) and **Kyle P Hurley** ([khurle6@students.towson.edu](mailto:khurle6@students.towson.edu)). *Physics and Mathematics within Pairs Figure Skating Jumps*. Preliminary report.

In the winter Olympic sport of figure skating, jumps are a required element for pairs skaters. We report on a secondary mathematics and physics activity based on kinematics and situated in the context of pairs side-by-side jumps. The activity was designed to address Common Core State Standards in Interpreting Functions and Next Generation Science Standards in Forces & Interactions. (Received September 19, 2016)

1125-I5-2162 **Jeffrey C Kallenbach\*** ([jkallenb@sienaheights.edu](mailto:jkallenb@sienaheights.edu)), 1247 East Siena Heights Drive, Adrian, MI 49221, and **Timothy H Husband** ([thusband@sienaheights.edu](mailto:thusband@sienaheights.edu)), 1247 E. Siena Heights Drive, Adrian, MI 49221. *Bean Bags and Basketball - Simple, Complete Experiments for the Introductory Statistics Classroom.*

The fairly recent approach to teaching the modern introductory statistics course walks the students through the entire statistical process (including simulation and hypothesis testing), in a variety of contexts, from the beginning of the course. In-class experiments allow us to discuss and develop, as a group, these concepts, as well as randomization and control. At the end of our units on paired data and consistency, we perform in simple

one-class period experiments that allow us to carry out the entire process, and at the same time engage in friendly competition between sections in pursuit of travelling trophies designating the most accurate and most consistent competitors. We outline our procedure, equipment, goals, and results of our experiences. (Received September 19, 2016)

1125-I5-2199 **Nicholas Gorgievski\*** ([nick.gorgievski@nichols.edu](mailto:nick.gorgievski@nichols.edu)), Center Road, PO Box 5000, Dudley, MA 01571. *The New NFL Overtime Rule: A Logistic Regression Analysis.*

In the National Football League, the coin toss winner was shown to have an advantage during the overtime period since the beginning of the 1994-1995 season. As a result, the NFL owners accepted a proposal to alter the overtime rule for postseason games beginning in the 2010-2011 season and for the regular season games beginning in the 2012-2013 season. In this session, I use logistic regression to examine the coin toss and its effect on the outcome of the overtime games under the new overtime rule. (Received September 19, 2016)

1125-I5-2378 **Mark C Ginn\*** ([ginnmc@appstate.edu](mailto:ginnmc@appstate.edu)), Department of Mathematical Sciences, Boone, NC 28608, and **Madison Combs, Amanda Lake Heath and Zachary Snipes.** *Expected Points in Appalachian State Football.*

The expected points function  $E(p, dd)$  gives the expected value of the next points scored in a football game given the current field position  $p$  and the current down and distance  $dd$ . This function can be used to analyze many different situations in a game such as when to punt or go for it on fourth down, whether to try on long pass on third and long r run a more conservative play and many other situations. While almost anyone would agree that the values of this function depend greatly on the team being analyzed it is very difficult to evaluate this function for a particular team because of the relatively small amount of data available (400 – 500 offensive plays in a season) for the large number of different input states (100 different field positions and  $> 40$  down and distance scenarios).

We compute values of this function for the Appalachian State football team for the 2015 season. We use various strategies to account for the lack of data and compare our results to other functions in the literature. (Received September 20, 2016)

1125-I5-2415 **Tom Tegtmeier\*** ([ttegt@math.utexas.edu](mailto:ttegt@math.utexas.edu)), The University of Texas at Austin, Department of Mathematics, Austin, TX 78712-1202. *Using the Oracle method to rank pitchers and batters.* Preliminary report.

The Oracle method (Balreira et al., 2014) introduced a league-wide common opponent into a Markovian ranking procedure. The Oracle can be used to simultaneously rank Major League Baseball pitchers and batters. This method produces both expected and surprising results. (Received September 20, 2016)

1125-I5-2682 **Charles Bergeron\*** ([chbergeron@gmail.com](mailto:chbergeron@gmail.com)), Albany College of Pharmacy and Health Sc., 106 New Scotland Ave, Albany, NY 12208. *What is the relative value of gold, silver, and bronze Olympic medals?* Preliminary report.

We investigate multi-objective ranking in the context of comparing country performances in sporting events such as the Olympics. For example, in the 2016 Summer Games in Rio de Janeiro, Croatia won 10 medals (5 gold, 3 silver, 2 bronze) and Canada won 22 medals (4 gold, 3 silver, 15 bronze). Which country displayed superior performance? Croatia won an extra gold, but Canada's total medal count is more than double Croatia's. Deciding this question raises the issues of *What is the value of a gold medal?* or *What is a gold medal worth relative to a silver or bronze?* The International Olympic Committee officially ranks countries according to the number of awarded gold medals, using silver and then bronze to break ties. This scheme is skewed, placing very high value on gold. In many countries, media outlets display medal tables according to total medal count, producing radically different results. This scheme is also skewed, assigning the same value to all medals. We hypothesize that it is possible to analyze medal table data to obtain a country ranking that is minimally controversial. Through formulating and solving optimization problems, we propose an objective scoring system. (Received September 20, 2016)

1125-I5-2891 **Conor Maley\*** ([cmaley17@wooster.edu](mailto:cmaley17@wooster.edu)), 1189 Beall Ave, Wooster, OH 44691. *Evolving Monkeys into Hawks: Analyzing Optimal Drafting Techniques Used for Daily Fantasy Football using Mathematical Modeling and Machine Learning.* Preliminary report.

The Daily Fantasy Sports industry has recently skyrocketed. Led by powerhouses FanDuel and DraftKings, the industry is estimated to be worth over a billion dollars. The games themselves have been declared "skill games," meaning there is an underlying strategy behind Daily Fantasy Sports. This anomaly can be seen simply by that fact that only a few top players typically win most of the cash prizes. In this project, we provide a deep exploration into the data science that lies behind choosing a team in Daily Fantasy Football. With team

salary restrictions, Daily Fantasy Sports present an interesting optimization problem. Users must make difficult decisions on which players to play on a given week. We investigate these decisions using analytics and machine learning to both model and analyze past data. (Received September 20, 2016)

1125-I5-2949 **Tyler Skorczewski\*** ([tskorczewski@cornellcollege.edu](mailto:tskorczewski@cornellcollege.edu)), 600 First Street SW, Mt. Vernon, IA 52314. *Modeling learning in youth archery.*

The National Archery in the Schools Program (NASP) began in Kentucky in 2003 and has rapidly expanded to several states to reach thousands of student archers from elementary school through high school. In a NASP tournament, student archers using standardized equipment shoot 30 arrows that each score between 0 and 10 (bullseye) giving a maximum tournament score of 300. While scores are an excellent metric to rank archer ability, they are a very limited measure of how well an individual archer is learning and improving. In this project we use archer data and a differential equation based model of a learning curve to quantify archer learning in a way independent of scores and to help us identify best practices in coaching archery. Moreover, it has the advantage of requiring very little prerequisite knowledge of students (calculus and statistics), which makes it an excellent candidate for individual student research projects and an early introduction to applied mathematics and data science. (Received September 20, 2016)

1125-I5-2979 **Jeffrey W Heath\*** ([jeffrey.heath@centre.edu](mailto:jeffrey.heath@centre.edu)), 600 W. Walnut St., Danville, KY 40422, and **Eric Murrell** and **Trevor Brewer**. *Basketball and Football Win Probabilities and the Point Spread.*

We examine how win probabilities in basketball and football correlate to the point spread. We then use these correlations to model the in-game win probabilities as a function of the point spread, the current point margin, and the current possession data. We present our win probability model on recent NCAA, NBA, and NFL data. (Received September 20, 2016)

1125-I5-3004 **Derrick R. Yam\*** ([dyam@skidmore.edu](mailto:dyam@skidmore.edu)), 815 N. Broadway, Skidmore College, Saratoga Springs, NY 12866, and **Michael J. Lopez**. *Quantifying the causal effects of conservative fourth down decision making in the National Football League.* Preliminary report.

Historically, it has been argued that NFL coaches are too conservative in attempting fourth downs. However, many empirical approaches looking at team decision-making are confounded by extraneous factors. For example, teams that decide to "go for it" on fourth down are inherently different from those that choose other play types. In many instances, teams are obliged to go for it because they are trailing on the scoreboard. As a result, inference on how all teams should behave on fourth down has required unjustifiable extrapolations. Using a data set featuring the last 11 years of play calls, we attempt to quantify the benefits of aggressive fourth down behavior in the NFL. Utilizing tools from causal inference and a nearest neighbor matching algorithm, teams that went for it ('treatment') are paired to those who did not go for it ('control') based on their probability of going for it, defined as the propensity score. After estimating each team's win probability before and after each play based on a random forest model, we approximate the additional number of wins that NFL teams could gain by implementing a more aggressive fourth down strategy. Results better inform decision-making in a high-stakes environment where standard statistical tools are informative but, to date, limited. (Received September 20, 2016)

1125-I5-3048 **Michael A. Furuto\***, [mfuruto@hawaii.edu](mailto:mfuruto@hawaii.edu). *An Analysis of 3 point shooting in the NBA, NCAA, and Olympics.* Preliminary report.

Engaging students in real-world problems is integral to developing their comprehension and appreciation of math. In the world of sports, math is ubiquitous, and participants will explore sports applications of 3 point shooting at different levels of basketball (NBA, NCAA, and Olympics). Content covered will include statistical and data analysis, problem solving, and the connection between math and basketball. The purpose is to share pedagogical approaches that have been effective, which can potentially lead to deepening student conceptual understanding, increasing procedural fluency, and solidifying skill proficiency. (Received September 20, 2016)

## *Mathematics Experiences and Projects in Business, Industry, and Government*

1125-J1-608 **William P Fox\*** ([wpxfox@nps.edu](mailto:wpxfox@nps.edu)), Department of Defense Analysis, Naval Postgraduate School, Monterey, CA 93943. *Mathematical Modeling of a Decision Planning Tool*. Preliminary report.

Ever been presented with a requirement for a decision –planning model to test plans prior to implementing. Here we present a model based upon a joint staff requirement for a planning tool that addresses redline issues. The draft model designed uses logistics regression and a logistics surface in 3 dimensions to address planning. Historical data and survey data is used to build the model. Additionally, TOPSIS and decision weights are used to obtain useful data in lieu of names of alternatives. After the model is build a predictive tool is developed where “real time” data is put into the model and the probability of a redline is calculated. It is up to the user to define the redline probability within the MS-Excel tool. (Received September 07, 2016)

1125-J1-999 **Abraham Levitan, Jessica Oehrlein\*** ([jessica.oehrlein@columbia.edu](mailto:jessica.oehrlein@columbia.edu)), **Laura Christakis** and **Shawn Ryan**. *Modeling of Gastrointestinal Stent Behavior*. Preliminary report.

We develop a model of the behavior of braided gastrointestinal stents under localized radial compression for use in the stent design process. Our approach is to develop an energy functional to calculate the energy in the stent given a radial profile. We then minimize this energy functional subject to a beltline constraint and calculate the resulting force response of the stent. This work was done under the sponsorship of Boston Scientific Corporation. (Received September 13, 2016)

1125-J1-1886 **James H Fife\*** ([jfife@ets.org](mailto:jfife@ets.org)), Educational Testing Service, 660 Rosedale Road, Princeton, NJ 08541. *Automated Scoring of Extended Text Responses to Mathematics Test Items*.

The immediate, automated scoring of constructed responses is an important and necessary feature for large-scale summative assessments, where efficient and cost-effective scoring is required, and for smaller-scale formative assessments, where immediate feedback is part of the learning process. A common framework for traditional paper-and-pencil mathematics tasks is to ask a question that has a simple numeric answer but to require the student to show his or her work. The scratch work can be read to make certain that the student has solved the problem correctly or to determine if the student deserves partial credit when the final response is incorrect. For computer-delivered assessments, the instruction to “Show your work” is not really appropriate, so instead students are often asked to explain their reasoning. These explanations must be human scored, adding time and cost to the assessment. In this talk, I will describe a project in which we used machine-learning techniques to score responses to computer-delivered mathematics tasks that requested an explanation or other extended text response. (Received September 19, 2016)

1125-J1-1970 **Kevin E. Burns\*** ([keburns@mc.edu](mailto:keburns@mc.edu)), P.O. Box 4025, Clinton, MS 39058, and **James D. Morgeson, Jason A. Dechant** and **Yazmin Seda-Sanabria**. *Exposure: A Decision Metric for Selecting Effective Sets of Security Upgrades at Dams*.

The U.S. Army Corps of Engineers (USACE) conducts Security Risk Assessments (SRAs) at dam projects. The Common Risk Model for Dams (CRM-D) provides a mathematically rigorous way to conduct SRAs. The CRM-D quantifies risk as the product of the probability of a successful attack, given it is attempted, and consequences. Referred to as conditional risk, this decision metric is the expected loss given a specified attack is attempted. A specified attack (consisting of an attacker type and an attack vector) carried out on a particular target comprises a scenario. The CRM-D considers three attacker types and thirty-two attack vectors. A dam with only a modest number of assets could thus have several hundred scenarios and, consequently, several hundred conditional risk estimates. Exposure is a decision metric which allows an analyst to aggregate conditional risk estimates across scenarios. An analyst can use exposure to compare risks by attack type, by target or for any set of scenarios. These comparisons can be used to determine a proposed set of security upgrades. A standard set of graphics and calculations based on exposure are introduced that summarize the current level of risk at a dam as well as the reduced level of risk should the set of security upgrades be implemented. (Received September 19, 2016)

1125-J1-2528 **Robert Chaney** and **Chanda Hughes\*** ([chughe08@leeu.edu](mailto:chughe08@leeu.edu)). *From the Classroom to the Corporate World: Sharing Internship Experiences*. Preliminary report.

Lee University has recently launched an internship program for mathematics majors. The main portion of the program involves working with local businesses on mathematical problems that arise in the industry. In this talk

we will give an overview of the program, introduce the problems we worked with, discuss the application of various mathematical processes to the problems and share the implications of our analyses. (Received September 20, 2016)

1125-J1-3024 **Bhagya Athukorallage\*** ([bhagya.athukorala@ttu.edu](mailto:bhagya.athukorala@ttu.edu)), Whitacre College of Engineering, Department of Mechanical Engineering, Lubbock, TX 79409, and **Darryl James**, Whitacre College of Engineering, Department of Mechanical Engineering, Lubbock, TX 79409. *Heat transfer analysis of road pavement system with phase change materials.*

Thermally-induced permanent deformation (rutting) is one main drawback faced by asphalt pavements over their life period which negatively affects the performance of roads. Phase change materials (PCMs) possess the ability to absorb a large amount of energy (latent heat) at a constant or narrow temperature range, and therefore the incorporation of phase change materials in asphalt pavement mixtures may help to regulate extreme temperature regions in pavement structures.

In this study, we consider a PCM- embedded pavement structure and assess its thermal performance through numerical simulations. The volume-averaged energy equation with phase change is used to analyze the transient temperature behavior in the integrated PCM pavement system. It is assumed that pavement surface is subjected to time dependent solar irradiation and convective heat fluxes. We explore the performance of the new PCM-embedded pavement system with varying PCM volume fractions. Our results show that a higher temperature drop through the pavement surface with PCM can be realized compared to a conventional pavement without PCM and the effective thermal conductivity of the PCM-embedded layer plays a key role in the heat transfer through the new pavement system. (Received September 20, 2016)

## Meaning Modeling in the First Two Years of College

1125-J5-207 **Aden Ahmed\*** ([aden.ahmed@tamuk.edu](mailto:aden.ahmed@tamuk.edu)), Department of Mathematics, Texas A&M University-Kingville, Kingsville, TX 78363, and **Khairul Islam** ([kislam@emich.edu](mailto:kislam@emich.edu)), Department of Mathematics, Eastern Michigan University, Ypsilanti, MI 48197. *On Implementing Meaningful Model Selection Criteria.*

We are surrounded by real-life data, and fitting models to real-life data are of the greatest interests while teaching modeling to beginner college students. It is possible that the fitted models are not adequate due to the lack of fitness or valid interpretability. Given a set of data, we might end up with different interpretations due to the use of different alternative models. What criteria should determine the best model for a given set of data? The answer to this question leads to the model selection criteria, which traditional text books never address. In this presentation, we will review various model selection criteria, which can be implemented successfully to help students adapt a meaningful model. We will also provide various examples and applications from real-life situations in order to demonstrate effectiveness of these model selection criteria. (Received September 20, 2016)

1125-J5-671 **Vlajko L Kocic\*** ([vkocic@xula.edu](mailto:vkocic@xula.edu)), Mathematics Department, Xavier University of Louisiana, 1 Drexel Dr., New Orleans, LA 70125. *Course "Mathematical Modeling in Life Sciences" at Xavier University of Louisiana.* Preliminary report.

The course "Mathematical Modeling in Life Sciences" was introduced few years ago at Xavier University of Louisiana. It is 2000-level course intended for mathematics and science majors interested in applied mathematics. The only prerequisite for the course is completion of Calculus I which present major challenge. In this talk we discuss several challenges and ways to overcome them such as limited mathematical background, diverse student population, interdisciplinary aspect of the course, selection of topics. (Received September 09, 2016)

1125-J5-1049 **Brian J Birgen\*** ([brian.birgen@wartburg.edu](mailto:brian.birgen@wartburg.edu)), 100 Wartburg College, Waverly, IA 50677. *An Alternative First Year Calculus Course: Modeling Calculus.*

Instead of the traditional Differential Calculus, we offer to our incoming college students a numerical based modeling with differential equations course. The students learn Euler's method and slope fields which do not require strong Algebra skills immediately and are analyzing simple population models by the end of the first month. Topics covered in the course include air resistance, rocket motion, bungee jumping, predator-prey, malaria just to name a few. Students are required to read journal articles and replicate their mathematical models.

We will discuss the benefits and pitfalls to such an alternate approach. (Received September 14, 2016)

1125-J5-1500 **Dan Kalman\*** ([kalman@american.edu](mailto:kalman@american.edu)). *A Discrete Approach to Continuous Logistic Growth.*

Logistic growth is a common modeling topic in courses that precede calculus. The discrete version is characterized by a difference equation of the form  $P_{n+1} = m(L - P_n)P_n$ , where  $m, L > 0$  are real parameters. These models are easily motivated as a refinement of exponential growth, and when  $0 \leq mL \leq 3$ , produce nicely behaved examples of limited growth. However, there is no general closed form solution to the difference equation, limiting the analysis methods accessible to precalculus students. Continuous logistic growth is defined by an equation of the form  $P(t) = \frac{A}{1+Bb^t}$ . These models are more easily analyzed in a precalculus context, but are harder to motivate.

This talk presents a synthesis of the two forms of logistic growth. It arises naturally as a refinement of the difference equation above. The refined difference equation is solvable by elementary methods, and the solutions are none other than continuous logistic growth curves. This development offers a meaningful example of successively refining a model by reconsidering its assumptions. As an added bonus, it inspires (re)discovery of a beautiful approach to solving the logistic growth differential equation. (Received September 17, 2016)

1125-J5-1632 **Suzanne Sumner\*** ([ssummer@umw.edu](mailto:ssummer@umw.edu)), 1301 College Avenue, Fredericksburg, VA 22401. *Environmental Applications: Introduction to Mathematical Modeling.*

The University of Mary Washington's Introduction to Mathematical Modeling course originated through an NSF grant to improve pre-service teacher preparation in STEM fields, so they become better mathematics and science teachers. This course focuses on the environment, i.e., every example describes some environmental issue, such as overpopulation, endangered species, pollution, limited resources, natural disasters, and epidemiology. The mathematical topics covered are linear regression, curve fitting, and difference equations. The first project involves finding the best-fit model (linear, exponential, power, or logarithmic) to a data set from Pfaff's Sustainability Math website, e.g., carbon emissions, global temperature, grain production, oil consumption, ozone depletion, or wind power production. For the second project, students create difference equations to describe environmental scenarios such as pollution, recycling, overpopulation, and invasive species. The third project studies the biology of infectious diseases, which makes it easier to model disease spread with a Susceptible-Infective-Removed difference equation model. Using pre-course and post-course surveys, classes have shown significant improvement in students' confidence in their mathematical abilities. (Received September 18, 2016)

1125-J5-1777 **Teresa D Magnus\*** ([tmagnus@rivier.edu](mailto:tmagnus@rivier.edu)), Dept. of Mathematics and Computer Science, Rivier University, 420 S. Main St., Nashua, NH 03060. *Less is More: Mathematical Modeling Experiences for non-STEM Majors.*

Mathematical modeling exposes students to how mathematics can be used to solve real-world, open-ended problems. This talk will share examples of some simply-stated questions which encourage students to make assumptions and choose their own strategies to solve a problem. Although the mathematical tools are not highly sophisticated, the lively mathematical discussions that ensue during the solving process and the refinement of solutions can lead to a deeper understanding of mathematics and dispel misconceptions about mathematics. No techniques or examples are presented in advance by the instructor. Instead the focus is on having students read and interpret the problem and determine the tools available to them. Less is provided, but more is gained. (Received September 19, 2016)

1125-J5-1807 **Melissa A Stoner\*** ([mastoner@salisbury.edu](mailto:mastoner@salisbury.edu)). *Calculus in Clinical Medicine: Using the Simulation Center to Model and Motivate Calculus and Differential Equations.* Preliminary report.

How can we utilize the medical simulation center on campus to enhance learning in mathematics class? Based on a student project, we have designed a differential equations model of the mechanical lung used in respiratory care. In collaboration with the respiratory therapy program, we have run simulations on the mathematical model comparing results to the model lungs in the simulation center. We then discuss the design of a project for the Differential Equations course regarding the mechanical lung, as well as the integration of the simulations in Calculus I to motivate the use of derivatives in medicine. We also share the results and outcomes of the project in a Fall Calculus class and the expansion for the Differential Equations class. (Received September 19, 2016)

1125-J5-2435 **Gregory D Foley\*** (foleyg@ohio.edu), 321D McCracken Hall, Athens, OH 45701-2979. *Modeling with Mathematics: A Second Course in a Quantitative Reasoning Pathway.* Preliminary report.

Modeling with Mathematics is a course designed for students who have completed a one-semester course in Quantitative and Statistical Reasoning. It may serve the needs of some mathematics majors and future mathematics teachers but is intended to be of general interest to all college and university students. Modeling with Mathematics is an innovative course in modeling using sequences, functions, and geometry. The MwM course is designed to fill a critical gap in the nation's collegiate mathematics course offerings. The traditional core topics of high school mathematics—algebra, geometry, functions, and trigonometry—are applied to solve genuine problems in the world around us. Motivating questions, engaging investigations, and a capstone modeling project are the heart of the course. The MwM course balances reasoning and sense making with communication and in-context problem solving. This talk presents a sampling of tasks from the course. (Received September 20, 2016)

1125-J5-2468 **Kelly Black\*** (kjblack@gmail.com), Dept of Mathematics, University of Georgia, Athens, GA 30605, **Guangming Yao** (guangmingyao@gmail.com), Dept. of Mathematics, Clarkson University, Potsdam, NY 13699, **Craig Wiegert** (wiegert@physast.uga.edu), Dept. of Physics, University of Georgia, Athens, GA 30605, and **Michael Ramsdell** (mramsde1@clarkson.edu), Dept. of Physics, Clarkson University, Potsdam, NY 13699. *COMPASS - Combining Mathematics and Physics to Raise Mathematical Achievement.* Preliminary report.

Increased retention of students in STEM disciplines is one vital part to increase the number of students who are able to succeed in STEM fields. Our efforts focus on the first year Calculus and Physics curriculum, and we explicitly bring these two closely connected areas together. The program has two parts. First is the identification of students who can benefit the most from a coordinated experience. This is done by identifying students strength in mathematics as well as physics prior to entry into the courses, and our focus is on students whose have a relative strength in physics and a relative weakness in mathematics skills. The second step is to bring the two courses together into a vibrant whole. This is done by making use of fundamental physics models, and our students are challenged to increase their analytic abilities through the development of physics based models and an exploration and analysis of the resulting models. (Received September 20, 2016)

1125-J5-2591 **Mary Parker\*** (mparker@austinncc.edu), **Hunter Ellinger** (hunter@ellinger.org) and **Lindsay Orlando** (lorlando@austinncc.edu). *Mathematics for Modeling.* Preliminary report.

In our liberal-arts course, students investigate how models can be used to find formulas fitting sets of (potentially noisy) empirical data, to evaluate the mathematical implications of rules, and to design rules that will produce desired effects. Building student skill in imaginative spreadsheet use is an important goal for the course. Students investigate debt and savings situations (rule-driven models) and build spreadsheets that allow them to investigate the effects of non-standard payments of various types. Students fit linear, exponential, and quadratic models to data and combine them to fit more complex situations, such as constrained growth. Our materials are open-source. (Received September 20, 2016)

1125-J5-2755 **Nathan Pennington\*** (nathanpennington@creighton.edu), Department of Mathematics, Hixson Leid Science Building, 2500 California Plaza, Omaha, NE 68178. *Integrating First Year Mathematics and Physics through a Problem-Based Modeling Course.*

Mathematical Modeling of the Physical World is a problem-based team-taught Calculus II and General Physics course aimed at incoming first-year students. The core of the course is a collection of open-ended modeling projects work on in groups throughout the semester. In this talk we discuss the structure of the course, finding compelling modeling problems, and the positives and negatives of building a course around modeling projects. (Received September 20, 2016)

1125-J5-2846 **Catherine Case\***, catherine.case@uga.edu, and **Tim Jacobbe**. *Statistical Modeling as a Thought-Revealing Activity.*

Traditional inference methods use theoretical probability distributions (e.g., Normal distribution, t distribution, chi-squared distribution) to model the outcomes that would occur by chance under the null hypothesis. However, in recent years, simulation-based inference methods have begun to replace or complement traditional methods in many introductory courses, resulting in a proliferation of possible models and representational systems to express the logic of inference. In this study, pairs of introductory statistics students were recorded as they used various models to reason about a statistical inference task. This session will highlight how students made their

thinking visible to each other through competing simulation models, thus challenging each other's statistical conceptions. In particular, as students proposed and evaluated different models – which involved both physical chance devices (e.g., coins, dice, spinners) and computer models – they spontaneously raised important conceptual issues. Examples include the distinction between random assignment and random selection; between initial randomization in data production and simulated re-randomization for inference; and between outcomes that occur “just by chance” and outcomes that are equally likely. (Received September 20, 2016)

## *Methods of Engaging Math Learners with Physical Impairments*

1125-K1-696 **Evelyn Sander\*** ([esander@gmu.edu](mailto:esander@gmu.edu)), Dept of Math Sci MS-3F2, 4400 University Dr., Fairfax, VA 22030. *3D-technological methods for teaching 2D-graphing to a blind student: a case study*. Preliminary report.

In this talk, I will discuss a case study of using the technology of 3D printing to teach a blind student in a Precalculus course. I will describe the prior background of the student, discuss some of the pedagogical techniques used by the instructor, show the tools that we have developed, and give some concrete examples of how we used these tools. The talk will include a detailed description of the use of both 3D-printed plots and a 3Doodler pen. It will conclude with a short discussion of some possible avenues for further development of 3D printed tactile graphs and related teaching materials.

This is joint work with Steve Schluchter. (Received September 09, 2016)

1125-K1-1497 **Rebekah Ann Gilbert\*** ([rgilber6@gmu.edu](mailto:rgilber6@gmu.edu)), Dept of Math Sci MS-3F2, 4400 University Drive, Fairfax, VA 22030. *Resources for teaching math students with physical impairments*. Preliminary report.

Teaching a math student with a physical impairment can seem like a daunting task the first time you do so. Fortunately, there are many resources available to direct your efforts. In this talk, I will outline the three types of resources that are most helpful with examples from when I taught a deaf student in a large-lecture section of Precalculus. First, I will discuss the offices and individuals within your institution that work to ensure an accessible learning environment and the ways in which they may be of service to you. Second, I will highlight a number of helpful online reports with guidelines and other recommendations. Finally, I will make the case that your best resource is your student (Received September 20, 2016)

1125-K1-1977 **Robert O. Shelton\*** ([robert.o.shelton@nasa.gov](mailto:robert.o.shelton@nasa.gov)), NASA Johnson Space Center, Mail Code ER7, 2101 NASA Parkway, Houston, TX 77058. *Communicating Mathematics Independent of Vision*. Preliminary report.

I lost most of my usable vision in 1960 when I was 11years old. The next 27 years as a student, researcher and professor of mathematics offered a unique perspective on the process of communicating mathematical examples, concepts, theory and practice when at least one party is missing a sense. Math was always easy for me, but as I progressed through undergraduate and graduate school, one fact that emerged was that vision is not necessary for almost all levels and areas of mathematics. Like most pedagogy, transfer of mathematical knowledge is a signals problem. The student needs information, a way to manipulate that information, and a way to return results of that synthesis as questions, solutions or proofs. Decades ago, my only options were human readers and Nemeth code for scratch work. It will be shown how modern technology can eliminate most of this part of the problem. A second aspect of the signals problem is finding representations for concepts which do not rely on vision. For example, simple sonification can convey rate of change with respect to time, whereas slope is a purely spatial representation of the same concept. An alternative non-visual method for attacking a vexing volume problem from elementary calculus will also be demonstrated. (Received September 19, 2016)

1125-K1-2074 **Stephen Liddle\*** ([sliddle@gmu.edu](mailto:sliddle@gmu.edu)), 9936 braddock road, Fairfax, VA 22032. *On being a scribe for a blind math student*. Preliminary report.

Starting in the Fall of 2015, I served as a scribe for a blind student as he progressed from Precalculus through Calculus 2. In this talk, I will discuss some of the pedagogical adjustments made by his instructors, the student, and me, as we all had to rethink how to effectively communicate mathematics in a non-visual manner. I will then describe the solutions to the problems I faced while serving as a scribe for quizzes, tests, and homework problems. Finally, I will discuss the organizational challenges of working between departments. (Received September 19, 2016)

1125-K1-2110 **Julia R Sykora\*** ([julia.sykora729@gmail.com](mailto:julia.sykora729@gmail.com)) and **Allison K Young**. *3D Mathematical Models For the Blind.*

Teaching mathematics to students with visual impairments provides numerous challenges. Many times, low-tech solutions can be found: pipe cleaners, puff paint, and rubber bands can all be excellent teaching aids. However, when students arrive in upper-level math classes it can be more difficult to illustrate these more complex concepts. Solids of revolution, typically taught in Calculus II classes, is a concept that provides many challenges to blind students. With the rise of 3D printing and 3D pens, it is becoming easier to make mathematical ideas more physically accessible to blind students. In this talk, we will explore the development of our ideas, the resulting 3D models, and a lesson plan to teach this topic in a way that both blind and sighted students can more easily understand. (Received September 19, 2016)

1125-K1-2410 **Kathryn J Montovan\*** ([kmontovan@bennington.edu](mailto:kmontovan@bennington.edu)), 1 College Drive, Bennington, VT 05201. *Making Real Analysis Accessible to the Visually Impaired - One Example.*

During Fall 2016, I taught real analysis using Inquiry-based Learning to a class that included a visually impaired student. This student was foreign, did not know Braille or Nemeth Braille, and exclusively used a Mac and its built-in text-to-speech and voice-to-text programs. In this talk, I will discuss the challenges we faced and the solutions we developed to make the course content and assignments accessible to the student and to allow him to fully participate in class. (Received September 20, 2016)

1125-K1-2970 **Gary C Blatto-Vallee\*** ([gcbntm@rrit.edu](mailto:gcbntm@rrit.edu)), NTID Science and Mathematics Department, 52 Lomb Memorial Dr, Rochester, NY 14623. *Teaching Mathematics to Deaf and Hard of Hearing Students in a Mainstream Setting: Tips, Tricks, and Strategies for Success*.

Since the Americans with Disabilities Act (ADA) was passed into law in 1990, the number of deaf and hard of hearing (DHH) students enrolling in colleges and universities across the US has been on the rise. With this new reality come new challenges for DHH students and their mathematics instructors, the vast majority of whom have little experience accommodating DHH persons. Providing a sign language interpreter or real-time captionist in the classroom is a necessary first step but cannot be the only resource used when teaching a DHH student. This presentation will discuss actions and resources that mathematics instructors can and should take to support DHH students in their courses. Research relevant to deaf education on the whole and to teaching math to DHH students specifically will form the foundation of this session. The presenter will share “instructor tips” based on his more than two decades of classroom experience as an interpreter and mathematics lecturer. Resources created by DeafTEC, a National Science Foundation Advanced Technological Education National Center of Excellence, aimed at improving educational outcomes for DHH students in STEM education will be presented. (Received September 20, 2016)

1125-K1-3077 **Heijung Kim\*** ([hei-jung.kim@pearson.com](mailto:hei-jung.kim@pearson.com)), Pearson, Boston, MA 02116. *Improving Math Accessibility with MyMathLab.*

See and hear how instructors can create more accessible coursework With MyMathLab. Thus print-disabled students can start, and continue to keep up, with the class-pace. In MyMathLab, questions that support the JAWS screen reader are identified and thus can be selected to create more accessible coursework. The student uses the JAWS screen reader to read the text and math and enter answers using the keyboard and a command line language for math syntax. In this session, we will share our work to support accessibility in MyMathLab with a look at our progress over the past several years with details on some lessons learned along the way. We will also share a short demonstration of MyMathLab and the JAWS screen reader to show how a student accesses their course materials and completes assignments. MyMathLab is a series of online courses that accompanies Pearson’s textbooks in mathematics and statistics. Since 2001, MyMathLab—along with MyStatLab and MathXL, have helped over 9 million students succeed at more than 1,900 colleges and universities. MyMathLab engages students in active learning—it’s modular, self-paced, and adaptable to each student’s learning style—and instructors can easily customize MyMathLab to better meet their students’ needs. (Received September 20, 2016)

## *Modern Data Sets for the Intro Statistics Classroom and Beyond*

1125-K5-126      **Jason T Shaw\*** ([jason.shaw@washburn.edu](mailto:jason.shaw@washburn.edu)), Department of Mathematics & Statistics, Washburn University, 1700 SW College Ave, Topeka, KS 66621. *State-by-State Correlation Between Religious Attitudes and Math ACT/SAT Scores*. Preliminary report.

In this paper, the author analyzes each state's average ACT scores along with state-by-state religious attitudes. Gallup regularly collects data on whether individuals are "Very Religious" (they find religion an important part of their lives AND they attend religious services every week or almost every week), "Nonreligious" (religion is not important to their lives AND seldom or never attend religious services), and "Moderately Religious," for which individuals meet just one criteria.

Both the percentages of Very Religious and Nonreligious individuals in a state are significant predictors of average Math ACT scores, even with accounting for other variables such as each state's median income and education spending.

When predicting Math SAT scores, Simpson's paradox will present itself when allowing for participation rates. (Received August 02, 2016)

1125-K5-487      **Khairul Islam\*** ([kislam@emich.edu](mailto:kislam@emich.edu)), Mathematics Department, Eastern Michigan University, Ypsilanti, MI 48197. *On Using SEER Data for Teaching and Research*.

Using real-life data in teaching helps students understand statistical methodology and its application better. Students understand how data leads to the proper interpretation and decision making. The Surveillance, Epidemiology, and End Results (SEER) Program of the National Cancer Institute (NCI) provides an excellent source of modern data relating to cancer incidence, prevalence and survival by gender, ethnicity and other cancer characteristics. The SEER program is the only comprehensive source of population-based data in the United States, which can be used for the Intro Statistics Classroom and beyond for undergraduate and graduate students' research. In this presentation, we talk about how to access SEER data, use SEER data in a statistics class, and involve students in research. (Received September 10, 2016)

1125-K5-1124      **Leon Kaganovskiy\*** ([leonkag@gmail.com](mailto:leonkag@gmail.com)), Touro College Mathematics Department, 1602 Ave J, Brooklyn, NY 11230. *Applications of Statistical R Package to Undergraduate Teaching and Research*.

In this presentation, I would like to share my teaching experience with Introductory and Intermediate Applied Statistics courses using R software. R is an incredibly powerful big data and research level software that at the same time can be used for teaching undergraduate students hands-on Data Analysis and Scientific Computing & Modeling. I plan to discuss teaching Intermediate and Applied Statistics as well as Bio-Statistics courses over the years. In addition, I will discuss two joint projects with Psychology and Medical faculty. (Received September 15, 2016)

1125-K5-1382      **Beverly L. Wood\*** ([woodb14@erau.edu](mailto:woodb14@erau.edu)) and **Carl Clark**. *Real Data is Messy... and Manageable*.

Using real data in an introductory statistics course is a delicate balance between reality and manageability. The internet is awash with data that is useful for students to answer questions of interest to them but it is not always formatted as neatly as textbook data. The ASA's recently endorsed {GAISE College Report 2016} points to the plausibility of considering multivariable thinking even if only at a rudimentary level. With both messy and multivariable data in mind, we present some activities/projects and sources for data to give introductory students the opportunity to engage with real data. (Received September 16, 2016)

1125-K5-1422      **Scott Strother\*** ([strother@carnegiefoundation.org](mailto:strother@carnegiefoundation.org)), 51 Vista Lane, Stanford, CA 94305. *Increasing Engagement by Using Modern Data Sets for Contexts in Introductory Statistics: Fostering Productive Struggle in Statway Lessons*.

Statway is a two-term course through which students both fulfill their developmental math requirements and achieve college-level math credit. A part of the Carnegie Math Pathways initiative, involving a network of 58 colleges nationwide, Statway has been shown to double students' success rates in half the time. A key element of this success is the deep engagement of network faculty working together to continuously improve the curriculum. Reflecting the 2016 GAISE, a focal area of this work is the improvement of Statway lesson contexts and, in particular, the use of modern data sets.

In this presentation we will share an exemplar lesson that was recently rewritten to employ sports-related data sets from 2014, including raw win/loss data and winning percentage, at home and away, for every team in four

professional sports. We will discuss how the use of these data sets supports students' developing understanding of the statistical analysis process and distributions, while engaging them in productive struggle in meaningful inquiry grounded in the context. Attendees will receive copies of the data sets and see examples of how students engage the lesson. We will also discuss the process that is employed to develop and test lessons using modern data sets in the Statway network. (Received September 16, 2016)

1125-K5-1652     **Michael D. Miner\*** ([michael.miner0@mycampus.apus.edu](mailto:michael.miner0@mycampus.apus.edu)). *Real Data: Collect Your Own Data and Use It!* Preliminary report.

As different methodologies emerge in delivering a statistics course that is meaningful and relevant to students across different learning environments, the impact of providing good data sets is a fundamental attribute to understanding statistical concepts. One method of collecting real data and using it to assist students in understanding statistical concepts is to simply collect your own data and use it throughout the class to present students with meaningful and relevant examples on how statistical processes can be applied. This presentation will show the effectiveness of collecting data from students in a statistics class and then using this data throughout the class to engage student's critical thinking skills and facilitate the learning and understanding of basic statistical concepts presented in the Introduction to Statistics course. (Received September 18, 2016)

1125-K5-1981     **Allan J. Rossman\*** ([arossman@calpoly.edu](mailto:arossman@calpoly.edu)). *Examples for Implementing the Revised GAISE Guidelines.*

The American Statistical Association updated its GAISE (Guidelines for Assessment and Instruction in Statistics Education) college report in 2016. The revised report retains the original six recommendations and adds two new emphases: one on multivariable thinking and one on the process of statistical investigations. This presentation will provide some examples and datasets for implementing the GAISE recommendations in introductory statistics courses. (Received September 19, 2016)

1125-K5-2244     **Dan Seth\*** ([dseth@wtamu.edu](mailto:dseth@wtamu.edu)), Department of Mathematics, WTAMU, Canyon, TX 79015. *Enhanced Student Learning in Elementary Statistics With Fresh Real Estate Data.* Preliminary report.

Technology labs and explorations have been integrated into Elementary Statistics at WTAMU the past 8 years. Most explorations incorporate fresh data that the students collect. The students taking the course over the past few years have been in pre-programs or are education majors. The big project the past few semesters has had students collect Real Estate Data. Groups collect home prices from current, within the past month, real estate adds in either Amarillo, Dallas, or Sante Fe using the Zillow site. They complete hypothesis tests at different significance levels comparing their data to a current stated average home price of their chosen city. Each group then collects data from at least two additional cities of their choice, anywhere in the world. This data is used to complete an ANOVA test for differences in real estate prices of their chosen cities. Summaries of novel student results will be presented. Other sample labs the students complete, with data they collect, will also be presented, e.g., a Chi-square test for Vehicle Types. Assessment results of added learning, increased retention, and improved attitudes due to student experience with labs that incorporate their fresh data will be presented. (Received September 20, 2016)

1125-K5-2524     **Robin H Lock\*** ([rlock@stlawu.edu](mailto:rlock@stlawu.edu)), Dept. of Math, CS and Stat, St. Lawrence University, Canton, NY 13617, and **Ivan Ramler** and **Choong-Soo Lee**. *Web Tools to Help Students Get Individualized Datasets on a Common Theme.* Preliminary report.

One challenge in having students do individual or group projects in a statistics course is finding suitable data. We may have students choose their own data, but that generally requires considerable guidance to find good data sources, get the data processed into a usable format, and be sure that the data characteristics are appropriate for the goals of the project. As an alternative, we might provide students with a good, appropriate dataset, but then they might all be working on the same data and lose a sense of ownership that one gets from being the first to analyze a particular dataset. A third approach is to have students work on individual datasets that have a common theme and structure. For example, we might have students each pick a favorite TV show, find ratings for past episodes, and see how ratings compare between different seasons; or get data on homes for sale in their hometown and build models to predict selling prices. Wouldn't it be nice if there were readily accessible tools that make it easy for students to obtain such individualized datasets and also tools to help an instructor deal with assessing projects where everyone is using a different but similarly structured dataset? We'll discuss examples of such tools in this session. (Received September 20, 2016)

1125-K5-2580 **Elizabeth Wilcox\*** ([elizabeth.wilcox@oswego.edu](mailto:elizabeth.wilcox@oswego.edu)), Elizabeth Wilcox, Mathematics Department, 398 Shineman Center, 30 Centennial Dr., Oswego, NY 13126. *Measuring Life: Data for introductory biostatistics*. Preliminary report.

While teaching the introductory biostatistics class, *Measuring Life*, at SUNY Oswego, I found several interesting data sets that provide connections to public health, education, and ecology. One example is a data set of public school immunization rates in California from the 2013-2014 academic year; another is a data set of records for the animals moving through a shelter system in Kentucky. Come see some of the labs I wrote based these data sets and learn how I found and cleaned the data for class purposes. (Received September 20, 2016)

1125-K5-2809 **Bob Guest\*** ([guest00@utk.edu](mailto:guest00@utk.edu)). *Portable Populations for Collecting Real Time Data Sets in the Classroom*. Preliminary report.

In my introductory statistics course students collect and analyze data from portable populations of 1,000 Scrabble tiles, multiple customized decks of Uno cards, standard dice and used dice from the craps tables of the Stratosphere casino. These populations allow for a variety of active learning experiences working with quantitative and categorical data. In some instances students are told the distribution of the population. With this information they calculate population parameters and investigate both probability and the impact of transformations of random variables on the population mean, standard deviation and distribution. Students also collect samples of different sizes and use their statistics to discover the nature of sampling distributions. Knowing the entire population allows students to see the connection sample statistics have to the parameters they are going to estimate or test. Altogether this enhances their understanding of the Central Limit Theorem and lays the foundation for the techniques they will use to estimate and test population parameters, including comparisons of two populations as well as Chi-squared goodness of fit tests and tests of independence. (Received September 20, 2016)

1125-K5-2909 **Joseph McCollum\*** ([jmccollum@siena.edu](mailto:jmccollum@siena.edu)), Siena college, QBUS Department, 515 Loudon Road, Loudonville, NY 12211, and **Nichole McCollum**. *Utilizing World Bank Data to Enrich the Learning of Students in all Levels of Statistics*. Preliminary report.

At Siena College, in upstate N.Y., the School of Business has its own Quantitative Analysis Department (QBUS). The role of this department is to teach the mathematically based courses, including business statistics, and to incorporate business applications into every lecture. In fall 2014 we enhanced our statistics course to include extra lab time to experiment with real-life data from the World Bank website. Students apply skills learned throughout the semester to a course long project that investigates natural world concerns. This activity of using real data enhances student engagement within the course and expands their knowledge of statistics. Throughout the length of the course students have specific due dates to check for understanding of course skills, allowing the teacher to use data to determine areas of weakness. The additional lab time can be used for re-teaching opportunities, 1:1 support to students and research time. We believe our data set engages students to think beyond the skills taught in the course and we would like to share our results and experiences. (Received September 20, 2016)

## *PIC Math and Preparing Students for Nonacademic Careers*

1125-L1-103 **Laura M Smith\*** ([lausmith@fullerton.edu](mailto:lausmith@fullerton.edu)), 800 N State College Blvd, California State University, Fullerton, Department of Mathematics - MH 154, Fullerton, CA 92831. *Designing an Industrial Project Course for Mathematics Majors*. Preliminary report.

After obtaining an industrial sponsor for an undergraduate course project, the structure of the course needs to be determined. There are many aspects to consider, including team dynamics, student background knowledge, instructor and sponsor feedback, and deliverables. Giving a well-structured course with set deadlines for the students, the sponsor, and the instructor has been key to completing six projects in two semester courses. I will discuss the organization of the two courses run, including the timelines and deadlines, the grade breakdowns, and sponsor involvement. Further, I will highlight the challenges and modifications to the course, as well as some positive student outcomes. (Received July 27, 2016)

1125-L1-1443 **Hyunju Oh** ([hoh@bennett.edu](mailto:hoh@bennett.edu)), 900 East Washington Street, Greensboro, NC 27401, and **Jan Rychtar\*** ([j\\_rychta@uncg.edu](mailto:j_rychta@uncg.edu)), 116 Petty Building, PO Box 26170, Greensboro, NC 27402. *Joint Program for PIC Math between Two Institutions*.

In this talk, we will present our successful joint program experiences for Preparation for Industrial Careers in Mathematical Sciences between Bennett College (one of only two women HBCU) and University of North

Carolina at Greensboro (UNCG) in spring 2016. Bennett and UNCG are in a consortium so students can take classes at each other institutions without any difficulty. Four students from Bennett and 12 students from UNCG have worked with Greensboro Police Department (GPD) on analysis traffic stop data to identify potential racial profiling. We will report how we organize class, mentor students, and establish close connection with GDP as one team. (Received September 17, 2016)

1125-L1-1464      **Zhifu Xie\*** ([zhifu.xie@usm.edu](mailto:zhifu.xie@usm.edu)), 118 College Drive #5045, Hattiesburg, MS 39406.  
*Mathematics Seminar class: undergraduate research in mathematics.*

In this presentation, I will share about my experience on running a PIC-Math program and its positive impact in our department. I will also share how I organize a mathematical seminar class centered on undergraduate research in mathematics with some financial supports. (Received September 16, 2016)

1125-L1-1683      **Dhanuja Kasturiratna\*** ([kasturirad1@nku.edu](mailto:kasturirad1@nku.edu)) and **Lisa Holden** ([holdenl@nku.edu](mailto:holdenl@nku.edu)).  
*Gathering Research Problems from Local Industries: Our Experience in the N. Kentucky and Cincinnati Area.*

In this talk, we'll recount our efforts to approach local industries in the hope of finding appropriate real-world problems for our students to consider in a spring, 2017 research course. Over the course of the summer and fall semester, we approached 15 companies and government agencies, received seven positive responses, and prepared three problems for our spring research course. We'll tell you how we did it and offer strategies for success as well as how to avoid some of the pitfalls we encountered. (Received September 18, 2016)

1125-L1-2141      **Stephen Pankavich\*** ([pankavic@mines.edu](mailto:pankavic@mines.edu)), 1500 Illinois Street, Golden, CO 80401.  
*Introducing Industrial Problems via Capstone Experience.*

The MAA's national PIC Math program aims to prepare undergraduates in the mathematical sciences for industrial careers by engaging them in research problems that come directly from industry. In 2015, we participated in the inaugural year of the program and introduced applied and industrial projects within the capstone course for the Computational and Applied Mathematics major at the Colorado School of Mines. We will discuss the general structure and background of this course, as well as the research activities of student groups, which culminated in part with a research article currently submitted for publication and two students receiving the 2015 Janet L. Andersen Award for Undergraduate Research in Mathematical or Computational Biology at MathFest. (Received September 19, 2016)

1125-L1-2215      **Chris Camfield\*** ([camfield@hendrix.edu](mailto:camfield@hendrix.edu)), Hendrix College, 1600 Washington Ave., Conway, AR 72032. *Please Come Back: Analyzing Alumni and Donor Data in my PIC Math Course.* Preliminary report.

Many lessons were learned while incorporating a PIC Math project into a Mathematical Models course. The students had the opportunity to analyze data collected by the college's Office of Advancement. Reports were given back to the college providing new insights into effective communication, event scheduling, and donor profiles. Industrial problem solving brought a new atmosphere to the classroom as the project continued to evolve in unexpected ways that required the students and instructor to remain flexible. (Received September 19, 2016)

1125-L1-2514      **Michael W Schroeder\*** ([schroederm@marshall.edu](mailto:schroederm@marshall.edu)). *Successes and Trials with PIC Math and Beyond.*

Marshall University participated in PIC Math for the first two years of the program. Running the program at my institution had its share of high points and problems in its implementation. In this talk, I will share the experiences we had with PIC Math and describe our pitfalls and successes with the program. I will close by describing how PIC Math has led to the creation of an interdisciplinary program at Marshall (still involving Mathematics, of course) that has gotten our math program a lot of local and state recognition. (Received September 20, 2016)

1125-L1-2601      **Zachary J Abernathy\*** ([abernathyz@winthrop.edu](mailto:abernathyz@winthrop.edu)). *PIC Math at Winthrop University: Finding Problems, Course Design, and Lessons Learned.*

The main goal of this talk will be to share my experiences teaching an industrial mathematics course at Winthrop University (a public comprehensive university serving approximately 5000 undergraduate and 1000 graduate students) as part of the NSF-funded PIC Math program sponsored by the MAA and SIAM. I will focus on the process of establishing contact with local industries, offer suggestions for course design to keep students engaged and manage team dynamics, and share several other tips such as acquiring data mining software. I will also

discuss various outcomes (for both myself and my students) that emerged from the course as well as lessons learned for improving the course in its next iteration. (Received September 20, 2016)

1125-L1-2652 **Christina M. Selby\*** (selby@rose-hulman.edu) and **Joseph A. Eichholz** (eichholz@rose-hulman.edu). *Learning Objectives and Assessment Techniques in a Preparation for Industrial Careers in Mathematical Sciences Course.*

The learning objectives in most mathematics courses can be assessed by determining if a student accurately solves a problem or writes a clear, logically correct proof. This talk will describe the learning objectives the authors developed for their Preparation for Industrial Careers in Mathematical Sciences course and the course artifacts used for assessing these objectives. The learning objectives fall in the areas of mathematical inquiry, collaboration, and professional communication. (Received September 20, 2016)

1125-L1-2864 **Steve M Cohen\*** (scohen@roosevelt.edu), Roosevelt University, Department of Mathematics & Actuarial Science, 430 S. Michigan Avenue, Chicago, IL 60605. *Achieving Balance in a PIC Math Course.*

I have taught a PIC Math class for the past two years, in which students work on a project, provided by a professional liaison, that has a mathematics component to it. Students need to figure out what the professional contact is asking of them, determine what mathematical tools they need (with help from me) and develop a plan to solve the problem. As the instructor, I want the students to develop their own take on what is being asked of them and how they might go about solving it. They need structure and guidance, but also freedom and autonomy to make mistakes, and a sense of responsibility to the professional contact. I will share some of the ways I worked on finding that balance and helping students to do their best work. (Received September 20, 2016)

1125-L1-3120 **Michael Dorff\*** (mdorff@math.byu.edu). *Students solving research problems from industry.*

PIC Math is a new program that prepares university mathematics students for careers in business, industry, and government. The main component of the program is a semester course. In this course the professor does little or no lecturing. Instead the professor guides the students as they work in groups on solving research problems from industry. For the course we provide sample syllabi, sample research problems from industry, sample student solutions to industrial research problems, and sample videos of student presenting their research. Also included in the PIC Math program are a 3-day summer training workshop for professors and an end-of-program conference at which the students present their research results. During this academic year there are 67 universities along with about 750 undergraduate students participating in PIC Math. (Received September 21, 2016)

1125-L1-3126 **Namyong Lee\*** (nlee@mmsu.edu). *How to find good industrial mathematics problems?*

Performing a group research in real world industrial problem is the core component of PIC Math (Preparation for Industrial Careers in Mathematical Sciences) course. However, many first time instructors of PIC Math course seem to have difficulty to find good industrial mathematics problems. In this talk, we present how we find many interesting industrial mathematics problems around us by recognizing "Math is everywhere!". We also present a few sample industrial mathematics problems which is suitable to our undergraduate students as well as to faculty research. (Received September 21, 2016)

## *Preparing Pre-service and In-service Teachers to Support the Common Core State Standards Assessments*

1125-L5-1515 **Cathy S Liebars\*** (liebars@tcnj.edu). *An Undergraduate Research Project that Investigated the Impact of Activities in a Mathematics Methods Course to Prepare Preservice Teachers for the CCSS.*

As an honors research project, an undergraduate taking a Mathematics Methods course designed and administered a survey at the beginning and end of the course to determine the impact of the course activities on her classmates' self-assessment of their preparation for the CCSS and their view of the Mathematical Practices. Results showed that the preservice teachers in this class overwhelmingly felt much more prepared to help their future students meet the CCSS after participating in the course activities. However, after having a better understanding of the CCSS, they also expressed more concern about the global implementation. Survey results and sample activities used in the methods class will be shared. The activities are designed to prepare preservice

teachers to understand the CCSS and Mathematical Practices and help their students meet these standards. (Received September 17, 2016)

1125-L5-2194 **Ryota Matsuura\*** ([matsuura@stolaf.edu](mailto:matsuura@stolaf.edu)), 1520 St. Olaf Avenue, Northfield, MN 55057, and **Sarah Sword, Al Cuoco and Glenn Stevens**. *Assessing Secondary Teachers' Algebraic Habits of Mind*.

*Assessing Secondary Teachers' Algebraic Habits of Mind* is a research project funded by the National Science Foundation since 2012. We are developing a long-term research program with the ultimate goal of understanding the connections between secondary teachers' mathematical knowledge for teaching and secondary students' mathematical understanding and achievement.

Our hypothesis is that teachers who not only possess strong content knowledge but also the *mathematical habits of mind* (MHoM) used by many mathematicians teach in a way that results in increased student learning and achievement. Recognizing the need for evidence-based research to refine and test this conjecture, we are engaged in a study centered on the following question: *What are the mathematical habits of mind that secondary teachers use, how do they use them, and how can we measure them?*

We are engaged in ongoing work to identify and precisely define MHoM, and to operationalize this framework into paper and pencil assessment problems that accurately and uniquely measure mathematical habits of mind for teaching. In this session, we will focus on the development of the assessment, sharing examples of problems and data from the field tests. (Received September 19, 2016)

1125-L5-2441 **Gregory D Foley\*** ([foleyg@ohio.edu](mailto:foleyg@ohio.edu)), 321D McCracken Hall, Athens, OH 45701-2979. *Professional Development in Mathematical Modeling to Support the Common Core*.

The Mathematical Modeling and Spatial Reasoning (Modspar) project develops technological, pedagogical, and content knowledge among teachers in Grades 9–12 via two yearlong courses: Modeling with Algebra and Modeling with Geometry. Modspar addresses discrete, continuous, and geometric models, the modeling process and spatial reasoning; creating and implementing cognitively demanding student tasks and assessments; using algebra, graphing, and geometry software; and in Modeling with Geometry, using physical models, including the Lénárt sphere. This session will describe these courses and their relationship to the Common Core State Standards for Mathematics and will present the results of the related research since 2009. The session will explain how the standards for mathematical practice are integrated into Modspar and how these courses could be adapted to preservice teacher preparation. (Received September 20, 2016)

1125-L5-2614 **Jennifer Bergner\*** ([jbergner@salisbury.edu](mailto:jbergner@salisbury.edu)), 1101 Camden Ave., Salisbury University, Salisbury, MD 21801. *Learning to Teach the Common Core through Undergraduate Research*.

This presentation describes a Research Experience for Undergraduates (REU) project funded by the National Science Foundation (NSF). The REU model prepares prospective teachers to approach the teaching of the Common Core State Standards with a researcher's mindset by making instructional decisions using classroom data. Each summer, prospective teachers in this REU project design instruction for small groups of students, implement it, document its impact on student learning, and use what is learned to inform future instruction. The eight-week instructional sequences they produce illustrate, test, and refine existing published research-based learning trajectories for the Common Core. I will share the REU model and its impact on prospective teachers and their faculty mentors. (Received September 20, 2016)

1125-L5-2976 **Cynthia O. Anhalt\*** ([canhalt@math.arizona.edu](mailto:canhalt@math.arizona.edu)), Mathematics Department, 617 N. Santa Rita Ave., Tucson, AZ 85750, and **Ricardo Cortez** ([rcortez@tulane.edu](mailto:rcortez@tulane.edu)), Mathematics Department, 6823 St. Charles Ave., New Orleans, LA 70118. *Mathematical Modeling in the Middle Grades ( $M^3$ ): A Professional Development Project for Grades 5-8 Teachers in Rural School Districts near the Southern US Border*.

We designed a year-long professional development project, Mathematical Modeling in the Middle Grades ( $M^3$ ), for teachers in rural communities with diverse student populations. The goal was to prepare teachers for implementing modeling tasks in their classrooms that focus on students' mathematics knowledge and leverage community contexts, resulting in successful student-created models. Participants of the  $M^3$  learned the modeling process and the value of engaging in modeling activities. Teachers revised their views of curriculum development by incorporating modeling activities that advantageously use their students' background knowledge as part of the solution process. This session will present a brief background on the  $M^3$  Project, the geographic region of the participating school districts, and its community contexts as settings for mathematical modeling tasks with which the teachers engaged. We will showcase sample modeling tasks, associated content standards and

mathematical practices from the Common Core, the work of teachers and students in grades 5-8, and the benefits derived from the contexts of the tasks. We will share teacher reflections of their professional growth and student learning in mathematical modeling. (Received September 20, 2016)

1125-L5-3064 **Brian J Lindaman\*** ([blindaman@csuchico.edu](mailto:blindaman@csuchico.edu)), Dept. of Math and Statistics, CSU Chico, 400 W. 1st St., Chico, CA 95929. *A Tool for Exploring Understanding of Rational Numbers.*

In the United States, the CCSSM has placed greater focus on building a unified understanding of rational numbers among different representations than have previous standards documents. Little extant research examines this particular facet of rational number understanding. Among a sample of middle school mathematics teachers, a series of research-based tasks was used to elicit the teachers' understandings of rational numbers. These same teachers were observed delivering instruction on rational number concepts, and comparisons were made between the interview responses and the classroom instruction. Results indicate a tendency by teachers to focus on differences within the set of rational numbers at the expense of similarities. The interview instrument has been used in preservice, inservice, and with K-12 students as a valuable assessment tool for gaining insight into the knowledge of rational numbers. The instrument, recommendations for inservice teacher professional development, and directions for future assessment research will be shared with session participants. (Received September 20, 2016)

1125-L5-3094 **Jana Talley\*** ([jana.r.talley@jsums.edu](mailto:jana.r.talley@jsums.edu)), 1400 Lynch St., Jackson, MS 39217, and **Lecretia A. Buckley** ([lecretia.a.buckley@jsums.edu](mailto:lecretia.a.buckley@jsums.edu)), 1400 J. R. Lynch St., Jackson, MS 39217. *Understanding the Impact of the Mathematics Advancement in Teaching through Professional Development (MAT-PD2) Program.*

The Mathematics Advancement in Teaching through Professional Development Phase II (MAT-PD2) program is a mathematics and science partnership among Jackson State University and high needs school districts throughout the state of Mississippi. It is aimed at enhancing teacher content knowledge and self-efficacy. The initiative goals include improved teacher practice through mathematics content enactment for the effective implementation of Common Core State Standards in middle and elementary mathematics classrooms. During an extensive three-week summer institute participants engaged in classroom simulations and inquiry-based activities designed to cultivate deep conceptual understanding of both K – 8 mathematics content and mathematical pedagogy. The professional development facilitators modeled research-based teaching strategies that the participants have been observed utilizing in their own classrooms. Outcome data results also showed that there was a statistically significant increase in teacher content knowledge and self-efficacy among participants. Ongoing data collection and analysis will consider the relationship between participant use of MAT-PD2 materials and strategies and student attitudes and content knowledge. (Received September 21, 2016)

## *Preserving and Writing the History of Mathematics Departments*

1125-M1-43 **Ronald L Merritt\*** ([ronald.merritt@athens.edu](mailto:ronald.merritt@athens.edu)), Athens State University, Dept. Mathematical, Computer and Natural Sci., 300 North Beaty Street, Athens, AL 35611. *The bicentennial history of the Athens State University Department of Mathematics: Its structure, curriculum and influence.*

Athens State University, located in Athens, Alabama, will celebrate its bicentennial in 2022. While it is known regionally for its historical significance, such as surviving threats of being burned to the ground during the American Civil War, little has been researched or documented specifically regarding the impact of the Department of Mathematics. Currently, the Department of Mathematics is a part of a broader division named the Department of Mathematical, Computer and Natural Sciences. This report examines the evolution of the Department of Mathematics from 1822 to 2016. During the past 200 years, who provided leadership for the department, who did the faculty teach and what were their research initiatives? What was the scope of influence the department has had locally, regionally or nationally? How has the curriculum modified over the last two centuries based on input from industrial or educational entities? Finally, what research foundation might the current department wish to invoke in order to perpetuate the research on the Department of Mathematics at Athens State University in the future? (Received June 16, 2016)

1125-M1-45      **John McCleary\*** ([mccleary@vassar.edu](mailto:mccleary@vassar.edu)), Department of Mathematics, Vassar College, Poughkeepsie, NY 12604. *Writing for Vassar College's Sesquicentennial*. Preliminary report.

This talk is a case study for the preservation and writing of the history of a mathematics department, in particular, Vassar College's department. The occasion was the 150th anniversary of the founding of the college. The audience was broad for such an event, calling for use of a wide range of materials. I will discuss how I went from available documents and artifacts to a written history, what I had to dig up, and the sort of questions prompted by the research. The web version of the history may be found at <https://math.vassar.edu/about/history.html>. (Received June 20, 2016)

1125-M1-111      **Thomas Philip Wakefield\*** ([tpwakefield@ysu.edu](mailto:tpwakefield@ysu.edu)). *Compiling a History of the Youngstown State Department of Mathematics and Statistics*.

To commemorate the 100th anniversary of the Ohio Section of the MAA, we assembled a history of the Department of Mathematics and Statistics at Youngstown State. Sources include school newspaper articles, undergraduate bulletins, a written history of the University and earlier compiled history of the Department, and interviews conducted with current and past members of the Department for an oral history collection. In this talk, we will discuss what we learned about the Department and share tips regarding the assembly of a history of a math department. (Received July 30, 2016)

1125-M1-409      **Jacqueline M Dewar\*** ([jdewar@lmu.edu](mailto:jdewar@lmu.edu)), Loyola Marymount University, 1 LMU Drive UH 2700, Department of Mathematics, Los Angeles, CA 90045, and **W. Scott Wright** and **Dennis G. Zill**. *Discovering Questions as Well as Answers When Writing a Departmental History*.

In the mid-1970s the Mathematics Department at Loyola Marymount University in Los Angeles, CA, underwent a radical change in governance. It transitioned from being run in the traditional manner of unilateral decisions (e.g., on hiring new faculty and assigning and scheduling courses) made by the department chairman to a very democratic process for decisions, one that has continued to the present time. The three faculty (co-authors of this paper) who were at the core of agitation for this change, Jacqueline M. Dewar, W. Scott Wright, and Dennis G. Zill, are now emeriti faculty, having retired between 2009 and 2013. In 2015, Dewar proposed that they write a history of the mathematics department going back to the mid-1960s, when the first of the three was hired, to document and preserve this history. As this abstract is being written, the project is not yet finished, but work on it to date has revealed some surprising facts about the department and the university. This paper will describe sources being used and questions that we never thought to ask, but for which we now have very interesting answers. Anyone in pursuit of a departmental history may want to ask these questions. (Received August 31, 2016)

1125-M1-433      **Satish C. Bhatnagar\***, [bhatnaga@unlv.nevada.edu](mailto:bhatnaga@unlv.nevada.edu). *Merits of the History of Mathematics Projects*.

A history of mathematics (HoM) course is the most unique course in any college catalog of mathematics courses. This feature applies to every facet of the course – its instruction, its instructors, and its material. Teaching/taking a HoM course without any hands-on experience is like teaching/taking a hardcore science course without providing/experiencing flavors of its lab. This paper is all about the genesis and completion of various projects that the students did in the HoM courses (MAT 714 and MATH 314) that I have taught during the last nine years. The projects deal with the histories of courses - from remedial to lower division courses; graduate students, gender based data on majors and minors, faculty and administrators etc. The idea of the HoM projects synchronized with the 50th anniversary celebration of UNLV in 2007. Two years ago, these projects were classified, compiled and organized in two 3-ring folders. The Special Collection Division of UNLV's Lied Library (digital) readily accepted the entire material. It is now available to any curious person and institutional researcher. (Received September 01, 2016)

1125-M1-434      **Peggy Aldrich Kidwell\*** ([kidwellp@si.edu](mailto:kidwellp@si.edu)), MRC671, NMAH, Smithsonian Institution, P.O. Box 37012, Washington, DC 20013-7012. *Preserving and Writing the History of Mathematics Departments – A Note on Museum Resources*. Preliminary report.

Most historians who write on the history of mathematics departments focus on written documentation. Colleges and universities also have a rich array of objects that can say much about the goals and practices of the mathematics departments that used them. Diverse resources, ranging from museum collections to manufacturer's catalogs to online web descriptions, exist for better understanding these materials. (Received September 02, 2016)

- 1125-M1-483 **Lawrence A D'Antonio\*** ([ldant@ramapo.edu](mailto:ldant@ramapo.edu)), 505 Ramapo Valley Rd., Mahwah, NJ 07430. *William Fogg Osgood and the Transformation of the Harvard Mathematics Department*. Preliminary report.

The American mathematician William Fogg Osgood played a central role in the movement of the mathematics department at Harvard from being primarily concerned with teaching towards being research oriented. Entering Harvard as an undergraduate in 1882, getting his doctorate in Germany, he then taught at Harvard from 1890 up to his forced retirement in 1933 (under somewhat scandalous circumstances). In this talk we look at the life and work of Osgood and how he impacted the tremendous growth in the mathematics department. (Received September 03, 2016)

- 1125-M1-563 **Walter J. Meyer\*** ([meyer1@adelphi.edu](mailto:meyer1@adelphi.edu)). *Stanford and Applied Mathematics: Getting Its Groove*.

Stanford University was founded in 1891 and, for the first few decades Stanford's mathematicians showed what appear to be mixed signals toward the role of mathematical applications at Stanford. This talk will concentrate on this issue up till 1938 and attempt to show that one could interpret what happened as resulting from a consistent point of view. The role of some major figures in the department will be touched on. (Received September 06, 2016)

- 1125-M1-1937 **William Dunham\*** ([bdunham@brynawr.edu](mailto:bdunham@brynawr.edu)). *Bertrand Russell at Bryn Mawr*.

Bertrand Russell, the famous mathematical philosopher and great contrarian, visited Bryn Mawr College on three occasions: in 1896, 1914, and 1943-44. We briefly describe these visits, which brought him scholarly recognition, many friends, two books, one wife, and (what else would we expect from Lord Russell?) more than a little controversy. (Received September 19, 2016)

- 1125-M1-2119 **Tina R. Hartley** and **V. Frederick Rickey\*** ([fred.rickey@me.com](mailto:fred.rickey@me.com)), 11 Stately Oaks, Cornwall, NY 12518. *The Calculus Curriculum at West Point in the Twentieth-Century*. Preliminary report.

Recent changes in the core curriculum at West Point lead one to ponder: Should students study calculus and if so, how much? Traditionally a leader in national undergraduate mathematical education, West Point is once again changing its curriculum. We will describe the current core mathematics curriculum and the purpose of the change. We will then step backwards through time to present other significant moments of change in the evolution of the curriculum, to include the effects of the late 20th century calculus reform movement, the impact of technology, the changes during the Sputnik era, and other key developments. Along the way we will present some interesting textbooks and highlight the key individuals who drove change. (Received September 19, 2016)

- 1125-M1-2859 **Noel F. Heitmann** ([nheitmann@millersville.edu](mailto:nheitmann@millersville.edu)), Millersville University, 1 S. George Street, P.O. Box 1002, Millersville, PA 17551, and **Michael G. Wismer\*** ([mwismer@millersville.edu](mailto:mwismer@millersville.edu)), Millersville University, 1 S. George Street, P.O. Box 1002, Millersville, PA 17551. *Millersville University Department of Mathematics: Preparing Mathematicians and Educators for More Than 150 Years*. Preliminary report.

Founded in 1855, Millersville University was established to meet the academic needs of central Pennsylvania. Since its initial function as Pennsylvania's first Normal (Teacher's) School, Millersville's mathematics department has grown and adapted to the diverse mathematical demands of our region, serving as a leader in mathematics education, pure mathematics, applied mathematics, and statistics. Through an exploration of historical records as well as interviews with former and current members of the department, this paper will describe how our mathematics department has faced the challenges of serving an evolving population and will share the stories of some of the key people who provided leadership to the department and university. (Received September 20, 2016)

- 1125-M1-3091 **Janet L. Beery\*** ([janet\\_beery@redlands.edu](mailto:janet_beery@redlands.edu)). *What's your angle? Exploring the history of your department one facet at a time*.

Accepting the premise that it is important to compile histories of our departments, how can you find something interesting to say about mathematics in a lesser-known college or university founded during the 20th century? And how can you motivate yourself to take on what might seem an unglamorous task? My answer: Find an angle that interests you. In my case, it was the women who have served as faculty in the department. You might be intrigued by this topic or by another group of faculty or students or by a single (and perhaps singular) personality connected with your department. If not its people, then what about its curriculum, the role of research (for faculty or students), or its relationship with other departments in your university or region over

time? I hope to convince you that exploring one interesting facet at a time can lead to a meaningful and increasingly comprehensive, but never complete, department history. (Received September 21, 2016)

## *Proofs and Mathematical Reasoning in the First Two Years of College*

1125-M5-147      **Eleftherios Gkioulekas\*** ([eleftherios.gkioulekas@utrgv.edu](mailto:eleftherios.gkioulekas@utrgv.edu)), University of Texas Rio Grande Valley, School of Mathematical and Statistical Science, 1201 West University Drive, Edinburg, TX 78539-2999. *On the denesting of nested square roots.*

We present the basic theory of denesting nested square roots, from an elementary point of view, suitable for lower-level coursework. Necessary and sufficient conditions are given for direct denesting, where the nested expression is rewritten as a sum of square roots of rational numbers, and for indirect denesting, where the nested expression is rewritten as a sum of fourth-order roots of rational numbers. The theory is illustrated with several solved examples. The proofs are simple, given in a formal and complete style, and can serve as excellent examples for introducing concepts of proof in lower-level coursework, such as proof by contradiction, proof by cases, and quantified statements. (Received August 04, 2016)

1125-M5-586      **Luis Valero-Elizondo\*** ([lvalero898@gmail.com](mailto:lvalero898@gmail.com)), Mexico, and **Karina Mariela Figueroa-Mora.** *An online smart, interactive, collaborative, multilingual database of mathematical theorems and proofs.*

By means of a graphical, user-friendly interface, we show how to access a free online database of mathematical theorems and proofs. Proofopia, which can be reached at <http://computo.fismat.umich.mx/prooftopia>, is a website hosting a database which everyone can browse, using several advanced search options to find the theorems you want. Wherever proofs are provided, you can study those proofs at your leisure, and you can also try to prove theorems, either by following a proof you previously studied, or by writing your own original proof. In either case, the computer will make sure that all steps taken in a proof are logically sound, so when the proof is complete, the user can be sure that it's correct. All users are also welcome to add new theorems (and proofs) to the database. This website has other useful features, such as multilingual support and social network capabilities, to name a few, and we are constantly adding new functionality. (Received September 07, 2016)

1125-M5-743      **Ahmed Benkhalti\*** ([benkhaaa@nmsu.edu](mailto:benkhaaa@nmsu.edu)), **John Selden** and **Annie Selden.** *The Role of Operable Interpretations of Definitions in Writing Proof Frameworks.*

Many mathematics departments have instituted transition-to-proof courses for second semester sophomores to help them learn how to create proofs to prepare them for proof-based courses in their later years. It is our understanding that many community colleges may want to begin teaching such courses. We have students start by writing a proof framework which is based on the logical structure of the theorem statement and associated definitions. Often there are two levels to a proof framework. Generating a first-level proof framework is often easy, provided the theorem is stated in the standard 'If ..., then ...' form. However, formulating a second-level proof framework requires knowing how to use the relevant mathematical definitions, that is, being able to put them in an operable form. For example, the definition of the inverse image of a set  $D$  under a function  $f : X \rightarrow Y$  is usually given as  $f^{-1}(D) = \{x \in X | f(x) \in D\}$ . However, in constructing a proof, one needs to be able to use this in an operable way: If  $a$  is an element of  $f^{-1}(D)$ , then one can say  $f(a)$  is an element of  $D$ , and conversely, if  $f(a)$  is an element of  $D$ , then  $a$  is an element of  $f^{-1}(D)$ . This may seem obvious for us, but it is not for some beginners. (Received September 10, 2016)

1125-M5-749      **Katherine E Stange\*** ([kstange@math.colorado.edu](mailto:kstange@math.colorado.edu)). *Standards-based grading and other experiences in a first proofs course.*

At the University of Colorado, Boulder, proof is first taught to mathematics majors in a course titled Introduction to Discrete Mathematics. I will discuss the background and purpose of the course, my varied experiences with this course, ranging from lecture to inquiry-based learning, and focus in particular on my experiences with standards-based grading and other aspects of assessment.

Standards-based grading, which has seen much recent interest in K-12 education, is based on the principle that grading should communicate to students their current mastery and progress with respect to well-defined standards. In particular, it is often adaptive, offering frequent opportunities to reassess (replacing earlier grades) on carefully articulated individual standards. (Received September 10, 2016)

1125-M5-769 **Igor Vladimirovich Minevich\*** ([igor.minevich@bc.edu](mailto:igor.minevich@bc.edu)), Boston College Mathematics Department, Maloney Hall 553, 140 Commonwealth Avenue, Chestnut Hill, MA 02467. *Intriguing Problems for Students in a Proofs Class.*

We will explore some engaging and challenging problems that will excite your students about doing proofs. These problems, involving real life scenarios that students can easily relate to, will teach students to use induction, numerical invariants, and the pigeonhole principle. (Received September 11, 2016)

1125-M5-1149 **Jacquelyn L Rische\*** ([jrische@hamilton.edu](mailto:jrische@hamilton.edu)). *Integrating "Introduction to Proofs" into Linear Algebra.* Preliminary report.

At Hamilton College, Introduction to Proofs and Mathematical Reasoning is integrated into Linear Algebra. The course is designed for first and second-year students that have completed Integral Calculus and covers an introduction to linear algebra. In this talk, I will discuss the methods I have used to integrate the teaching and practice of proof-writing into this course. (Received September 15, 2016)

1125-M5-1528 **Dean Gooch\*** ([dgooch@santarosa.edu](mailto:dgooch@santarosa.edu)), Mathematics, Shuhaw Hall, Santa Rosa Junior College, 1501 Mendocino Avenue, Santa Rosa, CA 95401. *Why Courses in Proofs and Mathematical Reasoning need to be Taught at Two-Year Colleges.*

Courses in proofs and mathematical reasoning need to be taught at all two-year colleges that have significant numbers of students who choose majors that will require upper division mathematics courses at their four-year transfer institutions. Transfer students often find that they are shut out of proofs based upper division courses at their transfer institutions until those students have taken a proofs and mathematical reasoning class. The resident non-transfer student seeking upper division proofs based courses is expected to have completed a proofs and mathematical reasoning course prior to their Junior year. This often puts the transfer student at the disadvantage of having to spend three years finishing that student's Junior and Senior years of college. The speaker will discuss the barriers that make it difficult to offer proofs and mathematical reasoning courses at two-year colleges and share why it is essential to offer these courses to our students. (Received September 17, 2016)

1125-M5-1648 **David Burns\*** ([burnsd@wcsu.edu](mailto:burnsd@wcsu.edu)). *Proofs: An Introduction to Reading, Analyzing and Writing Mathematics.* Preliminary report.

In fall 2013 the Western Connecticut mathematics department introduced a new foundations course titled "MAT 207 Proofs." This course, taken concurrently with Calculus II, is described as an introduction to higher mathematics and proof techniques with supporting content drawn from number theory. The course is divided into three units. The first is reading and analyzing definitions and theorems. The second is analyzing and writing proofs in various styles. The third concentrates on analyzing and proving content related to number theory. This talk will detail the course outline and activities. An added bonus is the assessment rubric designed to measure NCTM/CAEP Standard 2 for Mathematical Practices. (Received September 18, 2016)

1125-M5-1819 **Erin E Bancroft\*** ([eebancroft@gcc.edu](mailto:eebancroft@gcc.edu)). *Contraposition, Complements, Counterexamples, and Counting: Enumerative Combinatorics as an Introduction to Proof Course.*

The primary objective of an introduction to proofs course is for students to learn the skill and art of writing mathematical proofs. This is made difficult by the relatively little mathematics students know when they take the course, typically as sophomores. One option is to limit the course topics to those that do not require significant background knowledge. However, this can result in a course that feels fragmented or is uninteresting. Alternatively, students' first exposure to proof writing may come in a more unified course such as algebra or analysis. In this case, the abstractness of the subject and lack of a firm foundation can leave students floundering. In this talk I describe a framework for an introduction to proof course that addresses both of these concerns: enumerative combinatorics. Besides providing a natural structure for discussing sets, functions, and proof, framing the course in the context of enumerative combinatorics makes it engaging and accessible for students. I will also discuss how I structure the course, the textbooks and resources I use, and the evolution of the course to this point. (Received September 19, 2016)

1125-M5-2487 **Steven R LeMay\*** ([steven.lemay@uconn.edu](mailto:steven.lemay@uconn.edu)), 341 Mansfield Road, Unit 1009, Storrs, CT 06269, and **Fabiana A. Cardetti** ([fabiana.cardetti@uconn.edu](mailto:fabiana.cardetti@uconn.edu)), 341 Mansfield Road, Unit 1009, Storrs, CT 06269. *Introducing proof through argumentation: An analysis of K-12 tasks.*

In this talk we present the results of a study focused on engaging students in argumentation to support their growth as mathematical learners, which strengthens math majors' reading, comprehending, and writing proofs.

We identify five argumentation categories that promote the learning of argumentation skills and enrich mathematical reasoning at the undergraduate level. Using these categories, we present and discuss rich learning tasks in first- and second-year mathematics courses that meaningfully engage students in mathematical argumentation, in particular undergraduate mathematics majors, (Received September 20, 2016)

1125-M5-2515 **T. Alden Gassert\*** ([thomas.gassert@wne.edu](mailto:thomas.gassert@wne.edu)). *Tackling Intro to Proofs at CU Boulder*. In this presentation, I will share the course materials I created for my Discrete Math (Intro to Proofs) class at the University of Colorado Boulder. Rather than being content driven, my primary goals were for my students to come out of the semester with the ability to communicate mathematics effectively, both orally, and in writing (formal proof). I split class time approximately 50/50 between interactive lecture and group work. My in-class worksheets were focused on reinforcing definitions and helping students outline formal arguments. Homework was a mix of exercises and formal proofs, with mandatory rewrites for the proofs. On top of this, I required that all homework be written in LaTeX; I will share my methods and materials for teaching LaTeX as well.

All my course materials can be found at: [http://math.colorado.edu/~thga2182/Discrete\\_math/16S](http://math.colorado.edu/~thga2182/Discrete_math/16S). (Received September 20, 2016)

1125-M5-2642 **Livinus U Uko\*** ([luko@ggc.edu](mailto:luko@ggc.edu)), School of Science and Technology, Georgia Gwinnett College, Lawrenceville, GA 30043. *A student-friendly proof of the density of rationals*.

Many students struggle with proof courses, especially during their first two years of college. This causes much frustration for students and instructors alike. One way of improving this situation is to develop effective teaching techniques, especially those based on the active learning model. However, this approach will not achieve much if the proofs presented in the classroom are perceived by the students as really difficult. We believe that research on more effective proof-teaching techniques should be accompanied and complemented by research on the development of simpler and more incisive proofs that are easier for students to understand.

In this talk, I will present a simple sequence of results leading to a simple proof of the density of rational numbers that I have used with some success to reduce stress levels in my recent real analysis classes. (Received September 20, 2016)

1125-M5-3055 **Brandilyn Stigler\*** ([bstigler@smu.edu](mailto:bstigler@smu.edu)), 3200 Dyer Street, Box 0156, Dallas, TX 75275. *Use of Templates in Teaching Proof Writing*.

Developing the skill of reading and writing proofs is typically difficult for students when they first encounter mathematical logic. While students are able to master the skill of reading mathematical statements, knowing how to approach proofs is often overwhelming. We present proof templates for standard mathematical statements using logical operators and quantifiers. These templates provide students with the structure of their proof. With this as a starting point, students can usually progress to completion of the proof, which boosts their confidence. We also describe classroom activities that support the development of reading, speaking, and writing in the language of mathematics. (Received September 20, 2016)

## *Research in Undergraduate Mathematics Education (RUME)*

1125-N1-41 **Maarten McKubre-Jordens\*** ([maarten.jordens@canterbury.ac.nz](mailto:maarten.jordens@canterbury.ac.nz)), Private Bag 4800, Christchurch, Canterbury 8140, New Zealand, and **Erik Brogt** and **Annie Horton**. *Mathematical Maturity: How can it inform teaching and learning of mathematics?* Preliminary report.

Mathematical maturity is a ubiquitous, yet nebulous concept in mathematics. Often it is said that certain topics will be accessible for people with “sufficient mathematical maturity”, even to the point that it becomes an informal prerequisite. It appears linked to, but distinct from, the ability to do mathematics in an algorithmic sense, and tends to be a more holistic quality of a person.

This is a progress report on a scientifically motivated qualitative study of the concept of mathematical maturity, with an aim to develop and evaluate teaching strategies aimed at improving mathematical maturity and broader mathematical reasoning skills. The study is funded by Ako Aotearoa (National Centre for Tertiary Teaching Excellence in New Zealand). (Received June 15, 2016)

1125-N1-304 **Morgin Jones Williams\*** ([mjones137@student.gsu.edu](mailto:mjones137@student.gsu.edu)). *Mathematically Talented Black Women of Spelman, 1960s-2010s*.

Women of color in general and Black women in particular who pursue undergraduate and graduate degrees in mathematics are nearly invisible in the mathematics education research literature (Borum & Walker, 2012). The

majority of research published in the mid-to-late twentieth century that explored the mathematics education of women was limited not only by failing to explore the unique mathematical experiences of women of color but also by employing quantitative methodologies in positivist frames (see, e.g., Benbow & Stanley, 1980; Fennema & Sherman, 1977; Hyde, Fennema, Ryan, Frost, & Hopp, 1990). The purpose of this narrative inquiry project is to explore the mathematics teaching and learning experiences of Black women who earned an undergraduate degree in mathematics at Spelman College from the 1960s to 2010s. Three central questions guide the inquiry: (1) How did Black women construct their mathematical identities in Spelman's undergraduate mathematics program? (2) How did larger socio-historical and -political contexts and life experiences on and off campus help or hinder the construction of their identities as mathematicians? and (3) How did the relationships with other Spelman students, faculty, and staff impact their short- and long-term goals in the field of mathematics? (Received August 25, 2016)

1125-N1-361 **Mary E Pilgrim\*** ([mpilgrim@rams.colostate.edu](mailto:mpilgrim@rams.colostate.edu)), 1874 Campus Delivery, Fort Collins, CO, and **Ben D Sencindiver**. *Using Analytics to Better Understand Calculus Students' Weaknesses and Learning Behaviors*.

Calculus 1 has been and continues to be a key gateway course to STEM majors, which contributes to a loss of students in the STEM pipeline. While active learning is well established in the literature as a key component for student success, it is a challenge to consistently maintain such pedagogies at our institution for various reasons. Additionally, student-learning behaviors can mediate the impact of active engagement efforts in class and are particularly important factors in learning outside of class. Therefore, early identification of students whose learning behaviors are setting them up for failure is needed. The focus of our project is to leverage student-learning behaviors as predictors for early identification of students at risk of receiving a grade of D, F, or W in Calculus I. This research is being done by (1) using digital objects (electronic items in the learning management system with which students can interact) and (2) using data to develop an understanding of what learning behaviors students engage in. While we are still developing a model that will predict at-risk students, we will present initial findings on student behaviors with digital objects and how these behaviors (1) correlate with student performance and (2) reflect self-regulated learning behaviors. (Received August 29, 2016)

1125-N1-702 **Eyob S Demeke\*** ([edemeke@calstatela.edu](mailto:edemeke@calstatela.edu)), 5151 State University Dr, Dept. of Mathematics, Los Angeles, CA 90032. *How proficient learners of mathematics read proofs: An exploratory study*.

Research document that students have difficulty understanding the very concept of proof, identifying the role logic and definitions play in mathematical argumentation, and validating and constructing non-trivial proofs (Harel & Sowder, 1998; Inglis & Alcock, 2012; Moore, 1994; Selden & Selden, 2003).

Some researchers such as Weber (2012) conjectured that students difficulty comprehending proofs in part due the fact they lack effective proof reading strategies. Building on the work of Weber (2015) on proof reading strategies, in this talk I will discuss eleven effective proof reading strategies that came out of my interview study with mathematics doctoral students. I argue that students can use these strategies to enhance their proof comprehensions. Finally, I will briefly discuss the implication this research can have on mathematics pedagogy as well as mathematics education research. (Received September 09, 2016)

1125-N1-871 **Kedar Nepal\*** ([nepal\\_k@mercer.edu](mailto:nepal_k@mercer.edu)), 1501 Mercer University Drive, Department of Mathematics, Macon, GA 31207, and **Kailash Ghimire, Ramjee Sharma and Manoj Thapa**. *Do They Know What They Know or Do Not Know? A Report on Undergraduate Mathematics Students' Self-assessment Behaviors*. Preliminary report.

Research shows that low-achieving students are less able to accurately assess their own weaknesses. As a result, many might fail to see the need to explore the subject matter more deeply, in order to improve their conceptual understanding and procedural fluency. This study investigates undergraduate mathematics students' self-assessment behaviors. Students from a broad range of courses at three universities were asked to predict their expected grades on assignments, and these predictions were compared with the grades assessed by their instructors. They were also asked to justify their self-assessments if they did not give themselves full points. Preliminary results showed that students overall overestimate their grades. There was a significant difference between expected and actual grades. As test scores increased, the difference increased from negative to positive. Students in the B-range (between 80-89%) were the most accurate predictors. (Received September 12, 2016)

- 1125-N1-939 **Daniel Visscher\*** ([davissch@umich.edu](mailto:davissch@umich.edu)), Department of Mathematics, 2074 East Hall, 530 Church St., Ann Arbor, MI 48109-1043, and **Nina White** ([whitenj@umich.edu](mailto:whitenj@umich.edu)), Department of Mathematics, 2074 East Hall, 530 Church St., Ann Arbor, MI 48109-1043. *Math Anxiety in an Interactive Mathematics Classroom.*

According to previous research, pre-service elementary teachers have some of the highest levels of math anxiety among college students. The instruments used in these studies include items such as “Watching a teacher work an algebraic equation on the blackboard”, or “Listening to a lecture in math class”. The courses we teach for pre-service elementary school teachers follow an active and inquiry-based learning format. In class, students work on solving problems in groups, they present their solutions, and they evaluate the arguments that their peers present. Because the kinds of items above fail to capture much of the mathematical work students may be anxious about in an active learning and interactive classroom, we developed new Likert-scale questions to measure anxiety around doing mathematics of this kind.

In this talk, we present a set of math-practice themed items to supplement a standard MARS-based instrument, and we report on a validation study of the resulting instrument. We hope this will expand the notion of what constitutes math anxiety, and that it will also help to generate ideas about how to address math anxiety among pre-service elementary teachers at a time that the structure and meaning of math class is also shifting.

(Received September 13, 2016)

- 1125-N1-1158 **Khalid Bouhjar\***, School of Teacher Education, 1114 West Call Street, 2208L Stone Building, Tallahassee, FL 32306-4459, **Muhammad Haider**, School of Teacher Education, 1114 West Call Street, 2204E Stone Building, Tallahassee, FL 32306-4459, and **Christine Andrews Larson**, School of Teacher Education, 1114 West Call Street, G127 Stone Building, Tallahassee, FL 32306-4459. *Examining Students’ Procedural and Conceptual Understanding of Eigenvectors and Eigenvalues in the Context of Inquiry-Oriented Instruction.* Preliminary report.

This study examines students’ procedural and conceptual understanding as evidenced by their written responses to two questions designed to assess aspects of their understanding of eigenvalues and eigenvectors. This analysis draws on data taken from 126 students whose instructors taught using a particular inquiry-oriented instructional approach and 129 comparable students whose instructors did not use this instructional approach. In this proposal, we offer examples of student responses that provide insight into their reasoning and summarize broad trends observed in our quantitative analysis. In general, students in both groups performed better on the procedural item than on the conceptual item. Additionally, the group of students who were taught with the inquiry-oriented approach outperformed the group of students who were taught using other approaches. (Received September 15, 2016)

- 1125-N1-1274 **Elise Lockwood\*** ([elise.lockwood@oregonstate.edu](mailto:elise.lockwood@oregonstate.edu)), 064 Kidder Hall, Department of Mathematics, Oregon State University, Corvallis, OR 97330, and **Zackery Reed**. *Invited Talk: Students’ Meanings of a (Potentially) Powerful Generalized Representation in a Combinatorial Setting.*

In this talk, we provide two contrasting cases of students who solved a series of combinatorial tasks that were designed to facilitate generalizing activity. In these cases, each student generated what externally appeared to be the same representation – a general outcome structure that both students spontaneously developed. However, upon further examination, the ways in which the two students’ understood and subsequently used the general representation differed significantly. We seek to explain these differences by identifying two types of relating that emerged in the study, and by connecting this relating to Piaget’s notion of reflective abstraction. By comparing and contrasting these students, we gain insight into the kinds of activity that promote both efficacious generalizing activity and robust combinatorial reasoning. We conclude with implications and directions for further research. (Received September 15, 2016)

- 1125-N1-1294 **Irma E. Stevens\*** ([istevens@uga.edu](mailto:istevens@uga.edu)), 110 Carlton St., University of Georgia, Athens, GA 30602, and **Kevin C. Moore** ([kvcmoore@uga.edu](mailto:kvcmoore@uga.edu)), 110 Carlton St., University of Georgia, Athens, GA 30602. *A Case Study: When Graphs Contain Everything.*

A two-dimensional Cartesian graph represents the relationship between two quantities. To create a graph of two quantities given a dynamic situation, a productive way of reasoning involves conceiving the situation as entailing two quantities’ magnitudes and their covariation, choosing to orient these magnitudes orthogonally, and then uniting these magnitudes in a way that produces a graph. Researchers, however, have found that students develop ways of thinking about graphs—including iconic translations and seeing ‘graphs as wire’—that do not entail isolating relevant quantities. Over the course of a semester-long teaching experiment in which

we presented an undergraduate pre-service secondary teacher with graphing tasks using dynamic situations (e.g., Ferris wheel ride), we observed a student who first constructed and operated on a graph as if the graph contained everything in the situation (e.g., horizontal and vertical components, arc length). We will discuss how her ways of thinking about graphs made constructing and interpreting graphs problematic for her, and indicate how isolating quantities and reasoning quantitatively about magnitudes impacted her understanding of graphs. (Received September 16, 2016)

1125-N1-1314 **Natalie LF Hobson\*** (nhobson@uga.edu), Department of Mathematics, Boyd GSRC, University of Georgia, Athens, GA 30602-7403, and **Kevin C Moore**. *Exploring Experts' Thinking in Graphing Dynamic Situations*.

This study investigated two experts' reasoning abilities when tasked with drawing a graph that relates two varying quantities. Numerous researchers and policy makers have argued the importance of providing students opportunities to model dynamic situations; they have claimed that reasoning skills developed in such contexts are essential to problem solving topics in undergraduate mathematics. In the pursuit of developing models of student thinking, researchers have analyzed undergraduate students' and children's ways of reasoning while representing varying quantities. Less data, however, has been analyzed in regards to expert thinking in such tasks. In response, we used clinical interviews to investigate mathematics and mathematics education doctoral students. By comparing the activity of two of these experts, and corroborating previous researchers' findings, we identify particular complexities involved in the development of covariational reasoning. Particularly, we identify that conceiving amounts of change of quantities was challenging yet essential to the experts accurately reasoning about and graphically representing covarying quantities. (Received September 16, 2016)

1125-N1-1359 **Allison Dorko\*** (dorkoa@oregonstate.edu), 676 Waterville Road, Skowhegan, ME 04976. *Is it a Function? Generalising from the Single- to Multivariable Setting*.

This talk reports on a longitudinal study regarding how five calculus students generalised what it means to be a function from the single- to multivariable setting. Students answered tabular and graphical classification tasks three times over the span of their differential, integral, and multivariable calculus courses. One student generalised functions as that which represent patterns, which inhibited him from generalising the univalence criterion. The other four students correctly generalised univalence, but had to overcome stumbling blocks such as trying to use the vertical line test in R3 as parallel to the y-axis (as it is in R2). I will talk about what students attended to as they generalised, which includes patterns, the form of equations, the position of coordinate axes, particular coordinate points, and the idea of function as input-output. These results have implications for how we teach students about single-variable functions and what ways of thinking allow them to later build on that knowledge, as well as how we teach multivariable functions. (Received September 16, 2016)

1125-N1-1363 **Biyao Liang\*** (leungbiyao@uga.edu), 105 Aderhold Hall, Athens, GA 30602, and **Kevin C. Moore**. *Rate of Change as a Feature of Partitioning Activity: The Case of Lydia*.

Numerous researchers have argued that coordinating amounts of change of one quantity with equal increments of change in another is critical for students to understand rate of change. In a semester-long teaching experiment designed to investigate pre-service secondary mathematics teachers' graphing activity, we observed a student, Lydia, regularly coordinate the amount of change of one quantity with respect to another by considering equal partitions in the latter quantity. We illustrate that her thinking about rate of change, however, was constrained to carrying out particular partitioning activity (as opposed to a quantification of an operative, covariational relationship). Moreover, her way of thinking involved her carrying out the "same" activity between graphs and situations, the elements of which entailed figurative or sensorimotor features. We will conclude the presentation by discussing the mathematical consequences of her actions including how they constrained her reasoning with rate of change in novel graphing situations and representational systems. (Received September 16, 2016)

1125-N1-1393 **Naneh Apkarian\***, naneh.apkarian@gmail.com, **Dana Kirin**, dhasbach@pdx.edu, **Kristen Vroom**, vroom@pdx.edu, and **Progress through Calculus team**. *Active Learning Usage in Precalculus to Calculus 2*.

Student persistence in the STEM disciplines continues to be a national problem, and experiences in introductory mathematics courses have been identified as particularly problematic. Mathematics education research has shown the potential of active learning strategies for supporting student success and providing a more equitable learning environment (e.g., Freeman et al, 2014; Kogan & Laursen, 2014), resulting in calls for active learning in mathematics (ALM) to become part of postsecondary education (e.g., CBMS, 2016; Saxe et al, 2015). But to what extent is ALM already part of the national landscape? As part of an ongoing project investigating math programs across the country, information was obtained about the current usage of ALM in the Precalculus to

Calculus 2 (P2C2) sequence. Of surveyed departments, 45% report at least one ALM component in their P2C2 sequence, though the majority of courses are taught in a traditional lecture format. As part of our presentation we will provide details of where in the sequence ALM is used, to what extent, and what strategies are most common. Understanding that one size does not fit all, we will provide examples of P2C2 programs to illustrate specific ways in which departments have implemented ALM in their introductory mathematics sequences. (Received September 16, 2016)

1125-N1-1398     **Kevin Charles Moore\*** (kvcmoore@uga.edu), 110 Carlton St., Dept. of Mathematics & Science Education, UGA, Athens, GA 30602. *Graphing and Fostering Operative Thought.*

Piaget and mathematics education researchers have distinguished between thought that is dominated by sensorimotor experience and thought that is dominated by the coordination of mental actions. The latter, which Piaget defined as operative thought, is a hallmark of mathematical reasoning. Researchers have illustrated, however, that students are not receiving sufficient opportunities to construct ways of thinking constituted by operative thought. In such cases, the ways of thinking students construct have limited productivity for their study of undergraduate mathematics. In this talk, I illustrate operative thought in the context of students' ways of thinking for functions and their graphs while drawing comparisons with thought dominated by sensorimotor experience. Using data collected during clinical interviews and teaching experiments with undergraduate students, I also illustrate research and instructional design decisions that provide researchers and teachers opportunities to gain insights into the extent a student's ways of thinking entail operative thought. For example, I illustrate how using non-normative representational systems afford students' reflection on their actions and, hence, students' construction of ways of thinking dominated by operative thought. (Received September 16, 2016)

1125-N1-1475     **Darryl Chamberlain Jr.\*** (dchamberlain2@student.gsu.edu) and **Draga Vidakovic.** *A first lesson on proof by contradiction: Developing proof comprehension in a transition-to-proof course.* Preliminary report.

Proof is central to the curriculum for undergraduate mathematics majors. Typically, students are first introduced to formal proof in a transition-to-proof course. This course is designed to facilitate students' learning of various proof techniques in order to develop both the ability to construct mathematically correct proofs and a sound understanding of proof in general. Despite transition-to-proof courses, students continue to exhibit difficulties constructing and comprehending proofs in higher-level mathematics courses, such as Abstract Algebra and Analysis. In particular, proof by contradiction has been isolated as one of the most difficult proof methods for students to construct and comprehend. Yet, there is little research to date on student comprehension of proof by contradiction. The purpose of this paper is to report preliminary results on video recorded in-class teaching sessions with two sections of a transition-to-proof course during the Fall 2016 semester. Results from this study will provide empirical data that may be used to improve instruction on proof by contradiction in transition-to-proof courses. (Received September 17, 2016)

1125-N1-1588     **Nicholas Kirby** (kirbyn@apsu.edu), Department of Mathematics and Statistics, P.O. Box 4626, Clarksville, TN 37044, and **Jennifer Yantz\*** (yantzzj@apsu.edu), Department of Mathematics and Statistics, P.O. Box 4626, Clarksville, TN 37044. *What Were They Thinking? Students in College Algebra Confront Misconceptions by Analyzing Errors in Examples of Student Work.* Preliminary report.

Professors in typical College Algebra courses are often frustrated by the errors students make that are unrelated to new concepts but stem from a lack of basic algebra skills that should have been mastered in high school. Some of the most common errors involve prerequisite skills such as combining like terms, applying the distributive property of multiplication over addition, factoring polynomials, simplifying or combining rational expressions and applying rules of exponents when solving equations involving radicals. These errors prevent students from advancing their mathematical knowledge and may hinder their progress in modeling mathematically or engaging in problem-solving in a College Algebra course. In this study, the investigators sought to determine if remediation activities in which students examined and analyzed errors in the work of other students, accompanied by focused class discussions of the underlying mathematical concepts, would increase students' mathematical understanding and reduce the number of errors that they make. The quantity and pattern of errors on common assessments were analyzed and compared for a control and experimental section of College Algebra. We present preliminary data regarding the effectiveness of this intervention. (Received September 18, 2016)

1125-N1-1594 **Elena A. Cheser\*** ([cheser@hartford.edu](mailto:cheser@hartford.edu)), H154, 200 Bloomfield Avenue, West Hartford, CT 06117. *An Ongoing Assessment of the Effectiveness of the Hillyer College Bridge Program in Improving At-Risk First-Year Student Performance*. Preliminary report.

The present study is a continuation of an ongoing assessment which follows student performance in math from student entry into the Hiller College of the University of Hartford Summer Bridge Program to the final grade of the first academic semester college math course in Hillyer College. The results of the third consecutive Summer Bridge are presented, including the overall consolidated 3-year view, and impact on the first regular semester. A model developed in 2014 which maps and quantifies the critical path for academic success has been refined to reflect an emphasis on Bridge learning outcomes. Program effectiveness is assessed by: (a) determination of progress made during the Bridge Program as measured by comparing Pre- and Post-Bridge test results; (b) determination of student scores from MyMathLab (MML; math-specific software which is integrated into program and semester curriculum); (c) relationships between Post-Bridge test results and MML scores; (d) relationships between first semester math course grades and Bridge performance. Also, Bridge Program, 2016 students became members of the UHART First-Year Interest Groups (FIG) of the university learning community initiative, with the dual goals of improving learning/integrative thinking and creating community. (Received September 18, 2016)

1125-N1-1708 **Christine Andrews-Larson\*** ([cjlarson@fsu.edu](mailto:cjlarson@fsu.edu)), **Karen Keene** ([kakeene@ncsu.edu](mailto:kakeene@ncsu.edu)) and **Estrella Johnson** ([strej@vt.edu](mailto:strej@vt.edu)). *Supporting Instructional Change: The TIMES Project*.

Mounting evidence relates improvements in student outcomes to active learning approaches to undergraduate STEM instruction (e.g. Freeman et. al., 2014). The challenge of how to effectively support this kind of instructional change presents a pressing open question. We propose sessions in which projects aimed at supporting instructional change discuss their research and work.

In this session, we will characterize the NSF-funded TIMES instructional support model developed and being implemented to support instructors of abstract algebra, linear algebra, and differential equations in learning to teach in inquiry-oriented ways. The TIMES model consists of three instructional supports: instructional materials, a three-day summer workshop, and online workgroups held for one hour per week during the semester when instructors implement the instructional materials. We will present findings drawn from analysis of two years of survey, interview, and video data of summer workshops and online workgroups. Additionally, we report results from student content assessments developed for each content area, drawing on data collected from students in classrooms of participating and comparison instructors. (Received September 18, 2016)

1125-N1-1775 **Christine von Renesse\*** ([cvonrenesse@westfield.ma.edu](mailto:cvonrenesse@westfield.ma.edu)) and **Phil Hotchkiss**. *Supporting Instructional Change: The "Discovering the Art of Mathematics" Project*.

Mounting evidence relates improvements in student outcomes to active learning approaches to undergraduate STEM instruction (e.g. Freeman et. al., 2014). The challenge of how to support this kind of instructional change presents a pressing open question. We propose sessions in which projects aimed at supporting instructional change discuss their research and work. In this session, we will introduce the NSF funded project "Discovering the Art of Mathematics", see [www.artofmathematics.org](http://www.artofmathematics.org). Our project developed a wealth of instructional materials for teaching mathematics for liberal arts courses using inquiry, including online teaching resources such as teacher guides and classroom videos. Additionally, we have been offering traveling inquiry-based learning workshops for instructors at 2 and 4-year colleges and universities. We will present our results of 3 years of pre and post surveys measuring students' beliefs and attitudes, as well as our evaluations of the traveling workshops. (Received September 19, 2016)

1125-N1-1891 **Mairead Greene\*** ([mairead.greene@rockhurst.edu](mailto:mairead.greene@rockhurst.edu)). *Optimization: Assessing Student Understanding in an Inquiry Calculus Course*.

Mounting evidence relates improvements in student outcomes to active learning approaches to undergraduate STEM instruction (e.g. Freeman et. al., 2014). The challenge of how to support this kind of instructional change presents a pressing open question. We propose sessions in which projects aimed at supporting instructional change discuss their research and work. Optimisation is one of the richest concepts that we encounter in Calculus I. It requires multiple layers of thinking, familiarity with both concepts and skills and an ability to interpret mathematical concepts in applied situations. These components make it a very rich concept in which to study the development of student understanding and evaluate how successful we are at providing the situations necessary for students to develop that understanding. In inquiry Calculus materials I co-wrote with Dr. Paula Shorter ([www.iblcalculus.com](http://www.iblcalculus.com)), we utilize multiple representations to provide a wide variety of opportunities for our students to develop their understanding of optimization earlier than a typical Calculus I course. I will

present the results of a detailed assessment of the student work on optimization throughout our materials and illustrate how the assessment results are then used to improve student learning. (Received September 19, 2016)

1125-N1-1969 **Hayley Milbourne\*** ([hmilbourne@sdsu.edu](mailto:hmilbourne@sdsu.edu)). *The Lead TA Influence: A Case Study on How the Lead TA Influences the Teaching Practices of Other GTAs*. Preliminary report.

Across the nation, there is increased national interest in improving the way mathematics departments prepare their GTAs. In particular, this research focuses on how the mentor GTAs in the graduate teaching assistant program under consideration share effective teaching practices and how this affects changes in the teaching practice of GTAs. I will report preliminary results on how the focus of particular teaching practices of mentor GTAs (known as lead TAs) change over the period of one term through their participating in professional development. With an understanding of the differences and the similarities between the focuses of the lead TAs, an analysis of the differences between the Calculus I and II GTAs will become more apparent. The research that will be presented here represents the start of an increased understanding of how GTAs form their own teaching practices. (Received September 19, 2016)

1125-N1-2047 **Keith Weber\*** ([keith.weber@gse.rutgers.edu](mailto:keith.weber@gse.rutgers.edu)), 10 Seminary Place, New Brunswick, NJ 08901, and **Teo Paoletti, Dimitri Papadopoulos, Victoria Krupnik, Joe Olsen and Timothy Fukawa-Connelly**. *Teacher questioning in advanced mathematics lectures*.

In this paper, we categorize mathematics professors' questions in proof-oriented mathematics courses to analyze what types of participation opportunities are available for students in these courses.

We recorded 11 lectures from different mathematics professors teaching different proof-oriented courses. We flagged 1031 questions from these lectures. We used an open coding scheme to generate a list of ten categories of questions. We then re-coded the data set using these categories, achieving a high level of inter-rater reliability (94%).

We discuss two main findings in this report. First, although teacher questioning was common across lectures, the opportunity for student participation was limited. This was because most questions were classified as factual questions asking for a specific piece of information or questions asking for the next step in a computation or proof. The majority of the questions (59.5%) were answered by the professors. Most questions (80.3%) had a wait time of two seconds or less.

Second, we illustrate substantial variance in professor's question posing in these lectures. In some cases, entire proofs were routinely elicited from students. In others, there were hardly any genuine opportunities for student participation. (Received September 19, 2016)

1125-N1-2058 **Biyao Liang** ([leungbiyao@uga.edu](mailto:leungbiyao@uga.edu)), 105 Aderhold Hall, Athens, GA 30602, and **Carlos Castillo-Garsow\***. *Pre-service Secondary Teachers' Understandings of Central Angle and Inscribed Angle*.

Previous studies investigated students' multifaceted conceptions of angle and their difficulties with connecting angle measure to arcs or circles. Contributing to the existing literature, our study investigated three undergraduate students' thinking about angles in the context of circle geometry, specifically their conceptions of central angle and inscribed angle. The design of the study involved three tasks: a pre-test, a proof reading task that demonstrated the proof of the Inscribed Angle Theorem in either dynamic or static version, and a post-test (same as pre-test). Our analysis suggested that students had various conceptions of these angles that either supported or constrained their ability to complete the tasks. Particularly, conceiving the dynamic transformation of both angles, or identifying the common subtended arc shared by a central angle and inscribed angle was helpful for students to identify these angles, while considering angle as area or angle as ray pair constrained their thinking. We will conclude our presentation by discussing the educational implications of our findings. (Received September 19, 2016)

1125-N1-2166 **Sybilla Beckmann\*** ([sybilla@uga.edu](mailto:sybilla@uga.edu)), Department of Mathematics, Boyd Graduate Studies Building, University of Georgia, Athens, GA 30602. *Could a variable parts perspective on proportional relationships be useful in trigonometry, calculus, and probability?* Preliminary report.

Recent theoretical work in mathematics education identified a variable parts perspective on proportional relationships and noted that the perspective had been largely overlooked in mathematics education research. From this perspective, we view covarying quantities as consisting of fixed numbers of parts, where each part is the same size as the others, but the common size of the parts varies. Preliminary empirical work from an ongoing study suggests that the variable parts perspective is useful for generating and explaining equations in two variables, including for lines in a coordinate plane. Proportional relationships are an essential foundation for

undergraduate mathematics, yet we know that reasoning about these relationships is a challenge, especially in cases involving geometric similarity. A variable parts perspective is in some sense naturally suited to handling cases of geometric similarity, and may therefore be a valuable foundation for some undergraduate topics. This paper discusses how topics in trigonometry, calculus, and probability could be approached from a variable parts perspective. Topics include radians, degrees, and trigonometric ratios; difference quotients and their limit (the derivative); and probability distributions and the law of large numbers. (Received September 19, 2016)

1125-N1-2310 **Rachel Elizabeth Keller\*** ([rakeller@vt.edu](mailto:rakeller@vt.edu)) and **Estrella Johnson**. *Student support resources in first-year mathematics: Are they helping?*

The PCAST report has identified a critical shortage of STEM graduates and cited poor experiences in Calculus as a major factor in attrition of STEM-intending students. The present research reports trends in the pass rate across the introductory sequence (Precalculus, Calculus I, Calculus II) and investigates the viability of various student supports and resources with regard to reducing the rate of students who fail or withdraw. Using data collected from the NSF-funded Pathways Through Calculus project (DUE #1430540), the following research questions are investigated: Is there a relationship between robustness of support and pass rate? Can any relationships be identified between specific types of support and pass rate? Are these relationships invariant across institution types (i.e. MA versus PhD)? Preliminary results indicate that the quantity of available student supports and resources is uncorrelated with pass rate; however, when particular supports were considered individually, some significant relationships were seen. Of particular interest was the increase in pass rate by schools offering Calculus I in a differentiated format (i.e. some combination of Engineering Calculus, Elongated Calculus w/Precalculus, Calculus for non-Math majors, Honors Calculus, etc.). (Received September 20, 2016)

1125-N1-2426 **Wes Maciejewski\*** ([wesley.maciejewski@sjsu.edu](mailto:wesley.maciejewski@sjsu.edu)), Department of Mathematics and Statistics, One Washington Square, San Jose State University, San Jose, CA 95192-0103. *Choices Made by Students when Enacting Procedures*. Preliminary report.

Much of the emphasis in recommendations for mathematics education instruction has been on the teaching and learning of concepts. In practice, however, instructors at all levels of education do expect students to develop technical, procedural proficiency. This is especially true in tertiary education in which the clear majority of students enrolled in mathematics courses are studying towards a non-mathematics degree, taking mathematics primarily for technical skills required in their discipline-specific courses. The question then arises, how can procedures be taught in a non-rote, transferable, deep way? This presentation will focus on one aspect of a larger research program geared toward addressing the above question. A categorization of students' reasons for choosing one procedure over another in the face of choice will be presented. This will be tied to the literature on flexible procedural knowledge and expand on the emerging understanding of teaching for depth of procedural knowledge. (Received September 20, 2016)

1125-N1-2444 **R. Cavender Campbell\*** ([robert.campbell@mavs.uta.edu](mailto:robert.campbell@mavs.uta.edu)), 527 Button Dr., Mesquite, TX 75150. *Mathematical Problem Solving Practices: A comparison of a student in College Algebra to a student in Calculus*.

Intense focus on increasing science, technology, engineering, and mathematics (STEM) student retention in college requires more attention to mathematical problem solving (MPS) in critical gateway courses. To investigate the MPS practices of students in College Algebra and Calculus, we interviewed students in these courses as part of the MPS Item Development Project. This case study, of one student from each course, contrasts the MPS used by students at different stages of a STEM pathway. We discuss various aspects of their use of MPS and highlight a strong contrast in the use of representing and connecting strategies between the students interviewed. In addition, we explore the distinct use of algebraic notation and interpret the interplay between fluent use of algebraic notation and the observed MPS practices. In their respective courses, each student scored well and conveyed content understanding consistent with their current course level. We also offer considerations for teaching practice linked to observed MPS practices of these two successful students at different stages in their pathway to a STEM major. (Received September 20, 2016)

1125-N1-2483 **Aaron D Wangberg\*** ([awangberg@winona.edu](mailto:awangberg@winona.edu)), 322 Gildemeister Hall, Winona State University, Winona, MN 55987, and **Brian Fisher, Jason Samuels, Tisha Hooks and Elizabeth Gire**. *Supporting Instructional Change: The Raising Calculus to the Surface Project*.

Mounting evidence relates improvements in student outcomes to active learning approaches to undergraduate STEM instruction (e.g. Freeman et. al., 2014). The challenge of how to support this kind of instructional change

presents a pressing open question. We propose sessions in which projects aimed at supporting instructional change discuss their research and work.

The Raising Calculus to the Surface project (NSF DUE-1246094) utilizes physical manipulatives (e.g. surfaces and contour maps), measurement tools, and small-group activities in order to help students discover math concepts prior to a formal introduction by their instructor. Although a key feature of the instructional design was fostering student contributions to their small group and whole class, many instructors were attracted to the project for other reasons. Instructors participating in the project attended a 3 day summer workshop designed to help instructors become comfortable with the materials, and they could adjust both the content and number of activities they chose to incorporate into their course. We will present findings related to the impact of the project on instructor's beliefs and practices drawn from an analysis of instructor surveys, weekly reports, and interview data. (Received September 20, 2016)

1125-N1-2658 **Hyejin Park\*** (hjpark3@uga.edu), Athens, GA 30602. *One College Student's*

*Decision-Making in Selecting Proof Methods in Proof Construction.* Preliminary report.

The purpose of this study was to test how Schoenfeld's (2010) theoretical framework of decision-making consisting of three components – goals, orientations, and resources – can be extended to examine how and why college students chose a certain proof method to use among various proof methods when attempting to prove mathematical statements. To do this, I purposely looked at one student's decision-making behaviors in selecting proof methods while proving eleven statements given over two interviews. The interviews were conducted after the student was taught six proof methods (direct proof and proof by contrapositive, contradiction, cases, induction, and counterexample) through a transition-to-proof course. In the interviews, the student was also asked his thoughts about the proof methods, experiences with the methods, and proof method preferences. The results show that his knowledge of proof methods largely influenced his decisions except when he prioritized his preferences for particular methods or desire for efficiency. Such results show this framework is useful as a tool for explaining one's behaviors in such decision-making situations. (Received September 20, 2016)

1125-N1-2673 **Shiv Smith Karunakaran\***, Washington State University, PO Box 643113, Pullman, WA

99164, and **James Whitbread, Jr.** and **Abigail Higgins.** *Uses of neurocognitive measures to evaluate cognitive load during the mathematical proving process.* Preliminary report.

A recent special issue of ZDM (June 2016) made the case for increasing the interdisciplinary collaboration between researchers in the fields of mathematics education and cognitive neuroscience. Specifically, Ansari and Lyons (2016) argued for increasing the "ecological validity of the testing situations and specific [neurocognitive] tests used to measure mathematical processing" (pp. 379-380). To this end, Ansari and Lyons (2016) suggest that it would be useful to explore the use of lower-cost and less invasive neuroimaging methods such as Near Infrared Spectroscopy (NIRS). The study reported on in this talk serves as a "proof-of-concept" for the use of Frontal Near-Infrared Spectroscopy (fNIRS) to measure the level of cognitive load of the brain under mathematical processing. The talk will address the pros and cons of using neurocognitive measures, such as the fNIRS, to measure and examine the physiological stresses of the brain under the complex mathematical process of proving. (Received September 20, 2016)

1125-N1-2689 **Shawn Firouzian\*** (sfirouzi@ucsd.edu), 1 Miramar St, 929265, La Jolla, CA 92092. *Any*

*correlations among students' ways of thinking about the derivative and their abilities to solve the applied derivative problems.*

Students' understanding, thinking and difficulties with the derivative and their difficulties in solving applied problems have been the subject of rich research work. Very little research has examined students' ways of thinking about derivative through the lens of their work on applied questions. This presentation represents the results of two phases study looking at the correlation between students' thinking about the derivative and their abilities solving related rate, optimization and graphing problems. Survey data and clinical interviews were conducted to look at students' multiple ways of thinking and their work on applied derivative problems. "Multiple ways of thinking" refers to two or more ways of thinking about derivative (e.g., slope of the tangent line at a point on a function or instantaneous rate of change). Fine-grained analysis of the students' written surveys and clinical interviews revealed that students' having more than two ways of thinking about the derivative correlate to their higher abilities in solving applied derivative problems. (Received September 20, 2016)

- 1125-N1-2695 **Kathryn Rhoads\*** ([kerhoads@uta.edu](mailto:kerhoads@uta.edu)), **James A. Mendoza Epperson** and **R. Cavender Campbell**. *The Role of Justifying in Entry-Level Undergraduates' Mathematical Problem Solving*. Preliminary report.

The practice of justifying solutions or solution methods is central in mathematical problem solving (MPS). In the MPS Item Development Project, we created Likert-style survey items which capture five domains of MPS, including the domain of justifying. Campbell (2016) reported survey results from Fall 2015, indicating that Calculus I students scored significantly higher in justifying than College Algebra students ( $p < 0.05$ ), and in both courses, passing students scored significantly higher in justifying than non-passing students ( $p < 0.05$ ). In this presentation, we draw on data from refined and revised surveys administered in 2016 to over 640 students enrolled in College Algebra and over 500 students enrolled in Calculus I at a large, urban university in the southwest United States, as well as individual, task-based interviews with over 20 students in these courses. We explore justification in undergraduate students' MPS through qualitative analysis of interview data, and we further investigate the relationship between justification and course success through quantitative analysis of survey data. The results suggest possible directions for MPS teaching practice in entry-level undergraduate courses. (Received September 20, 2016)

- 1125-N1-2722 **Kristin M Frank\*** ([kristin.frank@asu.edu](mailto:kristin.frank@asu.edu)). *Examining the Role of Experiential Time in Students' Covariational Reasoning*.

To conceptualize variation a student must anticipate a quantity's measure taking on different values at different moments in time. While reasoning about time is essential for covariational reasoning, a student's conceptualization of time can inhibit him from engaging in covariational reasoning. In this paper I describe the results from ten clinical interviews with university precalculus students. I modeled the students' graphing schemes to detail the role of experiential and measured time in their reasoning. When students were able to substitute one quantity with their sense of experiential time they constructed a graph by tracking the continuous variation of the quantity's value while moving their pen left to right. When both quantities varied unsystematically so that neither could be substituted with experiential time, students coordinated measures of two quantities by plotting points and connecting the dots. Since covariational reasoning involves coordinating two varying quantities, anytime students reason with experiential time they do not engage in covariational reasoning. However, many problems involve quantities whose values increase monotonically. For students to engage in covariational reasoning we must re-think the types of relationships we ask students to reason about. (Received September 20, 2016)

- 1125-N1-2746 **Jonathan D Troup\*** ([jtroup@math.ou.edu](mailto:jtroup@math.ou.edu)). *Developing Students' Reasoning about the Derivative of Complex-Valued Functions with the Aid of Geometer's Sketchpad (GSP)*.

In this paper, I share results of a case study describing the development of two undergraduate students' geometric reasoning about the derivative of a complex-valued function with the aid of Geometer's Sketchpad (GSP). My participants initially had difficulty reasoning about the derivative as a rotation and dilation. Without the aid of GSP, they could describe the rotation and dilation aspect of the derivative for linear complex-valued functions, but were unable to generalize this to non-linear complex-valued functions. Participants' use of GSP, speech, and gesture assisted with discovering function behavior, generalizing how the derivative describes the rotation and dilation of an image with respect to its pre-image for non-linear complex-valued functions, and recognizing that the derivative is a local property. (Received September 20, 2016)

- 1125-N1-2995 **Duane Graysay\*** ([dtgraysa@syr.edu](mailto:dtgraysa@syr.edu)), 215 Carnegie Library, Syracuse, NY 13244. *Generality-construction processes of undergraduate students*.

This session will present results of a study examining the ways that undergraduate students in mathematics-intensive majors engage in the process of constructing mathematical generalities. The research design involved developing a framework for problematizing the construction of generalities, leading to the development of three types of tasks: characterizing tasks, in which individuals are asked to construct general statements regarding the elements of a given collection; populating tasks, in which one is asked to construct a collection of objects, each of which satisfies a given general statement; and reconstructing tasks, in which one is given a mathematical generality and asked to adapt it to a broader domain. Participants for this study were mathematics majors and mathematics education majors, each of whom were within three semesters of completing their degree programs. Major findings include descriptions of the varied approaches that were observed across participants and the impact of the use of specific cases as compared to the use of generic representations in the development of generalities. (Received September 20, 2016)

1125-N1-3098 **Zackery Kevin Reed\*** ([reedzac@oregonstate.edu](mailto:reedzac@oregonstate.edu)). *Student Generalizations from Finite to Infinite Dimensional Normed Spaces*. Preliminary report.

I recently conducted two teaching experiments investigating student generalization in real analysis settings, which focused on student reasoning about metric spaces. Normed Linear Spaces account for the majority of undergraduate students' initial glimpses at metric spaces. Such spaces are particularly interesting because students have the benefit of working with both a vector space structure and a topological structure created by the norms. Traditionally, undergraduate analysis courses have examined only finite dimensional vector spaces. From an analysis perspective, all finite dimensional spaces behave in the same way, and there are many nuances involved in the generalization to infinite dimensional metric spaces. The leap from finite to infinite dimensional normed spaces provides a natural setting for student-driven generalization to occur. In this preliminary presentation I report on student generalization of particular norms from a finite to infinite dimensional setting. I will explore nuances of students' initial generalizations and will discuss further directions that this research could go. (Received September 21, 2016)

## Revitalizing Complex Analysis

1125-N5-251 **Stephan Ramon Garcia\*** ([stephan.garcia@pomona.edu](mailto:stephan.garcia@pomona.edu)), Department of Mathematics, Pomona College, 610 N College Ave, Claremont, CA 91711. *If you believe in real numbers and matrices, then you believe in complex arithmetic!*

The introduction of the complex number system sets the tone for your entire complex-variables course. Important questions abound. Should complex numbers be presented as handed down by the gods? Is  $i$  just a symbol with magical properties? Should the properties of complex numbers be derived from more primitive principles? Many instructors are familiar with a matrix-based approach to  $\mathbb{C}$ . We push this perspective to its logical extreme and demonstrate that almost all properties of complex arithmetic follow from elementary linear algebra. (Received August 18, 2016)

1125-N5-1379 **Russell W Howell** ([howell@westmont.edu](mailto:howell@westmont.edu)), Dept. of Mathematics and Computer Science, Westmont College, Santa Barbara, CA 93108, and **David N Kyle\*** ([dkyle@westmont.edu](mailto:dkyle@westmont.edu)), Westmont College, Santa Barbara, CA 93108. *The Count of Monte Disco*. Preliminary report.

In their Pólya awarded paper of 2014, Brilleslyper and Schaubroeck characterized completely the *unimodular roots* (*i.e.*, zeros that lie on the boundary of the unit disk) of trinomials having the form  $p(z) = z^n + z^k - 1$ , where  $1 \leq k \leq n - 1$ . They then posed a challenge problem well-suited for an undergraduate research project: derive a formula that would count the number of *interior roots* (*i.e.*, zeros that lie inside the unit disk) of these trinomials. We present and prove such a formula. That the formula *counts* zeros inside the unit *disk* explains two of the three principle words of our title. To find out how *Monte* relates you must come to the talk! (Received September 16, 2016)

1125-N5-1547 **Kristin A. Camenga\*** ([camenga@juniata.edu](mailto:camenga@juniata.edu)), Juniata College, 1700 Moore Street, Huntingdon, PA 16652. *Harnessing student interest to present applications of Complex Analysis*. Preliminary report.

Complex Analysis is full of interesting applications that can excite student interest, but there is far more than can be included in a one semester course. As a first time teacher of Complex Analysis, I tried to help students explore applications by having students present individual projects that were closely connected to the content sequence and share smaller applications and solutions to problems throughout the semester, included in their grade as "personal initiative points." It was also my intent that this encourage upper level mathematics students to become more independent learners of mathematics. This presentation will report successes and challenges as well as any changes intended for the future. (Received September 17, 2016)

1125-N5-1948 **William W Johnston\*** ([bwjohnst@butler.edu](mailto:bwjohnst@butler.edu)). *Complex Analysis in the Transition (Proofs) Course*.

A surprisingly direct pathway through differential complex analysis can lead students in a transition course's single, manageable module to interesting and sophisticated applications. Of course, this pathway comes with an enjoyable travel through proofs and definitions fundamental to differential (but not integral) complex analysis. The presentation outlines one such single-minded focus on differential complex analysis in a proofs course and describes the results. (Received September 19, 2016)

1125-N5-2033 **Beth Schaubroek\*** ([beth.schaubroek@usafa.edu](mailto:beth.schaubroek@usafa.edu)). *Using applications to understand complex analysis concepts.*

In this talk, we will first discuss incorporating interactive labs in a complex variables course. Then we will discuss how application labs at the end of the semester provide a good review of much of the course material. In examining Dirichlet problems, students review the concepts of harmonic functions and the orthogonality of the real and imaginary parts of a function, as well as mapping properties of basic functions. While learning about iteration of functions, students review powers and roots of complex numbers and the amplitwist interpretation of the derivative. I suggest using these projects as both capstone and review material in an undergraduate complex analysis course. (Received September 19, 2016)

1125-N5-2055 **Michael A. Brilleslyper\*** ([mike.brilleslyper@usafa.edu](mailto:mike.brilleslyper@usafa.edu)). *Unimodular Roots of Trinomials and Connections to Cyclotomic Polynomials.* Preliminary report.

The trinomials  $p(z) = z^n + z^k - 1$  with  $1 \leq k \leq n - 1$  have unimodular roots (roots of modulus 1) if and only if  $6g$  divides  $n + k$ , where  $g = \gcd(n, k)$ . The factor of  $p(z)$  consisting of the unimodular roots has a particularly simple form and is equal to a cyclotomic polynomial whenever the prime factorization of  $g$  contains only 2's and 3's. If  $g$  contains other primes in its factorization, then the unimodular factor is divisible by a cyclotomic polynomial. In cases where  $g$  is a prime and  $g \geq 5$  we can express certain cyclotomic polynomials as a ratio of two trinomials. (Received September 19, 2016)

1125-N5-2228 **Zhengqing Chen\*** ([zhechen@clarkson.edu](mailto:zhechen@clarkson.edu)), Math Department, Box 5815, Clarkson University, 8 Clarkson Avenue, Potsdam, NY 13699, and **Scott Fulton.** *Complex analysis in action: Introducing the novel Fokas' transform method to our undergraduates.* Preliminary report.

Cauchy's theorem, residue calculus, contour integration (Jordan's lemma indispensable), are highlights in undergraduate Complex Analysis curriculum. We often feel we owe students a revisit of those critical topics. Here, we choreograph all the aforementioned into an act, with the stage being solving the Laplace equation. The act is "real" in the sense that it is from actual scientific research. Underlying is the novel Fokas' transform method for solving 2D PDEs, which uses complex analysis extensively. It is worth an introduction: Beyond broadening students' scope of view of Complex theory in application, it demonstrates how to put the knowledge together like a clockwork. In this talk, we first demonstrate the mechanism of Fokas' method with solving Laplace equations. Details include: novel integral solutions in a complex plane; conversion to classical series solutions (via residue calculus); limiting cases when semi-infinite domains become doubly infinite. (Analogy: Fourier series/transform) Next, we discuss the pedagogical values in introducing this method. One of its greatest inspirations is: sometimes a development in mathematics comes not from going more abstruse, but from stringing together familiar knowledge. (Received September 20, 2016)

1125-N5-2424 **Frank A. Farris\*** ([ffarris@scu.edu](mailto:ffarris@scu.edu)), 500 El Camino Real, Dept of Mathematics and Computer Science, Santa Clara, CA 95053. *The Domain-Coloring Algorithm and the Argument Principle.*

The domain-coloring algorithm allows us to visualize complex-valued functions on the plane in a single image, an alternative to before-and-after mapping diagrams. I will review the algorithm and then show a visual discovery/proof of the argument principle, which relates the count of poles and zeros of a meromorphic function inside a contour to the accumulated change in argument of the function around the contour. I connect these ideas to standard learning goals of courses in complex variables. (Received September 20, 2016)

## *The Scholarship of Teaching and Learning in Collegiate Mathematics*

1125-O1-346 **Adam T Heck\*** ([adamheck@southern.edu](mailto:adamheck@southern.edu)), Mathematics Department, Southern Adventist University, PO Box 370, Collegedale, TN 37302. *How I Flipped My Classroom and Why I am Sticking With It.*

The idea of a flipped classroom is a growing trend in education. Over the past couple of years I have transitioned from a traditional lecture style classroom (in-class lecture, out of class homework) to a flipped classroom style (out-of class lecture/homework followed by in-class reinforcement/applications). During the Spring Semester of 2016, I taught two sections of a College Algebra course (one flipped and one traditional). Students in both groups classes took identical exams. In this presentation I will discuss the exam results from both groups. I will

also discuss the means by which I made the change to a flipped classroom and the reasons I plan on sticking with it. (Received August 28, 2016)

1125-O1-402 **Victoria Brown, Adam Childers, Jan Minton, Hannah Robbins\***  
([robbins@roanoke.edu](mailto:robbins@roanoke.edu)), **Kristin Emrich** and **David Taylor**. *Curing the High DFW Rate in First Year Calculus*. Preliminary report.

Our department recently restructured our first calculus course to address the fact that it has perennially had one of the highest DFW rates at the college. Our solution was to offer both a one semester and a two semester version of calculus 1. Both versions cover the same calculus material as our previous calculus 1 course, and the two semester version also contains just-in-time algebra review. We use a placement test, along with SAT math scores and high school GPAs, to place students into the appropriate version. Additionally, we allow students to switch from the one semester version back to the two semester version after the first test.

The goal of our study is to see how successful this new course structure is in helping students succeed in first and second year calculus. We collected and analyzed students' grades, placement test scores, and final exams in the first two years of the restructured course, and compared them to data from the last year of our old version of calculus. Additionally, we collected grades for those students who continued on into our second calculus course. In this talk, we will give details about implementing this new calculus curriculum and discuss the preliminary results of our assessment. (Received August 31, 2016)

1125-O1-1276 **Jennifer Vandebussche\*** ([jvandenb@kennesaw.edu](mailto:jvandenb@kennesaw.edu)) and **Lake Ritter**. *The Effect of Required Office Hours on an Early Incentivized Remediation Program in Calculus I*. Preliminary report.

One author has been using an early incentivized remediation program in Calculus I for several years. The program assesses prerequisite skills early in the semester, and then allows students to retake the assessment if certain remediation activities are completed. The author has previously required office hours as part of the remediation activities, but it is unclear whether this is has been an important part of the intervention.

In this talk, we discuss preliminary results regarding an investigation into the effect of that office hours requirement. The authors each required one section to attend office hours as part of the remediation activities, and did not require the other section to. We investigate differences in both course outcomes (as measured by final grades and performance on selected common final exam questions) and attitudes toward office hours (as measured by pre- and post-surveys). (Received September 15, 2016)

1125-O1-1311 **Lisa J. Carnell\*** ([lcarnell@highpoint.edu](mailto:lcarnell@highpoint.edu)). *College teachers' beliefs: Teaching mathematics to students with learning disabilities*. Preliminary report.

Because of the Americans with Disabilities Act (ADA), more students with diagnosed learning disabilities are making the transition from high school to postsecondary education. This trend has potential implications for the college mathematics classroom. College mathematics teachers in North Carolina were surveyed regarding their perceived preparation to teach students in general education mathematics courses, as well as their attitudes toward students with learning disabilities in those classes. Among the findings, it was indicated that while most respondents felt at least somewhat confident in their ability to teach students with learning disabilities in math, the majority also felt they had not received adequate training to meet these students' needs (Received September 16, 2016)

1125-O1-1352 **Michael Weingart\*** ([weingart@math.rutgers.edu](mailto:weingart@math.rutgers.edu)) and **Alice W Seneres**  
([alice.seneres@echo.rutgers.edu](mailto:alice.seneres@echo.rutgers.edu)). *Flipping the liberal arts math classroom: improving learning, increasing verbal discourse*.

In Spring 2015, Rutgers implemented the flipped classroom model by introducing its first hybrid mathematics course. The professor created original materials for the hybrid course in the form of weekly videos and in-class scaffolding activities. The course was the primary liberal arts mathematics course offered at Rutgers; a traditional section was run in parallel to the hybrid section, with the same professor teaching both sections and administering comparable assessments. This enabled a comparative analysis of measurable differences in verbal discourse, student-instructor interaction, and overall learning results. Quantifying the verbal discourse that took place in the classroom showed a significantly higher amount of student discourse in the hybrid section. A sample excerpt from the hybrid course will illustrate both how students assist each other with the material and how the professor successfully becomes the 'guide on the side' to student learning. Finally, overall performance data were stronger in the hybrid section. This talk blends together the researcher's results with a first person account from the professor's point of view. (Received September 16, 2016)

1125-O1-1388 **Louis Gross\*** ([gross@nimbios.org](mailto:gross@nimbios.org)), University of Tennessee, National Institute for Math. & Bio. Synthesis, Knoxville, TN 37996, **Suzanne Lenhart**, University of Tennessee and NIMBioS, **Pamela Bishop**, NIMBioS, and **Kelly Sturner**, NIMBioS. *Assessing Impacts on Student Learning in Mathematics from Inclusion of Biological, Real-World Examples*. Preliminary report.

This project is working to develop and evaluate a concept inventory to assess student's mathematics abilities as affected by the use of real-world examples from the life sciences. The objective is to investigate whether life science students taking a course that places the calculus concepts in a biological context are more readily able to understand the underlying mathematical concepts, and apply them to other examples, than students who have not had this exposure to examples from the life sciences. This addresses a fundamental question in teaching mathematics: whether placing the mathematics in a concrete, real-world context helps students learn the mathematical ideas and enhances their skills in applying the mathematics. We will present our preliminary results from our concept inventory connecting calculus and quantitative biology. (Received September 16, 2016)

1125-O1-1661 **Paran Rebekah Norton\*** ([pfisch@clemson.edu](mailto:pfisch@clemson.edu)), **Karen High** and **William Bridges**. *Impact of Course Policy Changes on Calculus I DFW Rates*. Preliminary report.

This paper examines the impact of departmental policy changes on the trend in DFW rates for introductory calculus at a large research university. We defined three distinct policy periods: Traditional (2002-2005), Active Learning (SCALE-UP) (2006-2013), and Return to Traditional (2014-2015). Regression analysis showed DFW rates were increasing during the Traditional period, significantly decreased after the switch to SCALE-UP, remained fairly consistent during the SCALE-UP period, and then significantly increased during Return to Traditional. These trends were not consistent across demographic subsets of students. Specifically, white female STEM students typically had the lowest changes in DFW rates corresponding to policy changes, whereas non-white male non-STEM students had the highest changes in DFW rates. Individual trends for D's, F's, and W's were also analyzed. The two policy changes had the greatest influence on the trend in W rates. Potential factors that could influence a student to withdraw from the course were examined. Students who withdrew had midterm averages similar to F students during the SCALE-UP period, but their averages were significantly lower than the F students during Return to Traditional. (Received September 18, 2016)

1125-O1-1885 **Girija Sarada Nair-Hart\*** ([nairhaga@uc.edu](mailto:nairhaga@uc.edu)), OH. *What one must know about students' concept formation*.

During this presentation, the researcher will discuss prominent theories on concept formation and align them with what was observed during her own research on students' conceptual understanding of limits and continuity of rational functions. The results of the study that employed 'talk-aloud' problem solving interviews with 19 calculus students from a major university will also be discussed during this session. (Received September 19, 2016)

1125-O1-1954 **Laura J Schmidt\***, 410 10th Ave. E, University of Wisconsin - Stout, MSCS Department, Menomonie, WI 54751, and **Matt Corne** and **Todd Zimmerman**. *Effect of Belongingness Intervention on Student Performance*. Preliminary report.

Our investigation aims to determine whether addressing student issues of not feeling like they belong in a math or physics course will improve student performance in their course and subsequent courses. The courses studied are a remedial algebra course, an integrated pre-calculus/calculus course, and a statics course. Current research indicates that such interventions significantly impact student performance as well as increase usage of student support services. This research shows that low income, first generation, and minority students gain the largest benefits from belongingness interventions. In our investigation we will give students a series of quotes from previous students about how they coped with not feeling like they belonged and then ask the current students to write a short description of how they are dealing with their feelings of belonging in their course. We will look at student math/physics anxiety as well as self-efficacy by administering a survey at the beginning and end of the semester. If the belongingness intervention is shown to have significant impact on student performance after a single intervention, the intervention can be used in other classes to a similar end. Our presentation will discuss the details of our investigation and preliminary results. (Received September 19, 2016)

1125-O1-2161 **Sarah E. Hanusch\*** ([sarah.hanusch@oswego.edu](mailto:sarah.hanusch@oswego.edu)), Oswego, NY 13126. *Teaching Set Theory and Venn Diagrams with Embodied Cognition*.

Introductory set theory with Venn diagrams is a common topic for general education mathematics courses. This topic often causes difficulties for students because of the specialized notation involved. However, a lesson

utilizing embodied cognition resulted in significant learning gains for the students in a mathematics for pre-service elementary teachers course. In this session, I will present the lesson activity and the evidence of learning gains and student perceptions. (Received September 19, 2016)

1125-O1-2280 **Maria G Fung\*** (mfung@worchester.edu), Mathematics Department, Worcester State University, Worcester, MA 01602, and **Pamela Hollander** (phollander@worchester.edu), Education Department, Worcester, MA 01602. *Writing to Learn Intervention in an Algebra Course for K-8 Pre-service Teachers*. Preliminary report.

Pre-service teachers in an Algebra course are given a series of specific writing assignments—both formal and informal, that relate to the development of deeper conceptual understanding of the mathematical ideas in the course. The effect of this writing to learn intervention is considered through a pre- and post- surveys, results on assessments and interviews. (Received September 20, 2016)

1125-O1-2508 **Deb Carney\*** (dcarney@mines.edu), 1500 Illinois Ave., Colorado School of MinesAp, Applied Mathematics and Statistics, Golden, CO 80401, and **Rebecca I Swanson** (swanson@mines.edu), 1500 Illinois Ave, Colorado School of Mines, Applied Mathematics and Statistics, Golden, CO 80401. *Engineered Learning in Calculus at Colorado School of Mines*. Preliminary report.

In this talk we will discuss our flipped classroom model for a calculus II class and its focus on student communication. We will share preliminary evidence related to the following question. “Does improving students writing help improve student’s success in the course?” We will measure success by student attainment of higher level (Bloom’s) outcomes and student portrayal of better conceptual understanding. We will share preliminary results related to a pre/post communication assessment and analysis of student final exam work from current and prior semesters. Work will be judged using a communication rubric designed by the authors. (Received September 20, 2016)

1125-O1-2605 **Carrie J. Timmerman\*** (ctimmerman@comcast.net), 11087 Center St, Clayton, MI 49235. *Anticipatory Sets as a Method of Engagement in College Mathematics Classes*.

Many young adults start their college journey only to find that they are unable to fulfill their dreams due to the inability to pass general education requirements in mathematics. The pass rates on college entry exams and post-college licensure tests for math content is below the benchmarks set for these exams. Contributing to the problem is the inability to engage students and make connections between the subject and real life applications. One class of entry level college statistics students were taught using an anticipatory set while another class served as a control group. To assess the effectiveness of this method, a quasi-experimental comparison group pre-test/post-test design was implemented utilizing two intact groups of students from Siena Heights University. Gain scores on pre-post tests were used as measures of achievement and an attitude instrument was used to determine changes in attitude. The use of an anticipatory set in the lesson was used as the treatment for the experimental group. (Received September 20, 2016)

1125-O1-2654 **Lorraine F Dame\*** (lfdame@r.umn.edu), 111 South Broadway, Suite 300, Rochester, MN 55904, and **Aminul Huq, Bijaya Aryal** and **Xavier Prat-Resina**. *Increasing Student Knowledge Transfer from College Algebra Curriculum to Partner Disciplines*. Preliminary report.

In the Center for Learning Innovation at the University of Minnesota Rochester, mathematics and partner discipline faculty are collaboratively engaged in creating an integrated college algebra/physical sciences curriculum and using multidisciplinary teaching teams to deliver it. These efforts are strongly motivated by a need to increase the efficacy of knowledge transfer from college algebra curriculum to partner disciplines and to better prepare students identified as struggling with mathematics upon entry to a health sciences program. Evidence analyzed from this successful pilot program includes qualitative analysis of themes identified from individual student interviews during college algebra and student interviews after college algebra. Quantitative student outcomes in college algebra and outcome data from subsequent physical science courses was also collected and analyzed. Together with instructor observations, this evidence demonstrates a significant increase in college algebra knowledge transfer over a more traditional curriculum and delivery. (Received September 20, 2016)

1125-O1-2657 **Mihhail Berezovski\*** (mihhail.berezovski@gmail.com), 600 S. Clyde Morris Boulevard, Daytona Beach, FL 32114. *Implementation and evaluation of active learning elements and innovative strategies for learning and teaching in Calculus classes*.

High-enrollment courses with a high percentage of withdrawals sometimes referred to as “gateway” courses because they are also required as pre-requisites to degree program courses. Failure to pass these “gateway”

courses disrupts the natural progression of a degree program and often leads to high attrition rates. The focus of this research is to incorporate and enhance proven teaching practices in a course and assess their efficacy for improving learning and pass rates. This research reports on an innovative combined approach to teaching Calculus I and II. The designed method combine improved traditional practices with elements of innovative methods of modern education. The conceptual and practical forms of education provided through the strictly planned assignments and activities, such as polling open ended questions, bi-weekly two day workshops, provoking conceptual tests, hands-on homework projects, elements of flipped classroom and online assignments. The results of each activity and overall approach compared with traditional lecturing sections of the course. The analysis of the results of pre- test and post-tests are compared and presented. The general recommendation is formulated. The analysis of student's self-evaluation is performed. (Received September 20, 2016)

1125-O1-2743 **Belinda Pickett Edwards\*** ([bedwards@kennesaw.edu](mailto:bedwards@kennesaw.edu)), Department of Mathematics, 275 KSU Rd. NW, MD 1102, Kennesaw, GA 30152, and **B P Edwards**. *The Congruence between Instructor and Student Perceptions of Learner-Centered Teaching in Calculus I*.

Learner-centered (LC) teaching is an approach to teaching that is increasingly being encouraged in undergraduate gateway mathematics courses. There exists empirical evidence suggesting that LC pedagogical methods tend to improve student learning and increase academic success. While LC teaching has many reported benefits, research suggests it is not easily accomplished and there is often a mismatch between teacher and student expectations. The congruence between what instructors value and what students report doing in a LC environment is critical to understanding the value an instructor places on LC pedagogical practices such as student-engagement, technology use, peer-teaching, collaboration, and problem-solving. The question that guided the student was: What benefits and gains do students and their instructor report as a result of participating in a LC Calculus I course? Student and instructor reported benefits and gains were assessed using the Class-Level Survey of Student Engagement (CLASSE) survey results and qualitative methods of inquiry. The purpose of the study was to use the results to identify effective Calculus I LC pedagogical practices, shape/transform Calculus I teaching and learning, and inform the design of faculty development activities. (Received September 20, 2016)

1125-O1-2810 **James S Rolf\*** ([jim.rolf@yale.edu](mailto:jim.rolf@yale.edu)) and **John Hall**. *Long-term Learning Gains from an Online Bridge Program*. Preliminary report.

ONEXYS (Online Experiences for Yale Scholars) is a not-for-credit online bridge program for incoming students during the summer before their first year on campus. The program began in the summer of 2014 to address the question: "What can be done in the summer that will have impact on performance in quantitative reasoning courses during the year?" We utilized online videos, adaptive practice quizzes, challenging applications of math content, and undergraduates acting as coaches to answer this question. A year ago, short-term learning gains during ONEXYS 2015 were reported. We expand on those results by comparing first year GPA of students from that cohort with similar Yale students. New results from ONEXYS 2016, including short-term learning gains from pre/post tests and student attitudes from pre/post surveys and focus groups, are also presented. Results from ONEXYS 2015 and ONEXYS 2016 are then compared. (Received September 20, 2016)

1125-O1-2835 **Ashley Klahr\***, [aklahr@math.arizona.edu](mailto:aklahr@math.arizona.edu), and **Guadalupe Lozano**. *Understanding Gender Bias on Large Scale Precalculus Exams*.

It is often claimed that women are not as good at mathematics as men, but could discrepancies in the genders' mathematics ability be partially explained by test questions instead? Assessment questions that exhibit cultural, gender, and ethnic biases measure more than mathematics ability and may hence explain lower mathematics performance in particular groups (such as women). The goal of this paper is to analyze gender bias on large-scale, high stakes precalculus exams. Many people perform these types of analyses on formal standardized tests; however, very few schools perform these analyses on internal large-scale exams. In order to detect if there is any gender bias on any of the questions, I modeled the exam questions using Item Response Theory, IRT. I then used IRT based Differential Item Functioning to identify questions that performed differently for the genders. In this talk I will describe preliminary results of our analysis of gender-bias in a large scale, uniform precalculus final, including an exploration of differences in bias across two different versions of the exam differing only in the order of answer choices. Preliminary analyses suggest alternate versions of the exam had different questions that contained gender bias. (Received September 20, 2016)

1125-O1-2894 **Brett C Smith\*** ([brett.c.smith@yale.edu](mailto:brett.c.smith@yale.edu)), 109 Church St, Apt 302, New Haven, CT 06510, and **James S Rolf**. *Teaching Pre-Calculus through Gaming*. Preliminary report.

The Yale Math Department, the Yale Center for Teaching and Learning, the Yale Center for Health and Learning Games and PreviewLabs have collaborated to create a video game, Cartes, focusing on pre-calculus material,

exploring algebraic functions and their graphs. In particular, we are interested in the question, "How does playing Cartes impact student learning, motivation to learn, and confidence to learn?"

Our experimental design requires a two stage process. We will report on the first stage of our experiment in which we focus on understanding a) any differences between single player and multi-player use, b) student perception of learning and c) impact on student motivation to persist in learning in the context of gameplay. Our methods for this report include taking a stratified random sample of students from our differential calculus course and asking them to play both a single player version and a multi-player version of Cartes. Data will be collected in the form of focus groups and surveys.

The second stage will be assessment of implementation in a multi-section Differential Calculus course, to be reported at a later time.

Future work includes implementing the Cartes at local institutions, the University of Bridgeport and Housatonic Community College. (Received September 20, 2016)

1125-O1-2987 **John H. Johnson\*** ([johnson.5316@osu.edu](mailto:johnson.5316@osu.edu)). *Piloting an active learning course by a novice lecturer in a large enrollment calculus class.* Preliminary report.

As part of its two-year redesign project, the Ohio State University's Mathematics Department is supporting the introduction of active learning in its large enrollment first-year calculus courses. Ohio State's calculus program is "tightly coordinated": the syllabus, course calendar, all exams, and all homework assignments are prepared and selected by a course coordinator.

As a novice lecturer in this tightly coordinated environment, I piloted an active learning section of our Calculus I course. Based off surveys responses, students perceived that the active learning course was more active than our more traditional sections. Moreover, despite this difference in course organization and tight coordination, and as a novice Ohio State calculus lecturer, students' performance was comparable to the other traditional sections. (Received September 20, 2016)

## *Successful Implementation of Innovative Models for Developmental and General Education Mathematics*

1125-O5-628 **Kimberly J Presser\*** ([kjpres@ship.edu](mailto:kjpres@ship.edu)), Department of Mathematics, MCT 250B, 1871 Old Main Drive, Shippensburg, PA 17307. *Incorporating the Computer Lab in the Developmental Mathematics Classroom.* Preliminary report.

Our Developmental Mathematics Program has faced two main challenges in recent years: increasing enrollments and diminished student productivity outside of the classroom. Previously, our classes had been primarily lecture and paper and pencil group work with computer assignments for homework outside of the classroom. To address these challenges, we have restructured our classroom time to incorporate time in the computer lab. In addition, we have shifted tutoring availability to include hours in the lab; similar to the math emporium models at other schools. This talk will discuss various methods employed to use the lab effectively for the Developmental Mathematics classroom; pitfalls in the process and solutions; and measurable outcomes of successes and failures in our initial implementations. (Received September 08, 2016)

1125-O5-667 **Upasana Kashyap\*** ([upasana.kashyap@regiscollege.edu](mailto:upasana.kashyap@regiscollege.edu)), Regis College, 235 Wellesley St., Weston, MA 02493, and **Santhosh Mathew**, Regis College. *Design and Implementation of Corequisite Model in a Freshman Level Quantitative Reasoning Course.*

Many colleges and universities are increasingly seeking to alter the existing remedial education with corequisite model in which students enroll simultaneously in a credit-bearing course and a course that provides remediation. In this talk, we present design, implementation, and discuss how we facilitated this model in a freshman level Quantitative Reasoning course. We provide evidence of the success of this approach using data that compare student performance under corequisite model with the more traditional prerequisite model. Our study concludes that student performance and perceptions were significantly higher when they completed the course under corequisite model compared to the prerequisite model. (Received September 08, 2016)

1125-O5-738 **Dan L Ray\*** ([ray@carnegiefoundation.org](http://ray@carnegiefoundation.org)), 51 Vista Ln, Stanford, CA 94305. *The Carnegie Math Pathways: Structural, Curricular, and Pedagogical Innovation in Developmental Mathematics at Scale.*

Since 2010, 58 colleges have partnered with the Carnegie Foundation for the Advancement of Teaching to implement broad changes to their developmental mathematics course sequences and the way that these courses

are being taught. The Carnegie Math Pathways, Statway and Quantway, accelerate students who place into developmental math to and through college level mathematics courses in a single year. Statistics and Quantitative Reasoning are used in these pathways to provide much needed context to the mathematics being presented. Faculty who teach these courses are trained to facilitate their classes in a way that encourages student collaboration, fosters a growth mindset, and creates opportunities for productive struggle. To implement these curricular and pedagogical changes, the Pathways Network supports faculty and administrators with making the transition and provides ongoing professional development. Support for new faculty includes national, regional, and local workshops, mentoring from an experienced faculty member, and an online preparation course facilitated by a faculty mentor. This approach allows all of the networked colleges to be active players in ongoing improvement and refinement. We will provide evidence of the impact of these changes on student success rates. (Received September 10, 2016)

1125-O5-1230 **Keith E Mellinger\*** ([kmelling@umw.edu](mailto:kmelling@umw.edu)), University of Mary Washington, 1301 College Avenue, Trinkle Hall, Fredericksburg, VA 22401. *The ingredients for a successful liberal arts course in quantitative reasoning*. Preliminary report.

Following in the footsteps of Dot Sulock's inspiring course *Reality Math* taught at the University of North Carolina Asheville, *MATH 120: Quantitative Reasoning for the Sciences* has been redesigned at the University of Mary Washington to include inquiry-based learning activities, exploratory writing assignments, and student-led class discussions. This completely non-lecture course appeals to the liberal arts students with units on energy and electricity, health and fitness, and personal finance, along with a healthy emphasis on learning to use quantitative information to strengthen your writing. The talk will explain the delivery of the course, both in an online and a face-to-face environment. We provide many, many examples of the activities used to develop the students' quantitative skills while simultaneously providing them with concrete examples of how quantitative information can be used to understand contemporary issues such as wind power, carbon emissions, sugar in our diets, interest growth, and firearm deaths. Students leave the course as more informed citizens, able to use quantitative information to better support their own views on these important issues. (Received September 15, 2016)

1125-O5-1257 **Jan Orton Case\*** ([jcase@jsu.edu](mailto:jcase@jsu.edu)), JSU MCIS Department, 700 Pelham Road North, Jacksonville, AL 36265. *EdReady: A Low Stakes Alternative to Placement Testing*.

Jacksonville State University redesigned its developmental mathematics program from assessment based on a high stakes placement score into a placement "path" that uses a personalized learning plan to guide students in obtaining the math skills they need. At orientation, students take an online assessment using EdReady, an open source project sponsored by the Gates Foundation, the Hewlett Foundation, and member organizations. If the assessment identifies deficiencies, online learning materials are provided that have been designed specifically for JSU's courses. Students who are unable to complete the process independently enroll in a course that provides instruction, tutoring and continued assessment of the students' progress. The program was initiated in Fall 2015 with a control group placed by a traditional standardized test. Students placed by EdReady were significantly more likely to pass their subsequent math classes with grades of C or better. The results were especially pronounced for STEM majors. They were also significantly less likely to fail, to withdraw from class or to receive grades of incomplete. This presentation describes the process of changing the university culture of high stakes placement testing and offers evidence for a successful, low cost alternative. (Received September 15, 2016)

1125-O5-1358 **Kevin P Sukanek\*** ([ksukanek@utk.edu](mailto:ksukanek@utk.edu)), 227 Ayres Hall, 1403 Circle Dr., Knoxville, TN 37996-1320, and **Tracy L Cook** ([tcook@utk.edu](mailto:tcook@utk.edu)), 227 Ayres Hall, 1403 Circle Dr., Knoxville, TN 37996-1320. *Math Camp: Preparing Students for College Level Math*. Preliminary report.

The University of Tennessee Chancellor Jimmy Cheek requested an initiative from the Mathematics Department to announce at the White House Education Summit in January 2014. The Math Department proposed a summer program to focus on incoming freshmen who are not eligible to enroll in their first required math course for their major. The program focused on math competency, advising, and strategies to be a successful student. It was developed jointly with the UT Student Success Center, who promoted a strong sense of community. Math Camp, which does not provide college credit but allows students to place out of prerequisite classes, has run successfully for three summers, starting in 2014. We will describe pedagogical strategies and the curriculum, which covers College Algebra and Precalculus in three weeks prior to the start of the fall semester. We will present data on its success. (Received September 20, 2016)

1125-O5-1571 **Vesna Kilibarda\*** ([vkilibar@iun.edu](mailto:vkilibar@iun.edu)), Department of Mathematics and Actuarial Sci., 3400 Broadway, Gary, IN 46408, **Yuanying Guan** ([guany@iun.edu](mailto:guany@iun.edu)), Department of Mathematics and Actuarial Sci., 3400 Broadway, Gary, IN 46408, and **Xiaofeng Wang** ([wang287@iun.edu](mailto:wang287@iun.edu)), Department of Mathematics and Actuarial Sci., 3400 Broadway, Gary, IN 46408. *Active Learning in Developmental and General Education Mathematics Courses*. Preliminary report.

Strong evidence in research literature points to the success of active learning in the mathematics classroom Treisman (2009), Freeman et al. (2014), Levi et al. (2016). This research suggests that active learning has a disproportionately beneficial effect on members of minority groups and has the greatest effects in small ( $n \leq 50$ ) classes. Being a regional, commuter institution with a strategic goal of improving retention and graduation rates (45% minority students), it was natural that we developed active learning activities for general education and developmental courses.

We introduced an intervention in classes with 20 to 35 students. In activities that we created, students are required to explore problems in groups, test conjectures, develop equations and solutions, and explain their results. Instructors provide limited feedback on the spot and collect and grade one student's work, representative of the entire group.

In this paper we will share some of the activities we developed and show comparisons of DFW rates, scores on common midterm and final exams, class grades, and grades in the next STEM course between intervention classes and classes that did not use active learning. We will also compare intervention cohorts with previous semesters' cohorts. (Received September 18, 2016)

1125-O5-1902 **Abe Edwards\*** ([aedwards@msu.edu](mailto:aedwards@msu.edu)). *A New Angle on Assessing Quantitative Literacy Pathways*.

Since 2010, Cuyahoga Community College has been involved with implementing the Quantway program—a two semester pathway to college mathematics credit with an emphasis on quantitative literacy. Considerable in-house research indicates that this pathway has been largely successful, because it has allowed many students to achieve college mathematics credits. Yet, it has remained unclear what happens to these students after they complete the pathway. For example, are they able to draw on their mathematical and statistical knowledge in later academic experiences? This talk presents the culmination of a longitudinal research project to assess the role of QL instruction, from a totally new perspective. In particular, the talk focuses on the connections that former Quantway students made between tasks in their QL courses and tasks in subsequent general science courses. By focusing on connections that students themselves report, we begin to obtain a fuller picture what “success” looks like for a QL pathway. (Received September 19, 2016)

1125-O5-1938 **Peter G Hardy\*** ([peter.hardy@maine.edu](mailto:peter.hardy@maine.edu)), 111 South Street, Farmington, ME 04938. *Survivor Math - Incorporating a Semester-Long Research Project in Environmental Sustainability into an Introduction to Mathematics General Education Course*.

It is late October of 2016. Two weeks ago a coordinated series of terrorist attacks has effectively cut off the flow of oil from the Middle East. The world is in chaos. The United States government has declared a state of emergency for the entire country. Gasoline is being rationed and when it is available the prices are exorbitant. A blackout of the entire region began three days ago and there has been no word as to when electric service will be restored. The university has closed its doors and told all of its students to go home. You would love to go home, except that all of the gasoline in your car has been siphoned off and there is nowhere you can buy more to refill the tank. Luckily, you remember that your forward-thinking math professor predicted just such a scenario and began to prepare for it several years ago. Upon walking the few miles to his house you soon realize that your entire math class has had the same idea of joining him! Your task is to find a way to survive. Start with the short-term because there is no need for long-term planning if you don't make it through the winter. What are the basic necessities for human survival? How much of these resources will be needed to keep twenty-five of you alive for the next week? The next month? The next year? (Received September 19, 2016)

1125-O5-2414 **Wes Maciejewski\*** ([wesley.maciejewski@sjsu.edu](mailto:wesley.maciejewski@sjsu.edu)), Department of Mathematics and Statistics, One Washington Square, San Jose State University, San Jose, CA 95192-0103. *Developmental Math Students' Dispositions Towards Mathematics*.

Students enrolled in a developmental mathematics course comprise a large portion of many American college and university student bodies. Despite this, there is no clear consensus – among instructors and administrators, but also in the literature – of what constitutes an effective developmental mathematics curriculum, nor of how to deliver such a curriculum. I propose that this lack of consensus exists because we do not yet understand who takes developmental math courses. We do know that these students were deemed insufficiently prepared

for college study, perhaps by failing an entrance exam or by not having the requisite courses, but can we say anything deeper? Specifically, how do these students compare to their regular-stream college colleagues in terms of their dispositions and attitudes towards mathematics? In this talk I will present results from a survey offered at San Jose State University intended to address these questions above. It is hoped that these results can inform the design of effective developmental math curriculae and instruction. (Received September 20, 2016)

1125-O5-2554 **Veronica Ocampo\*** ([ocampod3@uwm.edu](mailto:ocampod3@uwm.edu)), 2979 s 45th St, Milwaukee, WI 5321. *Building Middle School Mathematics Foundational Skills Using the Environment as a Culturally Responsive Setting*. Preliminary report.

A considerable number of minority students arrive to middle schools mathematics classrooms with low mastery levels of foundational mathematics concepts and skills. Frequently, previous failed experiences with math might have produced negative perceptions of this content area. Outdoor classrooms are the setting that will bring students to natural areas and city parks to build and understand some of the prerequisite skills necessary for a successful performance in higher mathematics courses. This project presents the experiences of urban students as they interact with math problems and with their peers and nature in outdoor settings as they complete different seasonal mathematics problems. No matter if it is spring, summer, fall, or winter, the multiple connections with this natural context and the interaction with peers support students in understanding and mastering foundational mathematics topics, developing different solution strategies, and generating, communicating, and sharing the formal knowledge gained. Through focused seasonal activities, students experience how mathematics permeates every aspect and activity of their everyday lives, gaining a new appreciation and insight of mathematics as a content area. (Received September 20, 2016)

1125-O5-2634 **McKenzie Lamb\*** ([lambm@ripson.edu](mailto:lambm@ripson.edu)). *Quantitative Literacy for Non-Math Faculty: Challenges and Solutions*. Preliminary report.

Ripon College, a small, liberal arts school in Wisconsin, is currently designing and implementing a new curriculum, called "Catalyst". The core of this curriculum consists of five courses, each emphasizing a set of broadly useful, transferable skills. The goal is that any faculty member on campus should be able to teach any of these five courses. In Spring 2016, I led a committee tasked with designing a quantitative literacy course that could be taught by any faculty member. We will discuss the challenges, pitfalls, and rewards of designing such a course. The focus will be on coming to a consensus on the meaning of the term "quantitative literacy" and on helping non-math faculty to teach basic quantitative skills. (Received September 20, 2016)

1125-O5-2647 **Martha Makowski\*** ([martha.makowski@gmail.com](mailto:martha.makowski@gmail.com)), 612 W Church St Apt 41, Champaign, IL 61820. *Learning For or Through Problems?: Exploring Differentiating Experiences in a Problem-centered Developmental Math Class*. Preliminary report.

In the last five years, the use of problem solving and group work based materials in developmental mathematics classrooms has increased markedly. Success rates from early implementations are promising but provide little information about how developmental students experience these classes. Using grounded theory, this study contrasts the self-reported experiences of eight students in one problem-centered developmental mathematics class taught by a course designer to highlight emerging tensions between the experiences of high- and low-achieving students. Their experiences are supplemented with classroom audio of the students working in their groups and instructor interviews. At least some of the differences in the experiences of the high and low-achieving students can be understood by whether the students are learning algebraic content for the first time through problem solving or if they have had algebraic instruction before, and are thus learning algebra for problem solving. These results suggest that instructors of these classes should consider methods of deliberately supporting high-achieving students in understanding the needs of their low-achieving peers. (Received September 20, 2016)

1125-O5-2678 **Malissa M Peery\*** ([mpeery@utk.edu](mailto:mpeery@utk.edu)). *"REACT" to Improve Student Success Rates and Classroom Effectiveness—Reaching Excellence through Active Coordinated Teaching*. Preliminary report.

REACT courses provide graduate teaching assistants (GTAs) with a mentorship opportunity and combat poor success rates in gateway mathematics courses. I created lecture videos and employ an innovative flipped format where GTAs manage in class activities and assessments, gaining valuable pedagogical content knowledge. REACT students watch video lectures and take notes outside of class time. Lectures are not repeated in class. Students are expected to come to class prepared. In class, they work in groups on activities that improve their skills and help build connections between concepts in both College Algebra and Basic Calculus, two commonly paired courses. GTAs interact with students while they work, encouraging peer learning, clarifying details, answering questions and asking questions that help students put the pieces together. Students are assessed via

clicker and hand graded feedback on group activities, traditional quizzes and unit tests. The result is a more personal learning experience for the student and GTA. REACT students succeed at a higher rate than that of students in traditional sections as evidenced by departmental final exam data. In addition, student ratings of REACT GTAs are higher than those of non-REACT GTAs. (Received September 20, 2016)

1125-O5-2679 **April E Conner\*** ([aconner1@utk.edu](mailto:aconner1@utk.edu)). *An Effective Pathway for Implementation of an Active Coordinated Course*. Preliminary report.

In this presentation I will outline a comprehensive infrastructure for consistent implementation of a standardized course taught by a large number of instructors with varying teaching experience. The infrastructure is adaptable for any course design and scalable to accommodate any course size. It is currently being used to deliver two gateway mathematics courses in conjunction with a graduate teaching assistant/associate mentorship program. It is also being used to introduce courses to new faculty and to help faculty who want to improve their teaching skills and/or try innovative curricula. The cloud based framework is built around Google Drive using docs and sheets that allow real-time editing by the coordinator. Instant updates are available to all instructors of the course via the cloud. The Google Drive share capabilities allow the coordinator and instructors to communicate directly through comments and suggestions in the documents. The interactive master spreadsheet and course calendar are the epicenters of the infrastructure, allowing instructors easy access to all course materials from any internet capable device. I will explain how all of these components work together to successfully organize a course that is independent of a learning management system. (Received September 20, 2016)

1125-O5-2680 **Cara J Sulyok\*** ([csulyok@vols.utk.edu](mailto:csulyok@vols.utk.edu)). *A GTA's Perspective on Active Coordinated Teaching in a Mentorship Program*. Preliminary report.

How can a large university effectively use graduate teaching assistants (GTAs) with limited or no teaching experience to deliver multiple sections of a gateway mathematics course while maintaining a standardized curriculum? Traditional teaching responsibilities for new GTAs usually include grading or leading weekly recitations for large lecture courses that provide limited pedagogical learning opportunities for the GTA. Educators know that large lecture courses with recitations lack continuity and are less effective than small individual sections that provide more one-on-one interaction. Through an innovative mentorship program at the University of Tennessee, I participated in a successful year-long alternative to traditional course delivery using Malissa Peery's flipped College Algebra and Basic Calculus courses. I interacted with students and gained confidence as I learned to competently manage a classroom. As one of the first GTAs in the mentorship program, I was in a position to help Malissa refine the overall experience for future students and GTAs. I was also asked to pilot a version of the course in my second year. I hope my perspective allows other universities to see the value of such a program and consider providing a similar training experience for their new GTAs. (Received September 20, 2016)

1125-O5-2699 **Alfred Dahma, Timothy B. Flowers\*** ([flowers@iup.edu](mailto:flowers@iup.edu)) and **Yuliya Melnikova**. *Bridging Developmental Mathematics with College Algebra: A Study Using ALEKS and Homework Time Requirement*.

In the fall of 2014, Indiana University of Pennsylvania instituted a complete redesign for the developmental mathematics courses in order to improve student retention and the DFW rate. Through the redesign, the Intermediate Algebra course is now taught using a Web-based assessment and learning system (ALEKS) in an emporium-style setting. By using ALEKS and requiring a weekly time requirement, the DFW rate has improved, and the program is considered to be succeeding in its goals.

We are now focused on the follow-up course, College Algebra. In particular, we want to know how students are transitioning out of the developmental course and into their subsequent college mathematics courses. During the Spring 2016 semester, a study was designed to try to investigate this transition and ways to improve student success. Out of the 12 sections offered, 7 served as a control group and 5 as part of the experimental group. The experimental group was further split into two other groups. In this talk, we will give an overview of our experimental design and report on the results. (Received September 20, 2016)

1125-O5-2729 **Jean W. Richard\***, Department of Mathematics, BMCC, The City University of New York, 199 Chambers Street, New York, NY 10007, and **Daniela Bardac-Vlada, Susan Licwinko, Lucio M-G Prado** and **Bettina Hansel**. *Curriculum Innovations for Developmental Mathematics: Introductory Statistics with Algebra Workshops*.

Statistical concepts and models are among the most important for students majoring in Human Services, Criminal Justice and other non-STEM related fields. In most community colleges across the United States, students must be exempt from remedial algebra before registering for an introductory statistics course. For the majority of students, this presents a challenge since many do not successfully complete the remedial algebra course.

We proposed that concepts of elementary algebra be embedded in a traditional introductory statistics course, allowing students to fulfill the remedial course in the same semester and receive college credit for statistics. This course was piloted for four semesters. This talk outlines the curriculum of algebraic concepts developed in a traditional introductory statistics course and shares our teaching experience. Additionally, we will describe how the students benefit in terms of attendance, passing rates and acceleration towards graduation at Borough of Manhattan Community College.

(Received September 20, 2016)

1125-O5-2842     **Charity Watson\*** ([cwatson@clemsun.edu](mailto:cwatson@clemsun.edu)), Dept. of Mathematical Sciences, Clemson University, Martin O-312, Clemson, SC 29634, and **Eliza Gallagher** ([egallag@clemsun.edu](mailto:egallag@clemsun.edu)). *The Redesign of Precalculus at Clemson University.*

For STEM majors at Clemson, their score on a placement exam determines their path to calculus. Students with lower scores are placed in precalculus, a preparation course for the standard calculus sequence, and must pass this course before entering Calculus I. Before 2011, this precalculus course was run in a lecture/lab design where students attended a lecture 4 times a week and 2 lab sessions a week. Under this model, the precalculus pass rate ranged from 45-55% over the years. In 2011, the online learning system ALEKS was implemented in all precalculus sections. While there are still lecture and lab sessions, the lecture portion is now an asynchronous online component where the students work through the course material at a self-pace. The lab meetings now consist of targeted small group mini-lectures, independent work time, and assessment. By using ALEKS, students' learning paths and instruction have become more personalized while assessment and learning work together in a constant cycle. In the years since implementing this model, an increase in both pass rate (~70%) and retention in the calculus sequence has been seen. This presentation will focus on the implementation of ALEKS in precalculus and the resulting success that has been seen during and after the course. (Received September 20, 2016)

1125-O5-2927     **Rachel Maitra\*** ([maitrar@wit.edu](mailto:maitrar@wit.edu)), 550 Huntington Ave, Boston, MA 02115. *Projects in geometry for design students.*

This presentation describes the process and outcome of revamping a traditional geometry course for industrial and interior design students into a project-based exploration highlighting the relevance of geometry to design. Students completed projects and labs on topics including plane tilings, Voronoi cells, and involute/evolute curves. As a class, students collaboratively designed and built large tetrahedral kites and a polyhedral lampshades. The semester culminated in a final exhibit, for which each student designed an advertising poster and portfolio website. (Received September 20, 2016)

## Teaching Abstract Algebra: Topics and Techniques

1125-P1-71     **Paul E. Becker\*** ([peb8@psu.edu](mailto:peb8@psu.edu)), School of Science, Penn State Behrend, Erie, PA 16509, and **Mark Medwid**. *Matrix representations as a first topic in abstract algebra?*

Computer algebra systems are now widely available in college classrooms. In this environment, matrix representations of groups provide a unifying concept across abstract algebra, linear algebra, and geometry. Almost all finite groups encountered by undergraduates can be represented as multiplicative groups of concise, block-diagonal, binary matrices. Such representations provide simple examples for beginning a group theory course. More importantly, these representations provide concrete models for "abstract" concepts. We describe computer lab assignments which explore group actions, subgroups, normality, cosets, quotient groups, homomorphisms, isomorphisms, and kernels. (Received July 08, 2016)

1125-P1-177     **Mike Janssen\*** ([mike.janssen@dordt.edu](mailto:mike.janssen@dordt.edu)), 498 4th Ave NE, Sioux Center, IA 51250. *Specifications Grading in a First Course in Abstract Algebra.*

Specifications grading offers an alternative to more traditional, points-based grading and assessment structures. In place of partial credit, students are assessed pass/fail on whether or not they have achieved the learning outcomes being assessed on a given piece of work according to certain specifications, with limited opportunities for revision of non-passing work. This talk will describe the learning outcomes and specifications grading system I used in my Fall 2016 abstract algebra course, as well as student responses. (Received September 20, 2016)

1125-P1-322 **Jacqueline A Jensen-Vallin\*** ([jacqueline.jensen@lamar.edu](mailto:jacqueline.jensen@lamar.edu)). *Examples and Counterexamples in Abstract Algebra*. Preliminary report.

Inspired by the book *Counterexamples in Topology* and conversations with Robert Vallin, my abstract algebra course is designed around examples and counterexamples. In particular, my students are expected to memorize definitions, and are quizzed on these daily. During class time and as part of many homework assignments and all tests, students are expected to develop and explain examples to satisfy each new definition and counterexamples which fail to satisfy those definitions. For instance, students should be able to recall or construct a group which is abelian but not cyclic (or explain why no such group exists). This encourages students to understand definitions deeply in a way that helps students write stronger proofs. We will discuss these techniques, and examine some data and examples of student success during the Fall 2015, Spring 2016, and Fall 2016 semesters. (Received August 26, 2016)

1125-P1-418 **Stephen Lovett\*** ([stephen.lovett@wheaton.edu](mailto:stephen.lovett@wheaton.edu)), 501 College Avenue, Wheaton, IL 60187. *The Four Cs of Investigative Projects in Abstract Algebra*.

Since it often stands as a gateway between entry-level and upper-level, mathematics curricula often expect abstract algebra to play many roles: to deepen students' proof skills; to introduce structures in modern mathematics; to begin to develop a research mindset; and to teach specifically groups, rings and fields. The usual menu of weekly homework sets, quizzes, or in-class tests do not fully address all of these objectives. By emphasizing clarity, correctness, completeness, and creativity, investigative projects help students address many of the desired objectives of an abstract algebra course, including writing and research skills. This talk offers a report on eight years of fine-tuning the effectiveness of team-oriented investigative projects. By illustrating with actual student work, we discuss the evaluation categories, feedback and revision mechanisms, sample questions, a role in the assessment of the major, and usefulness for letters of recommendation. (Received September 01, 2016)

1125-P1-1433 **Michael D Smith\*** ([smithm@lycoming.edu](mailto:smithm@lycoming.edu)), Michael D Smith, Lycoming College, Campus Box 3, 700 College Place, Williamsport, PA 17701. *A Commutative but non-Associative Operation in the Game of SET*. Preliminary report.

SET is a card game played using a deck of 81 cards. Each card has a shape, color, filling, and number. Three cards make up a "set" if in all of these four attributes, the cards are either all the same or all different. The operation on the set of SET cards defined by  $A*B=C$  if and only if A, B, and C make up a "set" is an example of a binary operation that is commutative but not associative. This talk presents two interactive classroom activities that introduce this binary operation to students. (Received September 16, 2016)

1125-P1-1554 **Ben Blum-Smith\*** ([ben@cims.nyu.edu](mailto:ben@cims.nyu.edu)), Courant Institute of Mathematical Sciences, 251 Mercer St., Room 607, New York, NY 10012. *Group theory for middle schoolers and inservice teachers: close encounters with the abstract*.

We report on a pedagogical design paradigm, and several tasks based on it, used successfully to introduce group theory to middle school students and separately to inservice teachers. The paradigm consists of posing questions that can be engaged without much prior knowledge, but that force learners into a concrete encounter with the abstract. The tasks develop in an inquiry-based way the definitions of group and group isomorphism, and motivate cycle notation for computation in the symmetric group. Some of the tasks tap students' kinesthetic learning modality. (Received September 17, 2016)

1125-P1-1617 **Janet Heine Barnett\*** ([janet.barnett@csupueblo.edu](mailto:janet.barnett@csupueblo.edu)). *Read the masters! Learning abstract algebra via Primary Source Projects*.

The ring concept has deep historical roots that include the work of Richard Dedekind on algebraic number theory in which the concept of an ideal first appeared. Between 1871 and 1894, Dedekind published four versions of his theory of ideals, none of which simply revised an earlier paper. The mathematical insights resulting from these years of re-working and the clarity with which he expressed those insights make Dedekind a master well worth reading even today. This talk describes a 'Primary Source Project' (PSP), based on Dedekind's original writing, that can be used to learn elementary ring and ideal theory in today's abstract algebra classroom. Through guided reading of excerpts from Dedekind, students encounter his original motivations and develop their own understanding of these ideas by completing a set of tasks interspersed between those excerpts. Overviews of a companion PSP for learning group theory and of the pedagogical principles guiding the NSF-funded project *Transforming Undergraduate Mathematics via Primary Historical Sources* that is supporting development of a collection of PSPs for topics throughout undergraduate mathematics curriculum will also be provided. (Received September 18, 2016)

1125-P1-1638 **Tanya Cofer\*** ([tcofer@ccga.edu](mailto:tcofer@ccga.edu)). *A Visual and Intuitive Approach to the Teaching of the Always Even or Always Odd Theorem for Permutations.*

In the epic story of the insolubility of the general quintic polynomial and the birth of modern algebra, the study of the alternating group plays a key role. So why is it so hard to motivate the proof that all permutations in the symmetric group can be classified as even or odd? In this talk, I will outline a visual approach to this quintessentially algebraic proof that allows students in an abstract algebra course to use their intuition as a guide for understanding the theorem. By interpreting permutations as certain braid projections, I can provide both motivation for and proof of the “Always even or always odd” theorem from a topological viewpoint. (Received September 18, 2016)

1125-P1-1791 **Erica L Johnson\*** ([ejohnson@sjfc.edu](mailto:ejohnson@sjfc.edu)). *Confessions of an Abstract Algebra Noob.* Preliminary report.

This talk will focus on the speaker’s adventures in teaching Abstract Algebra for the very first time. The use of TIMES Inquiry-Oriented Abstract Algebra materials will be highlighted. Pitfalls, pratfalls, and prizes will be included. (Received September 19, 2016)

1125-P1-1964 **Carolyn Yackel\*** ([yackel\\_ca@mercero.edu](mailto:yackel_ca@mercero.edu)) and **Julie Beier** ([beierju@earlham.edu](mailto:beierju@earlham.edu)). *True/Sometimes True/False.* Preliminary report.

Abstract algebra is a terrific course overflowing with intriguing ideas. Teachers have the opportunity to help students learn to write mathematics carefully, which means lots of grading. This talk suggests an idea for a type of exam question that cleverly tests concepts while being very easy to grade! The title gives the idea. The examples in the talk will illustrate how to use the idea effectively and non-trivially. (Received September 19, 2016)

1125-P1-2331 **Matt Koetz\*** ([mkoetz1@naz.edu](mailto:mkoetz1@naz.edu)), 4245 East Avenue, Rochester, NY 14618. *Symmetry and IBL in Abstract Algebra.*

As a recent adopter of IBL in most of my classes, I was eager to try it out for the first time in Abstract Algebra. This talk will cover the benefits and challenges of generating my own notes, as well as the surprising success of having students build their own models to study symmetry groups. (Received September 20, 2016)

1125-P1-2450 **Katie Anders\*** ([kanders@uttyler.edu](mailto:kanders@uttyler.edu)). *Re-write and Re-submit: Multiple Attempts at Homework Problems in Abstract Algebra.* Preliminary report.

We will review a teaching technique from a two-semester undergraduate abstract algebra sequence in which students were allowed multiple submissions on a subset of homework problems. This practice provided constructive feedback both early and often, giving students the opportunity to improve and implement needed changes on a weekly basis. Our discussion will include student feedback and reviews of the method. (Received September 20, 2016)

1125-P1-2578 **Scott Zinzer\*** ([zinzer\\_s@wvwc.edu](mailto:zinzer_s@wvwc.edu)), 59 College Avenue, Buckhannon, WV 26201. *Teaching introductory group theory with the Rubik’s cube.* Preliminary report.

We describe techniques for incorporating the Rubik’s cube into a typical first-semester course in group theory. In this talk, we examine Rubik’s cube explorations motivating concepts in group theory, Rubik’s cube activities and applications illustrating results in group theory, and reflective Rubik’s cube questions and activities previewing more advanced topics in group theory. (Received September 20, 2016)

1125-P1-2612 **Kristi Meyer\*** ([kristi.meyer@wlc.edu](mailto:kristi.meyer@wlc.edu)), 8800 W. Bluemound Rd, Milwaukee, WI 53226. *Reading, Writing, and Rithmetic in the Abstract Algebra Classroom.*

”I have to write a paper?” ”I’m supposed to read a mathematics textbook?” ”You want me to talk about math?” Many of my students are comfortable with the idea of ”doing math.” But when they are asked to broaden their horizons and tackle other aspects of a liberal arts education - such as reading, writing, and speaking - they become much less comfortable. In this talk, I will detail several activities I use in Abstract Algebra to expose my students to these important liberal arts concepts. I will discuss grading of these activities and talk about student reaction to them. Student comments will be included, as well as an idea of how these activities fit into the course as a whole. (Received September 20, 2016)

1125-P1-2711 **Melanie Pivarski\*** ([mpivarski@roosevelt.edu](mailto:mpivarski@roosevelt.edu)) and **Steve Cohen** ([scohen@roosevelt.edu](mailto:scohen@roosevelt.edu)). *Concrete Algebra: Applying Knowledge From Abstract Algebra.*

Students are often curious about how the mathematics that they learn in class is applied in real life. To help them learn about this, our students complete a project where they describe where abstract structures occur in

the real world. Many of the students are planning to become high school teachers, so they need the ability to give mathematical examples and communicate them effectively. We describe this project and its implementation in a junior/senior-level abstract algebra course. (Received September 20, 2016)

1125-P1-2750 **Tom Langley\*** ([langley@rose-hulman.edu](mailto:langley@rose-hulman.edu)). *The probability that  $ab = ba$  and other adventures in commutativity in finite groups*. Preliminary report.

How close to Abelian can a non-Abelian group be? This question is the jumping-off point for an investigation into the structure of finite groups. We'll discuss a hands-on, discovery-based approach that lets students gather computer-generated data using GAP, conjecture wildly, and thusly armed embark on a journey through the lands of subgroups, centers, centralizers, and conjugates. We'll also discuss several generalizations and open questions. (Received September 20, 2016)

1125-P1-2771 **Bradford J Schleben\*** ([brad.schleben@belmont.edu](mailto:brad.schleben@belmont.edu)). *Strengthening the Narrative of an Abstract Algebra Course via Tutored Oral Exams and Other Techniques*. Preliminary report.

We discuss how tutored oral exams and problem-based learning help develop and reinforce conceptual understanding in students. In addition, we will examine techniques that provide support for improving students' ability to communicate mathematical concepts and proofs while reinforcing the value of these skills in assessments. Various methods, examples, and anecdotal evidence of effectiveness will be covered. (Received September 20, 2016)

1125-P1-2853 **Melissa Lindsey\*** ([melissa.lindsey@indwes.edu](mailto:melissa.lindsey@indwes.edu)), Department of Mathematics, Indiana Wesleyan University, 4201 S. Washington St, Marion, IN 46953. *Specifications Grading in Abstract Algebra*.

Write correct and intelligible proofs. Do routine problems involving groups and rings. Define basic terminology and paraphrase major theorems. In a nutshell, these are the three learning outcomes for the Modern Abstract Algebra course at Indiana Wesleyan University. In a traditional grading scheme involving points where students can earn partial credit, it is entirely possible for a student to earn a passing grade in this course without mastering any of the three learning outcomes. In her book, *Specifications Grading*, Linda Nilson outlines an argument for restoring rigor, motivating students, and saving faculty time by doing away with partial credit. In lieu of points and partial credit, she encourages faculty to clearly identify for students what it means to succeed on an assignment and grading each assignment as either meeting the criteria (pass) or not meeting the criteria (fail). In this talk, we will discuss the details of implementing specifications grading in the abstract algebra course described above. (Received September 20, 2016)

1125-P1-3026 **Jolie Roat\*** ([jolie.roat@cortland.edu](mailto:jolie.roat@cortland.edu)). *Using Semester Projects in Abstract Algebra*. Preliminary report.

In this talk, we will discuss the design and implementation of a semester long writing project in a first semester abstract algebra course. This project arose in an effort to provide students with a cumulative assignment that combines computation and theory as well as an opportunity to receive feedback focused on their writing and satisfy college writing requirements. (Received September 20, 2016)

1125-P1-3127 **A E Francis\*** ([afrancis@carroll.edu](mailto:afrancis@carroll.edu)), Math, Engineering and Computer Science, Carroll College, 1601 N Benton Ave, Helena, MT 59625. *Adopt your own group*. Preliminary report.

In this talk I will discuss a project-based strategy for teaching Abstract Algebra, where we ask students to adopt a group (and later a ring) near the beginning of the semester. As the semester progresses they learn more about the characteristics of their group, and hear from their peers about the characteristics of other groups. (Received September 21, 2016)

## *The Teaching and Learning of Undergraduate Ordinary Differential Equations*

1125-P5-1094 **Timothy A Lucas\*** ([timothy.lucas@pepperdine.edu](mailto:timothy.lucas@pepperdine.edu)). *Slopes: A Differential Equations Graphing Environment*.

A proper study of differential equations requires that students visualize solutions and analyze slope fields and phase planes. Although there is software that performs these functions (Maple, MATLAB or Mathematica),

these programs are expensive and students must invest a significant amount of time to learn the functions and proper syntax. *Slopes* is an interactive app that I have developed with faculty and students at Pepperdine University that allows students to explore numerical methods and graphical solutions to differential equations. The name of the app originates from interpreting the derivative as slope and most activities revolve around plotting slopes. One advantage of using the app is that iPads are highly portable and feature large touch screens that allow students to view and manipulate content easily. Research based on observations of mathematics courses at Pepperdine University has shown that students are more willing to collaborate and share their results when using tablets such as the iPad (Fisher, Lucas et al. 2013). The intuitive interface of *Slopes* invites students to fully immerse themselves in the world of differential equations so that they can understand the concepts from not only algebraic, but also graphical and numerical perspectives. (Received September 14, 2016)

1125-P5-1699 **Itai Seggev\*** (is+research@cs.hmc.edu), 2000 Trade Center, Champaign, IL 61801. *The Tautochrone: Times are the Same, Times are Different.*

The tautochrone problem is one of the classical problems of elementary physics: determine the shape for which the time of descent of a frictionless bead sliding from rest is independent of initial position. This problem found various geometric and analytic solutions from the early days of calculus until it led to the development of integral equations by Abel. But taking the next step of actually determining the motion of the beads leads to a treacherous path, where the pitfalls of energy and numerical methods rear their ugly heads. In this talk I describe my journey to create an animation of the tautochrone. This will serve as launching point to discuss traps that we often mention in passing to our students but then promptly forget ourselves. (Received September 18, 2016)

1125-P5-1824 **Audrey Malagon** (amalagon@vwc.edu), **Lydia Kennedy\*** and **Kristin Burney**. *Real time modeling illuminates mixing problems.*

Using powdered drink mix and stacked beverage dispensers, our students actually experience the mixing problems that are such standard fare in ODE courses. This seems to help them internalize the concepts to the extent that they perform better than previously on subsequent conceptual problems like multiple tanks and lake pollution. (Received September 19, 2016)

1125-P5-2073 **Thomas W. Judson\*** (judson@sfasu.edu). *ODE Reviews: A Repository of Reviews of Articles Related to the Teaching and Learning of ODEs.*

odereviews.org is a repository of contributed reviews of journal articles and software related to the teaching and learning of ordinary differential equations. Many articles describe interesting applications of ODEs that could be useful in a course on ODEs. We will give a short history of ODE Reviews, provide sample reviews, and describe how teachers and students can contribute to the repository. (Received September 19, 2016)

1125-P5-2101 **Baoling Ma\*** (baoling.ma@millersville.edu). *Student-Centered Teaching Strategies in Ordinary Differential Equations.*

A number of student-centered teaching strategies were adopted during a one-semester course in ordinary differential equations which I taught in the fall of 2015 and spring of 2016. These strategies were welcomed and well-received by the students according to their feedback and performance. In this talk, I will share various approaches I have used in teaching the differential equations course, describe my experience of implementing them in and out of the classroom, introduce the foundations that inspired the activities, and analyze the correlation between students' performance and these strategies. I would be interested in hearing what student-centered strategies other colleagues have been using in teaching similar math courses. (Received September 19, 2016)

1125-P5-2156 **P. Gavin LaRose\*** (glarose@umich.edu), Dept. of Mathematics, University of Michigan, 530 Church St., Ann Arbor, MI 48109-1043. *Engaged Learning in Large-enrollment Differential Equations through Computer Laboratory Materials.*

There is now substantial research evidence of the positive impact of engaged student learning in mathematics courses, to the point that many voices, including the CBMS, reasonably claim that it is a professional responsibility to promote engaged ("active") learning wherever possible. Left is the question of how to do this, especially in the context of large-enrollment courses taught in a traditional lecture format. In this talk we discuss the goals for and assessment of a course revision in our large-enrollment differential equations course, with specific emphasis on a set of computer laboratory assignments being used in the lab session associated with the course. We consider the goals and evaluation of these materials and their impact, and discuss the interaction between student engagement, computer use, and cooperative learning in their construction and effectiveness. (Received September 19, 2016)

1125-P5-2196 **Christopher S. Goodrich\*** (cgood@prep.creighton.edu), 7400 Western Ave., Omaha, NE 68114. *Reflections on Teaching a Combined Differential Equations/Linear Algebra Class.*

I will discuss my experiences teaching a one-semester, combined Differential Equations/Linear Algebra class. Particular attention will be focused on the inherent challenges in balancing the two curricula and, at the same time, the exciting opportunities that this offers. In addition, I will discuss some of the ways in which teaching both courses simultaneously can substantively enrich each individual course and, in particular, the students' understanding of ordinary differential equations. (Received September 19, 2016)

1125-P5-2297 **Brian Winkel\*** (brianwinkel@simiode.org). *Stay Tuned – Modeling in Differential Equations Courses.*

We present two rich illustrations of modeling to motivate learning in a differential equations course: (1) Tuned Mass Dampers to keep structures from destructive displacements in the presence of earthquakes, tremors, and wind forces and (2) Radio Tuning of an electrical circuit. We have found that application first is a great approach to motivate learning differential equations and we will illustrate this in these examples as well as point to other resources for doing so in SIMIODE - Systemic Initiative for Modeling Investigations and Opportunities with Differential (www.simiode.org) where there are resources for teaching and a community in which colleagues can communicate, collaborate, publish, teach, explore, contribute, etc. (Received September 20, 2016)

1125-P5-2299 **Patrice G Tiffany\*** (patrice.tiffany@manhattan.edu), Manhattan College, Mathematics Department, Manhattan College Parkway, Bronx, NY 10471, and **Rosemary C Farley** (rosemary.farley@manhattan.edu), Manhattan College, Mathematics Department, Manhattan College Parkway, Bronx, NY 10471. *A Modeling First Approach to Differential Equations Using SIMIODE.*

This presentation will introduce a modeling first approach to the differential equations course. Participants will be introduced to SIMIODE—Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations. SIMIODE is an online community of teachers and learners of differential equations who use modeling and technology throughout the learning process. This is a free community of those interested in the modeling first approach in differential equations and it provides a comprehensive and cohesive approach to studying differential equations.

The presenters will explain how they used their involvement in the SIMIODE community to introduce change in the differential equations classroom. We will explain how we used a computer algebra system together with modeling scenarios to transform our sections of differential equations. (Received September 20, 2016)

1125-P5-2555 **Heather A Moon\*** (hamoon@lsc.edu), 500 8th Ave, Lewiston, ID 83501, **Thomas J Asaki** (tasaki@wsu.edu), PO BOX 641067, Pullman, WA 99164-1067, **Marie Snipes** (msnipes@kenyon.edu), 103 College Rd, Gambier, OH 43022, and **Chris Camfield** (camfield@hendrix.edu), 1600 Washington Ave, Conway, AR 72032. *Wave Propagation Inspiring Techniques in Differential Equations.*

In this talk, we will discuss a classroom module that we have created to inspire techniques in ODEs. Our module uses a data application to reinforce the general idea of iteratively constructing solutions to differential equations and to motivate the level set method for solving partial differential equations. The data application that we use in our module is Wave propagation: We wish to understand and predict wavefront propagation in a physical medium. We consider the common case where wavefront speed is a function of position in the medium only (not a function of the wave size, shape of the wavefront, nor any wavefront history). We consider a tsunami traveling across an ocean, the spread of a wildfire across a landscape and light traveling through a refractive optical system. In this talk, we will share an outline of our module and how it can be implemented in an ODE course. (Received September 20, 2016)

1125-P5-2598 **Karen Allen Keene\*** (kakeene@ncsu.edu), PO Box 7801, North Carolina State University, Raleigh, NC 27695, and **Nicholas Fortune**. *What does it mean to find a solution to a system of differential equations? Hands-on and technology helps with the conceptualization.* Preliminary report.

In this presentation, we will present tasks used in an Inquiry-Oriented Differential Equations class that will help with conceptualizing the idea of a solution to a system of differential equations. Participants will work through a task that is hands-on that students have done in approximately 50 classrooms, and then see technology that supports student understanding of the notion and how it can be developed from the notion of solution to a single ODE. Work and video from students participating in this activity will be presented. Finally, the presenters

will discuss this tasks in the context of the bigger notion of inquiry-oriented teaching of differential equations. (Received September 20, 2016)

1125-P5-2668 **Ron Buckmire\*** (ron@oxy.edu), 1600 Campus Road, Department of Mathematics, Los Angeles, CA 90041, and **Treana Basu** (basu@oxy.edu), 1600 Campus Road, Department of Mathematics, Los Angeles, CA 90041. *An Analysis Of Various Effects Disaggregated By Gender Of Different Pedagogical Practices In An Introductory Differential Equations Course*. Preliminary report.

We shall present preliminary findings from our investigation into various effects of the implementation of different pedagogical practices in an introductory differential equations course. In particular, we analyze gender differences and perceived student success versus measurable student success between two roughly equal sections of the same course taught in the same semester with nearly identical curricular material (textbook, homework, midterms and final exam) at a liberal arts college. The two sections were subjected to two different treatments (pedagogical styles) while student learning was assessed using identical instruments. Results are disaggregated by gender (of both the students and the instructors). (Received September 20, 2016)

1125-P5-2742 **Charles Bergeron\*** (chbergeron@gmail.com), Albany College of Pharmacy and Health Sc., 106 New Scotland Ave, Albany, NY 12208. *Standards-based grading: An evaluation system that fosters meaningful knowledge acquisition and skills development*. Preliminary report.

Students are habituated to a grading system where each score is associated with a specific summative assessment, and a weighted average determines the final grade. In this talk, I present a very different scheme called Standards-Based Grading (SBG) that I have used for 3 years in my Differential Equations course. In my gradebook, each column is a standard. For example, *solve a first-order separable or linear DE* is a standard. Throughout the semester, frequent quizzes assess student abilities. The most recent assessment of a standard determines the student's score for that standard. This means that scores go up and down, and most quiz questions end up being formative. Additionally, there is a clearly-defined mechanism by which students may proactively remediate a standard. Under SBG, there is no value in cramming a standard the night before a quiz, because that standard could be quizzed again the following week. This system encourages meaningful learning and long-term retention of knowledge and skills, and rewards the grit that's required to succeed in a rigorous mathematics course. By the end of the semester, students are confident in their abilities, and walk into the final exam saying *Bring it on, Dr. Bergeron*. (Received September 20, 2016)

1125-P5-2752 **Paul D. Olson\*** (pdo2@psu.edu). *"Laplace Transforms or the Method of Undetermined Coefficients : which should be introduced first ? "*. Preliminary report.

When we teach how to solve linear , second order ODE's with constant coefficients , we have several choices . The Method of Undetermined Coefficients and the Method of Variation of Parameters are possible . We could also employ the use of Laplace Transforms . When working on an Initial Value problem , the Method of Undetermined Coefficients can be labor intensive in determining a particular solution . Laplace transforms use the initial conditions right away in solving the equation . But the building of the machinery of Laplace Transforms takes more time to develop . We shall consider the benefits and drawbacks of each approach . (Received September 20, 2016)

1125-P5-2756 **Christopher Towse\*** (ctowse@scrippscollege.edu) and **Eric Campbell**. *Construction and (some) classification of integer matrices with integer eigenvalues*. Preliminary report.

In spite of the provable rarity of integer matrices with integer eigenvalues, they are commonly used as examples for linear systems in introductory differential equations courses. We present a quick method for constructing such matrices starting with a given set of eigenvectors. In the case of repeated eigenvalues, we give some classification results along with additional fast constructions. (Received September 20, 2016)

1125-P5-2817 **Courtney L Davis\*** (courtney.davis2@pepperdine.edu) and **Timothy A Lucas** (timothy.lucas@pepperdine.edu). *Teaching Modeling Through Poster Projects in Differential Equations*.

In order to emphasize mathematical modeling within our curriculum, we have incorporated semester-long modeling projects into our Ordinary Differential Equations class. In these projects, the students develop and analyze differential equation models to investigate applications beyond mathematics. They choose a quantitative question to explore, make simplifying assumptions, develop model equations, conduct appropriate mathematical analysis, and interpret results in context. These projects are often the students' first encounter with creating their own mathematical models. Each group must fully analyze their models while demonstrating sufficient mastery of

analytical, graphical, and numerical techniques from the course. The semester-long projects culminate in the students creating conference-quality posters and presenting their work formally to each other and to faculty judges during a final poster session. Thus, they gain experience communicating their findings to both a lay and expert audience. We will share how judging and grading rubrics have evolved over six years of projects. We will also discuss how we use project assessment data to more broadly assess learning outcomes for our mathematics major. (Received September 20, 2016)

1125-P5-2836 **Chris Oehrlein\*** (cdoehrlein@gmail.com). *Exposure to Laplace Transforms Early in the Intro to ODE Course.*

The typical Introduction to Ordinary Differential Equations course taught to students after they have completed single-variable integral calculus (and maybe have seen some series and partial derivatives) presents standard first and second-order equations in the bulk of the term and gets to Laplace transforms and possibly series solutions or numerical methods later and sometimes in rushed fashion. Students never see discontinuous forcing functions until the end of the semester, and rarely do they make connections among different solution techniques. The presenter will show how introducing the unit step function and the Laplace transforms of polynomial and exponential functions immediately after first-order, linear differential equations provides opportunities to connect course objectives to each other and to topics the students see and use in physics and engineering courses. Early introduction to Laplace transforms also allows a chance for students to become more comfortable with the concept when it is revisited later in the course to explore second-order differential equations and applications of convolution. (Received September 20, 2016)

1125-P5-2879 **Michelle L Ghrist\***, HQ USAFA/DFMS, 2354 Fairchild Hall, Suite 6D2, USAF Academy, CO 80840. *Exploring the Solar System through Differential Equations and Vector Calculus.* Preliminary report.

I discuss a recently developed project that uses mathematics to compare Earth, Saturn, and Titan. One part of this project involves modeling projectile motion and air resistance; students are asked to develop and discuss several models of air resistance (to include researching various factors that affect drag) and then solve one particular system of first-order equations both analytically and numerically. The other part of the project requires students to model planetary surfaces and orbital trajectories via parameterizations and then use those parameterizations to estimate various physical quantities such as surface area, heat flux, and work. This project was implemented in an Engineering Mathematics course this semester; various parts could easily be incorporated into a Differential Equations and/or Multivariable Calculus course. (Received September 20, 2016)

1125-P5-3033 **Michael C. Barg\***, Dunleavy Hall, Rm. 339, Niagara University, NY 14109. *Find, Process, and Share: How an ODE Project led to Student Engagement in the Vidale-Wolfe Marketing Model.*

In a first undergraduate ODE class, one often learns about traditional problems in physics and biology. What does one do when the interests of their students lean more toward finance, economics, and marketing than the sciences? A project can help! To have students acquire the ability to interpret and communicate results, I introduced a reading/writing project into my course. The students are asked to Find, Process, and Share an article where relevant mathematics is discussed. One such project led to the Vidale-Wolfe marketing model, a model that appealed greatly to some of my students. While accessible mathematically at an elementary level (the ODE is first-order, linear and non-homogeneous), the context is non-standard, but quite alluring to “non-science” oriented students. Pushing slightly deeper into the model, one quickly encounters an interesting application of Green’s theorem! In this talk, I will describe the project and its implementation. I will discuss the Vidale-Wolfe model and suggest how it might enable students to see the value of some of their Calculus III knowledge. To demonstrate the efficacy of the project, data from my last three sections of the course will be presented. Additional anecdotal evidence and commentary from students will also be provided. (Received September 20, 2016)

## *Trends in Undergraduate Mathematical Biology Education*

1125-Q1-403 **Carrie Diaz Eaton\*** (ceaton@unity.edu) and **Emma Perry**. *CORaL: Diving into Calculus.* Preliminary report.

Marine Biology is increasingly relying on mathematical and computational skills for research and ecosystem management. Advances in technology such as remote sensing, unmanned submersibles, electronic tagging, and real-time monitoring have created a data rich environment. At Unity College we have recently updated the

Marine Biology curriculum to focus on the pressing environmental impacts on coral reefs and their ecology. Understanding the effect of climate change on coral reef ecosystems will require a systems thinking understanding and perspective to find solutions. While mathematics has been working towards incorporating more biological examples, these examples tend to serve a majority pre-med population, leaving work to be done to service the environmental professions. We explore potential intersections between our marine biology and calculus curriculum that may inform future curricular development for similar programs. (Received August 31, 2016)

1125-Q1-415 **Timothy D. Comar\*** (tcomar@ben.edu). *Getting Biocalculus Students to Apply Mathematics to Biology Through Active Learning.*

The two semester course sequence in biocalculus (calculus for the life sciences) is an great starting place for students in the life sciences to seriously begin applying mathematical and computational techniques to investigating biological problems and phenomena. This talk discusses a variety of active learning activities in the course sequence that lead students to develop the skills and understanding needed to be able to use mathematics to address biological problems. With a broad, underlying theme of population dynamics, the activities take the students from investigating the relationships of simple dynamical systems to data sets and exploring simple models to reading journal articles, working through computer laboratory projects, and eventually leading to the detailed study of a sophisticated model in a published journal article. Students with this preparation subsequently have completed successful research projects in mathematical biology as well as majors and research projects in related scientific disciplines. (Received September 01, 2016)

1125-Q1-644 **Milos Savic, OK** , and **Paxton Martin\***, 601 Elm St., Norman, OK 73019. *The Perceived vs. Actual Use of Mathematics in Medicine According to Pre-Medicine students and Practicing Physicians.*

This research was conducted to discover the perceived use of mathematics in the medical field according to undergraduate Pre-Medicine students versus the actual use of mathematics in the medical field according to practicing physicians. In particular, the researchers intended to discover the discrepancies between using mathematics in medicine and Pre-Medicine students' readiness to utilize mathematics in medicine. Therefore, the research question considered was: What is the actual use of mathematics in the medical field according to physicians and how does this compare with current pre-medical students' beliefs about the use of mathematics? Six current Pre-Medicine students and three practicing physicians were interviewed. The overall results were that physicians and pre-med students believe that mathematics is important in the field, but the physicians stated that higher-level mathematics such as calculus will be helpful for pre-med students, contrary to some pre-med students' answers. The results from this research may inform how mathematics education, particularly the requirements of mathematics courses, for future physicians can be improved. (Received September 20, 2016)

1125-Q1-1373 **James P Peirce\*** (jpeirce@uwlax.edu). *Turning an REU Investigation into Calculus II Projects.*

In our recent REU in Mathematical Ecology, a group of students investigated the prevalence of swimmer's itch in regional lakes. Swimmer's itch is an emerging disease caused by flatworm parasites that typically use waterfowls as definitive hosts. When parasite larvae mistakenly penetrate human skin they initiate localized inflammation that leads to intense itching and discomfort. Concerns about this issue have been growing recently due to an apparent increase in the occurrence of swimmer's itch and its subsequent impacts on recreational activities and revenues. Numerous media reports of swimmer's itch outbreak during the summer fostered our research student's enthusiasm for the project. Their peaked interest motivated a substantial modification of two existing writing projects for a second semester Calculus course. The projects used common Calculus II topics including approximate integration, first-order differential equations, and geometric series. In this talk, I will discuss the process of leveraging recent undergraduate research experiences to add life to old course projects. In particular, how a contemporary story captured the imaginations of classes of Calculus II students. (Received September 16, 2016)

1125-Q1-2129 **Dan Hrozencik\*** (dhro@att.net), 9501 S. King Dr., HWH 332, Chicago, IL 60628. *Senior Biomathematics Projects at Chicago State University.*

For several years the one-semester Math 4940 Mathematics Seminar has become the course where Chicago State mathematics majors begin and complete their senior thesis. In this course, students must select a topic, conduct research, write their results and present those results in a poster session open to the campus community. This talk will focus on several of the biomathematics projects conducted during this seminar during the last several years. Project topics discussed will include epidemiology and plant ecology. (Received September 19, 2016)

1125-Q1-2311 **Michael Kerckhove\*** ([mkerckho@richmond.edu](mailto:mkerckho@richmond.edu)), University of Richmond. *Picking and Choosing: Ten Lectures in Support of Planarian Tissue Regeneration.*

The setting is the opening module in an integrated science course for first-year undergraduates. Student background includes a high-school calculus course. The lab experiments involve amputations, chemical treatments affecting bio-electric signaling during regeneration of body parts, and measurement of behavioral recovery through the assessment of chemotactic ability in the freshwater planarian *S. mediterranea*. The challenges: to provide understanding of mathematical models for these biological processes, to reinforce Calculus I material, and to prepare students for module two physics. A meta-goal for the students and the structuring of assignments in order to achieve that goal will be discussed. (Received September 20, 2016)

1125-Q1-2379 **Sarah Hews\*** ([shews@hampshire.edu](mailto:shews@hampshire.edu)) and **Christina Cianfrani** ([ccns@hampshire.edu](mailto:ccns@hampshire.edu)). *An Integrated Sciences First Year Program at Hampshire College.*

The Integrated Sciences First Year Program (ISFP) is a new initiative at Hampshire College that presents a novel model for bringing science and mathematics students together to study relevant scientific research questions. It aims to challenge students to learn about complex systems and systems thinking, to improve quantitative skills, to make connections among fields of science, to design innovative collaborative projects, and to create a vibrant science community. ISFP, currently using a new Living Building on campus as a model system, consists of fall collaborative courses (ISFP I), a spring design projects course (ISFP II), and a summer research experience (ISFP III). ISFP I in particular presents an opportunity for science students to learn the benefits of modeling and for the mathematics students to learn applications of their skills. Students meet with their respective classes twice a week learning field specific skills and meet all together once a week to complete interdisciplinary labs where students collect and use data. This talk will focus on the first iteration of ISFP during the 2015-2016 academic year including design, implementation, and assessment of all parts of the program. (Received September 20, 2016)

1125-Q1-2991 **Rebecca E Gasper\*** ([rebeccagasper@creighton.edu](mailto:rebeccagasper@creighton.edu)). *Changing tracks: More Applied Courses Make a Med-Ready Major.*

Want to build a math bio course into your curriculum, or look at pre-med tracks available to your students? Our medium-sized university, with attached medical school, regularly sees 45-50% of freshmen have a pre-medical or pre-health career interest. We rebuilt our mathematics curriculum using the 2015 CUPM Curriculum Guide to Majors in the Mathematical Science and institutional goals including medical school readiness. Examples will be given of new interdisciplinary courses (and textbooks), active learning techniques, applied projects for calculus and above, and ways students can reflect on intersectional knowledge. Where available, we will share any results or data emerging from these new courses. A pre-med student and a mathematical biologist will be available for candid questions. (Received September 20, 2016)

1125-Q1-3008 **Raina S Robeva\*** ([robeva@sbc.edu](mailto:robeva@sbc.edu)), Department of Mathematical Sciences, Sweet Briar College, 134 Chapel Drive, Sweet Briar, VA 24595. *An Introduction to Mathematical Biology through Discrete Mathematics and Abstract Algebra.*

The entryway to mathematical biology through difference and differential equations has one significant disadvantage – many biology students are afraid of Calculus. Graphs and knots, on the other hand, feel natural to them – they remind them of the abundant cartoons populating their biology textbooks. The talk will highlight some relevant methods from discrete mathematics and abstract algebra appropriate for use in introductory courses in mathematical modeling and mathematical biology. As discrete and algebraic methods are being used successfully nowadays to address a wide range of research questions in biology including signaling networks and gene regulation, genome assembly, DNA, RNA and protein folding, drug resistance and control, ecological networks and food webs, this approach can also easily deliver topics and ideas for student research. We argue that using discrete and algebraic methods presents a viable and even preferable alternative to the Calculus-based approach. (Received September 20, 2016)

1125-Q1-3011 **Douglas Norton\*** ([douglas.norton@villanova.edu](mailto:douglas.norton@villanova.edu)), Department of Mathematics and Statistics, Villanova University, 800 Lancaster Avenue, Villanova, PA 19085. *A Modeling Course for Majors in the Life Sciences.*

The second iteration of a new course on mathematical modeling for majors in Biology and in Cognitive and Behavioral Neurosciences at Villanova University will have just been completed as the Joint Math Meetings begin. Students with Calculus experience, either from high school or in our newly redesigned one-semester Biocalculus course, have had the option to take a new course that consists of ideas of mathematical modeling applied across a range of areas of mathematics as well as in a variety of biological contexts. The course culminates in a

project from each student at the intersection of mathematical and biological areas they find interesting. In this presentation, we discuss the original edition of the course as well as the modifications introduced into the second edition based on input from three sources: student feedback from the first version of the course; experiences in a Faculty Mentoring Network on Bringing Research Data into Undergraduate Classrooms, sponsored by The Ecological Society of America (ESA) and Quantitative Undergraduate Biology Education Synthesis (QUBES); and participation in the Quantitative Biology Summer Institute, sponsored by the National Academies and others. (Received September 20, 2016)

1125-Q1-3092 **Yanping Ma\*** ([yma@lmu.edu](mailto:yma@lmu.edu)), CA. *The attitudes of students in calculus for life science toward Mathematics in their careers and some calculus applications in real life.* Preliminary report.

We developed an instrument to help identify the degree to which Life Science students' value mathematics and calculus skills in their prospective careers. There are five aspects measured, including positive attitude towards the importance and relevance of mathematics/calculus towards their future career, as well their level of confidence, anxiety, and enjoyment about mathematics/calculus. This type of tool would support the quantitative tracking of attitudinal changes on students in Life Science over time and attitudinal comparisons across various subpopulations, including effects based on modifications of pedagogy. The instrument has been validated, and we started to incorporate individual worksheets with real medical/health/chemistry related applications in the course. We will report some preliminary results collected by our instrument, and examples about our special worksheets in the meeting. (Received September 21, 2016)

## *Unexpected Topics for a Math Circle*

1125-Q5-567 **Martin J Strauss\*** ([martinjs@umich.edu](mailto:martinjs@umich.edu)). *Finite Projective Planes and Applications.* Preliminary report.

A finite projective plane is an incidence structure, consisting of a finite number each of lines and planes and an incidence relation indicating which points lie on which lines. We investigate applications and connections to error-correcting codes in engineering, linear perspective in art, and to finite field construction elsewhere in mathematics. (Received September 06, 2016)

1125-Q5-622 **Christopher Goff\*** ([cgoфф@pacific.edu](mailto:cgoфф@pacific.edu)), 3601 Pacific Ave, Stockton, CA 95211. *What's in a Logo?* Preliminary report.

After coming up with an eye-catching logo for the San Joaquin Math Teachers' Circle, we decided to spend a session studying it. Our investigation evolved into a journey covering over 2000 years of mathematics history, beginning with a lost work created by Apollonius, moving to a letter sent from Descartes to a princess, and ending up in the strange world of fractals. (Received September 08, 2016)

1125-Q5-830 **Sharon K. Robbert\*** ([sharon.robbertr@trnty.edu](mailto:sharon.robbertr@trnty.edu)), Trinity Christian College, 6601 W. College Drive, Palos Heights, IL 60463. *CryptoClue.* Preliminary report.

Elementary and middle school students may be drawn into interest in mathematics in different ways. Some children enjoy puzzles, games, and secret codes but may not realize how much mathematics is involved in encoding and decoding messages. Cryptography is an excellent tool to support mathematical learning for children at a variety of levels while the children have fun! As one assignment in a course on beginning cryptography for upper-class math and math education majors, college students used cryptography as a basis to design a fun math-based activity for middle school children. The students selected a variety of elementary cryptographic systems to create CryptoClue, a game loosely based on the board game Clue™. At the end of the semester, these same students led approximately 170 middle school math competition participants in playing their invented game. Components of CryptoClue, an overview of the mathematics within the game, extensions for continued mathematical study, simplifications for younger students, resources for learning more about cryptography, and intriguing short and long-term outcomes will be shared. (Received September 20, 2016)

1125-Q5-1011 **Mark C Hughes\*** ([hughes@mathematics.byu.edu](mailto:hughes@mathematics.byu.edu)). *Complex behavior from simple rules - cellular automata for Math Circles.*

Cellular automata are complex systems that are based on very simple sets of rules, and can be used to model real world phenomena in physics, biology, and engineering. Because they are so simple to define, they can easily be explained to young students (together with an engaging story), who then quickly begin to iterate 1-dimensional

automata using pencil and paper. Higher dimensional (2D) automata can also be demonstrated using software to show the complex patterns that arise.

From this students see first-hand how complex behavior can emerge from very simple rules, and can try to predict the patterns that will develop. They also see how real world systems can be modeled using simple mathematical rules. (Received September 14, 2016)

1125-Q5-1371 **Maria S. Nogin\*** ([mnogin@csufresno.edu](mailto:mnogin@csufresno.edu)), 5245 N Backer Ave, M/S PB 108, Fresno, CA 93740, and **Adnan H. Sabuwala** ([asabuwala@csufresno.edu](mailto:asabuwala@csufresno.edu)), 5245 N Backer Ave, M/S PB 108, Fresno, CA 93740. *The magical way to learn mathematics.*

In this talk we will demonstrate some of the card tricks that we teach to grade 5-10 students in the Math Circle at Fresno State. These tricks are purely mathematical in nature with no sleight of hand. Kids are always eager to learn why the tricks work and how to perform them – what a great incentive to learn the Pigeonhole principle, permutations, different base representations, and many other concepts! We will also share our website that contains descriptions of these and other magic tricks and (of course!) explanations of the mathematics behind them. (Received September 20, 2016)

1125-Q5-1404 **Celil Ekici** and **Christopher Plyley\*** ([christopher.plyley@uvi.edu](mailto:christopher.plyley@uvi.edu)), RR01 Box 10,000, Kingshill, VI 00850, and **Cigdem Alagoz** and **Angie Estien**. *Math Circles for Integrated STEM Learning Communities.*

Aligning with the Vision 2025 for mathematical sciences and following the legacy of Felix Klein, these novel math circle activities bring together mathematics and science teachers, mathematicians, high school students, and undergraduate peer mentors to study the dynamics of an invasive lionfish population that is threatening the local ecosystem. We consider alternative harvesting regimes and study their mathematical and scientific models using both continuous and discrete data. This approach allow teachers to practice mathematical modeling for an integrated STEM learning using population growth as a theme. Exemplified math circle activities provide support for teachers and students looking to develop disciplinary connections between discrete mathematics, precalculus, and calculus, using discrete and continuous models which have an impact on the local ecology and fishing industry. (Received September 16, 2016)

1125-Q5-1532 **James C Taylor\*** ([jtaylor@sfpref.org](mailto:jtaylor@sfpref.org)), 81 Calle Gurule, Santa Fe, NM 87505, and **Nicholas Bennett** ([nick@nickbenn.com](mailto:nick@nickbenn.com)), NM. *The Missing ‘M’ in STEM: A Math Circles & Modeling Approach.*

Experiencing STEM programs over the past couple of decades, it is clear that there is very little M in STEM, with that morsel of math being used in the service of science or computer science. In most cases, somebody just tells the students the math they should use, perhaps with some hand-waving, and that is incorporated into their project. Some people put the M under the STE, showing that math is actually the basis of it all, but that just promotes the notion that math is a utilitarian subject, a tool for solving other types of problems. There is none of the sort of creative and often deep mathematical thinking we promote and teach in mathematical circles, just more blind implementation of techniques and algorithms. Standard approaches don’t encourage—and sometimes actually discourage—mathematical thinking. This session takes the mathematical discoveries arrived at in a familiar math circle and demonstrates how they can be explored and visualized using computer modeling tools (and programming). This approach could just as well be used to investigate mathematics that leads into scientific experimentation in other areas—the key point here being that we start with the mathematical exploration that will lead to a deeper understanding of the physical or computational systems. (Received September 17, 2016)

1125-Q5-1923 **Anne M. Ho\*** ([aho@coastal.edu](mailto:aho@coastal.edu)), Conway, SC, and **Tara T. Craig** ([tcraig@coastal.edu](mailto:tcraig@coastal.edu)), Conway, SC. *Triangles, Squares, and Segregation: Introducing social issues through math.* Preliminary report.

Although a mathematics class is not the typical place for discussions on race and society, the fact that all students must take math means that it can be an ideal location to introduce students to important social issues and values. We adapt “Parable of the Polygons” (Vi Hart and Nicky Case), an online simulation on diversity and segregation, into an appropriate math circles session, which ran at the Coastal Carolina University Math Teachers’ Circle in December. The session is interactive, and offers multiple layers of content depending on the age and comfort level of students with conversations on social issues. These levels include: (1) a basic introduction to game theory, (2) an invitation for students to think about the benefits of diverse groups, and (3) a discussion of how individual biases, no matter how small, can lead to detrimental societal effects like segregation. (Received September 19, 2016)

1125-Q5-2067 **Ryan W. Morrill\*** (rmorrill@ualberta.ca). *A grid of liars.*

In this session we tackle an unusual variant of a familiar style of problem to give us a wide variety of puzzles. In this problem, we will assume liars always lie, and truthful people always tell the truth. We will also assume every person is either one or the other, and everyone knows what everyone else is. In an  $n \times n$  grid there stands a person in each cell. Each person states “There is exactly one liar adjacent to me.” What are all the possible different distributions of liars and truth tellers? This easy-to-state problem gives students a many avenues to explore and experiment, and it can inspire many other problems to investigate. (Received September 19, 2016)

1125-Q5-2122 **Angela Antonou\*** (aantonou@stfrancis.edu), **Rita Patel, Amanda Harsy, Dave Klanderma, Kristen Schreck** and **Amanda Snooks**. *Impact of the Southwest Chicago Math Teachers’ Circle on the Disposition of Teachers Toward Mathematics and Toward the Teaching and Learning of Mathematics*. Preliminary report.

The Southwest Chicago Math Teachers’ Circle is a recently formed math teachers’ circle focused on providing opportunities for middle and high school mathematics teachers in the southwest Chicago suburbs to grapple with deep and interesting mathematical problems which may, in many cases, relate to the topics which they teach in their classroom. The authors have attempted to determine the impact of participation in this program on the teachers’ dispositions toward mathematics, the teaching of mathematics, and their perception of the learning of mathematics. This talk will share some of the results gathered, thus far, from the participants. (Received September 19, 2016)

1125-Q5-2179 **Mary L. Garner\*** (mgarner@kennesaw.edu), 4621 Ivygate Circle, Atlanta, GA 30339, **Virginia Watson** (vwatson@kennesaw.edu), 1000 Chastain Road, Kennesaw, GA 30144, and **Beth Rogers** (mroger47@kennesaw.edu), 1000 Chastain Road, Kennesaw, GA 30144. *Dancing in Math Circles*.

Contra Dancing is a type of dance much like square dancing, usually performed to jigs or reels. Each contra dance begins with two long lines of dancers, everyone facing a partner. Two pairs of dancers form a square and execute a sequence of 8 figures for 64 beats of music; then each pair of dancers joins a new pair to execute the same sequence. The footwork consists of walking in time with the music, one step for each beat, so it’s easy to learn quickly and everyone in a math circle can be dancing within minutes! The dancing provides a very physical way to explore transformations of the plane and symmetries of the square, providing a context for discussing everything from simple rotations and reflections to matrices and group theory. It’s also a great way to break the ice in a new math circle! In this presentation, everyone will be dancing. (Received September 19, 2016)

1125-Q5-2262 **Japheth Wood\***, Bard College, Annandale-on-Hudson, NY 12504. *Math Circle Artifacts at the Bard Math Circle*.

Mathematical artifacts, physical objects with significant mathematical content, are something that students at the Bard Math Circle create, take home, and share. We’ll show several examples, from the Moebius Strip to the Singleton Calendar. Mathematical artifacts give students opportunity to learn math in an artistic and tactile fashion, and to disseminate mathematical ideas to others. (Received September 20, 2016)

1125-Q5-2355 **Amanda H Matson\***, 1550 Clarke Drive MS 1725, Dubuque, IA 52001, and **Jonas Meyer**, Loras College. *Pancakes, Music, and Games in MTC Dubuque*.

MTC Dubuque meets monthly during the academic year and hosts a four day workshop over the summer. We frequently make connections to the Common Core Math Practice Standards and have a different topic for questions and discussion each time we meet. We’ve discussed the pancake sorting problem, math and music with a new way of discovering the Fibonacci sequence, water flow with a guest speaker from a local watershed engineering group, and spend quite a bit of time on the mathematics behind games such as Spot It! and Set. I will share how we posed some of the problems to the group as well as how we interweave the Common Core Math Practice Standards throughout our discussions. (Received September 20, 2016)

1125-Q5-2811 **Angie Hodge\***, amhodge@unomaha.edu, and **Dylan Sorrell**, dsorrell@unomaha.edu. *Fold, cut, and problem solve: A Math Teachers’ Circle sampling*.

This session will describe several problems suited for a Math Teachers’ Circle that can be done using only paper and a scissors. These challenges do not require any algebra, but require critical thinking and problem solving skills. The audience will get a chance to engage in at least one of the problems. We will also share how to package the problems for a Math Teachers’ Circle session. (Received September 20, 2016)

1125-Q5-3088 **Tatiana Dezbah Shubin\*** ([tatiana.shubin@sjsu.edu](mailto:tatiana.shubin@sjsu.edu)), Department of Mathematics & Statistics, San Jose State University, San Jose, CA 95192-0103. *Different Angle*.

Turning the familiar into unusual is what fuels a good math circle session. We will present several examples of such "magic shifts" which lure participants into finding and investigating exciting new vistas. (Received September 21, 2016)

## Women in Mathematics

1125-R1-282 **Theresa N Martinez\*** ([martines@uiwtx.edu](mailto:martines@uiwtx.edu)), University of the Incarnate Word, 4301 Broadway, CPO 311, San Antonio, TX 78209. *Middle School Mathematics Day for Girls*.

This program is designed to assist girls with the transitions between middle/high school mathematics and to encourage colleges and universities to develop connections with middle schools and high schools. In the state of Texas, students are given a choice of 8th grade mathematics courses. In most cases this choice puts them on a track to either end high school with Algebra 1 or Calculus 1, students and parents are often unaware of these tracks. This program looks to show the use of mathematics in school and careers to encourage them to continue mathematics studies. Students are divided into 4 groups of 15-20 students which rotate between the various activities and speakers designed to encourage continued study of mathematics and introduce mathematics related careers. The activities are organized and supervised by faculty members, but the actual activity is run by undergraduate volunteers. The event targets 7th grade girls from the Young Women's Leadership Academy, an all-girls public school that is part of the San Antonio Independent School District (SAISD). The school is composed of 91% Hispanic, 7% African American, and 2% White (non-Hispanic) students with 92.6% of the total enrollment classified as economically disadvantaged. (Received August 23, 2016)

1125-R1-345 **Isabel M. Serrano\*** ([iserrano@csu.fullerton.edu](mailto:iserrano@csu.fullerton.edu)), 800 N State College Blvd, 154 McCarthy Hall, Department of Mathematics CSUF, Fullerton, CA 92834-6850. *Fullerton Mathematical Circle*.

The Fullerton Mathematical Circle is inspired by similar activities done in Central Europe over the last century where Universities offer enrichment programs to interested young gifted mathematicians. Our initiative is the only Mathematical Circle in the USA that draws its inspiration from the activities of *Gazeta matematică*, a monthly journal with continuous publication since 1895 by the Romanian Society of Mathematical Sciences. The project consists of mathematical sessions for gifted middle school and high school students. We will show how several research projects have developed starting from *Gazeta matematică's* problems to producing other published works, including several student posters and presentations at various MAA conferences. (Received August 28, 2016)

1125-R1-358 **Jitka Stehnova\*** ([stehnova@math.uchicago.edu](mailto:stehnova@math.uchicago.edu)), Department of Mathematics, 5734 S University Ave, University of Chicago, Chicago, IL 60637. *ExploreU Summer STEM Program*.

In this talk, I will provide most recent updates about our STEM Outreach Math and CS Summer Program, ExploreU, that started as a small project supported by MAA Tensor Grant for Women in Mathematics. In our third year of running, we were able to secure a full support from Transamerica Foundation via their corporate STEM grant. This support allowed us to expand ExploreU and add new exciting workshops including our newest addition in robotics workshops - an AI humanoid NAO. We are currently in our fifth year with plans for further expansion. I will highlight our development over the years and provide some insights on securing corporate grants for other STEM projects. (Received August 29, 2016)

1125-R1-721 **Violeta Vasilevska\*** ([violeta.vasilevska@uvu.edu](mailto:violeta.vasilevska@uvu.edu)), 800 W University Parkway, Orem, UT 84058. *Engaging Women in Extracurricular Math Activities*. Preliminary report.

Since 2007 the presenter has led outreach programs that engage high school and undergraduate female students through hands-on extracurricular math activities and encourage them to continue their education. In this talk the structure of the Utah Valley University outreach program Math Girls Rock! (<https://www.uvu.edu/math/mgr/>) that the presenter has been leading since 2011 will be discussed. This program is designed as a year-long Math Club for Young Women, offering weekly meetings with the undergraduate female students and a series of after-school meetings (2 times per semester) for the high-school girls. The program offers mentoring on two levels: the presenter mentors the undergraduate students who then mentor the high school girls through the designed hands-on projects. It engages both student groups in various fun and interesting math projects that show that math is exciting, beautiful and applicable to various other fields. In addition, the projects are designed to spark

student curiosity, to stimulate student inquiry, to get them to get “their hands dirty” with math, and to inspire them to want to learn more. Moreover, survey data on how learning math concepts in these types of informal settings affected the participants’ attitudes toward mathematics will be presented. (Received September 09, 2016)

1125-R1-773      **Qingxia Li\*** (qli@fisk.edu), 411 Annex Ave, Apt B4, Nashville, TN 37209. *Fisk University Math Club.*

The goal of Fisk University Math Club is to give students more practice and more skills by providing them a risk-free forum to have fun and socialize while practicing mathematics, motivate students to pursue a professional degree in math-related disciplines or career after their graduation, spur students’ interest towards mathematics, and help students improve their critical thinking and logical reasoning skills in STEM-related research projects. The club has invited faculty members and professionals, representing graduate school and industries in STEM fields, to inspire student’s passion for mathematics by giving math-related presentations and serving as mentors for club members on hands-on math projects. Club activities also stressed creative problem solving, with the primary purpose of increasing out-of-the-box thinking and risk-taking. A monthly math contest has been held to spur students’ interest towards mathematics. Data analysis shows that the club activities has increased retention and success rates in advanced mathematics courses. (Received September 11, 2016)

1125-R1-1324      **Carolyn Connell\*** (cconnell@westminstercollege.edu). *AWE+SUM Outreach Program: Challenges after 12 Years.*

The AWE+SUM program is a 4-day residential summer camp for entering 8th grade girls, held at Westminster College in Salt Lake City. This summer’s program was the 12th year of the program. AWE+SUM (“Attend Westminster, Explore Science, Use Math”) presents hands-on workshops, led by female math & science faculty. The program has the goal of showing that these subjects can be really fun, hopefully inspiring girls to consider careers in STEM fields. Girls from Title 1 schools and other underserved populations are given priority for admittance.

After years of the program, there are still challenges in providing the program. Funding for scholarships has not been the difficulty: several organizations provide support, such as the MAA’s Tensor Foundation and the American Association of University Women. The primary challenge for AWE+SUM has been the decision about which girls to admit each summer: even girls who are excelling academically need encouragement to pursue these fields, at this most difficult time in their lives. (Received September 16, 2016)

1125-R1-1488      **Sarah Schott\*** (schott@math.duke.edu), 120 Science Drive, 117 Physics Building, Durham, NC 27708, and **Emily Braley.** *Southeastern Conference for Undergraduate Women in Math.*

The Southeastern Conference for Undergraduate Women in Math (SCUWM) was created with the goal of increasing interest in and awareness of both mathematical careers and graduate school opportunities in the mathematical sciences among undergraduate women. Funded by a Tensor Women and Mathematics grant, Duke hosted the conference in the fall semesters of 2015 and 2016, after it was held at Clemson University. The two day conference includes invited talks by select female mathematicians, undergraduate talks, breakout sessions, a professional panel and a panel devoted to graduate school. While the conference is focused on undergraduates, graduate student involvement is key. Graduate students serve on the graduate school panel, give introductory level talks on their research, and are involved in the myriad details of conference planning. The benefits are twofold in that the conference is improved by their involvement, and they receive professional development that will aid them in their future careers. As a result, a larger effect of SCUWM is the development of a network that includes women at many different stages. Our hope is that this conference will continue encourage young women to pursue careers in mathematics as it is hosted at different universities in the Southeast. (Received September 17, 2016)

1125-R1-1752      **Jillian Folino\*** (jillianfolino@ircsd.org) and **Blair Madore** (madorebf@potsdam.edu). *Building the Pipeline From High School to College Mathematics.*

We like  $\pi$ ” is a math club at Indian River high school in northern New York. It was formed with support from SUNY Potsdam and an MAA Dolciani grant. The club’s program emphasizes the fun and excitement of mathematics - an approach that is particularly successful with women and other under-represented groups. By describing the club’s structure and activities we introduce the philosophy of this broad approach to building the pipeline to mathematics. (Received September 19, 2016)

1125-R1-1784 **Meghan De Witt\*** ([mdewitt@stac.edu](mailto:mdewitt@stac.edu)). *GEM: Girls Exploring Mathematics*. We describe an outreach program creating math clubs for girls in local high schools, sponsored by MAA/Tensor. Four undergraduate female math students are taught to design and prepare a lesson plan and then brought to two different local high schools where they teach an after school club. Topics include Platonic and Archimedean Solids, the Fibonacci Numbers and the Golden Ratio, and the mathematics of architecture. Each club meeting covers the necessary mathematics, highlights the history of the topic, and culminates in a creative art project utilizing their new-found knowledge. (Received September 19, 2016)

1125-R1-2477 **Yuliya Babenko\*** ([ybabenko@kennesaw.edu](mailto:ybabenko@kennesaw.edu)), Kennesaw State University, Math Building - Bldg. D, 1100 South Marietta Pkwy, MD # 9085, Marietta, GA 30060. *KWIM: struggles and successes*. Preliminary report.

In this talk we share our experience in establishing and developing Kennesaw Women in Mathematics (KWIM), an organization started in 2011 at KSU with the help of Tensor Grant. During its first six years of existence we introduced several rather successful initiatives, that became now traditional at KSU, such as Queens of Math Days, Infinite Horizons Lecture Series, Math Study Nights, etc.. We discuss these and other implemented ideas (what worked and what didn't work for us) for engaging and recruiting students, outreach and social activities, professional development seminars, search for external funding, etc. (Received September 20, 2016)

1125-R1-3019 **D. Natasha Brewley\*** ([dbrewley@ggc.edu](mailto:dbrewley@ggc.edu)), 1000 University Center Lane, School of Science and Technology, Lawrenceville, GA 30043, and **Alvina J. Atkinson** ([aatkinso@ggc.edu](mailto:aatkinso@ggc.edu)), 1000 University Center Lane, School of Science and Technology, Lawrenceville, GA 30043. *The MiA Scholars Program: Bringing an Interdisciplinary Mathematics Experience to Middle School Girls*.

The Georgia Gwinnett College Mathematics in Action (MiA) Scholars Program started their first ever mathematics summer program with funding from Dolciani Mathematics Enrichment Grant in 2012. The MiA Scholars Program takes academically talented or motivated middle school students on a week-long exploration of mathematics topics and activities. The program is directed and taught by Georgia Gwinnett College mathematics faculty, and uses real-world problems to introduce mathematically rich, open-ended interdisciplinary problems. Over the past four years of the program the number of girls that have participated has increased from one-third to more than half. In our presentation we will discuss the mission and goals of our program, the demographics of students served, and how girls' interest and participation in the MiA Scholars Program has informed how we plan for future programs. (Received September 20, 2016)

1125-R1-3115 **Junalyn P Navarra-Madsen\*** ([jnavarramadsen@mail.twu.edu](mailto:jnavarramadsen@mail.twu.edu)), Texas Woman's University, PO Box 4258867, Denton, TX 76204. *Keeping the Pipeline Full: A Woman Mathematician's Perspective*.

Texas Woman's University (TWU) is one of the largest US universities for women with more than 15,000 students (Graduate and Undergraduate). TWU recently received consecutive NSF S-STEM grants. The latest grant is labeled "Quantitative and Analytical Sciences for Academic Reinforcement and Success (QuASARS)" - NSF Grant 1154394. The author is a co-PI of these two grants and shares some information and insights on how to lower the attrition rate of female STEM majors in courses such as Calculus, Linear Algebra and Differential Equations using early intervention approaches, tutoring, mentoring which ultimately leads to building a learning community of successful career women in the STEM fields. (Received September 21, 2016)

## General Session on Algebra

1125-VA-426 **Victoria S Akin\*** ([takin89@gmail.com](mailto:takin89@gmail.com)). *An Algebraic Characterization of the Point-Pushing Subgroup*.

The *point-pushing subgroup*  $P(\Sigma_g)$  of the mapping class group  $\text{Mod}(\Sigma_{g,1})$  of a surface with marked point is an embedding of  $\pi_1(\Sigma_g)$  given by pushing the marked point around loops. We prove that for  $g \geq 3$ , the subgroup  $P(\Sigma_g)$  is the unique normal, genus  $g$  surface subgroup of  $\text{Mod}(\Sigma_{g,1})$ . As a corollary to this uniqueness result, we give a new proof that  $\text{Out}(\text{Mod}^\pm(\Sigma_{g,1})) = 1$ , where  $\text{Out}$  denotes the outer automorphism group; a proof which does not use automorphisms of complexes of curves. Ingredients in our proof of this characterization theorem include combinatorial group theory, representation theory, the Johnson theory of the Torelli group, surface topology, and the theory of Lie algebras. (Received September 01, 2016)

1125-VA-924 **Sam V Eastridge\*** (seastri@vt.edu), 208 Miller St., Christiansburg, VA 24073, and **Peter Linnell**. *First  $l^p$  Cohomology of Some Infinite Groups*. Preliminary report.

In this talk I will consider some common results about  $l^p$  cohomology for finitely generated groups, and see which of these results also apply for uncountable groups  $G$ . Many of the results do not hold even for the countably generated case. In particular, I intend to give some explanation as to the reason why even though the space  $H^1(G, l^p(G))$  is not even Hausdorff for  $G$  an infinite, amenable, finitely generated group, it vanishes for some (and perhaps all) uncountable amenable groups. I also intend to discuss the injectivity of some maps between cohomology groups, as well as look at some results on first cohomology groups with coefficients in  $c_0(G)$  and  $l^\infty(G)$ . (Received September 13, 2016)

1125-VA-1065 **J B Coykendall** and **B G Goodell\*** (bggoode@clemson.edu). *A Homological Approach to Factorization*.

An integral domain is said to be atomic if every element can be written as a product of irreducibles, but arbitrary integral domains cannot be assumed to be atomic. We study factorization in localizations of arbitrary (non-atomic) integral domains by using projections of groups of divisibility as a proxy by constructing a natural sequence of partially ordered groups. This sequence peels layers of atomicity away from arbitrary groups of divisibility like layers of an onion. In the direct limit, all that remains is the "antimatter" elements: a partially ordered group that has no minimal positive elements whatsoever and yet is generated by its positive elements. From this sequence, we can also obtain cochain complexes, we can compute their associated cohomology groups (whose properties detect factorization relationships in the underlying integral domain), and we can prove structure theorems. These structure theorems correspond to a variation on the idea of universal factorization. (Received September 14, 2016)

1125-VA-1155 **Rachel Victoria Barber\*** (rvb41@msstate.edu), P.O. Box MA, 410 Allen Hall, 175 President's Circle, Mississippi State, MS 39762. *Low-Dimensional Reality-Based Algebras*.

In this paper we introduce the definition of a reality-based algebra as well as a subclass of reality-based algebras, table algebras. We look at a specific reality-based algebra (RBA) of dimension 5 of the form  $\mathbb{C} \oplus M_2(\mathbb{C})$  and provide formulas for the structure constants of this algebra. We look at the structure constants and determine certain conditions when this RBA is a table algebra. (Received September 15, 2016)

1125-VA-1513 **Erik O. Hieta-aho\*** (eh991112@ohio.edu) and **Sergio Lopez-Permouth**. *Recognizing arbitrary rational functions amongst power series*. Preliminary report.

The talk will begin by reviewing topics introduced in a 2009 JPAA paper by Hou, Lopez-Permouth, and Parra-Avila on the characterization of (formal) rational power series over a commutative ring and the periodicity of coefficients. The aim of that paper was to characterize periodic rings; a ring was said to be periodic if all power series representing rational functions with a co-monic denominator have an eventually periodic sequence of coefficients are periodic. While focusing on only such rational functions makes sense in the context of fields, there may be a loss of generality in doing so. For that reason, we are currently pursuing the study of analogous questions regarding arbitrary rational functions. Emphasis is placed on finite rings as regular polynomials for those rings are easy to characterize. Among other results, we present a couple of somewhat surprising twists on familiar constructions regarding the embedding of fields of rational functions in power series rings as well as a generalization of the Kronecker criterion of rationality for functions given by power series. (Received September 17, 2016)

1125-VA-1663 **Chad Awtrey\*** (cawtre@elon.edu), **Jim Beuerle** and **Michael Keenan**. *When is a polynomial isomorphic to an even polynomial?*

Let  $f(x)$  be an irreducible polynomial with integer coefficients, and let  $K^f$  denote the stem field of  $f$ ; that is,  $K^f$  is the smallest subfield of the complex numbers that contains both the rational numbers and a root of  $f$ . If  $g(x)$  is irreducible, call  $f$  and  $g$  isomorphic if  $K^f$  and  $K^g$  are isomorphic as fields. In this talk, we focus on the question: if the degree of  $f$  is even, when is  $f$  isomorphic to an even polynomial  $g$ ? We discuss a simple method that answers this question and constructs such a polynomial  $g$  if it exists. We end with an application to computing Galois groups of polynomials of even degree. (Received September 18, 2016)

1125-VA-1712 **Babak Jabbar-Nezhad\*** (bjabbarn@uark.edu), 1 University of Arkansas, Fayetteville, AR 72701. *Defining equations of the multi-Rees algebra*.

In this talk we describe the defining equations of the multi-Rees algebra  $R[t_1 I_1, \dots, t_r I_r]$  over a polynomial ring  $R[T_1, \dots, T_s]$ , where there are some relations between  $I_i$ s. (Received September 18, 2016)

1125-VA-1841 **Grace McClurkin\*** (mccclurkin@math.utk.edu). *Extensions of the Congruence-based Zero-divisor Graph*. Preliminary report.

In this talk, I will extend the idea of the congruence-based zero-divisor graph, first introduced by Anderson and Lewis (2016), to other variations of the zero-divisor graph, such as the annihilator graph and the extended zero-divisor graph. In addition to defining the congruence-based graphs, examples and properties of these graphs will be examined. (Received September 19, 2016)

1125-VA-1919 **Kelli Marie Karcher\*** (kkarcher@vt.edu), 1743 Autumn Splendor Way, Blacksburg, VA 24060. *The Space of Biorders for Solvable Groups of Finite Rank*. Preliminary report.

A group is said to be biorderable if it has a total order invariant under left and right multiplication. These orders can be given a topology and is called the space of biorders on this group. There has been intensive study on the space of left-orders recently, but less on the space of biorders. We will focus on solvable groups to show under certain conditions the space of biorders is either finite or homeomorphic to the Cantor set. Furthermore, we will give a characterization in terms of the convex subgroups when a biorder is isolated for solvable groups of finite rank. (Received September 19, 2016)

1125-VA-2207 **Ryan Coopergard\*** (coope786@umn.edu) and **Marju Purin** (purin@stolaf.edu). *Truncated Path Algebras and Betti Numbers with Polynomial Growth*. Preliminary report.

In this talk, we will introduce a class of truncated path algebras in which the Betti numbers of a simple module satisfy a polynomial of arbitrarily large degree. We will give examples of such algebras where the  $i^{\text{th}}$  Betti number of a simple module  $S$  is  $\beta_i(S) = i^k$  for  $2 \leq k \leq 4$  and provide a method for constructing truncated path algebras where  $\beta_i(S)$  is a polynomial of degree four or less with nonnegative integer coefficients. In particular, we prove that algebras in this class can produce Betti numbers corresponding to any polynomial in a certain family. (Received September 19, 2016)

1125-VA-2230 **Christopher O'Neill\*** (coneill@math.ucdavis.edu), Department of Mathematics, UC Davis, One Shields Ave, Davis, CA 95616, and **Jacob Hartzler** (jacobhartzler@gmail.com), Mailstop 3368, Texas A&M University, College Station, TX 77843. *On the periodicity of irreducible elements in arithmetical congruence monoids*.

Arithmetical congruence monoids, which arise in the study of non-unique factorization, are multiplicative submonoids  $M_{a,b} \subset \mathbb{Z}_{\geq 1}$  consisting of all positive integers  $n$  satisfying  $n \equiv a \pmod{b}$ . In this talk, we examine asymptotic properties of the set of irreducible elements of  $M_{a,b}$ , and present a characterization in terms of  $a$  and  $b$  when this set forms an eventually periodic sequence. (Received September 20, 2016)

1125-VA-2476 **Nate Harman\*** (nharman@mit.edu), 77 Massachusetts Ave, Cambridge, MA 02139. *Model theoretic limits of categories and representations of diagram algebras*.

I will talk about how tools and ideas from model theory and first order logic can give new insight into the representation theory of certain diagram algebras such as the partition and Brauer algebras. In particular I will show how use this method to give new (and short) proofs of the semisimplicity conditions for these algebras. (Received September 20, 2016)

1125-VA-2547 **Nathaniel Bushek\*** (nbushek@alaska.edu), Department of Mathematics and Statistics, University of Alaska, Anchorage, SSB # 158 A, Anchorage, AK 99508. *Tensor product multiplicities and descent of line bundles to GIT quotients*.

A classic question of representation theory is to determine the multiplicity of one irreducible representation in the tensor product of two other irreducible representations. This question has been extensively studied and an interesting sub-question is how the multiplicities change as the irreducible representations change. It is known that this change is piecewise polynomial. However, explicit constructions of these polynomials have not been given. We aim to give such a construction through the use of geometric invariant theory on the triple product of flag varieties,  $X = G/B \times G/B \times G/B$ . In particular, we consider conditions when line bundles descend to  $X//G$ . When descent occurs we can use intersection theory to construct the multiplicity polynomial. While much is understood in this construction, some important questions remain unanswered. (Received September 20, 2016)

1125-VA-2548 **Bernadette Boyle\*** (boyleb7@sacredheart.edu), Sacred Heart University, 5151 Park Ave, Fairfield, CT 06825. *The Index of a Family of Gorenstein Numerical Semigroups in Four Generators*. Preliminary report.

In 2013, O. Veliche developed a closed formula to find the index of a complete intersection numerical semigroup ring with three generators. She also found methods to compute the index of these rings with more than three

generators; however, to use these methods one must first find other values, such as the Frobenius number and the order of elements. In this talk, we will discuss expanding Veliche's results to develop an explicit formula for the index of a family of Gornstein numerical semigroup rings in four generators which only requires knowledge of the generators of the ring. (Received September 20, 2016)

1125-VA-2611 **Andrew J. Kelley\*** ([kelly@math.binghamton.edu](mailto:kelly@math.binghamton.edu)). *Maximal subgroup growth of some groups.*

Let  $m_n(G)$  denote the number of maximal subgroups of a finitely generated group  $G$  of index  $n$ . Asymptotic bounds will be given for  $m_n(G)$  for certain metabelian groups  $G$ . (Received September 20, 2016)

1125-VA-2795 **Brian P. Johnson\*** ([bpjohnson@fgcu.edu](mailto:bpjohnson@fgcu.edu)), Department of Mathematics, Florida Gulf Coast University, 10501 FGCU Blvd. S., Fort Myers, FL 33965-6565, and **Katherine Cooper** ([k.cooper@uky.edu](mailto:k.cooper@uky.edu)), Department of Mathematics, University of Kentucky, 719 Patterson Office Tower, Lexington, KY 40506-0027. *Zero divisor graphs of commutative graded rings.*

We study a natural generalization of the zero divisor graph introduced by Anderson and Livingston, extended to commutative rings graded by abelian groups by considering only homogeneous zero divisors. We develop a basic theory for graded zero divisor graphs and present many examples. Finally, we examine classes of graphs that are realizable as graded zero divisor graphs and close with some open questions. (Received September 20, 2016)

1125-VA-2931 **Kimmy Cushman\*** ([cushkk80@oneonta.edu](mailto:cushkk80@oneonta.edu)). *Connecting the Algebraic Theory of Lie Algebra Spinor Representations to Applications in Physics.* Preliminary report.

This talk is a continuation of Jonathan Brown's talk introducing spinors from the point of view of an algebraist. We will investigate how the algebraic representation theoretic approach shines light on realizations in physical theories. We will explain how the Dirac equation describes the behavior of quantum particles such as electrons, and how the Dirac equation relies heavily on the algebraic objects, spinors, introduced by Elié Cartan. (Received September 20, 2016)

1125-VA-2939 **Jonathan Brown\*** ([jonathan.brown@oneonta.edu](mailto:jonathan.brown@oneonta.edu)). *The algebraic approach to spinor representation theory.*

This talk is the first talk in a two part series about spinor representations. Spinor representations play an important role in describing the fundamental properties of many particles in physics such as electrons. Mathematically spinor representations are certain finite dimensional irreducible representations of orthogonal Lie algebras. In this talk I will explain the algebraic approach to spinor representation theory. I will briefly go over the classification of finite dimensional irreducible representations of orthogonal Lie algebras via highest weight theory, and explain which highest weight finite dimensional representations are spinor representations. Next I will define Clifford algebras and I will explain how to embed orthogonal Lie algebras into Clifford algebras. Finally I will show how to use highest weight theory to decompose the Clifford algebra as a direct sum of irreducible orthogonal Lie algebra representations, which turn out to be the spinor representations. In the second talk of the series Kimmy Cushman will explain the significance of spinor representations in physics. (Received September 20, 2016)

1125-VA-2992 **Tucker L. Dowell\*** ([tucker.dowell@pop.belmont.edu](mailto:tucker.dowell@pop.belmont.edu)) and **Brad Schleben.** *Counting Elements of Particular Orders in the Symmetric Group.*

The number of elements that square to the identity in the symmetric group  $S_n$  is determined by a well-known recursion. We study a generalization of this question: for which values of  $k$  are there exactly  $k$  elements such that  $x^k$  is the identity? Given any  $n$ , we determine the greatest  $k < n!$  and the least  $k > 1$  that satisfy this condition. (Received September 20, 2016)

## General Session on Analysis

1125-VB-79 **Mr Deepak Kumar\*** ([deepak.kumar@lpu.co.in](mailto:deepak.kumar@lpu.co.in)), Department of Mathematics, Lovely Professional University, Phagwara, Punjab, India, and **Dr Sumit Chandok** ([chandok.sumit@gmail.com](mailto:chandok.sumit@gmail.com)), School of Mathematics, Thapar University, Patiala, Punjab, India. *Almost  $\alpha$ -type  $f$ -weak contractive mappings in partial metric space and fixed points.*

In this paper, we introduce new concept of almost  $\alpha$ -type  $F$ -weak contractive mappings in the frame work of partial metric space. Then, sufficient conditions for the existence and uniqueness of fixed points are established

for almost  $\alpha$ -type  $F$ -weak contractive mappings in partial metric space. The obtained results generalize the existing results in the literature. (Received July 20, 2016)

1125-VB-302      **Shusen Ding**, Seattle University, Seattle, WA 98112, **Guannan Shi\*** (shign@hit.edu.cn), No.92 Westdazhi St. Nangang District, Harbin, 150001, Peoples Rep of China, and **Yuming Xing**, No.92 Westdazhi St. Nangang District, Harbin, 150001, Peoples Rep of China. *Higher integrability of iterated operators on differential forms.*

In this paper, we first prove the local higher integrability and higher order imbedding theorems for the iterated operators defined on differential forms. Then, we prove the global higher integrability and higher order imbedding inequalities for these operators. Finally, we demonstrate applications of the main results by examples. (Received August 26, 2016)

1125-VB-516      **Chandra Kethi-Reddy\*** (chan.dra@knights.ucf.edu), 2122 Westbourne Dr, Oviedo, FL 32765. *Resolving the Unsolvable and Graphing the Infinite.* Preliminary report.

Infinite numbers are reinterpreted as numbers which count and differentiate the "speeds" at which non-finite sets produce new mathematical entities, i.e., the "stream rate" of infinite sequences. This makes the infinite more intuitive and concrete to analyze. I provide a new interpretation of why the continuum hypothesis is independent from ZFC. On a related note, I argue it is possible to construct a procedure akin to a Dedekind cut using logarithms to generate infinities between the aleph-numbers. Log-log graphs are repurposed to create a new coordinate system that extends the Cartesian version by tiling it, allowing one to graph new functions, including those with surreal and surcomplex values. The theory of paraconvergent series is also introduced, with a conjecture that every divergent series has a paraconvergent solution. A paraconvergent solution is one that, like Grandi's series, provides a solution to non-converging infinite series. I challenge the implicit assumption that divergent series use an infinite number of binary addition operations. I argue instead that they only use a single infinitary operation. Thus, the study of algebraic structures built with infinitary operations can lead to considerable progress in the general analysis of functions. (Received September 07, 2016)

1125-VB-787      **Jeremy Trageser\*** (jtrageser@gwu.edu) and **Xiaofeng Ren.** *Discontinuous Local Minimizers to a Class of Semilinear Integral Equations.*

In this work we study a nonlocal bistable equation which arises as the Euler-Lagrange equation of a nonlocal van der Waals type functional. We compare nonlocal interactions given by Green's functions of second-order differential equations and investigate stationary points with multiple discontinuous interfaces. We consider criteria under which the function does not admit a stable stationary point of a finite number of discontinuities. We also provide criteria which ensures the existence of a stable stationary point. (Received September 11, 2016)

1125-VB-820      **Leonardo Pinheiro\***, lpinheiro@ric.edu, and **Kit Chan.** *Chaotic Extensions of General Operators in Hilbert Spaces.*

In this talk we show that every bounded linear operator defined on a closed infinite-codimensional subspace of an infinite-dimensional Hilbert Space can be extended to a chaotic operator. (Received September 12, 2016)

1125-VB-1315      **Bryan Dawson\*** (bdawson@uu.edu). *A New Extension of the Riemann Integral.*

Using the hyperreals, a new extension of the Riemann integral is introduced in which every bounded function is integrable and for which there exists a function  $g : [0, 1] \rightarrow \mathbf{R}$  simultaneously satisfying (1)  $g$  is integrable, (2)  $g$  is unbounded on every subinterval of  $[0, 1]$ , and (3)  $g$  is identical to its average value function. (Received September 16, 2016)

1125-VB-1441      **Muhenned A Abdulsahib\*** (mabdulsa@uark.edu), 900 North Leverette Avenue, Apartment # 303, Fayetteville, AR 72701. *Hartogs Domain and the Diederich-Fornaess Index.*

The Diederich-Fornaess index has played a crucial role in studying regularity for the Bergman projection in Sobolev spaces, as shown by Kohn (1999), Harrington (2011), and Pinton-Zampieri (2014). In this talk, we will discuss the Diederich-Fornaess index on Hartogs domains. (Received September 16, 2016)

1125-VB-1613      **Gregory AE Vaughan\*** (gavaugh@iupui.edu). *On the Convergence of the Positive Roots of Recursively Defined Polynomials.*

Given seed polynomials  $Q_0(x) = q_0$  and  $Q_1(x) = q_1$ , we recursively define a sequence of polynomials for  $n > 1$  by

$$Q_n(x) = Q_{n-1}(x) + x^k Q_{n-2}(x)$$

We show that for  $x_0 > 0$ , there is a monotonically increasing sequence of integers  $n_i$  with  $n_1 > 0$  and a sequence of positive real numbers  $x_{n_i}$  such that  $Q_{n_i}(x_{n_i}) = 0$  and  $\lim_{n_i} x_{n_i} = x_0$  if and only if

$$2q_1(x_0) = \left(1 - \sqrt{1 + 4x_0^k}\right) q_0(x_0).$$

Equivalently, when  $q_0(x) \neq 0$ , the limits of the roots occurs when

$$\frac{q_1(x_0)}{q_0(x_0)} = \frac{1 - \sqrt{1 + 4x_0^k}}{2}$$

where the left side only depends on the seed polynomials and the right side only depends on the recurrence.

This extends results from B. Alberts (2011) and D. Thompson (2013). (Received September 18, 2016)

1125-VB-1753 **Patrick Cummings\*** (patrickc@bu.edu) and **C. E. Wayne**. *Modified Energy Functionals and the NLS Approximation*.

We consider a model equation that captures important properties of the water wave equation. We discuss a new proof of the fact that wave packet solutions of this equation are approximated by the nonlinear Schrödinger equation. This proof both simplifies and strengthens previous results of Wayne and Schneider so that the approximation holds for the full interval of existence of the approximate NLS solution rather than just a subinterval. Furthermore, the proof avoids the problems associated with inverting the normal form transform by working with a modified energy functional motivated by Craig and Hunter et. al. (Received September 19, 2016)

1125-VB-2036 **Waleed K. Al-Rawashdeh\*** (walrawashdeh@mttech.edu), Montana Tech, West Park Street, Butte, MT 59701. *Schatten Class Weighted Composition Operators on Generalized Fock Spaces  $\mathcal{F}_\phi^2(\mathbb{C}^n)$* .

Let  $\psi$  be an entire self-map of the  $n$ -dimensional Euclidean complex space  $\mathbb{C}^n$  and  $u$  be an entire function on  $\mathbb{C}^n$ . A weighted composition operator induced by  $\psi$  with weight  $u$  is given by  $(uC_\psi f)(z) = u(z)f(\psi(z))$ , for  $z \in \mathbb{C}^n$  and  $f$  is entire function on  $\mathbb{C}^n$ . Let  $\phi \in C^2(\mathbb{C}^n)$  be a real valued function on  $\mathbb{C}^n$  such that  $mw_0 < dd^c\phi < Mw_0$  holds uniformly pointwise on  $\mathbb{C}^n$  for some positive constants  $m$  and  $M$  where  $d$  is the usual exterior derivative,  $d^c = \frac{i}{4}(\bar{\partial} - \partial)$ , and  $w_0 = dd^c|\cdot|^2$  is the standard Euclidean Kähler form on  $\mathbb{C}^n$ . The generalized Fock space  $\mathcal{F}_\phi^2(\mathbb{C}^n)$  consists of all entire functions  $f$  on  $\mathbb{C}^n$  such that  $\|f\|_{2,\phi}^2 = \int_{\mathbb{C}^n} |f(z)|^2 e^{-2\phi(z)} dv(z)$  is finite. In this talk, we characterize the Schatten  $p$ -class of weighted composition operators acting on the generalized Fock space for  $0 < p < \infty$ . (Received September 19, 2016)

1125-VB-2045 **Timothy I Myers\*** (tim-myer@hotmail.com). *Lebesgue Integration on a Banach Space with a Schauder Basis*.

This talk will feature the construction of a Lebesgue measure and integral on any Banach space with a Schauder basis. This theory has the advantage that the integral is computable from below as a limit of Lebesgue integrals on Euclidean space as the dimension  $n \rightarrow \infty$ , so that we may evaluate infinite dimensional quantities by means of finite dimensional approximation. We will discuss applications to Feynman's path integral and Gaussian measure. (Received September 19, 2016)

1125-VB-2057 **Kit C. Chan** (kchan@bgsu.edu) and **David Walmsley\*** (byrned@bgsu.edu). *A Constructive Approach to the Universality Criterion for Semigroups*.

A constructive approach to the Universality Criterion leads to some classical results, such as the universality of a Blaschke product for a sequence of composition operators. Using this approach, we also obtain some new results for the operator algebra of a Hilbert space. (Received September 19, 2016)

1125-VB-2542 **Lisa De Castro\*** (ldecastro@flsouthern.edu), Florida Southern College, Department of Mathematics, 111 Lake Hollingsworth Drive, Lakeland, FL 33801. *An extension of "Positive  $H^{1/2}$  Functions are Constants"*. Preliminary report.

In 1967 J. Neuwirth and D. J. Newman published a one page paper entitled "Positive  $H^{1/2}$  Functions are Constants". This theorem can be extended to finitely connected domains with no isolated boundary points. For these domains, the sufficient conditions can be relaxed so that functions need only be non-negative on one boundary curve for the result to hold. (Received September 20, 2016)

1125-VB-2569 **Ashok Aryal\*** (aaryal@ksu.edu), Mathematics Department, 138 Cardwell Hall, 1228 N. 17th Street, Manhattan, KS 66506. *Mean Value Theorem for general divergence form elliptic operators*.

The Mean Value Theorem (MVT) is one of the most fundamental theorems in the study of harmonic functions. The proof most often seen assumes twice differentiability of the functions, and it is not easy to follow. In his Fermi lectures in 1998, Caffarelli gave a short, intuitive proof which makes minimal assumptions of regularity

on the functions in question. In the same notes, he observed that although his proof looked like it needed to use some of the nicer properties of the Laplacian, in fact all he really needed was to be able to solve an appropriate obstacle problem in order to extend everything to general divergence form elliptic PDEs. In this talk, after giving some background of Caffarelli's proof of the MVT, and I will begin to discuss the obstacle problem and give some description of the essential elements of the proof of the MVT in general divergence form. (Received September 20, 2016)

1125-VB-2716 **Tom McNamara\*** ([thomas.mcnamara@swous.edu](mailto:thomas.mcnamara@swous.edu)), Mathematics Department, 100 Campus Drive, Weatherford, OK 73096. *Calculations with Generating Functions*. Preliminary report.

Given a sequence  $a_n$ , we say that the formal power series  $f(x) = \sum a_n x^n$  is the generating function for the sequence  $a_n$ . We will show how to create the generating functions for several classes of sequences. We will also indicate how to extend this idea to find generating functions for sequences of functions. (Received September 20, 2016)

1125-VB-2779 **Jason Hong Jae Park\*** ([hongjae79@gmail.com](mailto:hongjae79@gmail.com)), 701 South Mount Vernon Ave, San Bernardino, CA 92410. *A random measure algebra under convolution*.

The study of convolutions has become one of the most interesting areas of analysis. In this talk, a convolution of second order random measures on a locally compact abelian group is considered. With a newly defined (random) convolution, we construct a ring and a normed algebra of second order random measures. (Received September 20, 2016)

1125-VB-3073 **Prem M. Talwai\*** ([pmt55@cornell.edu](mailto:pmt55@cornell.edu)). *A Trace Operator for the Laplacian on the Sierpinski Gasket*.

As our world grows increasingly irregular, it is imperative that we reformulate our mathematical models to accurately describe this inherent natural roughness. The classical "smooth" analysis on manifolds is inadequate for a comprehensive study of the dynamic fractal phenomena that permeate nature. In the last two decades, a theory of analysis on fractals has been developed that centers on the construction of a Laplacian on "rough" sets such as the Sierpinski gasket SG (also called the Sierpinski triangle). This project characterizes the trace of the domain of this Laplacian operator ( $\text{dom } \Delta$ ) to the boundary of the gasket. In particular, if SG is embedded in  $\mathbb{R}^2$  such that its base coincides with the unit interval  $I$ , and  $u \in \text{dom } \Delta \cap C(SG)$ , we apply a biharmonic spline approximation scheme and a novel convergence condition for the Laplacian to elucidate the regularity properties of  $u|_I$ . We show that the trace of  $\text{dom } \Delta$  is contained in the Besov space  $B_{\alpha}^{2,2}(I)$  with  $\alpha = \frac{\ln 3}{\ln 4} + \frac{1}{2}$ . (Received September 20, 2016)

## General Session on Applied Mathematics

1125-VC-40 **Alrazi M Abdeljabbar\*** ([aabdeljabbar@pi.ac.ae](mailto:aabdeljabbar@pi.ac.ae)), P.O Box 2533, The Petroleum Institute, Abu Dhabi, United Arab Emirates. *Exact solutions to a generalized (3+1)-dimensional nonlinear partial differential equations*.

Based on Hirota bilinearization method, the Pfaffian technique is employed to construct an explicit exact solutions to 3+1- dimensional nonlinear partial differential equations of KP type and Jimbo-Miwa type. New coupled system will be introduced using a procedure called Pfaffianization. (Received June 15, 2016)

1125-VC-44 **Michael A Norton\*** ([michaelanortonster@gmail.com](mailto:michaelanortonster@gmail.com)), 12006 Waterside View Dr. #12, Reston, VA 20194. *Magnetic constant determined; diffuses uncertainty and integrates scales of measure*.

A magnetic constant of  $2.52 \times 10^{11}$  unities/field (M) updates our calculus. To measure small-scale but large-volume chemistry variances, M/amu of an element outputs the element's magnetic oscillation, or m-point. Eg. (H m-point  $\times 2$ ) + (O m-point  $\times 1$ ) = (M-point for water). The constant unifies neutrino scale, mRNA, DNA, chromosomal, molecular, atmospheric, and astronomical fields of unity in time and space, derived from in-field data, Einstein and company's Standard model, modern computing applications, Stephen Hawking's modern cosmic background temperature equation, and a technological progression series based on standard milestones of human development. It can be used to predict cures for disease, efficacy and diagnostic differentials of existing medicine and therapies, anthropogenic and other climate change variables, the point at which lightspeed travel will be accomplished on Earth, locations of other intelligent civilizations, molecular composition and reciprocity among large-body cosmological objects, and neuronal, cellular, and consciousness quanta in humans generally and

specially. The constant can be used to verify existing mathematical relationships, mechanisms, and equations, and advance the field along with its other co-dependent fields. Efficient! (Received June 19, 2016)

1125-VC-152 **Jennifer A. Crodelle (Kile)\*** ([kilej@rpi.edu](mailto:kilej@rpi.edu)), Amos Eaton 301, RPI, 110 8th St, Troy, NY 12180, and **Gregor Kovacic** and **David Cai**. *The Role of Electrotonic Junctions between Excitatory Neurons in the Cortex*. Preliminary report.

Global oscillations in the brain have been linked to synchronized neuronal activity, which has been shown to contribute to cognitive processes such as perception, motor performance, learning and memory. Electric coupling through gap junctions may facilitate the emergence of synchronized oscillations, and influence their properties. Gap junctions between inhibitory neurons in the mammalian cerebral cortex have been well studied, but electrical synapses between excitatory, pyramidal neurons, or electrotonic junctions, have only recently been discovered experimentally. In this study, we follow experimental data to construct a detailed, comprehensive model with both synaptic and electric coupling for both excitatory and inhibitory neurons using a modified version of the Hodgkin-Huxley equations. We organize the neurons on a grid to capture the highly structured spatial properties of a network containing both synaptic and gap-junction connections, and to ensure that the probability of neurons being coupled is dependent on their location within the network. Using this model, we find that the addition of gap junctions between inhibitory neurons creates oscillations in the network, and further show that sparse pairs of electrotonic junctions are optimal for tighter oscillations. (Received September 16, 2016)

1125-VC-190 **Laxmi P. Paudel\*** ([laxmi.paudel@asurams.edu](mailto:laxmi.paudel@asurams.edu)), 1447 US Highway 19 South, Unit 1-C, Leesburg, GA 31763, and **Joseph Iaia**. *Porous Medium Equation and Its one parameter family of solutions with degenerate interface*. Preliminary report.

We reduce the porous medium equation into second order ordinary differential equation and prove the existence of one parameter family of solutions. The solution has degenerate interface that advances at a constant speed. We show that the interfaces occur under a very general initial conditions and the solutions are stable under certain class of perturbations. We also discuss the relevance of the solution to the flow of a thin layer of fluid on a horizontal surface under the action of gravity. (Received August 10, 2016)

1125-VC-267 **Buddhi R Pantha\*** ([bpantha@abac.edu](mailto:bpantha@abac.edu)), Department of Science and Mathematics, Abraham Baldwin Agricultural College, Tifton, GA 31794, and **Suzanne Lenhart** and **Judy Day**. *Optimal control applied to a differential equation model for an anthrax epizootic*.

Anthrax is a fatal disease caused by a gram positive, spore forming bacteria called *Bacillus anthracis*. The disease is endemic to several national parks and one of the main causes of herbivore decline. Most anthrax infected animals face inevitable death and each infected carcass deposits massive number of spores into the surrounding environment. Thus, controlling new infections through vaccination and eliminating spread through proper carcass disposal are the only feasible ways to effectively control the disease when an outbreak occurs. In this talk, a system made up of parabolic partial differential equations together with ordinary differential equations will be presented and effect of the two most commonly used controls of vaccination and carcass disposal on disease transmission will be investigated. Some numerical results will also be presented. (Received August 21, 2016)

1125-VC-337 **Tania Hazra\*** ([thazra@crimson.ua.edu](mailto:thazra@crimson.ua.edu)), 1103 14th Street, Apt 26A, Tuscaloosa, AL 35401. *Stable Operator Splitting Method for Free Energy Calculation of One Atom Model*. Preliminary report.

The work explores the stability impact of the novel unconditionally stable operator splitting methods for solving the time dependent nonlinear Poisson-Boltzmann (NPB) equation for the electrostatic analysis of solvated biomolecules. In a pseudo-transient continuation solution of the NPB equation, the nonlinear term is analytically integrated, so that the difficulties in direct treatment of the strong nonlinearity can be bypassed.

There are several methods to solve the NPB equations. We are interested in Alternating Direction Implicit (ADI) schemes and Locally-One Dimensional (LOD) schemes. The ADI methods are known to be conditionally stable, although being fully implicit. On the other hand, LOD scheme is computationally less expensive and LOD scheme based on implicit Euler integration is more stable than ADI schemes.

It has been observed that there is a noticeable difference between the linearized Poisson Boltzmann Equation result and ADI-based NPB results in steady state solutions. That motivated us to analyze the ADI schemes and later on LOD schemes to detect the factor for which the stability is getting affected. (Received August 27, 2016)

- 1125-VC-677 **Jashmon Rana\*** (jr937963@cameron.edu), 2800 W Gore Blvd, Lawton, OK 73505, **Gokul R Kadel** (gkadel@cameron.edu), 2800 W. Gore Blvd, Lawton, OK 73505, and **Narayan Thapa** (nthapa@cameron.edu), 2800 W. Gore Blvd, Lawton, OK 73505.  
*Comparison of Numerical Solutions of Advection-Reaction-Dispersion Model.*

Advection-Reaction-Dispersion model is instrumental to study transportation of chemical or biological contaminants through subsurface aquifer systems. To better understand the movements of contaminants in porous media, we develop an Initial Boundary Value Problem (IBVP) and solve both analytically and numerically. In particular, solutions from Forward Time Central Space scheme, Backward Time Central Space scheme, Crank-Nicolson scheme, and Finite Element method are implemented. Numerical experiments are presented and error analyses are carried out. (Received September 20, 2016)

- 1125-VC-678 **Nirjal Shrestha\*** (ns931479@cameron.edu), 2800 W. Gore Blvd, Lawton, OK 73505, and **Ioannis K. Argyros** (ioannisa@cameron.edu), 2800 W. Gore Blvd, Lawton, OK 73505.  
*Convergence of Iterative Methods under Weak Conditions.*

The convergence order of iterative methods defined on the real line is usually determined using higher derivatives and Taylor expansions although these derivatives do not appear in the methods. In the present study, we show convergence of some popular iterative methods using only hypotheses on the first derivative. This way we expand the applicability of these methods. We use the computational order of convergence of the method. Numerical examples are also presented to show that our results can be used to solve equations in cases that the results in earlier studies cannot be used. (Received September 09, 2016)

- 1125-VC-688 **Yimin Zhong\*** (yzhong@math.utexas.edu), **Rongting Zhang** (rzhang@math.utexas.edu) and **Kui Ren** (ren@math.utexas.edu). *FMM Preconditioner for Radiative Transport Equation with isotropic coefficients.*

We propose in this work a fast numerical algorithm for solving the equation of radiative transfer (ERT) in isotropic media. The algorithm has two steps. In the first step, we derive an integral equation for the angularly averaged ERT solution by taking advantage of the isotropy of the scattering kernel, and solve the integral equation with a fast multipole method (FMM). In the second step, we solve a scattering-free transport equation to recover the original ERT solution. Numerical simulations are presented to demonstrate the performance of the algorithm for both homogeneous and inhomogeneous media. This is a joint work with Kui Ren and Rongting Zhang. (Received September 09, 2016)

- 1125-VC-713 **Mingchao Cai\*** (cmchao2005@gmail.com), 1128 E Belvedere Av, Apt B, Baltimore, MD 21239. *Fast solvers for poroelastic models.*

Poroelastic models have been widely used in geoscience and biomechanics. We consider to apply staggered-grid Finite Difference method to discretize the quasi-static poroelastic model. Our goal is developing preconditioners to accelerate the convergence rate of the Krylov subspace methods applied to the resulting saddle point problems. The approximation of the inverse of the Schur complement is derived by using a Fourier analysis approach. For constant-coefficient problem, we show that if exact Poisson solvers are employed, the preconditioned system has clustered eigenvalues values which are robust with respect to mesh refinement and parameters. The analysis reflects the stability of the stagger-grid Finite Difference discretization and the effects of boundary treatment. Numerical experiments are provided to verify the effectiveness of the proposed preconditioners. (Received September 09, 2016)

- 1125-VC-763 **Kulwinder Singh\*** (kbg111@gmail.com), India, and **Rajneesh Kumar**. *Stresses in Micropolar thermoelastic Elastic Solid due to Ramp-type increase in Thermal and Normal Loading.*

This article deals with the study of thermoelastic interactions of fractional order theories in micropolar elastic solid, whose boundary is subjected to (i) ramp-type heating (ii) ramp type normal loading. The generalized theories of thermoelasticity of integer and fractional order with one relaxation time are used to investigate the problem. After developing a mathematical model, Laplace and Fourier transform have been used to find a common solution corresponding to different theories. The inversion of transforms is obtained numerically and results obtained are compared graphically. Some particular cases of interest are also deduced from the present study (Received September 11, 2016)

1125-VC-801 **Samundra Regmi\*** (samundra.regmi@cameron.edu), 2800 W Gore Blvd, Lawton, OK 73505, and **Parshuram Budhathoki** (pbudhath@cameron.edu), 2800 W. Gore Blvd, Lawton, OK 73505. *Diffie-Hellman key exchange protocol and its software implementation.* Preliminary report.

In 1976, Whitefield Diffie and Martin Hellman introduced the first practical method for establishing a common key between two parties over an unsecured communication channel, which is also known as Diffie-Hellman key exchange protocol. This is one of the widely used protocols in different cryptographic algorithms (ex. DES, AES, HMAC). So, implementation of this protocol is of high interest. In this project we are primarily focused on software implementation of Diffie-Hellman key exchange protocol using Python. We can find many work related to this area in the literature. We will also study these existing implementations and will try to optimize software implementation of this protocol using Python. (Received September 12, 2016)

1125-VC-1269 **Wenjing Zhang\*** (wenjing.zhang@ttu.edu), **Lindi Wahl** and **Pei Yu**. *Recurrent Viral Infection May Need No Exogenous Trigger.*

Recurrent viral infection is characterized by short episodes of high viral reproduction separated by long periods of relative quiescence. This recurrent pattern is observed in many persistent infections, including the “viral blips” observed during chronic HIV infection. Previous works model viral blips by incorporating either stochastic components or forcing terms as immune stimulation. We present an established 4-d HIV antioxidant-therapy model which exhibits viral blips, take advantage of dynamical systems theory and bifurcation theory to reinvestigate the 4-d model, and show that an increasing, saturating infectivity function contributes to the recurrent behavior. A hypothesis for the existence of viral blips in a deterministic in-host infection model is proposed and employed to derive the simplest (2-d) blips-generating infection model. The corresponding paper, titled *Viral Blips May Not Need a Trigger: How Transient Viremia Can Arise in Deterministic In-Host Models*, was published in *SIAM Review* in 2014. (Received September 15, 2016)

1125-VC-1281 **Yinlin Dong\*** (yinlin@uta.edu), University of Texas at Arlington, Department of Mathematics, Arlington, TX 76019, and **Chaoqun Liu**. *Solving Poisson's Equations Using Buffered Fourier Spectral Method.* Preliminary report.

We propose a numerical method based on fast Fourier transform (FFT) algorithm to solve elliptic partial differential equations. To illustrate our buffered Fourier spectral method (BFSM), we begin with solving ordinary differential equations for one variable. By implementation of buffer zone, we will make the source function and its derivative periodic on the boundaries, so that FFT can be applied on the extended domain. Once we obtain the numerical results, we will delete the buffer zone and recover the original solution. Compared with the regular FFT, our method can improve the error order from  $10^{-4}$  to  $10^{-9}$  when the grid size  $N = 128$  with little extra computation cost. We then apply BFSM to solve Poisson's equations with non-periodic boundary conditions. As shown in examples, our method has gained high order accuracy and less computation time compared with the second order finite difference method. The method will be further used for simulation of transitional and turbulent flows. (Received September 15, 2016)

1125-VC-1310 **Sharee Brewer\*** (snbrewer1@my.fisk.edu), 411 Annex Ave, Apt B4, Nashville, TN 37209, **Taylor Swett** (tswett26@my.fisk.edu), 411 Annex Ave, Apt B4, Nashville, TN 37209, and **Qingxia Li** (qli@fisk.edu), 411 Annex Ave, Apt B4, Nashville, TN 37209. *A predator-prey model for the ecological system in a lake with the effect of acid rain.*

Acid rain decreases the pH level in a lake and the lake acidity has an effect on organisms and can reduce their body size, reproduction capacity, egg viability and mortality rate. In this paper, a predator-prey model consisting of algae (cyanobacteria), an herbivore (*Daphnia magna*), and a predator (yellow perch) is developed and the coefficient functions governing dynamics of growth rate and death rate of organisms with respect to pH value in this model are determined. Parameter values in this model are adopted from experimental data in published references. This presentation is based on a summary report on an ecological modeling problem from Graduate Student Math Modeling Camp at Rensselaer Polytechnic Institute in 2009. (Received September 16, 2016)

1125-VC-1313 **Ghanshyam Bhatt\*** (gbhatt@tnstate.edu), 4409 Kirkbrook ct, Nashville, TN 37221. *Incoherent Matrices for Compressed Sensing.* Preliminary report.

The maximum of the absolute values of the inner products of the columns of a matrix is known as the coherence of that matrix. In applications, namely in compressed sensing, the matrix in question is not a square matrix, it is a fat matrix. The set of columns of the matrix is said to be equi-angular if the absolute values of these inner products is a constant for any pair of columns. These are highly desirable matrices but difficult to construct in higher dimensions. The lowest coherence is given by the well known Welch bound. Under mild conditions, these

columns form a frame. The optimal (with lowest coherence) ones are known as the Grassmanian frames. In this talk, we look at these columns as a frame and talk about reducing the coherence of the frame. (Received September 16, 2016)

1125-VC-1325 **Annela R Kelly\*** (adamannika@cox.net), 10 Stanley Avenue, Barrington, RI 02806. *Multiplayer Fibonacci Nim*. Preliminary report.

Multiplayer game analysis is widely used in computer games, business models, and conflict situations. This area of mathematics was developed by John Nash, Nobel Prize winner and the inspiration for the movie "A Beautiful Mind". My presentation analyzes multiplayer Fibonacci Nim game with collaboration among players that are selected before the play and mathematically determined the winning strategies. I investigate the multiplayer Fibonacci Nim with alliance system using n-by-n player preference matrix that was defined by Krawec. The talk will summarize the results for the Fibonacci Nim with standard alliance matrix. (Received September 16, 2016)

1125-VC-1337 **Timo Heister** and **Muhammad Mohebujjaman\*** (mmohebu@g.clemson.edu), Clemson, SC 29634, and **Leo Rebholz**. *Efficient Numerical Methods for Magnetohydrodynamics Flow*.

Firstly we propose, analyze, and test a  $\theta$ -timestepping-method for MHD which decouples the system into two Oseen problems at each timestep, yet maintains unconditional stability with respect to the time step size. The proposed method chooses  $\theta \in [0, 1]$ , dependent on the viscosity  $\nu$  and magnetic diffusivity  $\nu_m$ , so that unconditionally stability is achieved and gives temporal accuracy  $O(\Delta t^2 + (1 - \theta)|\nu - \nu_m|\Delta t)$ . In practice  $\nu$  and  $\nu_m$  are small, and so the method behaves like second order. We show the  $\theta$ -method provides excellent accuracy in cases where usual BDF2 is unstable. We also proposed another algorithm for computing flow ensembles under uncertainties in initial or boundary data. The ensemble average of  $J$  realizations is approximated through a clever algorithm that, at each time step, uses the same matrix for each of the  $J$  system solves. Hence, preconditioners need built only once per time step, and the algorithm can take advantage of block linear solvers. Additionally, an Elsässer variable formulation is used, which allows for a stable decoupling of each MHD system at each time step. We prove stability and convergence of the algorithm, and test it with two numerical experiments. (Received September 16, 2016)

1125-VC-1437 **Xin Luo\*** (xluo@crimson.ua.edu) and **Min Sun**. *Development of Modal Interval Algorithm for Solving Continuous Minimax Problems*.

While there are a large variety of effective methods developed for solving more traditional minimization problems, much less success has been reported in solving the minimax problem. Continuous minimax problems can be applied to engineering, finance and other fields. Miguel Sainz in 2008 proposed a modal interval algorithm based on his semantic extensions to solve continuous minimax problems. We developed an improved algorithm using modal intervals to solve unconstrained continuous minimax problems. A new interval method is introduced by taking advantage of both the original minimax problem and its dual problem (called maxmini problem). The new algorithm is implemented in the framework of separate as well as uniform partitions of the search domain. Various improvement techniques including more bisecting choices, sampling methods and deletion conditions are applied to make the new method more powerful. Preliminary numerical results provide promising evidence of its effectiveness. (Received September 16, 2016)

1125-VC-1448 **Ryan Joseph Nicely\*** (rnicely@asu.edu), School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287, **Ali S Cole**, School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287, and **Mohammed Moustaoi**. *Wave-Induced Momentum Transport through a Non-Uniformly Stratified Thin Layer near the Tropopause*.

This research gives a study of the transmission and reflection of momentum induced by atmospheric gravity waves generated by mountains. The wave-induced transport of momentum through a stably stratified thin layer near the tropopause is controlled by the change in stability above and below the layer, the thickness and the flow. The thickness of the tropopause and the change in stability across it is found to play a major role in regulating the amount of wave transmitted and reflected. This role is evidenced by deriving analytically and numerically the coefficients of transmission and reflection under various representative profiles of stability. The coefficients are computed from the least squares method. The results demonstrate how the stability in the troposphere, tropopause and the stratosphere modifies wave transmission across the tropopause. (Received September 16, 2016)

1125-VC-1450 **Alexandra S Cole\*** (ascole4@asu.edu), School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287, and **Ryan J Nicely** and **Mohamed Moustauoui**. *A Numerical Simulation of Mountain Waves*. Preliminary report. In this talk I will present atmospheric dynamics associated with gravity waves as they propagate through the atmosphere, specifically focusing on the propagation of these waves through the troposphere and stratosphere. Gravity waves impact large scale atmospheric dynamics and can contribute to the general circulation at upper atmospheric levels. I will focus on a real case scenario of mountain waves. For the real case, I will analyze data from a simulation using a high-resolution numerical prediction model. The simulation will be conducted over Owens Valley, CA, a mountainous region, over a 48-hour period in March, 2006, using multi-nested domains. The results illustrate the impact of the change in stability above and below the tropopause on the amount of momentum flux transmitted to upper atmospheric levels. (Received September 16, 2016)

1125-VC-1452 **Duane C Harris\*** (dharri31@asu.edu), School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287, and **Mohamed Moustauoui** and **Collin Kofroth**. *Impact of Stability above, below and within the Tropopause on Mountain Wave-Induced Momentum Transfer to the Stratosphere*. Preliminary report.

An analytical study of upward propagating gravity waves generated by flows over topography in the presence of non-uniform stratification is presented. These waves are partially transmitted and reflected as they reach the tropopause depending on the behavior of the stability profile in the upper troposphere and lower stratosphere. The tropopause is viewed as a thin stably stratified layer characterized by a sharp stratification with high values of stability in its vicinity compared to layers above and below as reported in some observations. The dynamics of gravity waves under the Boussinesq approximation is governed by Taylor-Goldstein equation. Analytical solutions of this equation are derived for typical stability profiles with different values below, within and above the tropopause. It is found that the amount of waves transferred to the stratosphere is significantly impacted by the stability profile. (Received September 16, 2016)

1125-VC-1527 **Parshuram Budhathoki\*** (pbudhath@cameron.edu), 2800 W Gore Blvd, Lawton, OK. *Elliptic Curve based RFID authentication scheme and its software implementation*. Preliminary report.

Elliptic Curve Cryptography (ECC) is one of the active areas of research because of its shorter key sizes. Radio Frequency Identification (RFID) has been widely used techniques in different applications, such as medical drugs management. In this project we are going to focus on RFID authentication scheme using ECC. We will also look over its software implementation using Python. (Received September 17, 2016)

1125-VC-1551 **Connor D Lincoln\*** (cdlincol@asu.edu), School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287, and **Carl Gardner**. *Numerical Simulation of the Protostellar Jet HH24 C/E*.

I will describe a model of the evolution of the supersonic protostellar jet HH24. We use a positivity preserving, third order WENO scheme for calculations across discontinuities, parallelized with OpenMP and MPI. The Hubble Telescope image of HH24 depicts large, ellipsoidal cavities which are created by the jet. We show that these cavities are being created primarily by the pulsing of the jet, rather than by a constant jet flow. (Received September 17, 2016)

1125-VC-1578 **Suzanne M. O'Regan\*** (smoregan@ncat.edu) and **Danielle L. Burton**. *Leading indicators of bifurcations in ecological systems*. Preliminary report.

Many ecological systems exhibit bifurcations. The bifurcation may be anticipated because prior to reaching the dynamical threshold, the system gradually loses stability ('critical slowing down'). Signatures of critical slowing down may be detectable through summary statistics, but the influence of environmental and demographic stochasticity on statistical patterns is unclear. Here, we consider the simplest possible models that exhibit transcritical, saddle-node and pitchfork bifurcations. Noise was assumed to be either additive, multiplicative or demographic in nature. In each case, linearization of the resulting stochastic differential equation leads to an Ornstein-Uhlenbeck process for the fluctuations around equilibrium. We derived expressions for leading indicator statistics, including variance, autocorrelation and power spectrum. Trends in leading indicators of each bifurcation depend on noise type. The ability to classify trends of summary statistics for a broad class of ecological models enhances our understanding of how critical slowing down manifests in systems approaching a transition. (Received September 18, 2016)

1125-VC-1592 **Collin Kofroth\*** ([ckofroth@asu.edu](mailto:ckofroth@asu.edu)) and **Don Jones**. *Global Existence of Solutions to Shallow Water Equations with Alternative Frictional Operators*.

The shallow water equations (SWE) model flow in a geographically-balanced fluid. These equations are of particular interest for atmospheric scientists and oceanographers. In order to help justify the schemes used in these models, one must consider when/if solutions exist to the SWE, and if they exist, the length of time that they exist. Mathematically, they are often equipped with the Laplace operator (diffusion). However, since damping and bi-harmonic operators also carry relevance in application, we will discuss the global existence of solutions to the SWE using these operators under initial data constraints by the use of energy estimates. (Received September 19, 2016)

1125-VC-1625 **Hieu Nguyen\*** ([nguyen084@connect.wcsu.edu](mailto:nguyen084@connect.wcsu.edu)), 23 Scuppo Rd, Unit 5-1, Danbury, CT 06811, and **Julia Yu** ([juliayu1@live.com](mailto:juliayu1@live.com)), **Sarah Zhao** ([brightheart@gmail.com](mailto:brightheart@gmail.com)) and **Xiaodi Wang** ([xiaodiwang1@yahoo.com](mailto:xiaodiwang1@yahoo.com)). *Pseudo Quantum Steganography and M-Band Wavelet based Denoising in Color Barcode*. Preliminary report.

Due to their high data capacity, color barcodes have garnered much interest from researchers and corporations. In our research, we looked to enhance the applications of a color barcode. First, we stacked multiple color channels together to create a color barcode with a higher capacity. The resulting algorithm creates a color barcode with greater capacity than the black-and-white QR code. We then used this color barcode as host data and embedded secret information, such as images and audio, into the color channels of the barcode through the use of M-Band Wavelets and pseudo-quantum steganography. Finally, we investigated the denoising of a noisy, unreadable color barcode; the denoising of such a barcode was accomplished through the use of Patch Group Prior based Denoising (PGPD) and wavelets. This denoising algorithm was capable of removing heavy noise from the color barcodes, as seen when evaluated by the peak signal-to-noise ratios (PSNR) of various noisy and denoised images. Ultimately, we have achieved a higher capacity and robustness of a color QR code through the stacking of color channels together with the use of new steganography and denoising algorithms. Hopefully, such research could contribute to future commercial use of the color barcode. (Received September 18, 2016)

1125-VC-1633 **Ahmed T. Ali\*** ([math@uta.edu](mailto:math@uta.edu)), 411 S. Nedderman Drive, 478 Pickard Hall, Arlington, TX 76019-0408, and **Li Ren-Ceng**. *A Bisection Method for the Banded Hyperbolic Quadratic Eigenvalue Problem*.

It is well-known that the eigenvalues of a Hermitian matrix in a given interval can be approximated within a predefined error tolerance using the bisection method as a direct application of the Sylvester's Law of Inertia. In this thesis, we will develop a bisection method for the hyperbolic quadratic eigenvalue problem (HQEP) which is guaranteed to have  $2n$  real eigenvalues for a problem of size  $n$ . A number of numerical methods are available to solve HQEPs. Matlab's `polyeig` uses the QZ algorithm on the problem after linearizing it to a pencil of size  $2n$ . Another approach is by finding a solvent matrix. Both approaches ignore any banded structure of the problem. For the tri-diagonal HQEPs, an approach to approximate the eigenvalues by efficiently solving the characteristic equation was also proposed. The method can't be applied to higher banded HQEPs efficiently. Our method will avoid converting the HQEP to a definite pencil of order  $2n$  by working on the HQEP directly taking into consideration any banded structure of the problem. Our method can be applied to large banded HQEPs and produces more accurate eigenvalue approximations compared to the approaches stated. (Received September 18, 2016)

1125-VC-1654 **Ram C Neupane\*** ([neupaner@uwv.edu](mailto:neupaner@uwv.edu)), 800 W. Main Street, Whitewater, WI 53190. *Connecting Regional-scale Tree Distribution Models with Seed Dispersal Kernels*.

Regional scale forest distribution models take climate and geographic variables as input and are therefore helpful for long-term decision support and climate adaptation planning. Generally local processes of germination and seedling survival are resolved probabilistically with explanatory variables such as elevation, and regional presence-absence data. Without having detailed site-level mechanistic processes, these models accurately reflect the fate of seedlings after seeds have arrived at a site. How seeds are distributed in these models, however, is far more problematic since it is difficult to accurately parameterize dispersal models using large-scale presence-absence data, particularly for actively dispersed tree species. The challenge is that variables conditioning vertebrate seed are not represented in large scale distribution models, and in fact vary much smaller than the smallest pixel size for the distribution model. The homogenized seed dispersal kernel (HSDK) offers a tool to make use of this scale separation. In this paper we develop scenarios for seed dispersal on landscape scales, linking small-scale variables with dispersal probabilities on large scales as predicted by HSDKs. (Received September 18, 2016)

1125-VC-1678 **Bal K. Khadka\*** (bkhadkka@gmc.edu), 221 New Petersburg Dr, Apt D, Martinez, GA 30907, and **Spyros M. Magliveras**. *Techniques in Lattice Basis Reduction*.

The credit on *reduction theory* goes back to the work of Lagrange, Gauss, Hermite, Korkin, Zolotarev, and Minkowski. Modern reduction theory is voluminous and includes the work of A. Lenstra, H. Lenstra and L. Lovasz who created the well known LLL algorithm, and many other researchers such as L. Babai and C. P. Schnorr who created significant new variants of basis reduction algorithms. In this paper, we propose and investigate the efficacy of new optimization techniques to be used along with LLL algorithm. The techniques we have proposed are: i) *hill climbing (HC)*, ii) *lattice diffusion-sub lattice fusion (LDSF)*, and iii) *multistage hybrid LDSF-HC*. The first technique relies on the sensitivity of LLL to permutations of the input basis  $B$ , and optimization ideas over the symmetric group  $S_m$  viewed as a metric space. The second technique relies on partitioning the lattice into sublattices, performing basis reduction in the partition sublattice blocks, fusing the sublattices, and repeating. We also point out places where parallel computation can reduce run-times achieving almost linear speedup. The multistage hybrid technique relies on the lattice diffusion and sublattice fusion and hill climbing algorithms.

(Received September 18, 2016)

1125-VC-1827 **Ramanjit K. Sahi\*** (sahir@apsu.edu), **Chenchutta D. Jackson** and **Samuel N. Jator**. *Using continued fractions with logarithmic basis functions to overcome singular points via a nonlinear one-step method*. Preliminary report.

Many numerical methods based on rational interpolation have been successful in finding numerical solution of singular initial value problems (IVPs). In this study, we develop a special rational interpolation function expressed as a finite continued fraction with logarithmic basis functions to solve singular IVPs. Numerical experiments performed show that the nonlinear one-step method based continued fractions is more accurate than improved Euler's method. (Received September 19, 2016)

1125-VC-1847 **Benjamin D Ritz\*** (ritzbd@clarkson.edu). *Hybrid Optimization for Mixed-Integer Nonlinear Problems via a Genetic Algorithm and Implicit Filtering*.

Many design problems in engineering and the sciences can be posed as mixed-integer nonlinear programming (MINLP) problems, requiring both real-valued variables and integer variables. This formulation is non-differentiable. A binary genetic algorithm (GA) is one derivative-free method that could be applied. However, GAs are weak on a local scale and can require many function evaluations to identify an optimum. To address this weakness, hybrid optimization can be used. Hybrid optimization involves combining methods to capitalize on the strengths of each method while attempting to overcome weaknesses in functionality. This work explores the hybridization of a genetic algorithm with implicit filtering, which expands the capabilities of implicit filtering to MINLP problems. Implicit filtering is a derivative-free optimization method for solving noisy problems that uses an adaptive stencil to approximate the gradient of the objective. We demonstrate the hybrid algorithm on a suite of test problems and an application to simulation-based modeling in water resources. In addition, sensitivity analysis is performed on optimization parameters to examine problem-dependent parameter selection. (Received September 19, 2016)

1125-VC-1932 **Peter Jantsch\*** (pjantsch@gmail.com), 208 Ayres Hall at, 1403 Circle Dr., Knoxville, TN 37996. *Accelerating stochastic collocation methods for PDEs with random coefficients*.

Sparse grid stochastic collocation (SC) methods are a valuable tool for solving problems in uncertainty quantification, yet they suffer from a dramatic increase in costs in high-dimensions. In this talk, we apply SC methods to solve partial differential equations (PDEs) with random coefficients, and exploit multilevel and hierarchical structure in the spatial and stochastic approximation schemes to drastically improve the computation efficiency of the method. We provide a thorough analysis of the savings, and present numerical examples of our methods for both linear and non-linear random PDEs. (Received September 19, 2016)

1125-VC-1942 **Danielle Sienko**, **Paul Ache** and **Yun Lu\*** (lu@kutztown.edu), Mathematics Department, Kutztown University, Kutztown, PA 19530, and **Francis Vasko** and **Ted Witryk**. *An undergraduate uses O.R. to improve her university's final exam schedules*.

In this talk, we describe how an undergraduate student analysed the final exam schedules at Kutztown University in order to reduce student conflicts defined to be a student having three exams scheduled on the same day. The approach she developed, based on a balanced bin packing algorithm, was tested using actual data from the Fall 2015 semester and resulted in a 42% reduction in student conflicts. This approach, because of its simplicity and intuitive appeal, was widely accepted by the Kutztown University faculty and administrators and is implemented for the Fall 2016 semester at Kutztown University. (Received September 19, 2016)

1125-VC-1971 **Katelyn Leisman\*** (plaisk@rpi.edu), **Gregor Kovacic** and **David Cai**. *The Effectively Linear Behavior of the Nonlinear Schrödinger Equation.*

The linear part of the Nonlinear Schrödinger Equation (NLS) ( $iq_t = q_{xx}$ ) has dispersion relation  $\omega = k^2$ . We don't expect solutions to the fully nonlinear equation to behave nicely or have any kind of effective dispersion relation like this. However, I have seen that solutions to the NLS are actually weakly coupled and are often nearly sinusoidal in time with a dominant frequency, often behaving similarly to modulated plane waves. In fact, highly nonlinear solutions behave more and more effectively linearly. (Received September 19, 2016)

1125-VC-1992 **Rebecca C Conley\*** (rconley@saintpeters.edu), 121 Glenwood Ave, Loyola Hall Room 25, Jersey City, NJ 07306, and **Tristan J Delaney** and **Xiangmin Jiao**. *High-Order Adaptive Extended Stencil Finite Element Method (AES-FEM) on Tangled Meshes.* Preliminary report.

The finite element methods (FEM) are widely used for solving partial differential equations, but they are severely dependent on element quality. Additionally, they can fail if the mesh is tangled, that is, any of the elements are inverted (i.e. the Jacobian is negative). Adaptive Extended Stencil Finite Element Method (AES-FEM) is a generalization of FEM, which is insensitive to mesh quality. We propose an extension of AES-FEM that is stable on tangled meshes. AES-FEM achieves this by replacing the traditional basis function (hat functions) with generalized Lagrange polynomial basis functions, which are computed using weighed least squares. Numerical results demonstrate that AES-FEM improves the accuracy and stability of traditional FEM for 2D and 3D elliptic PDEs and a linear elasticity problem. (Received September 19, 2016)

1125-VC-2017 **Janak R Joshi\*** (janakrajjoshi@my.unt.edu), 2413 West Prairie street apt#19, Denton, TX 76201, and **Joseph Iaia**. *Existence of Solutions for semilinear problems with prescribed number of zeros on exterior domains.*

In this paper we prove the existence of an infinite number of radial solutions of  $\Delta(u) + f(u) = 0$  with prescribed number of zeros on the exterior of the ball of radius  $R > 0$  centered at the origin in  $\mathbb{R}^N$  where  $f$  is odd with  $f < 0$  on  $(0, \beta)$ ,  $f > 0$  on  $(\beta, \infty)$ .

(Received September 19, 2016)

1125-VC-2043 **Karel Marshall**, **Paul Vienhage\*** (paul.vienhage@emory.edu), **Heather Barcomb** and **William A. Black**. *War-Gaming Applications for Achieving Optimum Acquisition of Future Space Systems.*

In 2014, the federal government spent nearly half a trillion dollars on contractor projects. Motivated by the Space Modernization Initiative and the Defense Innovation Initiative, the Department of Defense wants to develop an algorithm to optimize the acquisition of new technologies. This Unified Game-based Acquisition Framework Advanced Game-based Mathematical Framework makes use of game theory, probability and statistics, non-linear programming and mathematical models to model negotiations between governmental agencies and private contractors. This project focuses on generating the optimum Program and Technical Baseline (PTB) solution and its corresponding acquisition strategy with associated contract incentives for the Fixed-Price Incentive Firm and Fixed Price Seal Bid contract types. An "optimum solution" is obtained by compromising the system and acquisition objectives to achieve low lifecycle cost/total ownership cost, innovative design, decreased acquisition time, while meeting warfighter needs. This project culminates in a collection of MATLAB (MathWorks) programs which model contractor and governmental interactions. The newly developed strategy shows strong convergence to nash equilibrium values for the bidding games and successful selection of PTB solutions. (Received September 19, 2016)

1125-VC-2084 **Mary Barker\*** (mary.barker@go.tarleton.edu), 328 w collins st. Apt 4, stephenville, TX 76401. *Cleaner Air Through Parallelized Simulations of Novel Mathematical Models of Gas-Surface Interactions.*

A new particle based theory of thermophoresis is used to model gas-surface interactions from Random Billiard Dynamical Systems (RBDS). This particular RBDS model has been expanded from the typical single-particle model to accommodate high density particle simulations that are necessary for more complex interactions to model thermophoretic effects. Due to the high number of simple calculations required at each time step of simulation, the model is ideal for parallelizing. A parallel algorithm is introduced that models gas particles in a 3 dimensional simulation. The main objective for this work is to improve the model for thermophoresis being used for pollution control applications. Some results are presented that demonstrate the agreement of theory with experimental result. In addition, the runtime speedup from the serial to the parallel algorithm is shown. Finally, future developments are discussed. (Received September 19, 2016)

1125-VC-2154 **Aaron H Chen\*** (ahc232@cornell.edu). *Neural codes, undecidability, and a new class of local obstructions.*

Given an intersection pattern of open sets in Euclidean space (which can be described precisely by a hypergraph), is it possible to tell if there is an arrangement so that the open sets are convex? This question may appear combinatorial/topological in nature, but surprisingly has applications as a mathematical model for spatial cognition motivated by neuroscience research on certain neurons called “place cells.” Using topological methods, we prove that the notions of a neural code being locally good and a good cover code are in fact equivalent, and that the corresponding decision problem is undecidable. We also present an intermediate criterion that is stronger than being locally good but still weaker than convexity by considering collapsibility of links of missing codewords. (Received September 19, 2016)

1125-VC-2178 **Davood Damircheli\*** (dd1424@msstate.edu), 410 Allen Hall, 175 President’s Circle, Department of Mathematics and statistics, Mississippi State University, Starkville, MS 39759. *Pricing of boundary-linked assets by stochastic boundary value problems by using a new adaptive multiple shooting methods.*

With the growing sophistication of financial markets, investors are demanding new, more complex options products, tailored to their needs. In particular, there is an increasing number of financial assets whose values are contractually linked at certain periods of time, such as leases and rental agreements. An illustrative example is the English real estate lease market. The value of lease assets can be formulated by a second-order boundary value stochastic differential equation.

An adaptive multiple shooting method is proposed for solving the stochastic boundary problem. We illustrate the effectiveness of this approach on several standard test problems selected from the literature and compare it with other existing methods. We apply these ideas to study the valuation of boundary-linked assets and their derivatives.

Furthermore, we value boundary-linked derivatives using Malliavin calculus and Monte Carlo methods. We apply these ideas to value European call options of boundary-linked assets.

(Received September 19, 2016)

1125-VC-2264 **Serifenur Cebesoy\*** (scebesoy@ankara.edu.tr), Döğol Street, Department of Mathematics, Faculty of Science, Ankara University, 06100 Ankara, Turkey. *Spectral Singularities of the Impulsive Difference Equations.* Preliminary report.

We consider the problem of locating spectral singularities of a general complex point interaction for the second order discrete equations. We also determine the eigenvalues, examine the special cases where the point interaction is  $P$ -,  $T$ - and  $PT$  – *Symmetric*. (Received September 20, 2016)

1125-VC-2388 **Robert Erra\***, EPITA, 14/16 rue Voltaire, 94270 Le Kremlin Bicetre, France, and **Marwan Burelle, Alexandre Letois** and **Mark Angoustures**. *Strategies and tactics to approximate the diameter and the center of a graph or a point set.* Preliminary report.

We consider two problems: (1) the Graph Problem: we want an approximation of the diameter and the center of a graph, sparse and very large; and (2) the Point Set Problem: idem for a very large set of high dimensional points. For both problems, we consider data from real-world datasets, so, here very large means: hundred of millions of nodes/points. Points are high dimensional and known algorithms are quite expensive if the dimension is high, for the Graph Problem classical exact algorithms can not be used because of their cubic complexity. These two problems are becoming very interesting, the Graph Problem is concerned with the *web graphs* and the Point Set Problem appears when we want to measure the quality of clusters obtained with algorithms like k-means or KNN. For both problems we need to be able to compute the eccentricity of a node or a point. These distinct problems are algorithmically similar: *i.e.* the best algorithms that solves these problems follows quite identical strategies and tactics. So, we will present and compare the best known strategies and tactics and we will show how to adapt algorithms developed for one of the problem to the other problem. We will give results with experiments on real datasets of very large size. (Received September 20, 2016)

1125-VC-2429 **Mehtap Lafci\*** (mlafci@ankara.edu.tr), Ankara University, Faculty of Science, Department of Mathematics, Dogol Street, 06100 Besevler, Ankara, Turkey, and **Huseyin Bereketoğlu**. *Qualitative analysis of the solutions of a partial differential equation with piecewise constant arguments.*

We consider a partial differential equation with piecewise constant arguments. We prove the existence and uniqueness of the solutions of this equation. We also study the conditions of oscillation, instability and asymptotic stability of the solutions.

(Received September 20, 2016)

1125-VC-2531 **Nar Rawal\*** (nar.rawal@hamptonu.edu) and **Tung Nguyen**. *Coexistence and Extinction in Time-Periodic Volterra-Lotka Type Systems with Nonlocal Dispersal.*

We will talk something about coexistence and extinction of time periodic Volterra-Lotka type competing systems with nonlocal dispersal. Such issues have already been studied for time independent systems with nonlocal dispersal and time periodic systems with random dispersal, but have not been studied yet for time periodic systems with nonlocal dispersal. In this talk, the outlines of the proof of the main theorems about relations between the coefficients representing Malthusian growths, self regulations and competitions of the two species will be presented which ensure coexistence and extinction for the time periodic Volterra-Lotka type system with nonlocal dispersal. (Received September 20, 2016)

1125-VC-2536 **David C. Carothers\*** (carothdc@jmu.edu), Department of Mathematics and Statistics, James Madison University, Harrisonburg, VA 22807. *Polynomial systems of differential equations and functions with removable singularities.* Preliminary report.

We examine the relationship between a class of analytic functions characterized by the singularity of polynomial ODEs for which they are a solution (Flanders, *Functions not satisfying implicit, polynomial ODE*, J. Differential Equations **240**) and first order systems of polynomial differential equations as studied by Parker, Sochacki, et. al. (e.g. Sochacki, *Polynomial ODEs - Examples, Solutions, Properties*, Neural, Parallel & Scientific Computations **18**). (Received September 20, 2016)

1125-VC-2553 **Thomas Bellsky\***, 326 Neville Hall, Orono, ME 04469. *Stalking methods for ensemble Kalman filter covariance inflation.*

The artificial inflation of ensembles is a common technique in ensemble data assimilation whereby the ensemble variance is increased in order to prevent deviation of the ensemble from the truth. Various techniques for inflating ensembles exist in the literature. This talk will discuss shadowing and stalking methods and our implementation of stalking techniques as a method of ensemble inflation. We will also offer results from a low order chaotic system supporting the use of stalking methods. (Received September 20, 2016)

1125-VC-2558 **Timothy Robertson\*** (robertsont@andrews.edu) and **Joon Hyuk Kang**. *Coexistence conditions for nonlinear reaction-diffusion population models.*

We consider a nonlinear partial differential equation population model. Conditions for positive steady-state solutions are provided for three cases: two cooperating species, multiple species with unchanging interactions, and multiple species with varying interactions. We allow for variations such as the interaction changing from competition to cooperation. (Received September 20, 2016)

1125-VC-2571 **Celeste R Vallejo\*** (cvallejo@uf1.edu) and **James E Keesling**. *Using Little's Law in Stochastic Modeling.* Preliminary report.

The ability to analyze a stochastic network or a queue is useful for many applications. This type of analysis can be applied to systems in such fields as epidemiology or business. In order to analyze these systems, it is necessary to make certain limiting assumptions that facilitate computation. This talk will introduce a new way to approach these systems. This new approach utilizes a theorem from operations research called Little's Law. Little's Law is typically used to determine an unknown parameter in a queuing system. In this talk, I will use Little's Law to begin the analysis of a system rather than as an aide in calculation. (Received September 20, 2016)

1125-VC-2656 **Curtis Taylor Peterson\*** (kostelich@asu.edu), Arizona State University box 871804, Tempe, AZ 85287, and **Wenbo Tang**. *Periodic Advection-Diffusion-Reaction Systems.*

Many physical systems, such as the Belousov-Zhabtinsky reaction or biogeochemical cycles, exhibit oscillatory behavior in the concentration fields. The dynamics is more complex when this chemical behavior is coupled with background fluid motions. To capture the scenarios where small-scale stimuli trigger large-scale chemical oscillations, such as phytoplankton blooms, and understand if there is favorable fluid regions that enhance or

inhibit the reactions, we study these systems via numerical simulations of advection-diffusion-reaction systems that have the simplest chemical limit cycle behavior. By triggering the reaction in coherent vortices and high stretching regions, we find different time scales where homogenization takes place. (Received September 20, 2016)

1125-VC-2683 **Ivanti Galloway\***, 3658 McConnell Rd. Apt. 3B, Greensboro, NC 27405, and **Brandon Joyce, Christopher Pritchard and Ashely Jones**. *Analysis of Individual Greensboro Officers' Stopping Patterns Using Propensity Scores*.

In October 2015, a New York Times article highlighted a disparity between the proportion of black versus non-black drivers pulled over in traffic stops in Greensboro, NC. In response to these allegations, we examined 563 individual officers in the Greensboro Police Department (GPD) to determine if the driver's race played a role in their traffic stops. We used propensity score weighting, which compared an officer's particular stops to similar stops made by peers. This method was based on RAND Corporation's study for the Cincinnati Police Department. For our purposes, two stops were similar if they occurred for the same reason at a similar time of day and at a similar location in town. After applying our propensity score weights, we conducted a false discovery rate analysis. In this analysis, 10 out of the 563 officers had z-statistics that indicated racial bias against black drivers. These results are based off of 295,228 stops that occurred between January 1, 2009 and September 30, 2015. (Received September 20, 2016)

1125-VC-2813 **Gary R. Engler\***, Department of Mathematical Sciences, Stevens Institute of Technology, Hoboken, NJ 07030, and **Kenneth R. Blaney**. *A Network-Induced Multi-Neuronal Spike Train Metric*. Preliminary report.

The activity of a network of neurons can be characterized by the timing of individual spikes for each neuron in the network. These spikes represent the time that a communication signal is generated and subsequently sent to other neurons. A metric which encodes the relationships within networks of artificial spiking neurons is introduced in this talk and applications to optimization of network structure of this metric and the associated metric space are discussed. Learning has been shown to be correlated to the altering of edge weights in the underlying network. The applications of the metric on a network while learning is taking place is discussed as a method to measure the effects of the active learning algorithm on the activity of the network. (Received September 20, 2016)

1125-VC-2841 **Manoj K Thapa\*** (manoj.thapa@gsu.edu), School of Computing and Mathematics, Georgia Southwestern State University, Americus, GA 31709. *Numerical Study about the Origin of the Flow Chaos in Late Boundary Layer Transition*.

The transition process from laminar to turbulent flow in boundary layers is a fundamental problem in modern fluid mechanic. It has been the subject of intense research for over a century due to its great importance to various engineering applications. Our current understanding of the problem is far from complete. In this paper we will discuss recent Direct Numerical Simulation (DNS) results on the nature of late flow transition. Preliminary observations from our current project on mechanism of flow chaos in late boundary transition also will be briefly highlighted. (Received September 20, 2016)

1125-VC-2845 **Indika Gayani Udagedara\*** (udagedig@clarkson.edu), 8, Clarkson avenue, Dept. of Mathematics, Clarkson university, Potsdam, NY 13699, and **Brian Helenbrook and Aaron Luttmann**. *Improved Probabilistic Principal Component Analysis for Application to Reduced Order Modeling*.

A fundamental application of classical PCA is reduced order modeling (ROM), where the principal components are computed from a training data (TD) set and used as a reduced basis for the data space. In our previous work, we made a ROM for stochastic systems by projecting rapidly calculated (but noisy) data onto the basis vectors to obtain rapid, accurate predictions. All of these steps were done using techniques that do not account for noise in the data and  $L_2$  projection to create the reduced representations of the noisy data. In this work, stochastic approaches are used. The PPCA is used to generate the basis, which then allows estimating the noise in the TD. The standard PPCA has also been improved so that the variance of the basis expansion coefficients when representing the TD can be estimated. Another benefit of PPCA is to use model selection (MS) criteria to automatically truncate the basis used for the ROM. A new statistical approach for the projection step is used. The holistic approach gives a fully stochastic method for computing a ROM from noisy TD, determining the ideal MS, and projecting noisy test data onto the ROM. We demonstrate our framework on synthetic data and on an application to computing radiation transport with MC simulations. (Received September 20, 2016)

1125-VC-2882 **Amanda J Mangum\*** (amangum@niagara.edu), 5795 Lewiston Road, Niagara University, NY 14109, and **Mansoor Haider**, NCSU, Dept. of Mathematics, Box 8205, Raleigh, NC 27695. *Comparison of Three Clustering Algorithms, K-Means, Self-Organizing Maps, and Relational Self-Organizing Maps, on Porcine Atherosclerotic Tissues*. Preliminary report.

Atherosclerosis is a cardiovascular disease in which plaque accumulates along the wall of an artery, increasing the risk for heart attack or stroke. Acoustic Radiation Force Impulse (ARFI) is an ultrasound imaging technique in which acoustic waves are focused at a point, causing displacement of the tissue that is then tracked over time to measure elastic and viscoelastic material properties from the imaging data. We investigate the results of three data clustering algorithms, K-Means, Self-Organizing Maps (SOMs), and Relational SOMs to ARFI imaging of porcine atherosclerotic plaques. In this context, we hope to cluster images based on similar patterns in the data set. Based on the dimension, size and scope of image patterns considered in this work, the clustering configuration used for each clustering algorithm considered was a 3x3 lattice of nine neurons. We will discuss metrics that were used to compare the performance of these three clustering methods on the data set. (Received September 20, 2016)

1125-VC-2902 **Collin Kofroth\*** (ckofroth@asu.edu), **Duane Harris** and **Mohamed Moustouai**. *Numerical Solutions of the Taylor-Goldstein Equation for Gravity Waves Propagating through the Tropopause Inversion Layer*.

A numerical study of wave-induced momentum transport across the tropopause in the presence of a stably stratified thin inversion layer will be discussed. This layer consists of a sharp increase in static stability within the tropopause. The wave propagation is modeled by numerically solving the Taylor-Goldstein equation, which governs the dynamics of internal waves in stably stratified shear flows. The waves are forced by a flow over a bell shaped mountain placed at the lower boundary of the domain. A perfectly radiating condition based on the group velocity of mountain waves is imposed at the top to avoid artificial wave reflection. A brief validation for the numerical method through comparisons with the corresponding analytical solutions will be provided. Then, the method will be applied to more realistic profiles of the stability and wind shear to study the impact of these profiles on wave propagation through the tropopause. (Received September 20, 2016)

1125-VC-2912 **Israel Ncube\***, Alabama A & M University, Department of Mathematics, 4900 Meridian Street North, Huntsville, AL 35762. *Stability in a scalar differential equation with multiple, distributed time delays*.

We consider a linear scalar delay differential equation (DDE), consisting of two arbitrary distributed time delays. We study the stability of the trivial solution as well as the explicit effects of a specified distribution on some qualitative features of the DDE. (Received September 20, 2016)

1125-VC-3009 **Rommel G Regis\*** (rregis@sju.edu), Saint Joseph's University, Department of Mathematics, 5600 City Avenue, Philadelphia, PA 19131. *Simplex Gradients and Generalized Simplex Derivatives*. Preliminary report.

Given a function  $f$  defined on the Euclidean space  $R^n$  and a set  $X$  of vertices of a simplex in  $R^n$  (i.e.,  $X$  is a set of  $n+1$  affinely independent points), the simplex gradient of  $f$  with respect to  $X$  is the gradient of a linear model that interpolates the data points on the surface of  $f$  that correspond to  $X$ . The simplex gradient is important in the area of derivative-free optimization and it is used in the design of practical optimization algorithms. This talk will present the basic mathematical properties of simplex gradients and more general simplex derivatives. (Received September 20, 2016)

1125-VC-3118 **Pallavi Mishra\*** (mshrpal@gmail.com), Department of Mathematics, Indian Institute of Technology, Kharagpur, Kharagpur, 721302, India, and **Dharmendra Kumar Gupta**. *A new algorithm for finding valid permutations for solving Sudoku puzzles*.

A Sudoku puzzle of rank  $n$  involves a grid of size  $n^2 \times n^2$  partitioned into  $n \times n$  distinct blocks for some positive integer  $n$  in which the task is to fill each cell of the grid so that each row, each column and each block contain the integers 1 through  $n^2$  exactly once. Its solution is known as a Sudoku square of rank  $n$ . Permutations are called S-permutations if they are derived from  $n^2 \times n^2$  S-permutation matrices. A Sudoku square of rank  $n$  has one to one correspondence with a set of mutually disjoint S-permutations of cardinality  $n^2$ . In this paper, a new integer programming formulation is given for a Sudoku puzzle. Then, a new algorithm is proposed to generate the valid permutations for each block so as to solve the puzzles. Firstly, all the S-permutations are generated for the given puzzle. Then, the algorithm generates valid permutations for each block which is verified using the relation between Sudoku squares and a set of disjoint S-permutations.

Keywords: Sudoku puzzles; Sudoku squares; S-permutation matrices; S-permutations (Received September 21, 2016)

## General Session on Assessment

1125-VD-539 **Kevin G Brown\*** (kbrown@southern.edu), Southern Adventist University, Mathematics Department, PO Box 370, Collegedale, TN 37315. *Does Grading Homework Improve Student Performance?*

Homework has always been an integral part of mathematics courses. For years I believed that giving students careful/thoughtful/constructive feedback on homework papers was as vital as students doing the work. As it turns out, it's not. A study was done to examine student performance by comparing two sections of the same course; one in which homework was turned in and graded and one in which no homework grading was done. Two sections of introductory statistics per semester for two semesters were used as experimental groups and two sections in one semester were used as a control group. In the experimental groups, one section turned in homework papers to be graded and the other section did not. In the control group, homework was graded for both sections. Various statistical tests were used to examine test averages, overall course averages, and withdrawal rates. It was found that there is no statistically significant difference between any of these features when homework is graded and when it is not. While I still believe that doing homework is vital, it seems as though giving feedback in the form of grading is not helpful to student performance. As a result I have started using online homework assignments; the feedback is instant, but not as thoughtful. (Received September 06, 2016)

1125-VD-1788 **Jeff Barton** (jbarton@bsc.edu), **Bernadette Mullins** (bmullins@bsc.edu), **Doug Riley** (driley@bsc.edu) and **Maria Stadnik\*** (mstadnik@bsc.edu). *A case study of major assessment at a small liberal arts college.* Preliminary report.

With increased scrutiny by society and accrediting bodies, assessment of major programs has changed significantly in the past few years. At small schools, which often lack resources and expertise in assessment, practices can vary widely. In this talk, we discuss how one small liberal arts school has approached the assessment question for the mathematics major. We describe the student learning outcomes for the program, how they are measured, and what the department has done to ensure the consistency and reliability of the data. We also discuss how the department has used the assessment process to motivate change to the major program. (Received September 19, 2016)

1125-VD-1865 **Girija Sarada Nair-Hart\*** (nairhaga@uc.edu), OH 45102. *Who is on the other end?*

Do you use online testing as a method of formative assessment? If so, don't you wonder who is on the other end taking your test? Perhaps face-to-face examinations might alleviate that concern. How do you require proctored examinations for online courses? How do you proctor face-to-face tests in your online courses? During this presentation, based on her own experience, the presenter will share the challenges of assessing online learning. A discussion on implementing effective assessment techniques and practical, reliable, and low cost ways to administer proctored examinations will also be a part of this session. (Received September 19, 2016)

1125-VD-1913 **Jason R Elsinger\*** (jelsinger@shc.edu). *Using mastery-based assessment in a precalculus course.*

Mastery-based assessment differs from traditional exams by only granting credit for a problem if its solution is completely correct. Otherwise the student can retry a similar version, up to a finite amount of attempts. In spring 2016 semester, I used mastery-based assessment in a precalculus course, a course many students at Spring Hill College struggle in due partly to its abundance of information. Throughout the semester I collected the result of each attempt students took for each problem. Using this data, I will discuss how one can measure student improvement and discuss other aspects of improvement I observed. Final exam grades are also compared with previous semesters using traditional exams. (Received September 19, 2016)

1125-VD-2189 **Peri A Shereen\*** (pshereen@carleton.edu). *A hybrid approach to standards based grading.* Preliminary report.

Learning is a process that happens through revised efforts, while assessment is an evaluation of students' progress towards our pre-determined learning outcomes. Standards based grading is a useful tool for instructors to bridge the two. In particular, two components of standards based grading are significantly useful: a points-free grading method that evaluates students' progress towards mastery of a concept and the opportunity it allows students to make multiple attempts to demonstrate proficiency. Hence, standards based grading is a great resource that

encourages a growth mind-set. If you have always wanted or want to adopt standards based grading into your classes, but have been overwhelmed, then this talk might just be for you. I will discuss my experiences with a low-stake approach to adopting this method in my classes. Specifically, I will explain how and why I chose to use only the standards based approach with quizzes. (Received September 19, 2016)

1125-VD-2281 **Perry Y.C. Lee\*** (plee@kutztown.edu), Lytle Hall 267, Department of Mathematics, Kutztown, PA 19530, and **Padraig M McLoughlin** (mcloughl@kutztown.edu), Lytle Hall 265, Department of Mathematics, Kutztown, PA 19530. *Results From The On-Going Flip-IBL Study – Comparison of Traditional and F/IBL (Flipped and Inquiry-Based Learning) for 'Large' College Algebra – Classroom Settings Reboot*. Preliminary report.

During the past two academic years (2014 – 2016), an assessment study to obtain student-learned outcome data to multi-sections of College Algebra classrooms (both 'large' and 'small') was conducted. During these two academic years, the lead author incorporated the Flipped (or inverted) and the Inquiry Based Learning (IBL) approaches (or the F/IBL method) into each of his 'large' and 'small' classrooms. Other 'large' College Algebra classrooms were taught predominantly using the traditional lecture-style methods. A summary of assessment data based on student-learned outcomes from these multi-sections of College Algebra classrooms is presented.

The assessment instrument has been changed effective Fall 2016 semester; thus, the study has been 'rebooted.' Student scores were collected by administering assessments twice during Fall 2016: the pre-assessment and the post-assessment using Educational Testing Service's (ETS) standardized Intermediate Algebra Skills Assessments (IAS). Previously, the Elementary Algebra Skills (EAS) was used in the pre- and post-assessments. A summary of assessment data based on student-learned outcomes from multi-sections of College Algebra classrooms shall also be presented. (Received September 20, 2016)

1125-VD-2588 **Zephyrinus C Okonkwo\*** (zephyrinus.okonkwo@asurams.edu), Department of Mathematics and CS, 504 College Dr, Albany, GA 31705, **Anilkumar Devarapu** (anilkumar.devarapu@asurams.edu), Department of Mathematics and CS, 504 College Dr, Albany, GA 31705, and **Robert S owor**. *Using collaborative pedagogy and assessment instruments to enhance student achievement in College Algebra at Albany State University*.

University System of Georgia requires that all students in the state colleges and universities garner common intellectual experiences by taking a set of courses called core courses. At most Georgia Systems Universities, MATH 1111-College Algebra is a core requirement, hence multiple sections of this course are offered every semester. And in some colleges, there is little or no collaboration between instructors of College Algebra. In this paper, we present some of the positive outcomes of faculty collaboration focusing on pedagogy and the sharing of assessment instruments which leads to improved student achievement in the course. (Received September 20, 2016)

## General Session on Geometry

1125-VE-76 **Noureen Khan\*** (noureen.khan@unt.edu), 7400 University Hills Blvd, Dallas, TX 75241. *On Path Width and Bridge Index of Virtual Knots*.

We define path-width of planar graphs corresponding to a virtual knot. The focus is to compare their characteristics with the bridge index of virtual knots. We show that for pseudo prime knots, the bridge index is equivalent to the path width of graphs under virtual generalization. (Received July 11, 2016)

1125-VE-890 **Daniel J Freese\*** (dfreese3@liberty.edu), 5706 Wainwright Avenue, Rockville, MD 20851. *Intrinsic Surfaces of Revolution*.

We generalize the Enneper surface by studying surfaces in Euclidean space parametrized on an annular domain, whose first fundamental form and principal curvatures are independent of rotation and principal curvature directions depend only on the angle of rotation. We show that, in general, surfaces whose first fundamental form and shape operator have the above properties and satisfy the Gauss and Codazzi equations must have constant mean curvature, and their principal curvature directions rotate at constant speed. Using Bonnet's Theorem, we construct and classify the minimal case for these surfaces. We also numerically construct the general, nonzero constant mean curvature surfaces, which Smyth has classified. (Received September 12, 2016)

1125-VE-1002 **Bangyan Wen, Yi Lin and Zengxiang Tong\*** (ztong@otterbein.edu), Department of Mathematical Sciences, Otterbein University, Westerville, OH 43081. *A New Set of Axioms for Metric Geometry.*

This paper proposes a new set of axioms for Metric geometry, which consists of (1) five primitive concepts (Point, Field, Distance, Direction, and Continuum Set of Points), (2) five existence postulates (Point, Sphere, Fields without Boundary, Fields with Boundary, and Continuum Set of Points), and (3) five relationship axioms: (a) Pair of Points 3-Statuses: Separation, Coincidence, Empty (b) Concentric Spheres 3-statuses: inner, upon, outer; (c) Iso-Spheres 3-Statuses: Separation, Tangency, Intersection, (d) Two planes 3-statuses: parallel, Intersection, Coincidence (e) The segments of two lines cut by three parallel planes are propositional.

\*The new set of axioms has the following advantageous features:

(1) The straight lines and planes will have constructive definitions and non-Euclidean geometry will be understood differently. For example: A straight line is an ordered set of all tangent points of all spheres with centers at two given different points.

(2) It can combine different branches of Metric geometry, such as Euclidean geometry, affine geometry, Projective geometry, and vector geometry, etc., and make theoretical levels very clear.

(3) It is rigorous, concise, easy to understand, and suitable in geometry education. (Received September 13, 2016)

1125-VE-1254 **Jerry Lodder\*** (jlodder@nmsu.edu), New Mexico State University, Mathematical Sciences, Dept. 3MB, Box 30001, Las Cruces, NM 88003. *Distance in Geometry.*

A college course in plane or solid geometry is today taught from a highly polished system of axioms developed over more than two millennia, beginning with Euclid's *Elements*. In this talk we return to the geometric roots of the *Elements* and show how the Pythagorean Theorem can be taught and made transparent via the ancient Greek view of area. The Pythagorean Theorem and the area results required for its proof depend on the Euclidean Parallel Postulate (EPP), with the Pythagorean Theorem itself giving us the modern distance formula between two points. Should the EPP fail, how would distance be determined? We briefly examine the work of Felix Klein on non-Euclidean geometry for determining a "scale" for distance measure in hyperbolic geometry. These two topics, the Pythagorean Theorem and distance in hyperbolic geometry, form the material for two curricular modules for teaching geometry, based on primary historical sources. (Received September 15, 2016)

1125-VE-1451 **Iwan Praton\*** (iwan.praton@fandm.edu), Department of Mathematics, Franklin & Marshall College, Lancaster, PA 17604. *Minimal tilings of the unit square.*

Tile the unit square with  $n$  smaller squares and compute the sum of the side lengths of the small squares. We determine the minimum value of this sum for each  $n \neq 2, 3, 5$ . (Received September 16, 2016)

1125-VE-1897 **James Case\*** (jcase666777@aol.com). *Who Really Proved the Isoperimetric Theorem?*

It was known to Pythagoras more than 2500 years ago that, among all simple closed curves of given arc length, the circle encloses the greatest area. Subsequent authors, including Zenodorus (200 BC – 140 BC), were able to prove uniqueness for "the isoperimetric problem" without proving existence: The maximizing curve, if there is one, can only be the circle. But proof that the circle does indeed solve the problem was apparently lacking until Weierstrass (in or about 1870) used his "sufficiency test" from the calculus of variations to validate the classical result. Accordingly, Weierstrass usually receives credit for completing the proof of the isoperimetric theorem. It will be argued that, although his book *On Isoperimetric Figures* is lost, Zenodorus almost surely supplied the missing proof. (Received September 19, 2016)

1125-VE-2032 **Jonathan M. Clark\*** (jclar121@vols.utk.edu), 7238 Austin Park Lane, Knoxville, TN 37920, and **Lauren J. Clark** (dr.jenevaclark@utk.edu), 7238 Austin Park Lane, Knoxville, TN 37920. *Seeing the Light: Connecting Conic Section Representations Using Flashlights and Parametric Functions.*

Common Core State Standards Initiative calls for geometry students to translate between geometric descriptions of conics and the equations describing them. For example, students should derive the equations of an ellipse or hyperbola given the foci (CCSS 2012). In this talk, we will explore pedagogical tools for inciting investigations into understanding conic sections by asking how students could make connections between the different descriptions of conics. Specifically, we will review an undergraduate research project undertaken which sought to answer the question of how the foci are related to the intersecting cone and plane, which employed the "rotating flashlight" visualization in a mathematical form. This led to generating a map  $C : [0, 2\pi) \rightarrow \mathbb{R}^2$  which defined the set of all conic sections in terms of the parameters associated with the intersecting cone and plane, and

provided a derivation for the explicit relationship between these descriptions. We will then discuss how one may generalize this pedagogical technique to other research endeavors. (Received September 19, 2016)

1125-VE-2825 **Matt Sunderland\*** ([msunderland@gradcenter.cuny.edu](mailto:msunderland@gradcenter.cuny.edu)), 365 Fifth Avenue, New York, NY 10016. *Random walks on Gromov hyperbolic spaces*. Preliminary report.

A random walk on a separable Gromov hyperbolic space converges to a unique point on the boundary at infinity with probability one when two of the possible steps are in independent "hyperbolic directions." In particular, the random walk escapes from the origin at an at least linear rate.

This rate of escape is known to approach an exactly linear rate exponentially fast when (1) the step lengths are "exponentially" unlikely to be long, and (2) the set of steps satisfies a condition known as acylindricity.

We extend the result to the non-acylindrical case. (Received September 20, 2016)

1125-VE-2869 **Alfredo Villanueva\*** ([villanuevaa@savannahstate.edu](mailto:villanuevaa@savannahstate.edu)), 3219 College St, Department of Mathematics, Savannah, GA 31404. *3-Ellipses on Spheres*.

A traditional ellipse is a closed curve in the plane, where the sum of the distances from each point of this curve to two fixed points (foci) is constant. A 3-ellipse is a closed curve in the plane with 3 foci. This notion of 3-ellipses was introduced by the Scottish physicist and mathematician James Clerk Maxwell. Here we are extending this definition from the plane to a sphere; a 3-ellipse is a curve in the sphere, where sum of the distances from each point to three fixed points is constant. We also show explicit examples of these curves. These are the grounds to generalize n-ellipse curves (n foci) on Riemannian manifolds. (Received September 20, 2016)

## General Session on Graph Theory

1125-VF-192 **Brian G. Kronenthal\*** ([kronenthal@kutztown.edu](mailto:kronenthal@kutztown.edu)), **Felix Lazebnik** and **Jason Williford**. *On the uniqueness of some girth eight algebraically defined graphs*.

In this talk, we will discuss algebraically defined bipartite graphs. Indeed, let  $\mathbb{F}$  denote a field, and consider the bipartite graph with partite sets  $P = \mathbb{F}^3 = L$  such that  $(p_1, p_2, p_3) \in P$  and  $[\ell_1, \ell_2, \ell_3] \in L$  are adjacent if and only if  $p_2 + \ell_2 = p_1 \ell_1$  and  $p_3 + \ell_3 = p_1 \ell_1^2$ . This graph has girth eight, and of particular interest is whether it is possible to alter these equations by replacing  $p_1 \ell_1$  and  $p_1 \ell_1^2$  with other bivariate polynomials to create a nonisomorphic girth eight graph. In addition to discussing some results related to this question, we will also explain the connection between algebraically defined graphs and the point-line incidence graphs of generalized quadrangles, which partially motivates the study of the objects in this talk. (Received August 10, 2016)

1125-VF-311 **P Ye\*** ([pye@ung.edu](mailto:pye@ung.edu)), 2511 Education Way, Oakwood, GA 30566, and **M D Norton**. *The pharmaceutical Supply Chain*. Preliminary report.

This paper is going to create a bridge between some basic understandings of graph theory and how those principles are used in the creation of a pharmaceutical supply chain used by businesses. The Shapley value is used in the supply chain with a custom definition as it can be applied in a few different ways depending on what needs to be solved to find the most efficient route. In the end a pseudo pharmaceutical supply chain example will be created to demonstrate the decision making process that happens and all of the options that have to be weighed. (Received September 16, 2016)

1125-VF-362 **Ian M Hill\***, [hillim@dukes.jmu.edu](mailto:hillim@dukes.jmu.edu), and **Josh E Ducey**. *The Critical Group of  $KG(n,2)$* . Preliminary report.

Let  $KG(n,k)$  denote the graph whose vertices are the subsets of size  $k$  of a set of size  $n$ , where two vertices are adjacent if they are disjoint. This is the Kneser Graph. We will look at the critical group of  $KG(n,2)$  and take a combinatorial approach through a "chip-firing game" to prove that it is isomorphic to a particular direct sum of cyclic groups. (Received August 29, 2016)

1125-VF-523 **Jian Cheng\*** ([jiancheng@math.wvu.edu](mailto:jiancheng@math.wvu.edu)), 320 Armstrong Hall, PO Box 6310, Morgantown, WV 26506, and **Y. Lu, R. Luo** and **C.-Q. Zhang**. *Shortest Circuit Covers of Signed Graphs*.

A shortest circuit cover  $\mathcal{F}$  of a bridgeless graph  $G$  is a family of circuits that covers every edge of  $G$  and is of minimum total length. The total length of a shortest circuit cover  $\mathcal{F}$  of  $G$  is denoted by  $SCC(G)$ . For ordinary graphs (graphs without sign), the subject of shortest circuit cover is closely related to some mainstream areas, such as, Tutte's integer flow theory, circuit double cover conjecture, Fulkerson conjecture, and others. For signed

graphs  $G$ , it is proved recently by Máčajová, Raspaud, Rollová and Škoviera that  $SCC(G) \leq 11|E|$  if  $G$  is  $s$ -bridgeless, and  $SCC(G) \leq 9|E|$  if  $G$  is 2-edge-connected. In our paper this result is improved as follows,

$$SCC(G) \leq |E| + 3|V| + z$$

where  $z = \min\{\frac{2}{3}|E| + \frac{4}{3}\epsilon_N - 7, |V| + 2\epsilon_N - 8\}$  and  $\epsilon_N$  is the negativness of  $G$ . As a corollary, we prove that  $SCC(G) \leq \frac{14}{3}|E|$ . The above upper bounds can be further reduced if  $G$  is 2-edge-connected with even negativness. This is joint work with Y. Lu, R. Luo and C.-Q. Zhang from West Virginia University. (Received September 05, 2016)

1125-VF-574 **Katherine F. Benson** and **Daniela Ferrero\*** (dferrero@txstate.edu), Department of Mathematics, Texas State University, San Marcos, TX 78666, and **Mary Flagg, Veronika Furst, Leslie Hogben** and **Violeta Vasilevska**. *Nordhaus-Gaddum bounds for the power domination number of a graph.*

In 1956, Nordhaus and Gaddum gave lower and upper bounds for the sum and the product of the chromatic number of a graph and that of its complement, in terms of the order of the graph. Since then, similar bounds have been proposed for other graph parameters and interesting results have been derived from them. In this talk we present Nordhaus-Gaddum type bounds for the power domination number of a graph. (Received September 06, 2016)

1125-VF-597 **Jason J Moliterno\*** (moliternoj@sacredheart.edu), Department of Mathematics, 5151 Park Avenue, Sacred Heart University, Fairfield, CT 06825. *Maximal outerplanar graphs whose algebraic connectivity is at most one.*

The Laplacian matrix  $L$  for a graph  $G$  is the matrix  $L = D - A$  where  $D$  is the diagonal matrix of the vertex degrees and  $A$  is the traditional adjacency matrix. The Laplacian matrix is positive semidefinite with eigenvalues  $0 = \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ . The eigenvalue  $\lambda_2$  is known as the algebraic connectivity  $a(G)$  of a graph. In this talk, we investigate the algebraic connectivity of maximal outerplanar graphs. We outline a proof that shows that if  $G$  is a maximal outerplanar graph on  $n \geq 12$  vertices, then  $a(G) \leq 1$  where equality holds on exactly two maximal outerplanar graphs on 12 vertices. The proof relies heavily on vertex labellings. (Received September 07, 2016)

1125-VF-842 **Murong Xu\*** (xumurong@math.wvu.edu), 7205 University Commons Dr., Morgantown, WV 26505, and **Janet Anderson, Suohai Fan, Hong-Jian Lai** and **Xiaoxia Lin**. *An extremal problem in digraph connectivity.*

A digraph  $D$  is strong if for any pair of vertices  $u, v \in V(D)$ ,  $D$  always contains a  $(u, v)$ -dipath. The strong arc connectivity of a digraph  $D$ , denoted by  $\lambda(D)$ , is the minimum number of arcs whose removal results in a non-strong digraph. If we just count the number of arcs in a digraph, can we predict that  $D$  contains a subdigraph with high strong arc connectivity? We define  $\bar{\lambda}(D) = \max\{\lambda(H) : H \subseteq D\}$ . Given an integer  $k > 0$ , a strict digraph  $D$  is  $k$ -maximal if  $\bar{\lambda}(D) \leq k$  but adding any arc which is not in  $D$  will surely create a subdigraph with strong arc connectivity at least  $k + 1$ . Mader [Math. Ann. 1971] and Lai [JGT 1990] studied the extremal size of undirected  $k$ -maximal graphs. In this project, we determine that if  $D$  is a  $k$ -maximal digraph on  $n > k$  vertices, then

$$\binom{n}{2} + (n-1)k + \lfloor \frac{n}{k+2} \rfloor \left(1 + 2k - \binom{k+2}{2}\right) \leq |A(D)| \leq k(2n - k - 1) + \binom{n-k}{2}.$$

Consequently, if  $|A(D)| > k(2n - k - 1) + \binom{n-k}{2}$ , then  $D$  must have a nontrivial subdigraph  $H$  such that the strong arc connectivity of  $H$  is at least  $k + 1$ . (Received September 12, 2016)

1125-VF-872 **Wing Hong Tony Wong\*** (wong@kutztown.edu), Department of Mathematics, Kutztown University of Pennsylvania, 15200 Kutztown Road, Kutztown, PA 19530, and **Brian Kronenthal**. *When "Flow Free" is Played on a Torus.*

"Flow Free" is a game played on smart phones. In this game, there are several colored dots in a square grid, and the task is to draw pipes to connect pairs of dots of the same color to create a flow, subject to the following two constraints: the pipes must cover all the squares in the entire grid, and pipes cannot cross or overlap each other.

In this talk, we will extend the game from square grids on a rectangular board to an L-shaped board, as well as to a torus and a higher dimensional torus. We will discuss some sufficient conditions to configure the colored dots so that there is a flow between them. This project is closely tied with Hamiltonian-connected and Hamiltonian-laceable graphs. (Received September 12, 2016)

1125-VF-954 **Eammon J Hart\*** (eammonhart@mail.adelphi.edu), 86 Kamda Blvd., New Hyde Park, NY 11040, and **Branden Stone**. *Excluded Minors for Families of Graphs*. Preliminary report.

In 2004, Roberston and Seymour culminated a 20 publication long series with a proof of Wagner's conjecture, which said any minor closed family of graphs has finitely many excluded minors. My research gives a system for generating large (though not necessarily comprehensive) collections of excluded minors for certain finite families of graphs. I then provide a system for generating all of the excluded minors of the non-minor closed collection of families of graphs with maximum degree  $n$ . I then will conclude with a discussion of graphs that are excluded under some, but not all three of the the minor operations. We will primarily discuss exclusion under vertex and edge deletion. (Received September 13, 2016)

1125-VF-1023 **Darren Narayan\***, School of Mathematical Sciences, Rochester Institute of Technnology, Rochester, NY 14612, **Rigoberto Florez**, Department of Mathematics and CS, Charleston, SC 29409, **Ruth Lopez**, Department of Mathematics and Statistics, California State University, Long Beach, Long Beach, CA 90840, and **Jacob Worrell**, Dept. of Psychological and Brain Sciences, Indiana University, Bloomington, IN 47405. *A graph theoretic analysis of co-branding in social networks*.

Social networks such as Twitter and Instagram have an underlying graph structure that change as relationships are created or ended. In some cases people merge their sites and present their relationship as a single unit. This can be modeled through edge contraction where two adjacent vertices  $u$  and  $v$  in a graph  $G$  are merged to form a single vertex  $uv$ . The edges incident to  $uv$  in the new graph are the edges incident to either  $u$  or  $v$  (but not both) in the original graph. We examine the impact of edge contraction on graph properties such as betweenness centrality, closeness centrality, and leverage centrality. (Received September 14, 2016)

1125-VF-1086 **Abdel-Rahman Amr Madkour\*** (madkour@stolaf.edu), 1500 St. Olaf Avenue, Northfield, MN 55057, and **Phillip Nadolny** (nadolnyp@stolaf.edu). *Finding Minimal Spanning Forests in a Graph*.

In the computation of multidimensional persistent homology, a popular tool in topological data analysis, a family of planar graphs arises. We have studied the problem of partitioning these graphs in a way that will be useful for parallelizing the persistent homology calculation. Specifically, we desire to partition an edge-weighted, undirected graph  $G$  into  $k$  connected components,  $G_1, \dots, G_k$ . Let  $w_i$  be the weight of a minimum spanning tree in component  $G_i$ . For our purposes, an ideal partition is one that minimizes  $\max\{w_1, \dots, w_k\}$ . This problem is known to be NP-hard in the case of general graphs and we are unable to find this specific problem in the graph partitioning literature. We propose two approximation algorithms, one that uses a dynamic programming strategy and one that uses a spectral clustering approach, that produce near-optimal partitions in practice on a family of test graphs. We present detailed descriptions of these algorithms and the analysis of empirical performance data. (Received September 14, 2016)

1125-VF-1095 **John Asplund\*** (jasplund@daltonstate.edu), 650 College Dr., Dalton, GA 30720, and **N. Bradley Fox** (foxb@apsu.edu). *Counting cycles in the graphs of overlapping permutations*.

It is a common enough practice to count the number of cycles in a graph. One can even be fancy about it and count the number of cycles of a certain length. Soon after beginning this process, most will find frustration in counting all of the cycles in a general graph of a particular length. This leads quickly to counting the number of cycles in classes of graphs, and we do just that! In this talk we look at the graph of overlapping permutations which is an analogue to the De Bruijn graph. In this graph, we will talk about some of the difficulties involved in counting cycles on this graph, some of the success stories, and where one can go from here. (Received September 14, 2016)

1125-VF-1376 **John Villalpando\***, 60 West Olsen Road, Thousand Oaks, CA 91360, and **Vesta Coufal, Karri Fogel, Aparna Higgins, William Higgins, Rob Ray and Kathie Yerion**. *Trees for Given Values of the Span and Icap for  $L(2,1)$ -Colorings*.

An  $L(2,1)$ -coloring of a graph is a labeling of the vertices using non-negative integers such that adjacent vertices differ in label by at least 2 and distance two vertices differ in label. A well studied invariant of  $L(2,1)$ -colorings, the span denoted by  $\lambda$ , is the smallest integer  $k$  for a given graph such that there exists an  $L(2,1)$ -coloring of the graph using only non-negative integers less than or equal to  $k$ . An  $L(2,1)$ -coloring of a graph is irreducible if reducing the label on any vertex violates an  $L(2,1)$ -coloring condition. The invariant icaps, denoted  $\kappa$ , is the least number of color classes required to create an irreducible  $L(2,1)$ -coloring on a given graph. For any tree  $T$  it is known that  $\Delta + 1 \leq \kappa \leq \lambda + 1$  and  $\lambda \in \{\Delta + 1, \Delta + 2\}$  where  $\Delta$  is the maximum degree of the tree. Thus, there are only three possible values for  $\kappa$ ;  $\{\Delta + 1, \Delta + 2, \Delta + 3\}$ . Then for the two possible values of the span,

we consider the three possible values of the  $\text{icap}$ , determine if there exist a tree with the two specified values of the invariants, and provide a family of such trees if any exist. (Received September 16, 2016)

1125-VF-1377 **Michael D. Barrus\*** ([barrus@uri.edu](mailto:barrus@uri.edu)) and **John Sinkovic**. *Uniqueness in labelings of tree-depth-critical graphs.*

A  $k$ -ranking of a graph  $G$  is a labeling of the vertices of  $G$  with values from  $\{1, \dots, k\}$  such that any path joining two vertices with the same label contains a vertex having a higher label. The tree-depth of  $G$  is the smallest value of  $k$  for which a  $k$ -ranking of  $G$  exists. The graph  $G$  is critical if every proper minor of  $G$  has smaller tree-depth than  $G$  has.

Focusing on  $k$ -rankings of critical graphs, we define a graph  $G$  to be 1-unique if for every vertex  $v$  in  $G$ , there exists an optimal ranking in which  $v$  is the unique vertex with label 1. We explore the seemingly close relationships between 1-uniqueness and criticality, showing that (contrary to an earlier conjecture) not all critical graphs are 1-unique, though many are. We show that for graphs that are 1-unique, we have useful tools to more easily conclude criticality, construct larger critical graphs, and prove conjectured properties of critical graphs. (Received September 16, 2016)

1125-VF-1543 **Doug Chatham\*** ([d.chatham@moreheadstate.edu](mailto:d.chatham@moreheadstate.edu)), Department of Mathematics and Physics, Morehead State University, Morehead, KY 40351. *Dragon placement problems.* Preliminary report.

In Shogi, a Japanese relative of Chess, a *dragon king* is a piece that moves like a rook and king and a *dragon horse* is a piece that moves like a bishop and king. For each piece, we form a graph with vertices being the squares of an  $n \times n$  board and with two vertices adjacent iff the piece can go from one square to the other in a single move. In this talk, we discuss the independence number, domination number, and independent domination number for the  $n \times n$  dragon kings graph and the  $n \times n$  dragon horses graph. (Received September 17, 2016)

1125-VF-1745 **Peter Maceli\*** ([peter.maceli@canisius.edu](mailto:peter.maceli@canisius.edu)). *Coloring graphs and their complements.* Nordhaus and Gaddum showed that for any graph the sum of its chromatic number together with the chromatic number of its complement is at most one more than the number of vertices in the graph. The class of graphs which satisfy this upper bound with equality have long been understood, however not much beyond this initial case is known in terms of characterizing graphs via this sum of complementary chromatic numbers. In this talk, we will discuss how adopting a more structural approach to this general problem leads to an interesting method of graph decomposition, which in turn allows one to generalize and extend several previous results. (Received September 19, 2016)

1125-VF-1842 **Dimitri Plessas\*** ([plessas@nsuok.edu](mailto:plessas@nsuok.edu)) and **Tien Chih** ([tien.chih@newberry.edu](mailto:tien.chih@newberry.edu)). *A Categorical Reformulation of the Reconstruction Conjectures.*

In the standard category of (undirected) graphs, the graphs are restricted to not have multiple edges and are allowed at most one loop at any vertex. By instead considering the category of all graphs, which allows for multiple edges and loops, we establish a categorical version of the First Noether Isomorphism Theorem. This categorical version uses kernel and cokernel pairs and fails to hold in the standard category of graphs. A direct consequence of this result is a reformulation of the vertex and edge reconstruction conjectures in terms of graph homomorphisms. We show that reconstructability in both conjectures is equivalent to the existence of a graph homomorphism satisfying a single equation. (Received September 19, 2016)

1125-VF-1946 **Zhenming Bi\*** ([zhenming.bi@wmich.edu](mailto:zhenming.bi@wmich.edu)). *Rainbow Hamiltonian-Connected Graphs.*

A graph  $G$  is Hamiltonian-connected if every pair of vertices of  $G$  are connected by a Hamiltonian path that contains every vertex of  $G$ . A graph is edge-colored if each of its edges is assigned a color (where adjacent edges can be assigned the same color). A path  $P$  in an edge-colored graph is a rainbow path if no two edges of  $P$  are colored the same. An edge coloring of a Hamiltonian-connected graph  $G$  is a Hamiltonian-connected rainbow coloring if every two vertices of  $G$  are connected by a rainbow Hamiltonian path. The minimum number of colors in a Hamiltonian-connected rainbow coloring of  $G$  is the rainbow Hamiltonian-connection number  $\text{hrc}(G)$  of  $G$ . If  $G$  has order  $n$  and size  $m$ , then  $n - 1 \leq \text{hrc}(G) \leq m$ . The rainbow Hamiltonian-connection number is investigated for the Cartesian product of complete graphs and of odd cycles with  $K_2$ . As a result of this, both the lower bound  $n - 1$  and the upper bound  $m$  for  $\text{hrc}(G)$  are shown to be sharp. Several results and open questions are presented in this area of research. (Received September 19, 2016)

1125-VF-1989 **Megan Cream\*** ([mcream@spelman.edu](mailto:mcream@spelman.edu)). *Chorded Pancyclicity.*

Historically there have been many results concerning the existence of certain types of cycles in graphs. A graph property of particular interest is pancyclicity, that is, the property of a graph containing a cycle of every

possible length, from three to the order of the graph. In this talk we define a new graph property called chorded pancyclicity and we investigate a density condition and forbidden subgraphs in claw-free graphs that imply this new property. Specifically, we consider a degree-sum condition and we forbid certain paths and triangles with pendant paths as subgraphs. Further, we extend J. A. Bondy's meta-conjecture on pancyclic graphs to a meta-conjecture on chorded pancyclic graphs. This is joint work with Ronald J. Gould, Kazuhide Hirohata, and Victor Larson. (Received September 19, 2016)

1125-VF-2300 **Elizabeth Donovan\*** ([edonovan@murraystate.edu](mailto:edonovan@murraystate.edu)). *An informative invariant: the neighborhood degree list.*

The neighborhood degree list (NDL) is a graph invariant that refines information given by the degree sequence and joint degree matrix of a graph and is useful in distinguishing graphs having the same degree sequence. We show that the space of realizations of an NDL is connected via a switching operation. We then determine the NDLs that have a unique realization by a labeled graph; the characterization ties these NDLs and their realizations to the threshold graphs and difference graphs. (Received September 20, 2016)

1125-VF-2368 **Jennifer Diemunsch, Nathan Graber, Lucas Kramer, Victor Larsen\*** ([vlarsen@kennesaw.edu](mailto:vlarsen@kennesaw.edu)), **Lauren Nelsen, Luke Nelsen, Devon Sigler, Derrick Stolee** and **Charlie Suer**. *Color-blind index, computational complexity, and hypergraphs.*

A not necessarily proper edge-coloring on a graph yields a color palette  $\bar{c}(v) = \{a_i \dots, a_k\}$  for each vertex  $v$  where  $a_i$  is the number of edges incident to  $v$  with color  $i$ . We reorder  $\bar{c}(v)$  for every  $v$  in non-increasing order to obtain the *color-blind partition*  $c^*(v)$ . When the color-blind partition forms a proper vertex labeling, we say that the edge-coloring is *color-blind distinguishing*, and we let  $\text{dal}(G)$  be the smallest number of colors necessary for a color-blind distinguishing edge-coloring.

In this talk, we examine the problem of determining  $\text{dal}(G)$  for graphs of low degree, and show its connection with computational complexity theory and hypergraph coloring. We show that, for general graphs, determining  $\text{dal}(G)$  is NP-complete even when it is known that  $\text{dal}(G) \in \{2, 3\}$ . However, we can use known results from hypergraph coloring to deal with regular bipartite graphs. (Received September 20, 2016)

1125-VF-2627 **Miranda Bowie\*** ([mbowie@una.edu](mailto:mbowie@una.edu)), **Louis Sewell** and **Anne Sinko**. *Set-Sized Packing on Graphs*. Preliminary report.

For a graph  $G$ , the packing number,  $\rho(G)$ , is defined to be  $\max\{|S| : S \subseteq V(G) \text{ and } |N[v] \cap S| \leq 1 \text{ for each } v \in V(G)\}$ . Notice that for every vertex in  $V(G)$  there is a restriction on the number of vertices in the packing set  $S$  which lie within that vertex's closed neighborhood. Set-sized packing extends the notion of packing beyond restrictions for individual vertices to collections of vertices. We define the set-sized packing number  $\rho_{X(c_1, c_2, \dots, c_t, \dots)}(G)$  to be the maximum cardinality of a set  $S \subseteq V(G)$  such that, for each set of  $k$  vertices, there are no more than  $c_k$  vertices of  $S$  in the union of their closed neighborhoods. An introduction to set-sized packing will be discussed along with preliminary results. (Received September 20, 2016)

1125-VF-3021 **Janet Fierson\*** ([fierson@lasalle.edu](mailto:fierson@lasalle.edu)), Dept. of Mathematics and Computer Science, La Salle University, 1900 W. Olney Ave., Philadelphia, PA 19141, and **Eric Frazier III**. *Computer-aided investigation of coloring graphs under rainbow connection*. Preliminary report.

Given a graph  $G$  and a positive integer  $k$ , the  $k$ -coloring graph of  $G$  is constructed by creating a vertex for each proper  $k$ -coloring of  $G$  and inserting an edge between vertices in the coloring graph whose corresponding colorings of  $G$  differ in exactly one position. Originally applied to vertex coloring and edge coloring, the concept of the coloring graph has recently been applied to rainbow connection. In an edge-colored graph, a path is said to be rainbow if no two of its edges share a color; a graph is rainbow-connected if a rainbow path exists for every pair of vertices.

Incorporating technology into the research process in the area of coloring graphs under rainbow connection has facilitated the discovery of structures, the formulation of conjectures, and the investigation of variations on the rules for coloring the original graph  $G$  and inserting edges in the coloring graph itself. In addition, studying random graphs of various types has led to a better sense of what is "typical" of coloring graphs under rainbow connection. In this talk, we present theoretical results and the specific ways in which technology played a role in their discovery. (Received September 20, 2016)

## General Session on History of Philosophy of Mathematics

1125-VG-27 **Ram U. Verma\*** (verma99@msn.com), Denton, TX 76205. *Chain Rule - A Wonderful Mind Imaging*. Preliminary report.

The general perception for the Chain Rule is based on the principle of symbolic cancellation which connects (conceptually) the derivative of a function with the derivative of a composite function in our calculus instruction. In fact, the Chain Rule is a more deeper imagination of human minds rather than just a symbolic cancellation procedure, this what (maybe) Newton and Leibniz had in mind more than three centuries ago. (Received May 25, 2016)

1125-VG-545 **Andrew J Simoson\*** (ajsimoso@king.edu), 1349 King College Road, Bristol, TN 37620. *Extrapolating Plimpton 322—the most famous ancient mathematical artefact*.

Labeled one through fifteen, the clay tablet Plimpton 322 contains the short and long (diagonal) integer-valued sides of Pythagorean triples. We give a proof-without-words for the ancient algorithm of reciprocal pairs of regular sexagesimal numbers used to generate the terms on the tablet and answer two natural questions. Why is the first triple (119, 120, 249)? And—if the table were extended—what would the sixteenth triple have been? This material is suitable for inclusion in a history of mathematics class, a discrete mathematics course, or for a mathematics appreciation course. (Received September 06, 2016)

1125-VG-649 **James V Rauff\*** (jrauff@millikin.edu), Department of Mathematics, Millikin University, 1184 W. Main St., Decatur, IL 62522. *The Algebra of Marriage: An Episode in the History of Applied Group Theory*.

In 1949, André Weil contributed a mathematical appendix to Claude Lévi-Strauss's landmark book, *The Elementary Structures of Kinship*. In this appendix, Weil (one of the Bourbaki mathematicians) used group-theoretic techniques to model Australian marriage systems. Weil's paper marked the beginning of mathematical anthropology. This paper describes Weil's analysis of marriage systems and traces the uneasy history of the application of group theory to kinship studies. (Received September 08, 2016)

1125-VG-873 **Eric B Kahn\*** (ekahn@bloomu.edu). *A First Attempt at a History of Mathematics Course: Mathematics and General Education*.

The general education program at Bloomsburg University recently was rewritten and moved from a system based on academic categories, to one based on student learning objectives. The speaker took this opportunity to develop a history of mathematics course which would strengthen his department's curriculum while broadening mathematics majors' opportunities to earn general education credit. In this talk, we will discuss the structure of this history of mathematics course along with particular assignments that make it applicable to earn general education credit in communication and information literacy goals. (Received September 12, 2016)

1125-VG-1760 **Daniel J. Curtin\*** (curtin@nku.edu), Dept of Mathematics & Statistics, Northern Kentucky University, Highland Heights, KY 41099. *Euler and the Problem of Surface Area*.

Leonhard Euler in his paper (E133, 1750) *De superficie conorum scalenorum, aliorumque corporum conicorum* (On the surface area of a scalene cone and of other conical bodies) amends earlier solutions by Leibniz and Varignon to the problem of the surface area of a scalene cone, i.e., a cone whose base is a circle, but whose vertex is not over the center of the circle; and then extends the approach to other base curves. The speaker has translated this paper and will discuss Euler's solution. (Received September 19, 2016)

## General Session on Interdisciplinary Topics in Mathematics

1125-VH-751 **Mark A. Krines\*** (krinesm@ripson.edu). *The Reflection Principle and Bertrand's Ballot Theorem on Three Alternatives*.

Bertrand's Ballot Theorem concerns the probability that in a two-candidate election, the winning candidate is always ahead of the losing candidate during the counting of the votes. In this presentation, we consider this problem on three alternatives in answering two questions. One, what is the probability that the entire ranking of the three alternatives at the end of the election is maintained (including ties) throughout the counting of the ballots? Two, what is the probability that the winning alternative never trails either of the losing candidates throughout the counting of the ballots? Our methods for answering these questions are based in extending a version of the reflection principle that can be used to prove Bertrand's Ballot Theorem. (Received September 10, 2016)

1125-VH-1217 **Mariama J. Abramson\*** ([mariama.abramson@cameron.edu](mailto:mariama.abramson@cameron.edu)), 2800 W Gore Blvd, Lawton, OK 73505, and **Jean-Jacques Kengwoung-Keumo** ([jkengwou@cameron.edu](mailto:jkengwou@cameron.edu)), 2800 W. Gore Blvd, Lawton, OK 73505. *Disparities in Cutaneous Melanoma Hazard Rates between Whites and Black/African Americans in the U.S.A. from 1973 through 2014*. Preliminary report.

There are five different types of skin cancers and melanoma is one of them. Melanoma occurs in the melanocytes (melanin producing cells). Melanoma spans pre-dominantly in the skin but can also be found in the ears, eyes, gastro-intestinal tract, and oral and genital mucus membranes. Melanoma is the deadliest leading cancer diagnosed in the U.S.A. It occurs predominantly among Whites (95%), followed by Hispanics, American Indians/Alaskan Natives (AI/AN), Asians/Pacific Islanders (API), and African Americans/Blacks (Wu, 2011). Melanoma cases have increased more rapidly than all other cancers combined over the past few decades. This has resulted one in five Americans developing melanoma in the course of a lifetime and death of one American almost every single hour. The results of Kengwoung-Keumo et al. (2016) indicate that Blacks had higher survival rates than Whites and women had higher survival rates than men in both racial groups. We are interested in investigating whether these results still hold on hazard rates. (Received September 15, 2016)

1125-VH-1591 **Marat R Mamedbakov\*** ([maratm77@gmail.com](mailto:maratm77@gmail.com)), 153 Seabrook Road, Hyannis, MA 02601, and **Alexander N Nepp** ([anep@urfu.ru](mailto:anep@urfu.ru)), 19 Mira, Yekaterinburg, 620002. *The Regression Analysis for the Influence of Religion upon Several Economic Indicators*.

The influence of religion on economic activity was studied by early economists such as Adam Smith, but modern economists have done a little research on the subject. However, it is clear that the impact of religion on the affairs of nations is important, particularly in the last 40 years. This presentation will briefly present the best papers which study the influence of religion on economics. Then, empirical research (including two regression models sampling across more than 40 countries) will be presented. This research analyzed the impact of religion (Protestantism and Catholicism) on economic indicators such as GDP (Gross Domestic Product) and HDI (Human Development Index). The impact of the religions will be count as the “religiosity” of various of faiths. The results showed us that the level of “religiosity” is statistically highly significant. Religiosity is the level of importance of particular religion in different states. The main sources of data are WVS, Euro-Barometer, Pew Research Center and others. Moreover, the empirical impact of Protestantism and Catholicism is different on economic indicators. (Received September 18, 2016)

1125-VH-1593 **Aramayis Orkusyan** and **Adnan H. Sabuwala\*** ([asabuwala@csumfresno.edu](mailto:asabuwala@csumfresno.edu)), 5245 N Backer Ave, M/S PB 108, Fresno, CA 93740, and **David Wu**.  $\pi$ : *Billiards, Physics, and Mathematics*. Preliminary report.

In this talk, we will present a famous billiards ball problem in physics. We will show its surprising connection to  $\pi$  and answer many related questions. We will also pose some open problems. In this talk, we use undergraduate linear algebra, trigonometry, basic complex variables, and should be accessible to any undergraduate student in mathematics. (Received September 18, 2016)

1125-VH-2362 **Brittany Bannish\*** ([bbannish@uco.edu](mailto:bbannish@uco.edu)). *Modeling the role of inhibitors in blood clot degradation*.

Fibrinolysis is the enzymatic degradation of blood clots. Understanding how blood clots degrade is important from both physiological and clinical standpoints. We will discuss our stochastic multiscale model of fibrinolysis, and how the presence of fibrinolytic inhibitors affects the progression of degradation. Specifically, we study the direct inhibitor  $\alpha_2$ -antiplasmin and the indirect inhibitor thrombin-activatable fibrinolysis inhibitor (TAFI). We show that the presence of these inhibitors affects the amount of plasmin (the main fibrinolytic enzyme) produced and how this reduction in plasmin slows the effective diffusion of other important enzymes through the clot. Results of this work have implications for stroke drug development. (Received September 20, 2016)

1125-VH-2662 **Anastasia B Wilson\*** ([anawilson@augusta.edu](mailto:anawilson@augusta.edu)). *Modeling Protein Adsorption in Multimodal Membranes*.

The emergence of biopharmaceuticals, and particularly therapeutic proteins, as a leading way to manage chronic diseases in humans has created a need for technologies that deliver purified products efficiently and quickly. Towards this end, there has been a significant amount of research on development of porous membranes used in chromatographic bioseparations. In this presentation, we focus on modeling high-capacity multimodal membranes developed by Husson and colleagues in the Department of Chemical and Biomolecular Engineering at Clemson University.

We will present results from numerical simulations using the advection-diffusion-reaction equation to model these membranes. We will focus specifically on different models for protein adsorption incorporating both instantaneous and non-instantaneous adsorption. We will also present a brief analysis of the breakthrough curves obtained from the numerical simulations and a comparison to experimental data. (Received September 20, 2016)

1125-VH-2667 **Jeff Poet\*** ([poet@missouriwestern.edu](mailto:poet@missouriwestern.edu)), 4525 Downs Drive, CSMP, Saint Joseph, MO 64507, and **Laurie Heyer**. *Optimizing the Search Space for New Biological Riboswitches – An Applied Combinatorics Problem*.

In an interdisciplinary research project with biologists, our team of undergraduates was asked to optimally design a large library of related genetic devices. The team developed several results, anticipating the unspoken potential restrictions of the biologists. (Received September 20, 2016)

1125-VH-2730 **Cynthia L Stenger\*** ([clstenger@una.edu](mailto:clstenger@una.edu)) and **James A Jerkins**. *Using computer programming to improve mathematical thinking*. Preliminary report.

As colleagues in a Mathematics/Computer Science department, we found that many of our undergraduates were not able to participate successfully in the full range of STEM course offerings. In response to this need, we formed a partnership with regional high school teachers and developed a strategy for explicit instruction in mathematical abstraction and generalization. Our instructional design is grounded in a theory of mathematical learning that uses computer programming to induce students to build the mental frameworks needed for understanding a math concept. The design includes writing mini programs to explore a mathematical concept, finding general expressions in the code, making conjectures about the relationships among general expressions, and writing logical arguments for the conjectures. We share results from a longitudinal study of 106 middle and high school math teachers attending professional development workshops employing this teaching method over a period of 3 years. Our results indicate the teachers showed significant improvement and maintained the improvement over subsequent sessions. (Received September 20, 2016)

1125-VH-2895 **Andrew E Long\*** ([longa@nku.edu](mailto:longa@nku.edu)), 495 Rossford Ave, Ft. Thomas, KY 41075, and **Steven Wilkinson** ([wilkinson@nku.edu](mailto:wilkinson@nku.edu)), **Madison Culbertson** ([culbertson1@mymail.nku.edu](mailto:culbertson1@mymail.nku.edu)) and **Laura Farro** ([farrol1@mymail.nku.edu](mailto:farrol1@mymail.nku.edu)). *Teasing climate signals from one hundred year-old seasonal data of Nova Scotia*. Preliminary report.

Starting in 1892 and continuing for 30 years, Alexander MacKay, Superintendent of Public Schools in Nova Scotia, directed a program of citizen science for the collection of seasonal data on plants and animals. Students in hundreds of schools amassed the data. MacKay, a brilliant scientist as well, summarized the collected data and published the summaries in Nova Scotia's premier science journal.

We pick up where MacKay left off, using his summary data. We incorporate meteorological data, sea ice data, etc. to tease out the impact of climate on changes in first appearance of flowers in five plant species (MacKay's favorites).

We built a complete data set for 23 years of MacKay data, dealing with region changes and missing values through weighted averages and imputation based on the Singular Value Decomposition (SVD).

Then we focus on physical aspects of place (e.g. winds and ocean currents) as well as climatic factors (e.g. air temperatures and Arctic sea ice extent, precipitation and snowfall) to model First Appearance as a function of latitude, longitude, air temperature, and sea ice extent. We use linear and non-linear regression, and application of a tensor variant of the SVD.

We hope that our work might shed light on how plants will respond in a changing climate. (Received September 20, 2016)

1125-VH-2936 **Anilkumar Devarapu\***, Department of Mathematics and CS, 504 College Dr, Albany, GA 31705, **Zephyrinus C Okonkwo**, 504 college Dr, Albany, GA 31705, and **Manjuladevi Gottapu**. *Similarity Solutions For a Class of Mixed Convection Heat Transfer Problems*.

This research article deals with similarity solutions for unsteady mixed convection flow. The parameter that characterizes mixed convection flow is  $Gr$ , where the Grashof number ( $Gr$ ) and the Reynolds number ( $Re$ ) represent the vigor of the natural convection and forced flow effects, respectively. The limiting case of  $\frac{Gr}{Re^n} \rightarrow 0$  and  $\frac{Gr}{Re^n} \rightarrow \infty$  correspond to the forced and natural convection limits, respectively. The non-linear coupled partial differential equations governing the mixed convection flow have been solved using different similarity methods, namely self-similar, semi-similar and non-similar transformation methods. We will show the detail analysis of these methods. (Received September 20, 2016)

1125-VH-2948 **Vani Cheruvu\*** ([vani.cheruvu@utoledo.edu](mailto:vani.cheruvu@utoledo.edu)), Department of Mathematics and Statistics, The University of Toledo, Toledo, OH 43606. *Wavelet Regularization for Numerical Solution of Laplace equation in an arbitrary shaped domain.*

Interior Dirichlet problem for the Laplace equation in an arbitrary shaped domain is considered. Analytic continuation is used to embed the given domain into a circular domain resulting in an inverse problem. The ill-conditioning associated with the inverse problem is dealt with wavelet regularization. In this talk, we present the idea and conclude with numerical results. (Received September 20, 2016)

1125-VH-2960 **Manjuladevi Gottapu\*** ([manjulanaidu23@gmail.com](mailto:manjulanaidu23@gmail.com)), Department of Mathematics and CS, Albany, GA 31705, and **Robert S Owor**. *Testing and Refining Dynamic Statistical Penetration Testing Security Indices.*

Dynamic and accurate measures for the security of computer systems is still an open question for which much research continues to be carried out. Between 2010 and 2015, researchers at Albany State University, developed theoretical Penetration Testing Statistical Indices for determining the security of a computer system. While much work has been done on the theoretical framework, these indices are yet to be practically tested and refined. In this research, a set of information assurance statistical measures will be tested for computability and practical reliability. Among the content areas for which statistical measures will be tested are reconnaissance, vulnerabilities, threats, attacks and defenses. The analysis of the statistical measures will be done from an attacker's point of view. Refinements and practical adaptations will be made on the statistical measures based on the practical results obtained. Finally a report of findings will be published. (Received September 20, 2016)

1125-VH-2964 **Lisa Driskell\*** ([liskel@coloradomesa.edu](mailto:liskel@coloradomesa.edu)). *Interdisciplinary Team Teaching: The Good, the Bad, and the Beautiful.*

With a desire to emphasize the interdisciplinary component of the student learning outcomes, my institution has established a "Milestone" course to serve as a capstone to the general education requirements. The Milestone courses are topics-oriented, team-taught courses at the 200-level. I will share my experience of teaching with a partner in Biology to a general audience and I will highlight the experiences of others on my campus. I will also provide details on our course, Numbers and Patterns in Nature, and its development from conception to implementation, including the integration of substance from both disciplines into the course activities and assignments. (Received September 20, 2016)

## General Session on Linear Algebra

1125-VI-154 **Rene Ardila\*** ([rene-ardila@uiowa.edu](mailto:rene-ardila@uiowa.edu)). *Factorization Properties of Graph Correspondences.*

In the field of operator algebras, graph correspondences are the source of a great amount of research activity. We study the factorization properties of semigroups formed by graph correspondences. By expressing these semigroups in terms of matrices, we show how to turn questions about correspondence factorization into questions about matrix factorizations. I will give examples of the great variety of factorization behaviors found in semigroups of graph correspondences: from unique factorization to extremely non-unique factorization. (Received August 04, 2016)

1125-VI-470 **Máximo De Jesús Santana\*** ([msantana22@uasd.edu.do](mailto:msantana22@uasd.edu.do)), Instituto de Matemática, Universidad Autónoma de Santo Domingo, Ciudad Universitaria, 10105 Santo Domingo, Dominican Rep, **Rafael Bru** ([rbru@mat.upv.es](mailto:rbru@mat.upv.es)), Institut de Matemàtica Multidisciplinar, Universitat Politècnica de València, 46022 València, Spain, **María Teresa Gassó** ([mgasso@mat.upv.es](mailto:mgasso@mat.upv.es)), Institut de Matemàtica Multidisciplinar, Universitat Politècnica de València, 46022 València, Spain, and **Isabel Giménez** ([igimenez@mat.upv.es](mailto:igimenez@mat.upv.es)), Institut de Matemàtica Multidisciplinar, Universitat Politècnica de València, 46022 València, Spain. *Combined Matrices of sign regular matrices.*

The combined matrix of a nonsingular matrix  $A$  is the Hadamard (entry wise) product  $C(A) = A \circ (A^{-1})^T$ . Since each row and column sum of  $C(A)$  is equal to one, the combined matrix is doubly stochastic when it is nonnegative. In this work, we study the nonnegativity of the combined matrix of sign regular matrices, based upon their signature. In particular, a few coordinates of the signature  $\varepsilon$  of  $A$  play a crucial role in determining whether or not  $C(A)$  is nonnegative. (Received September 02, 2016)

1125-VI-937 **Youngsoo Kim\*** (kimy@mytu.tuskegee.edu) and **Byunghoon Lee** (blee@mytu.tuskegee.edu). *Zero-Sum Coefficient Derivations in Three Variables of Triangular Algebras.*

It is known that a Jordan derivation of a triangular algebra is a derivation and that a Lie derivation or a Lie triple derivation of a triangular algebra is in the standard form, that is, the sum of a derivation and a central map. Generalizing the notions of Jordan or Lie derivations, one obtains  $f$ -derivations where  $f$  is a multilinear polynomial of several variables. Benkovič showed that if the sum of the coefficients of  $f$  is nonzero, every  $f$ -derivation of triangular algebras is a derivation (under mild assumptions).

We considered the case when the sum of the coefficients of  $f$  is zero and  $f$  is a multilinear polynomial of three variables. We will present sufficient conditions on the coefficients to ensure an  $f$ -derivation of a triangular algebra is a derivation or in standard form. We will also present a special case that has not been resolved yet and is still an open problem. (Received September 13, 2016)

1125-VI-1138 **Göran Bergqvist\*** (gober@mai.liu.se), Department of Mathematics, Linköping University, 58183 Linköping, Sweden. *Envelopes that bound the spectrum of a matrix.*

The real part of any eigenvalue of a matrix  $A$  is less or equal to the largest eigenvalue of its Hermitian part  $H(A)$ . Applied to  $\exp(-iv)A$  for all  $v$ , the spectrum of  $A$  is also contained in an infinite intersection of  $v$ -rotated half-planes, an intersection that equals the numerical range  $F(A)$ . Adam, Psarrakos and Tsatsomeros showed that using the two largest eigenvalues of  $H(A)$ , a cubic curve that restricts the location of eigenvalues can be constructed and, using the idea of rotations, the envelope of such cubic curves defines a region inside  $F(A)$  that still contains the spectrum. In contrast to  $F(A)$ , the new region is not necessarily convex or connected. Here we present a generalization of their method and show how new restricting curves for the spectrum can be found if one utilizes more than two eigenvalues of  $H(A)$ , and how the envelope of such curves bounds a new smaller region for the spectrum. (Received September 15, 2016)

1125-VI-1844 **Laurie Zack\*** (lzack@highpoint.edu), NC. *Mathematical Rankings of an FBI Drug Ring.* Various mathematical approaches of ranking were used to determine the level of importance of every member in a FBI drug ring using phone records. The phone data consisted of call logs over a three year span. This talk looks at which methods were used and which methods provided the best results. (Received September 19, 2016)

1125-VI-2335 **J Donato Fortin\*** (dfortin@jwu.edu), Johnson & Wales University, 801 West Trade St, Charlotte, NC 28202. *Wow Them: Achieve the Maximum Error in Ill-Conditioned Systems.*

Ill-conditioned linear systems ( $Ax = b$ ) admit potentially violent perturbations of the solution from relatively small changes to the coefficient matrix ( $A$ ) or the RHS ( $b$ ). Unexpected results evoke the "wow" factor and illustrate the potentially disastrous effects of observation, approximation, or round-off error. The way is shown to achieve the maximum error in case of small perturbations to either the coefficient matrix or the RHS. Explanations are via the singular value decomposition (SVD), but applications are accessible for any level course in which linear equations are solved. (Received September 20, 2016)

1125-VI-2512 **Joshua Boone\*** (joshua.boone@lmunet.edu). *Generalized Cyclotomic Polynomials and Projective Order.* Preliminary report.

We introduce  $\Phi_n(x, y)$ , the generalized cyclotomic polynomial, and derive some identities. An application to  $2 \times 2$  matrices of projective order  $n$  is given as well. Examples are provided throughout. (Received September 20, 2016)

1125-VI-2955 **Cara D. Brooks\*** (cbrooks@fgcu.edu), 10501 FGCU Blvd South, Fort Myers, FL 33965, and **Alberto A. Condori**, 10501 FGCU Blvd South, Fort Myers, FL 33965. *A Criterion for Normality.* Preliminary report.

In this talk, we discuss a distance formula that can be used as a criterion to determine normality of a matrix. We also establish some consequences related to norm behavior, pseudospectra, and unitary equivalence. (Received September 20, 2016)

1125-VI-2961 **Hashim A Saber\*** (hashim.saber@ung.edu), Mathematics Department, University of North Georgia, Oakwood, GA 30566. *Implementation of Nested Dissection Method Using Block Elimination.*

In this paper, we consider the problem of solving an  $n^2 \times n^2$  sparse positive definite system  $Ax=b$ , arising from the use of finite difference methods to solve an elliptic boundary value problem on an  $n \times n$  mesh where  $n=(2^k)-1$ .

The large sparse linear system can be solved directly in an efficient way using nested dissection method, originally proposed by Alan George. This paper demonstrates two algorithms for finding orderings using a version of the nested dissection method which leads to block Gaussian elimination of the matrix  $A$ . Implementation of these algorithms is pursued and the issue of storage and execution time trade-offs is discussed. (Received September 20, 2016)

1125-VI-3136 **Justin D Marks\*** ([marksj@gonzaga.edu](mailto:marksj@gonzaga.edu)). *Tangent Bundle Algorithms for Averaging Point Clouds on Grassmann and Stiefel Manifolds.*

In this talk, we introduce two iterative algorithms for determining mean representatives for a cluster of points on the Grassmann manifold and on the Stiefel manifold. Each of the algorithms depends upon a pair of maps parametrized by the underlying manifold, and we appeal to the characterization of the tangent and normal space at each point. These pairs of maps allow one to move from the manifold to a given tangent space and vice-versa. Iterating in a manner similar to the Karcher mean algorithm, we observe rapid convergence to a fixed point which acts as a mean for the cluster. (Received September 21, 2016)

## General Session on Logic and Foundations

1125-VJ-1927 **Kaethe Minden\*** ([kminden@gradcenter.cuny.edu](mailto:kminden@gradcenter.cuny.edu)). *Split Principles.*

In this talk I will introduce various “Split Principles”, which posit the existence of a sequence which in some sense “splits” any large set into two unbounded pieces. We will see that the failure of a particular split principle tends to characterize some large cardinal property; in particular weak compactness, ineffability, measurability and supercompactness can each be characterized in terms of the failure of a split principle. The initial idea of a split principle came about recently in the joint work of Fuchs, Gitman, and Hamkins. The content of the talk is the result of exploring the idea further with Gunter Fuchs. (Received September 19, 2016)

1125-VJ-1928 **Erin Carmody, Victoria Gitman and Miha E. Habič\*** ([mhabic@gradcenter.cuny.edu](mailto:mhabic@gradcenter.cuny.edu)). *A Mitchell-like order for Ramsey cardinals.*

The Mitchell order on a measurable cardinal  $\kappa$  orders the normal measures on  $\kappa$  based on the degree of measurability that  $\kappa$  retains in their respective ultrapowers. We introduce a Mitchell-like order for Ramsey and Ramsey-like cardinals, ordering collections of filters witnessing the large cardinal property. This order and its associated ranking have all the desirable properties of the classical Mitchell order on measurable cardinals and neatly stratify the large-cardinal hierarchy in the region of Ramsey cardinals. We show that the order behaves robustly with respect to forcing extensions and give a forcing construction to precisely control the rank of a given cardinal. (Received September 19, 2016)

1125-VJ-2308 **Cyrus F Nourani** ([acdmkrd@gmail.com](mailto:acdmkrd@gmail.com)) and **Patrik Eklund\*** ([peklund@cs.umu.se](mailto:peklund@cs.umu.se)), Umea, Sweden. *Term Functors and Signature Product Models: A Brief.* Preliminary report.

Theorem 1 (Nourani 2014) There is a generic functor on the category the omitting  $n$ -types realizing a direct product model. Let us forward with  $n$ -types and positive local realizability. The set of all complete  $n$ types over  $T$  is denoted  $n(T)$ . Newer areas are term functors direct product algebra category; Objects are term functors and morphisms are natural transformations on representation presheaves (Nourani 2006). Let us call these  $nD$ -type embedding categories, or  $F$ -Type categories. Theorem 2 There are embedding functors from  $F$ -Type to the direct product category realizing a filter for the product algebra trees on  $nD$ types. Let there be a functor category defined on power set  $(T < \text{Sigma} >)$ .  $T(\text{Sigma})$  is the free trees on signature  $\text{Sigma}$ . This can be glimpsed with a monoidal category. The adjunction is with the  $n$ -type product signature category, natural transformations  $\text{Hom}$  functors and left adjoint forget full functors category to the  $n$  type product signature category. The adjunction functors being the functor  $F$ , forgetful to the product signature  $n$ -type category with  $G$  the embedding functor from the  $n$ -type category on the product tree signatures to the Powerset category. Proposition  $F.G$  is a Monad on pair product signature  $n$ -type. Remark : The above is a filter monad. (Received September 21, 2016)

1125-VJ-2463 **Shehzad Ahmed\*** ([sa066513@ohio.edu](mailto:sa066513@ohio.edu)). *Club Guessing in Prikry Models.* Preliminary report.

If  $\mu$  is a singular cardinal with  $\mu^+$  Jónsson, then club guessing ideals on  $\mu^+$  and scales on  $\mu$  seem to be, on the surface, quite related. So one might ask whether or not there is a deep reason for this phenomenon. The purpose of this talk is to probe at the relationship between scales and club guessing. In particular, we will do this by

way of Prikry extensions, as scales are relatively well understood there. Time permitting, we will show how to produce a club guessing sequence so that it avoids scales in a technical sense. (Received September 20, 2016)

1125-VJ-2942 **Ron Taylor\*** (rtaylor@berry.edu), Department of Math & CS, Berry College, Mount Berry, GA 30149, **Carter Smith** (carter.smith@vikings.berry.edu), G.W.W. School of Mechanical Engineering, Georgia Tech, and **Michael Papazian** (mpapazian@berry.edu), Department of Religion and Philosophy, Berry College. *Separable equivalence*. Preliminary report.

The notion of separable equivalence is an extension of logical equivalence where we do not consider the order of the values in a truth table. In particular we say that two statements are separably equivalent if they have the same ratio of true and false values in a truth table. In this talk we provide a conceptual and theoretical development of this new concept. We also show how this idea of separable equivalence can serve as an intuitive and accessible way to provide a theory of possibility in propositional logic. (Received September 20, 2016)

1125-VJ-3013 **Hakim J. Walker\*** (hjwalker@gwu.edu), 801 22nd Street NW, Room 724A, Washington, DC 22202. *Effective Categoricity of Infinite Directed Graphs and Trees*. Preliminary report. Two computable structures  $\mathcal{A}$  and  $\mathcal{B}$  are *computably isomorphic* if there exists a computable isomorphism from  $\mathcal{A}$  to  $\mathcal{B}$ . Furthermore, we say that  $\mathcal{A}$  is *computably categoric* if every two computable copies of  $\mathcal{A}$  are computably isomorphic. Significant work on computable categoricity (or effective categoricity) has been done for a variety of mathematical structures, including linear orders, abelian groups, Boolean algebras, and many others.

We introduce the notion of a  $(2,1):1$  structure, which consists of a countable set  $A$  (usually the natural numbers) together with a function  $f : A \rightarrow A$  such that for every element  $x$  in  $A$ ,  $f$  maps either exactly one or exactly two elements of  $A$  to  $x$ . These structures extend the notions of injection structures,  $2:1$  structures, and  $(2,0):1$  structures studied by Cenzer, Harizanov, and Remmel, all of which can be thought of as infinite directed graphs. In this talk, we investigate various computability-theoretic properties of  $(2,1):1$  structures, provide conditions under which such structures are (and are not) computably categoric, and present some interesting examples. (Received September 20, 2016)

## General Session on Mathematics and Technology

1125-VK-1570 **Boyan Kostadinov\*** (bkostadinov@citytech.cuny.edu), NYC College of Technology, 300 Jay Street, Mathematics Department, Brooklyn, NY 11201. *R is not only for Data Science: Visualizing Art Patterns Coded in R*.

We present several visualization projects, which we developed for our STEM students interested in coding. The projects are designed to mix programming, mathematics and experimentation, and engage students' creativity by appealing to their artistic side for creating art patterns inspired by mathematics. The mathematics behind the projects makes use of finite weighted sums of complex exponentials, out of phase logarithmic spirals to visualize galactic arms in spiral galaxies, 2D and interactive 3D Lissajous figures, the Mandelbrot set and other fractal systems, and contour projections of 2D surfaces superimposed over their own heat-maps for creating complex art patterns. All projects are designed to be implemented in 10-20 lines of vectorized code, using the high-level, open-source, free programming language R, a popular software in industry and academia for doing data science. We hope that familiarity with R could improve students' chances of getting internships and full-time jobs. These projects were created for the "Code in R" program that we developed at City Tech, CUNY, supported by a MSEIP Grant P120A150063 from the Department of Education. (Received September 18, 2016)

1125-VK-1979 **Ratna Khatri\*** (rkhatri3@masonlive.gmu.edu), George Mason University, Attn: Department of Mathematical Sciences, 4400 University Drive, Fairfax, VA 22030, and **Evelyn Sander** (esander@gmu.edu), George Mason University, Attn: Department of Mathematical Sciences, 4400 University Drive, Fairfax, VA 22030. *Mathematical Education and 3D Printing in the GMU Math Maker Lab*. Preliminary report.

3D Printing is revolutionizing the production industry. It has made prototyping time-saving and accessible to people. In this talk, we present our experiences of using 3D Printing as an instructional tool in mathematics courses. We describe details of how we have successfully incorporated 3D printing in teaching calculus and upper-level undergraduate math courses, in a combination of in-class demonstration materials and hands-on learning. We have also included 3D printing as part of our outreach activities, through which we expose middle and high school children to the application of 3D printing in mathematics. We additionally facilitated the math learning experience of a blind student using 3D printing through the use of tactile graphs. The talk will include

examples of 3D printed models which have enhanced student understanding and intuition of mathematical ideas. (Received September 19, 2016)

1125-VK-2321 **Nermin Bayazit\*** (nbayazit@fitchburgstate.edu), **Draga Vidakovic** (dvidakovic@gsu.edu) and **Pier Junor Clarke** (pjunor@gsu.edu). *“Active” vs “Looking Active” in a Fully Online Mathematics Class: Word of Caution*. Preliminary report.

In our research to understand the argumentative knowledge construction in an online mathematics course, the case of Kevin and Helen challenged us with our initial perception of “active learners”. In order to better understand the knowledge construction process we wanted to focus on the students who were the “most active” and/or “most present” on the discussion board. The impetus was that they would give us more access to their thinking. As we closely analyzed their “presence in the course” we realized the substantial differences in their participation and it became critical for us to re-conceptualize “active presence” in an online course using Weinberger and Fisher (2006)’s framework. During this presentation, we will share our preliminary definition of an “active learner in an online course,” some of the indicators to identify “active looking learners” and a potential way to tackle this issue in an online mathematics course. (Received September 20, 2016)

1125-VK-2496 **Eric J Kostelich\*** (kostelich@asu.edu), School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287. *High-Performance Computing for Undergraduate and Graduate Mathematics Students*.

I will describe two courses in high-performance computing, one aimed at undergraduates and the other at first-year graduate students in applied mathematics. The informal goal is to help students “hit the ground running” in internships or thesis work that involves a substantial amount of numerical computing. The undergraduate course emphasizes software development skills in a Linux environment with applications in numerical linear algebra and ordinary differential equations. Topics include shell scripting, C/C++ and Fortran programming, the make utility, and libraries like LAPACK and ODEPACK. The graduate course includes some of these same topics plus a survey of models of parallelism, including vectorization; multithreading with OpenMP; distributed memory programming with the Message Passing Interface; and partitioned global address space environments (e.g., coarray Fortran and Unified Parallel C). I will include examples of programming exercises and other topics that are suitable for lower-division programming courses. (Received September 20, 2016)

1125-VK-3023 **Mojtaba Moniri\*** (m-moniri@wiu.edu). *Pairs of close cycle-points in a logistic map: 5-periodicity or 10?*

The logistic map with parameter  $r$  is defined on  $[0, 1]$  as  $f_r(x) = rx(1 - x)$ . Most parameters  $r \in (3.570, 4)$  exhibit a chaotic orbit for an arbitrary initial point  $x_0 \in (0, 1)$ . However the interval  $(3.739, 3.744)$  is inside a period-5, 10, 20, 40, etc., region (with successive period-doubling bifurcations). Here we are concerned with the particular parameter  $r = 3.74$  to clarify on some misleading literature, like half of a dozen items which we mention, that claim an attracting 5-cycle. For  $x_0 = 0.5$ , some relied on, just as early as, the 100 – 110th terms, all up to only six decimal digits. We give a Mathematica code which outputs the 100,000,000th term followed by the next 19, all with 22 digits (in  $\approx 3$  minutes on an ordinary desktop). It indicates two cycles of length 10. The five pairs of counterpart numbers agree up to 13 – 15 digits. Generally speaking, and in similar situations, one could challenge this via two attacking approaches. One is to iterate more in an effort to see if we can overcome disagreements: can the digits be stabilized and discrepancies go away (10 to 5 undertake)? In parallel, one can zoom out on the digits in an effort to see if the current agreements persist, or rather differences show up (10 to 20 undertake). (Received September 20, 2016)

## General Session on Modeling and Applications

1125-VM-153 **Heidi A. Berger\*** (heidi.berger@simpson.edu), 701 N C Street, Indianola, IA 50125, and **Emma C. Christensen, Nick J. Joslyn** and **Madeline M. Kersten**. *Optimization of Down Syndrome Specialty Care Clinic Locations using Operations Research*. Preliminary report.

As the field of healthcare continues to grow and evolve, specialized and coordinated care for extraordinary conditions becomes more and more necessary. Down syndrome, one of these extraordinary conditions, is a chromosomal disorder that is often accompanied by other severe medical conditions that affect an individual’s overall health. Currently, there are 71 Down syndrome specialty care clinics in 34 states in the U.S. that serve less than 10% of the eligible population. Using operations research, specifically  $p$ -median and maximal covering models, we found that the current placement of these 71 clinics adds to the problem of inaccessibility. Our

research presents the optimal locations for Down syndrome specialty care clinics in the continental United States and offers potential future clinic locations given the current placement. (Received August 04, 2016)

1125-VM-270 **L Patton\*** ([critique@vt.edu](mailto:critique@vt.edu)), Virginia Tech, 220 Stanger St., Blacksburg, VA 24061-0126. *Time-frequency methods for parameter estimation using gravitational waves*. Preliminary report.

"On September 14, 2015 at 09:50:45 UTC the two detectors of the Laser Interferometer Gravitational-Wave Observatory [LIGO] simultaneously observed a transient gravitational-wave signal" (<http://dx.doi.org/10.1103/PhysRevLett.116.061102>). The LIGO researchers concluded that the signal emanates from a binary black hole (BBH) system. The LIGO researchers use an estimate of the chirp mass of the system (the initial mass minus the mass radiated as gravitational waves) to estimate the parameters of the BBH system. I present and analyze a "time-frequency" method from Chassande-Mottin and Flandrin (App. and Comp. Harmonic Analysis) to model a range of expected values for the chirp mass, and the effect of changing values on the time-frequency method for detection of gravitational waves. (Received August 22, 2016)

1125-VM-332 **Layachi Hadji\*** ([lhadji@ua.edu](mailto:lhadji@ua.edu)), Mathematics Department, 345 Gordon Palmer Hall, Tuscaloosa, AL 35487, and **C. Taber Wanstall**, Mathematics Department, 345 Gordon Palmer Hall, Tuscaloosa, AL 25487. *A new model of the convective stability of geological carbon sequestration*.

The convective stability associated with carbon sequestration is modeled by an unstably stratified basic profile having a step function density with top heavy carbon saturated layer overlying a lighter carbon free layer. The resulting configuration mimics that of the Rayleigh-Taylor problem without the free interface. We carry out a linear stability analysis to derive the instability threshold parameters for two sets of boundary conditions. First, an upper boundary that is maintained at a concentration  $C_0$  and a lower boundary that is impervious to mass flow. Second, we consider an upper boundary that is nearly impermeable and a lower boundary that is impervious to mass flow. We solve for the minimum thickness of the carbon-rich layer at which convection sets in. The threshold instability conditions are found to be much lower than those corresponding to continuous stratification. The investigation of the second case is extended to the nonlinear regime, the analysis of which leads to the determination of a uniformly valid super critical steady solution. Our model accounts for anisotropy in both diffusion and permeability and chemical reactions between the  $CO_2$  rich brine and host mineralogy. (Received August 27, 2016)

1125-VM-347 **Tiffany Jann\*** ([tiffanyjann@berkeley.edu](mailto:tiffanyjann@berkeley.edu)) and **Erin Boggess** ([erin.boggess@my.simpson.edu](mailto:erin.boggess@my.simpson.edu)). *Reverse Engineering Functional Brain Networks from fMRI Data Using Probabilistic Boolean Networks*. Preliminary report.

The brain functions by communicating information across regions, and neurological diseases can alter the way these regions communicate. To characterize brain disorders and eventually propose systematic approaches to diagnosis, we should study the brain as a system and consider both its structure and dynamics. In the present work, we proposed a pipeline to reverse engineer static functional brain networks and dynamic mathematical models from fMRI data. Using probabilistic Boolean networks (PBNs) as our mathematical framework, our pipeline iterates through several steps, each with non-trivial aspects to resolve. We studied these steps using fMRI data generated from in silico networks, and successfully validated steps (1-2). For step (1), reverse engineering static functional brain network from fMRI data, we applied 44 reverse engineering methods and proposed a way to combine top-performing methods such that the result outperformed any individual method. In step (2), discretizing fMRI data into Boolean states, we proposed a novel metric to benchmark and rank 11 discretization methods. For step (3), inferring dynamical models, our preliminary studies are consistent with results from our proposed metric from step (2), and future work should focus on the validation of dynamic models. (Received August 28, 2016)

1125-VM-482 **Shota Kurtanidze\*** ([shotkurtanidze@gmail.com](mailto:shotkurtanidze@gmail.com)), apt. 41, 67a Ketevan Tsamebuli ave., 0120 Tbilisi, Rep of Georgia. *Securing FingerPrint Data By RSA algorithm*.

In many organizations fingerprint sensor is used to determine if person has access to this organization. It is reasonable to forbid storing fingerprints by organizations. We developed solution which allows organization to verify person's fingerprint without storing fingerprints in organization's database. We offer to use plastic cards which stores the encrypted version of the fingerprint of the person and for encryption the RSA algorithm private key of the Organization is used. When person's identity is to be verified, organization scans his/her fingerprint, then retrieves the encrypted fingerprint from person's plastic card, decrypts it with its public key and compares with the scanned fingerprint of the person and by this the correspondence of person's fingerprint

with the encrypted fingerprint stored on the plastic card is established. As for identification of the person, his RSA private key is used. Person's fingerprint is stored on the plastic card as encrypted by the private key of the Person. The public key of the person is stored in organization's database. When retrieving user's fingerprint from the plastic card, organization has to decrypt it by the public key of the person and by this the identification of the person is conducted. (Received September 18, 2016)

1125-VM-654 **Jack A. Ryan\*** ([jjryan23@vols.utk.edu](mailto:jjryan23@vols.utk.edu)). *Betting Better on Broadway: the Application of Statistical Matrix Theory to the Prediction of the Tony Awards for Best Play and Best Musical.*

In this work, we employ methods previously used to rank sports teams to analyze particular quantifiers pertaining to art. The research uses statistical principles and matrix theory to predict which show will win a Tony Award in a given season. Using weekly statistics including weekly gross profit, percentage of seats sold, average ticket price, top ticket price, and number of seats sold, we rank Tony Award nominees from the past ten years using two algorithms: the Colley and Massey methods. The method and quantifier combination that most often successfully yields accurate results is used to predict the winners of the 2016 Tony Awards for Best Play and Best Musical. (Received September 08, 2016)

1125-VM-663 **Giang Tran\*** ([gtran@math.utexas.edu](mailto:gtran@math.utexas.edu)) and **Rachel Ward.** *Exact Recovery of Chaotic Systems from Highly Corrupted Data.*

Learning the governing equations in dynamical systems from time-varying measurements is of great interest across different scientific fields. This task becomes prohibitive when such data is moreover highly corrupted or sensitive to initial conditions, for example, due to the recording mechanism failing over unknown intervals of time. In this work, we consider continuous time dynamical systems where each component of the governing equations  $f$  is a multivariate polynomial of maximal degree  $p$ ; we aim to identify  $f$  exactly from possibly highly corrupted measurements. As our main theoretical result, we show that if the system is sufficiently ergodic that this data satisfies a strong central limit theorem (as is known to hold for chaotic Lorenz systems), then the governing equations  $f$  can be exactly recovered as the solution to an L1 minimization problem – even if a large percentage of the data is corrupted by outliers. Numerically, we apply the alternating minimization method to solve the corresponding optimization problem. Through several examples of 3D chaotic systems and higher dimensional hyper-chaotic systems, we illustrate the power, generality, and efficiency of the algorithm for recovering governing equations from noisy and highly corrupted measurement data. (Received September 08, 2016)

1125-VM-676 **Junkoo Park\*** ([jpark15@ggc.edu](mailto:jpark15@ggc.edu)). *A refined Gaussian Network Model and Its Application to Biological Structures.*

Gaussian Network Models (GNM) have shown promise in predicting the B-factors of amino acids, a data point which is obtained in X-ray crystallography experiments. In order to improve protein modeling, there have been attempted modifications to the model for obtaining better correlation between experimental B-factors and predicted B-factors from the model. However, many of them don't consider including amino acid interaction preferences but rather focused on the physical properties of bonds. In this talk, a refined Gaussian Network Model reflecting the inherent physical characteristics of biological structures is presented and compared to the previously modified Gaussian Network Model. (Received September 09, 2016)

1125-VM-789 **William Patrick Noland\*** ([wpnoland@noctrl.edu](mailto:wpnoland@noctrl.edu)), 3741 Evensong Drive, Union, KY 41091, **Dylan Matthew Smith** ([dylan.m.smith@uconn.edu](mailto:dylan.m.smith@uconn.edu)), 74 East Avenue, Glen Cove, NY 11542, and **Seth Selken** ([sselken@iastate.edu](mailto:sselken@iastate.edu)), **Monica Swartz** ([mswartz@smith.edu](mailto:mswartz@smith.edu)), **Marcus Battraw** ([marcusbattraw@gmail.com](mailto:marcusbattraw@gmail.com)) and **Sergei Fomin** ([sfomin@csuchico.edu](mailto:sfomin@csuchico.edu)). *Modeling Tsunami Run-Up and Draw-Down on the Beach.*

Previous literature has investigated the run-up and draw-down of tsunami waves on a one-dimensional, constant-sloped beach, but existing solutions are complex and computationally unwieldy. Our research aims to establish a simpler model while still obtaining accurate results. We do so by using a quasi-linear theory derived from the nonlinear shallow-water wave equations. The main difficulty in solving this problem is the moving boundary associated with the shoreline motion, which we eliminate by applying a substitution to the spatial variable. A key feature of any tsunami problem is the presence of the small parameter  $\varepsilon = \eta_0/h_0$ , where  $\eta_0$  is the characteristic amplitude of the wave and  $h_0$  is the characteristic depth of the ocean. Due to the presence of this small parameter, the problem can be essentially linearized using the method of perturbations and then solved analytically via an integral transformation. The resulting explicit solution enables us to swiftly predict the behavior of the wave using an essentially linear model. Testing the accuracy of our model against the numerical solution obtained

using Mathematica reveals minimal discrepancy. This project supported by the NSF award DMS – 1559788. (Received September 11, 2016)

1125-VM-1418 **Anthony Bonifonte\*** (abonifon@gatech.edu), **Turgay Ayer**, **Benjamin Haaland** and **Peter Wilson**. *A Continuous Time Stochastic Model to Optimize Blood Pressure Treatment Decisions*.

Antihypertensive drug treatment can control elevated blood pressure and reduce the risk of future cardiovascular outcomes. We develop a data-driven stochastic model of blood pressure progression that generalizes Brownian motion by modeling the change in blood pressure per unit time as a Gaussian mixture distribution. This model addresses the question of what thresholds at which to initiate antihypertensive treatment and the optimal intensity. Our main finding is initiation and intensity decisions depend jointly on systolic and diastolic pressure. The methods are generalizable to other chronic diseases with continuous valued measurements. (Received September 16, 2016)

1125-VM-1447 **Kevin Winseck\*** (kevin.winseck@asu.edu), School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287, and **Edward Martinez** and **Scott Mahan**. *Obstacles and Boundaries in Flocking Behavior*. Preliminary report.

Flocking behavior observed in birds, fish and insects can be modeled by a system of differential equations. This talk will focus on a model in which agents' behaviors are determined by equations describing alignment, attraction, and repulsion. Of particular interest is the inclusion of parameters describing boundary conditions. We will explore the effects of obstacles within the domain, specifically if these obstacles prevent emergence of a flock. The existence of flocking is quantified by examining the variance in velocity of all the agents, with a flock forming as variance in velocity approaches zero. (Received September 16, 2016)

1125-VM-1470 **Edward Martinez\***, School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287, **Scott Mahan**, School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287, and **Kevin Winseck**, School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287. *Optimized Control of Flocking Models*. Preliminary report.

In flocking models, agents interact according to a set of rules which lead them to an equilibrium in velocity. We examine how different rules affect flocking behavior. Our model consists of a 3-zone Cucker-Smale model with self-propulsion. We introduce a novel optimized control that aims to minimize an appropriate energy functional in the system. For instance, to facilitate flocking, the energy is taken as the variance in velocity of the system. We use a Lagrangian formulation to solve numerically the optimization problem. The numerical implementation uses a combination of forward/backward Euler's method. The control greatly reduces the variance in velocity and also in position creating a more coherent flock. (Received September 17, 2016)

1125-VM-1533 **Scott Mahan\***, School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287, **Edward Martinez**, School of Mathematical & Statistical Sciences, Box 871804, Arizona State University, Tempe, AZ 85287, and **Kevin Winseck**, School of Mathematical & Statistical Sciences, Box 871704, Arizona State University, Tempe, AZ 85287. *Sparse Control and Disruptive Behavior in Biological Flocking Models*.

The Cucker-Smale model aims at mimicking the flocking behavior of biological agents. This research extends that model and explores how optimal control theory can enhance flocking behavior. One application involves sparse control: how many agents must be controlled to successfully form a flock? The study also focuses on implementing a controlled adversarial agent who serves to disrupt the flocking behavior. I will show results that the number of controlled agents is negatively correlated with flocking time and that disruptive behavior indeed prevents flocking under certain conditions. These findings may broaden the scope of agent-based models and their applications to mathematical biology. (Received September 17, 2016)

1125-VM-1548 **Kasie G Farlow\***, kasie.farlow@dc.edu, and **Desmond Cummins**, **Joseph Pedersen** and **Brian M Sadler**. *n-Section Querying Methods for Target Estimation on an Interval*. Preliminary report.

We consider a point estimation problem to approximate the location of a unique scalar value target in a known partitioned interval using a twenty questions like game. At each step in the game a noisy oracle is queried as to which set the target resides in. We estimate the target by modeling it as a scalar valued random variable with a prior distribution. We follow a Bayesian type approach in that after each iteration or query we find a posterior distribution of the random variable. We develop new querying techniques by extending the probabilistic bisection method from Jedynek, et, al., and Waeber, et al., to a method which partitions the unit interval into  $n$  regions

of equal probability with  $n \geq 2$ . Several different models are considered for the oracle's noisy response using an  $n$ -ary channel, including modifications to roughly model human query response error. We analyze this method in terms of entropy, variance and mean square error and find optimal querying policies to maximize the expected entropy reduction per query. Our Simulations show that the variance and mean square error of the posterior probability distribution associated with the estimated target's location converges to 0. (Received September 17, 2016)

1125-VM-1580 **Ayush Prasad\*** (ayush.prasad520@topper.wku.edu). *Using Individual Patient Data to Quantify a Mathematical Model for the Interactions of Matrix Metalloproteinases and Their Inhibitors in a Wound*. Preliminary report.

Because the medical treatment of diabetic foot ulcers remains a challenge for clinicians, a quantitative approach using patient data and mathematical modeling can help researchers understand the physiology of the wounds. In this work, we extend a previously developed mathematical model describing the interactions among matrix metalloproteinases, their inhibitors, extracellular matrix, and fibroblasts (Krishna et al., 2015). In the previous work, the model was curve-fitted to the averaged data of patients with diabetic foot ulcers from Muller et al. (2008), and the model parameters were estimated using ordinary least-squares. The model and parameter values were then analyzed using global and local sensitivity analyses, which were used to describe how sensitive each parameter value of the model was to changes in the system. This work uses the individual patient data obtained from Muller for curve-fitting a modified model using similar techniques from the previous work. The goal of this work is to quantify and understand differences between patients in order to predict future responses and individualize treatment for each patient. (Received September 18, 2016)

1125-VM-1773 **Danilo R. Diedrichs\*** (danilo.diedrichs@wheaton.edu), **Kaile Phelps** and **Paul A. Isihara**. *Quantifying Communication Effects in Disaster Response Logistics: A Multiple Network System Dynamics Model*.

Complementing the importance of adequate relief supplies and transportation capacity in the first two weeks of post-disaster logistics, efficient communication, information sharing and informed decision-making play a crucial yet often underestimated role in reducing wasted material resources and loss of human life. We propose a discrete dynamical systems model to model the transportation of different commodities from multiple relief suppliers to disaster sites across a network of limited capacity. The physical network is overlaid with the communication network to model information delays and communication breakdowns between agents. The cost in human lives and the monetary cost are measured separately. Simulation results highlight quantitatively how communication deficiencies and indiscriminate shipping of resources result in material convergence and shortage of urgent supplies observed in actual emergencies. The model provides an example of a simple, objective, quantitative tool for decision-making and training volunteer managers in the importance of a smart response protocol. (Received September 19, 2016)

1125-VM-1810 **Kelly Rooker\*** (krooker@vols.utk.edu) and **Sergey Gavrilets** (gavrila@tiem.utk.edu). *Modeling the evolution of female sexual signaling*.

A long-standing evolutionary mystery surrounds female ovulation signs. Even among just primates, why do some species have substantial sexual swellings and/or bright colorations around their genital regions, while other species are like humans, in which there are no signs of ovulation visible? What is the evolutionary purpose behind not just these signs, but also this great variation seen among species? Here we examine the evolutionary trade-offs associated with ovulation signaling using evolutionary game theory and mathematical modeling. In particular, we use adaptive dynamics to construct two models, one a mean-field approximation and the other an individual-based model, in order to determine equilibria values and their stability. Our modeling results show convergence to stable equilibria (typically unique) with female sexual signaling increasing or decreasing in the population depending on parameter conditions. We find that increasing the impact of infanticide can increase ovulation signaling, but despite any benefit visible ovulation signs have in protecting a female's offspring, ecological costs (e.g. increasing group size and/or increasing physiological costs of having ovulation signs present) can still surpass any benefit, leading to a state of concealed ovulation. (Received September 19, 2016)

1125-VM-1883 **Arielle Gaudiello\*** (arielle@ucf.edu), 4393 Andromeda Loop N, Orlando, FL 32816, and **Zhisheng Shuai** (zhisheng.shuai@ucf.edu), 4393 Andromeda Loop N, Orlando, FL 32816. *A Mathematical Model for the Human Papillomavirus (HPV) with a Case Study in Japan*. Preliminary report.

The human papillomavirus (HPV) is a sexually transmitted infection prominent among young adults across the world. The self-clearing infection is a predecessor to numerous cancerous cells, ranging from cervical to penile to

esophageal cancers. We develop an ordinary differential equation model, where the total population is divided into seven classes: juveniles ( $J_k(t)$ ), susceptibles, ( $S_k(t)$ ), infectious ( $I_k(t)$ ), with  $k = f, m$  for females and males respectively, and vaccinated females ( $V_f(t)$ ). We investigate the existence and stability of the disease-free equilibrium and endemic equilibrium. We discuss applications to ongoing issues in Japan, where government-based vaccination programs have terminated and vaccination rates have plummeted. (Received September 19, 2016)

1125-VM-2020 **Kodwo Annan\*** (kannan@ggc.edu), School of Science & Technology, Georgia Gwinnett College, Lawrenceville, GA 30043. *Stability and Time-scale Analysis of Malaria Transmission in Human-Mosquito Population.*

More realistic human-mosquito mathematical model in which re-infected asymptomatic humans are considered is presented. The Next Generation Matrix technique is used to construct epidemiological threshold known as the reproduction number. Locally and globally asymptotically stable disease-free equilibrium conditions for the model are established. Possible time-scale of events for model transition from non-endemic to endemic are analyzed. Results show that the buildup of the latent asymptomatic humans at steady state is the main dynamics of malaria in the endemic region. (Received September 19, 2016)

1125-VM-2184 **Carynne Litcher** and **Chunhui Yu\*** (chunhui.yu@farmingdale.edu), Mathematics Department, Farmingdale State College, SUNY, 2350 Broadhollow Road, Farmingdale, NY 11735. *Shortfall risk in long term hedging with short-term futures contracts on multi-commodity case.*

In this paper, we study the strategies to reduce shortfall risk in long-term hedging with short-term futures contracts on multi-commodity case. We re-evaluate these strategies in Glasserman's work and introduce new hedging strategies in order to minimize average risk. (Received September 19, 2016)

1125-VM-2369 **Brooks K Emerick\*** (brooks.emerick@trincoll.edu), 300 Summit Street, Hartford, CT 06106. *The effects of parasitoid migration on stability of discrete-time host-parasitoid population dynamic models.* Preliminary report.

Extensive work has been done on analyzing host-parasitoid interactions using discrete-time models, the most notable of which is the Nicholson-Bailey model. Our research focuses on a semi-discrete framework in which the host-parasitoid interactions are characterized by a continuous-time model. The continuous dynamic allows us to incorporate intricate behaviors of the host-parasitoid interaction such as host-feeding, egg load capacity, or migration. This paper incorporates migration of parasitoids between two locations. We find that in the simplest case, when the migration rates are constant, the model is unstable yielding diverging oscillations similar to the Nicholson-Bailey model. However, when we consider one-way migration, i.e. a no-return scenario, coexistence between hosts and parasitoids occurs. A similar stability region arises when we consider an instant transportation of parasitoids between the two locations. In this work, we present analytic and numerical results that describe the region in parameter space in which coexistence among the two species is possible. This parameter space is characterized by two factors: the number of viable larvae per adult host and the fraction of host larvae present at the initial location each year. (Received September 20, 2016)

1125-VM-2416 **Jerome Goddard II\*** (jgoddard@aum.edu), **A. Barnett, D. Harrell** and **R. Shivaji** (shivaji@uncg.edu). *Modeling habitat fragmentation at the landscape level via reaction diffusion equations.*

Habitat fragmentation affects a population in two key aspects, namely, the size of fragmented patches of habitat and the quality of inferior habitat surrounding the patches, called the matrix. Ecologists have confirmed that an organism's survival in a system is often linked to the size of the patches, quality of its surrounding matrix, and distance between patches. In this talk, we will model the effects of habitat fragmentation at the landscape level using a reaction diffusion system. We will explore dynamics of the model via study of the model's positive steady state solutions. Our results are obtained through a time map analysis (quadrature method) and Mathematica computations. We will briefly explore the biological implications of these results. (Received September 20, 2016)

1125-VM-2474 **Germaine Kamleu Ndouma\*** (3008010@myuwc.ac.za), Cape Town, South Africa, **Lorna Holtman**, Cape Town, South Africa, and **Bingwen Yan**, Cape Town, South Africa. *Analytical model for assessing the knowledge of statistical procedures amongst postgraduate students.*

Over the past decades, university students experienced considerable difficulties in applying the knowledge of statistical concepts that they learnt in their previous courses. In this study, we work with the South African

Higher Education system. In this context, many strategies were developed to redress issues of reparation and social imbalances inherited from apartheid and to reconstruct a comprehensive educational quality framework. This study proposes an analytical model to assess the knowledge of statistical procedure amongst postgraduate students in academic research environment with the new Higher Education (HE) system. The results indicate that confusion and frustration characterised the attitude of students during the selection of suitable statistical test. (Received September 20, 2016)

1125-VM-2535 **Nicole M Panza\*** ([npanza@marion.edu](mailto:npanza@marion.edu)), Francis Marion University, Department of Mathematics, PO Box 100547, Florence, SC 29505-0547. *Modeling Three-Wave Follicle Dynamics in the Menstrual Cycle*. Preliminary report.

A nonlinear differential equation model which represents the hormonal regulation of the menstrual cycle with three follicle waves per cycle is presented. Follicle waves have been reported in women by Baerwald et al. (2003). Typically two or three waves occur per cycle. The model exhibits three waves of antral follicles during a woman's cycle using a Follicle Stimulating Hormone threshold function. The three-wave cycle is simulated by modeling six reproductive hormones. (Received September 20, 2016)

1125-VM-2692 **Lidia Smith\*** ([lidia.smith@blinn.edu](mailto:lidia.smith@blinn.edu)), 2423 Blinn Blvd., Bryan, TX 77802. *Probabilities in a Sensor Network*.

A problem of interest in engineering is placement of sensors for detection of events. For a water distribution system such events could be water contamination. Optimization problems that have as goal maximizing the probability of event detection given resource constraints are formulated as integer linear programming problems. The mathematical modeling starts with viewing the water system as a directed graph with vertices corresponding to junctions and edges corresponding to the pipes. Probabilities are associated with edges based on the known flow through the pipes. In this article we will discuss the equivalence between two different formulations of the optimization problem based on properties of expected value. (Received September 20, 2016)

1125-VM-2871 **Betsy Heines\*** ([ehaines@vols.utk.edu](mailto:ehaines@vols.utk.edu)), 227 Ayres Hall, 1403 Circle Drive, Knoxville, TN 37996. *Assessing the Economic Tradeoffs Between Prevention and Suppression of Forest Fires*. Preliminary report.

Decades of successful fire suppression efforts have led to the overgrowth of today's forests. As a result, there has been an increase in severe wildfire events and total fire suppression spending. Management of a forest prior to a fire, through techniques such as mechanical thinning, prescribed fire, etc., can help decrease the severity of a fire and the cost to suppress it. We formulate systems of differential equations to represent the dynamics before and after a fire. The time of fire occurrence is given as a random variable with a specified distribution. Using optimal control methods, we determine an optimal management expenditure schedule for a forest before a fire occurs and optimal suppression costs of a fire once it occurs. (Received September 20, 2016)

1125-VM-2921 **Zahava C Wilstein\*** ([zwilstein@berry.edu](mailto:zwilstein@berry.edu)), Berry College, Mathematics Department, 2277 Martha Berry Highway, Mount Berry, GA 30149, and **Daniel Alligood, Valerie McLure and Austinn Miller**. *A Mathematical Model of Biomechanical and Chemical Influences on Hypertension*. Preliminary report.

Hypertension is a prevalent problem in the world, especially in the United States. According to the CDC, 1 in 3 Americans is classified as pre-hypertensive. Modeling hypertension may provide insights into critical aspects of its development. While typical mathematical models focus on the physical mechanics of hypertension, our model also incorporates prevailing cell signaling pathways thought to contribute to arterial remodeling. Building off of experimental work on hypertension in rats, we model the biological processes thought to be involved in the development of chronic hypertension with a system of algebraic and delay differential equations. Numerical simulations provide results that align well with available experimental data for normal and spontaneously-hypertensive rats. (Received September 20, 2016)

1125-VM-2937 **Christopher Strickland\*** ([wstrick@live.unc.edu](mailto:wstrick@live.unc.edu)), **Nadiah Kristensen** and **Laura Miller**. *Modelling the Spread of Parasitoid Wasps from Point Release*.

Parasitoid wasps are critical for biological pest control and are increasingly being used in agriculture to protect crops via release. However, due to their small size (often less than 1 mm), movement and long-distance dispersal of these wasps have long been poorly understood and likely underestimated. Recent data collected by Kristensen et al. (2013) on the wind-borne dispersal pattern of *E. hayati* (0.7 mm long) provides a new and significant opportunity to develop a detailed multi-scale model for the initial spread of small invasive insects and biological control introductions.

In this talk I will present a new model for parasitoid wasp dispersal from point release. The model is derived from underlying stochastic processes and, as a special case of the Fokker-Planck equation, is fully deterministic. The Python implementation of this model is capable of running month long simulations on the scale of 150 km<sup>2</sup> while maintaining a resolution of 25x25 m, all within a minute on a common workstation. Speed is an essential component of our model because it permits fitting parameters to data using a Bayesian framework. Evaluation of the model in comparison with multi-scale, first-release data reveals the critical role of non-linear air flow for dispersal further than 1 km. (Received September 20, 2016)

1125-VM-2954     **Tugba Karabiyik\*** ([tugba@shsu.edu](mailto:tugba@shsu.edu)), Lee Drain Building, Room 420, 1900 Avenue I, Huntsville, TX 77340, and **Umit Karabiyik** ([umit@shsu.edu](mailto:umit@shsu.edu)), 1803 Avenue I, Academic Building 1, Room 214, Huntsville, TX 77340. *Multi-armed Bandit Problem in Digital Forensics.*

Multi-armed Bandit (MAB) problem is a well-known problem in probability theory and machine learning. It is a problem of decision making when a gambler has multiple slot machines to play. Each machine looks the same but has an independent unknown probability of success and yields a reward. The gambler faces with a conflict in terms of which machine to play, how long to play each machine and in what order to play the machines in order to maximize the total reward. Gittins index is a theorem which gives an optimal solution for maximizing the expected discounted reward for MAB.

Digital forensics (DF) is a branch of forensics science which deals with the investigation and recovery of digital information found in digital devices which are mostly found at crime scenes and belonging to suspects. These devices are searched for evidence using specific techniques and software tools which often perform similar tasks on a given device such as evidence acquisition and validation.

The objective of this research is to create a mathematical model for DF tool selection for particular tasks when multiple tools perform the same task as a MAB problem. We also aim to adapt Gittins Index solution to our problem in order to rank the tools independent of their history of successes and failures. (Received September 20, 2016)

1125-VM-2980     **Chris McCarthy\*** ([cmccarthy@bmcc.cuny.edu](mailto:cmccarthy@bmcc.cuny.edu)), Borough of Manhattan Community College CUNY, Department of Mathematics, 199 Chambers Street, New York, NY 10007. *Mathematical models of condensation, adsorption, and filters.*

I will discuss kinetic models of condensation and adsorption, as well as models of filters based on those mechanisms.

These mathematical models have been developed in support of our interdisciplinary lab group, which is centered at BMCC/CUNY (City University of New York). Our group conducts research into bio-remediation of heavy metal contaminated water via filtration. The filters are constructed out of biomass, such as spent tea leaves. The spent tea leaves are available in large quantities as a result of the industrial production of tea beverages. The heavy metals bond with the surfaces of the tea leaves (adsorption).

The models involve differential equations, stochastic methods, and recursive functions. I will compare the models' predictions to data obtained from computer simulations and experimentally by our lab group.

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1125-VM-2983     **Hansapani Sarasepa Rodrigo\*** ([sarasepa@mail.usf.edu](mailto:sarasepa@mail.usf.edu)), Department of Mathematics and Statistics, 4202 E Fowler Ave, CMC342, Tampa, FL 33620, and **Chris P Tsokos** ([ctsokos@usf.edu](mailto:ctsokos@usf.edu)). *Artificial Neural Network Model for Predicting Lung Cancer Survival.*

The object of our present study is to develop a piecewise constant hazard model using an artificial neural network (ANN) to capture the complex shapes of the hazard functions, which cannot be achieved with conventional survival analysis models like Cox proportional hazard. We proposed a more convenient approach to piecewise exponential artificial neural network model (PEANN) created by Fornili et al. to handle a large amount of data. In particular, it provides much better prediction accuracies over both the Poisson regression and generalized estimating equations. This has been demonstrated with lung cancer patient data taken from Surveillance, Epidemiology and End Results (SEER) program. The quality of the proposed model is evaluated using several error measurement criteria. (Received September 20, 2016)

1125-VM-3139 **Matthew Joshua Buhr** ([matthew\\_buhr2@hotmail.com](mailto:matthew_buhr2@hotmail.com)), 1519 n 122nd plz, Apartment 2, Omaha, NE 68154, and **Jose Flores\*** ([matthew\\_buhr2@hotmail.com](mailto:matthew_buhr2@hotmail.com)), 1519 n 122nd plz, 2, omaha, NE 68154. *Spider Monkeys in Fragmented Landscapes: A Discrete Mathematical Model*.

We implement a discrete model to study the population dynamics of *Ateles hybridus* in a single patch. Since data suggest a population level of under one thousand inhabitants, a discrete model is the most suitable. The different patches resemble a landscape which has been fragmented over the past few years particularly in Colombia. Given the population, the population is divided into categories by sex: male and female. Furthermore, the population is broken down so that the female population is broken into subgroups: adult females and young females, to account for an age of reproductive ability. Additionally, females are the dispersing sex in spider monkeys. In our population, a young female acquires its reproductive ability around their seventh year, at which point they disperse from their group or family in search of another group where they will spend their reproductive life. We also consider the possibility that their new patch has an unfit operational sex ratio in which some females who make a poor decision on a new patch may never reproduce. We analyze equilibria, and modify parameters to simulate different initial conditions in a real-life model to conclude how to best handle spider monkey populations. (Received September 21, 2016)

## General Session on Number Theory

1125-VN-34 **Jay Lawrence Schiffman\*** ([schiffman@rowan.edu](mailto:schiffman@rowan.edu)), 604-36 South Washington Square # 815, Philadelphia, PA 19106. *Variations on The Harmonic Series*. Preliminary report.

The Harmonic Series represents one of the most important series studied in the calculus classroom. This paper will explore variations on the divergent harmonic series (which is asymptotic  $\log n$ ) including the harmonic series of primes (which diverges even more slowly and is asymptotic  $\log \log (n)$ ) and the harmonic series of twin primes converging to Brun's constant. In addition, we will explore the harmonic series of Fibonacci and Lucas numbers (both series converge) and some harmonic series of figurative numbers including the harmonic series of square, triangular, pentagonal, hexagonal and octagonal numbers which all converge. The deployment of Mathematica will aid in our explorations. (Received June 06, 2016)

1125-VN-321 **Juan Orozco\*** ([jorozco@theexcelguru.com](mailto:jorozco@theexcelguru.com)), 4101 Aplomado Falcon Cove, Austin, TX 78738. *Advances in the Goldbach and Twin Primes conjectures*. Preliminary report.

This paper introduces some advances in several open problems in number theory, particularly the Goldbach Conjecture and the Twin Prime Conjecture incorporating Mertens' third theorem and the twin prime constant. The approach is generalized for algorithms resulting in an Euler product like  $\prod_p \left(1 - \frac{x}{p}\right)$ . (Received September 06, 2016)

1125-VN-541 **M Affouf\*** ([maffouf@kean.edu](mailto:maffouf@kean.edu)), 1000 Morris Ave, Union, NJ 07083. *Iterative limit of numbers with digit reversals*. Preliminary report.

We consider a class of recreational integer numbers with operations on the digit reversal and their iterations, we identify the limit and limit cycles of these iterations, some of their properties and related open questions (Received September 06, 2016)

1125-VN-686 **Jeremiah Bartz\*** ([jeremiah.bartz@und.edu](mailto:jeremiah.bartz@und.edu)). *Cyclic Patterns in Digital Root Series*. Preliminary report.

Let  $b, n \in \mathbb{Z}_{>0}$  with  $b \geq 2$ . The *base b digital root* of  $n$ , denoted  $\rho_b(n)$  is defined as the single digit obtained by the addition of the corresponding digits of  $n$  expressed in base  $b$  (and process repeated if needed). The *base b digital root series*  $\{n_i\}$  beginning at  $n_1 \in \mathbb{Z}_{>0}$  is defined recursively for  $i \geq 2$  by  $n_i = n_{i-1} + \rho_b(n_{i-1})$ . In this talk, we discuss a cyclic pattern observed by V.S. Kumar for  $b = 10$  and its extensions to certain other values of  $b$ . (Received September 09, 2016)

1125-VN-690 **Jathan W. Austin\*** ([jwaustin@salisbury.edu](mailto:jwaustin@salisbury.edu)), Department of Mathematics & Computer Science, Salisbury University, 1101 Camden Avenue, Salisbury, MD 21801. *Fibonacci Numbers in PTPMs*. Preliminary report.

A matrix for a linear transformation that maps Pythagorean triples into Pythagorean triples is called a Pythagorean-triple-preserving matrix (PTPM). In this talk, we will see how to construct PTPMs whose entries can be expressed in terms of Fibonacci numbers. (Received September 09, 2016)

1125-VN-703 **Steven R Edwards\*** ([sedwar77@kennesaw.edu](mailto:sedwar77@kennesaw.edu)), Kennesaw State University, Math Building-Building D, 1100 S. Marietta Pkwy SE #9085, Marietta, GA 30060, and **William Griffiths** ([wgriff17@kennesaw.edu](mailto:wgriff17@kennesaw.edu)), Kennesaw State University, Math Building-Building D, 1100 S. Marietta Pkwy SE #9085, Marietta, GA 30060. *Binomial Sums That Generate Doubly-Recursive Sequences.*

We begin by considering sums of the form  $\sum_{j=0}^m (-1)^j \binom{m}{j} \binom{n-j}{m+k}$ , for integers  $n$  and  $k$ , and non-negative integers  $m$ . These sums are then used to generate doubly-recursive sequences. We examine the structure of these sequences and the relationship between the sequences. (Received September 09, 2016)

1125-VN-1301 **Chatchawan Panraksa\*** ([chatchawan.pan@mahidol.edu](mailto:chatchawan.pan@mahidol.edu)), 999 Phutthamonthon 4 Road, Salaya, Nakhonpath 73170, Thailand. *Parametrization of Four-Periodic Points of Rational Quadratic Functions.*

Let  $\phi_c(x) = x^2 + c$  with  $c$  rational. We study points of period four, i.e., the points  $x$  such that  $\phi_c(\phi_c(\phi_c(\phi_c(x)))) = x$ . We show that the points form Galois conjugate pairs. Moreover, by using Groebner basis, we can parametrize the points of period four. We also discuss irreducible factors of polynomials where the roots are the points of period four. (Received September 16, 2016)

1125-VN-1412 **R. Scott Williams\*** ([rwilliams77@uco.edu](mailto:rwilliams77@uco.edu)), University of Central Oklahoma, 100 North University Drive, Box 129, Edmond, OK 73034. *Level compatibility in the passage from modular symbols to cup products.*

In some sense, there is a correspondence that relates the geometry of the modular curve  $X_1(M)$  and the arithmetic of the cyclotomic field  $\mathbb{Q}(\zeta_M)$ . More specifically, for a positive integer  $M$  and an odd prime  $p$ , R. Sharifi defined a map  $\varpi_M$  from the first homology group of the modular curve  $X_1(M)$  with  $\mathbb{Z}_p$  coefficients to a second Galois cohomology group over  $\mathbb{Q}(\mu_M)$  with restricted ramification and  $\mathbb{Z}_p(2)$ -coefficients which provides one direction of this correspondence. T. Fukaya and K. Kato showed that if  $p|M$  and  $p \geq 5$  then  $\varpi_{Mp}$  and  $\varpi_M$  are compatible via the map of homology induced by the quotient  $X_1(Mp) \rightarrow X_1(M)$  and corestriction from  $\mathbb{Q}(\mu_{Mp})$  to  $\mathbb{Q}(\mu_M)$ . In this talk, we will show that for a prime  $\ell \nmid M$ ,  $\ell \neq p \geq 5$ , the maps  $\varpi_{M\ell}$  and  $\varpi_M$  are once again compatible, albeit now via a particular combination of the standard degeneracy maps along with corestriction. (Received September 16, 2016)

1125-VN-1487 **Yasanthi Kottogoda\*** ([ykottogoda@newhaven.edu](mailto:ykottogoda@newhaven.edu)). *Exploring the characteristics of modulo one sequences.* Preliminary report.

Modulo one sequences is a special category of sequences of pairwise relatively prime positive integers where an old idea of working with modulo arithmetic simultaneously in order to speed up computer arithmetic is used in the construction of these sequences. Let  $m_1, m_2, \dots, m_t$  be positive integers that are pairwise relatively prime. Set  $M = m_1 m_2 \cdots m_t$  and  $M_i = M/m_i$ . Then the sequence is defined as a mod one sequence if  $M_i \equiv 1 \pmod{m_i}$  for each  $i$ . I also discuss the necessary and sufficient conditions for the existence of the sequence and prove that there are such sequences of arbitrary length. Additionally, results on classification of the mod one sequences based on its length with the examples obtained from observations using MATHEMATICA will be discussed. (Received September 17, 2016)

1125-VN-1509 **Thomas J. Osler\*** ([osler@rowan.edu](mailto:osler@rowan.edu)), 233 Pomona Ave, Glassboro, NJ, NJ 08028. *A random walk and the Riemann hypothesis for children.* Preliminary report.

The celebrated Riemann hypothesis (RH) is usually described as follows: "The complex roots of the Riemann zeta function all have real part equal to  $1/2$ ." To explain RH in these terms requires considerable effort. You must first define the zeta function, extend the zeta function to the entire complex plane except for the point at 1, as well as other considerations. However, there are at least 21 other "equivalent" statements of RH that are simpler. That is there are statements that are true, if and only if RH is true. One of these, given by Edmund Landau in his 1899 doctoral thesis at the University of Berlin can be interpreted (almost) in a very simple way. Here it is: (RH for children) Randomly pick a positive integer  $N$ . Find the prime factorization of  $N$  and call  $w(N)$  the number of primes in this factorization (counting multiplicity). For example,  $w(12) = 3$ . Then RH is equivalent (almost) to the statement that  $w(N)$ , when  $N$  is randomly selected, has equal probability of being even or odd. How simple, and how seemingly obvious! We show how this is strongly related to a random walk in one dimension and allows us to interpret Landau's precise equivalent RH statement. (Received September 17, 2016)

1125-VN-1882 **Dorin Ervin Dutkay** and **Isabelle Kraus\*** ([izzy.kraus@knights.ucf.edu](mailto:izzy.kraus@knights.ucf.edu)), University of Central Florida, Department of Mathematics, 4393 Andromeda Loop N., Orlando, FL 32816-1364. *Scaling of Spectra of Cantor-Type Measures and Some Number Theoretic Considerations.*

We present some relations between number theory and spectral measures related to the harmonic analysis of a Cantor set. Specifically, we discuss ways to determine when an odd natural number  $m$  generates a complete or incomplete Fourier basis for a Cantor-type measure with scale  $g$ . The analytical question is turned into a problem about the dynamics of two maps on the set of integers. We find ways to determine if a number generates a complete Fourier basis in terms of its prime decomposition, or in terms of its base  $g$  expansion (Received September 19, 2016)

1125-VN-1925 **J. Larry Lehman\*** ([1lehman@umw.edu](mailto:1lehman@umw.edu)), Department of Mathematics, University of Mary Washington, 1301 College Avenue, Fredericksburg, VA 22401. *A Formula for the Number of Solutions of an Arbitrary Quadratic Congruence.*

Let  $f(x) = ax^2 + bx + c$  be a primitive polynomial and let  $m$  be a positive integer. We establish a formula for the number of solutions of the quadratic congruence  $f(x) \equiv 0 \pmod{m}$  that depends only on  $\Delta(f) = b^2 - 4ac$  and on the prime factorization of  $m$ . The proof of this formula is a straightforward inductive argument, using the concept of a  $p$ -seeding polynomial for  $f(x)$ , in cases where  $p^2$  divides  $\Delta(f)$ . (Received September 19, 2016)

1125-VN-1945 **Vadim Ponomarenko\*** ([vponomarenko@mail.sdsu.edu](mailto:vponomarenko@mail.sdsu.edu)), 5500 Campanile Dr., San Diego, CA 921827720, and **Jason Thoma**. *Gelfand's Question in Different Bases.*

Gelfand's 50-year old question (actually several related questions) about the leading digits of  $(2^n, 3^n, \dots, 9^n)$  was answered in a lovely paper by Eising, Radcliffe and Top, appearing in 2015 in the American Mathematical Monthly. We extend these results to different bases. (Received September 19, 2016)

1125-VN-1983 **Joshua Zelinsky\***, [zelinsky@gmail.com](mailto:zelinsky@gmail.com). *Integer Complexity and  $p$ -Adic Expansions of Rational Numbers.*

Define  $\|n\|$  to be the *complexity* of  $n$ , the smallest number of 1's needed to write  $\|n\|$  using an arbitrary combination of addition and multiplication. John Selfridge showed that  $\|n\| \geq 3 \log_3 n$  for all  $n$ , and Guy noted the trivial upper bound that  $\|n\| \leq 3 \log_2 n$  for all  $n > 1$  by writing  $n$  in base 2. An upper for almost all  $n$  was provided by de Reyna and Jan Van de Lune. We discuss better upper bounds and how further improvements relate to understanding the  $p$ -adic expansions of rational numbers of the form  $-1/m$  for various  $m$ . (Received September 19, 2016)

1125-VN-2044 **Nicholas Heiner** ([heiner@hendrix.edu](mailto:heiner@hendrix.edu)) and **Duff Campbell\*** ([campbell@hendrix.edu](mailto:campbell@hendrix.edu)). *Generalizing the convergent to a simple continued fraction.*

If we write the simple continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$  as  $[a_0; a_1, a_2, \dots]$ , then the convergents are well-known as  $\frac{h_n}{k_n}$ , where  $h_n = a_n h_{n-1} + h_{n-2}$  and  $k_n = a_n k_{n-1} + k_{n-2}$  for  $n \geq 0$ ; the four numbers  $h_{-1} = 1$ ,  $h_{-2} = 0$ ,  $k_{-1} = 0$ ,  $k_{-2} = 1$  set "initial conditions" so that the first two convergents are, appropriately,  $\frac{h_0}{k_0} = a_0$  and  $\frac{h_0}{k_0} = \frac{a_0 a_1 + 1}{a_1} = a_0 + \frac{1}{a_1}$ . Nicholas Heiner made it his senior project to generalize these initial conditions, and thus  $h_{-1} = b$ ,  $h_{-2} = a$ ,  $k_{-1} = d$ ,  $k_{-2} = c$  are arbitrary. Several interesting theorems were discovered, as well as other conjectures which remain as yet unproven. (Received September 19, 2016)

1125-VN-2079 **Sungkon Chang\***, Armstrong State University, Department of Mathematics, 11935 Abercorn St, Savannah, GA 31419. *Generalization of Pythagorean Triples.*

The problem of parametrizing Pythagorean triples can be generalized as we change the 90-degree angle to other angles such as 60 degrees. The sides of triangles with the angles of 60 degrees or 120 degrees are called Eisenstein triples when they are all integers. We introduce more examples of triangles with integer sides and a fixed angle, and their parametrizations. (Received September 19, 2016)

1125-VN-2128 **Ji Young Choi\*** ([jychoi@ship.edu](mailto:jychoi@ship.edu)), 1871 Old Main Dr., Shippensburg, PA 17257.  
*Monotonically Increasing Digits.* Preliminary report.

A digit that is monotonically increasing from right to left in a string of digits is called a monotonically increasing digit. In this talk, we will count the number of monotonically increasing digits in the  $n$ -ary representation of an integer, for any integer  $n > 1$ . Then, we will calculate the digit sum of a multiple of  $n - 1$  in the  $n$ -ary representation. (Received September 19, 2016)

1125-VN-2260 **Erhan Gürel\*** ([egurel@metu.edu.tr](mailto:egurel@metu.edu.tr)), Middle East Technical University, Northern Cyprus Campus, TZ-32, Kalkanlı, 99738 Güzelyurt, Turkey. *A note on the products*  
 $((m + 1)^2 + 1)((m + 2)^2 + 1) \dots (n^2 + 1)$  and  $((m + 1)^3 + 1)((m + 2)^3 + 1) \dots (n^3 + 1)$ .

We prove that for any positive integer  $m$  there exists a positive real number  $N_m$  such that whenever the integer  $n \geq N_m$  neither the product  $P_m^n = ((m + 1)^2 + 1)((m + 2)^2 + 1) \dots (n^2 + 1)$  nor the product  $Q_m^n = ((m + 1)^3 + 1)((m + 2)^3 + 1) \dots (n^3 + 1)$  is a square. (Received September 20, 2016)

1125-VN-2296 **Jacqueline Anderson\*** ([jacqueline.anderson@bridgew.edu](mailto:jacqueline.anderson@bridgew.edu)), Bridgewater, MA 02325, and **Spencer Hamblen, Bjorn Poonen** and **Laura Walton.** *Local Arboreal Galois Representations.*

Let  $f(z) = z^\ell - c$  be a separable polynomial over a field  $K$  complete with respect to a discrete valuation  $v$  and with residue field of characteristic  $p$ , and let  $a \in K$ . Let  $f^n(z)$  denote the  $n$ -th iterate of  $f$ . We examine the Galois groups and ramification groups obtained from the extensions of  $K$  containing all of the roots of the polynomial  $f^n(z) - a$  in both tame ( $p \nmid \ell$ ) and wild ( $\ell = p$ ) cases, and show how these groups depend on the valuation  $v(c)$ . (Received September 20, 2016)

1125-VN-2322 **E. M. Sanger\***, 244 Mathematics Building, Buffalo, NY 14260, and **T. W. Cusick.**  
*Rotation Symmetric Bent Boolean Functions in  $n = 2p$  Variables.* Preliminary report.

There is a well known conjecture that there are no homogeneous Rotation Symmetric bent Boolean functions of degree greater than 2. In this talk we will use Hadamard matrices to characterize the RotS bent functions of degree 2. We will then extend this idea to higher degrees in both the homogeneous and nonhomogeneous cases when  $n=2p$ ,  $p$  a prime. (Received September 20, 2016)

1125-VN-2393 **Florian Bouyer, Edgar Costa, Dino Festi, Christopher Nicholls** and **Mckenzie West\*** ([westm@reed.edu](mailto:westm@reed.edu)). *On the arithmetic of a family of degree-two diagonal K3 surfaces.*

Let  $Z$  be a typical degree-two K3 surface of the following family

$$w^2 = ax^6 + by^6 + cz^6 + dx^2y^2z^2.$$

We explicitly compute the geometric Picard lattice and its Galois structure with the eventual goal to determine whether or not there is a Brauer–Manin obstruction to rational points on a surface in the family. (Received September 20, 2016)

1125-VN-2440 **Elizabeth M. Reid\*** ([emreid2@buffalo.edu](mailto:emreid2@buffalo.edu)), 101 Hendricks Blvd, Amherst, NY 14226, and **Thomas Cusick.** *Affine equivalence classes of 2-rotation symmetric cubic Boolean functions.*

A Boolean function in  $n$  variables is 2-rotation symmetric if it is invariant under even powers of the cyclic permutation  $\rho(x_1, \dots, x_n) = (x_2, \dots, x_n, x_1)$  of the variables, but not under the first power. We call such a function a 2-function. A 2-function is said to be monomial rotation symmetric (MRS) if it is generated by applying powers of  $\rho^2$  to a single monomial. In 2014 Cusick and Johns developed the theory of cubic MRS 2-functions in  $2n$  variables generated by a monomial  $x_1x_r x_s$  with  $1 < r < s$  and  $r$  and  $s$  not both odd. They gave a complete description of the affine equivalence classes for these functions. Here, we develop the theory further by determining the smallest group that acts on the set of all these cubic MRS functions to give the affine equivalence classes. (Received September 20, 2016)

1125-VN-2465 **Jonathan Webster\*** ([jwebste@butler.edu](mailto:jwebste@butler.edu)) and **Andrew Shallue.** *New ideas for tabulating Baillie-PSW pseudoprimes.* Preliminary report.

This talk will discuss how the ideas behind the algorithm to find strong pseudoprimes to many bases may be applied to the tabulation of Baillie-PSW pseudoprimes. Such a composite number  $N$  is simultaneously a base-2 Fermat pseudoprime, a Lucas pseudoprime with respect to a certain quadratic polynomial with discriminant  $D$ , and satisfies  $(D|N) = -1$ . There is a 620 USD reward for such a number. (Received September 20, 2016)

1125-VN-2665 **Michael Chou\*** ([michael.chou@uconn.edu](mailto:michael.chou@uconn.edu)). *Growth of torsion points on elliptic curves from  $\mathbb{Q}$  to the maximal abelian extension of  $\mathbb{Q}$ .*

Torsion of an elliptic curve over a number field is finite due to the Mordell-Weil theorem. However, even in certain infinite extensions of  $\mathbb{Q}$  we have that torsion is finite. Ribet proved that torsion over the maximal abelian extension of  $\mathbb{Q}$  of an elliptic curve with coefficients in  $\mathbb{Q}$  is finite. In this talk, we show that the size of such torsion subgroups is in fact uniformly bounded. Further, we give a classification of all possible torsion structures an elliptic curve defined over  $\mathbb{Q}$  can obtain when base extended to the maximal abelian extension of  $\mathbb{Q}$ . (Received September 20, 2016)

1125-VN-2686 **Samuel N Edwards\*** ([edwasa01@gettysburg.edu](mailto:edwasa01@gettysburg.edu)). *Comparing the Restricted Critical Number and Size of Weakly Zero Sum-Free Sets.*

We define a weakly zero- $h$ -sum-free set as a set where no  $h$ -termed sum of distinct elements from the set equals 0. Given a group  $G$  and a non-negative integer  $h$ , we investigate the maximum size of a subset of  $G$  that is weakly zero- $h$ -sum-free, denoted  $\tau(G, h)$ . On a similar hand, we define the restricted  $h$ -critical number of a group  $G$  to be the minimum value  $m$  such that the restricted  $h$ -fold sumset of all  $m$ -subsets spans the entire group; this value is denoted  $\chi(G, h)$ . These two distinct entities have a specific relationship— that is,  $\tau(G, h) \leq \chi(G, h) - 1$ . We analyze the situations where  $\tau(G, h)$  is strictly less than  $\chi(G, h) - 1$ . (Received September 20, 2016)

1125-VN-2687 **Bir Kafle\*** ([bkafle@pnw.edu](mailto:bkafle@pnw.edu)), 1401 S. US 421, Westville, IN 46391, and **Florian Luca** and **Alain Togbé**. *On the  $x$ -coordinates of Pell equations which are Fibonacci numbers.* Preliminary report.

Let  $d > 1$  be a positive integer which is not a perfect square. Consider the Pell equation  $x^2 - dy^2 = \pm 4$ . All its positive integer solutions  $(x, y)$  are given by

$$\frac{x_n + y_n \sqrt{d}}{2} = \left( \frac{x_1 + y_1 \sqrt{d}}{2} \right)^n,$$

for some positive integer  $n$ , where  $(x_1, y_1)$  is the smallest positive integer solution. In this talk, we will show that there is at most one value of the positive integer  $x$  participating in the above Pell equation which is a Fibonacci number, when  $d \geq 2$ . In case  $d = 2$ , and 5, we have exactly two values of  $x$  being members of the Fibonacci sequence.

Joint work with Florian Luca and Alain Togbé. (Received September 20, 2016)

1125-VN-2705 **Andres Israel Zumba Quezada\*** ([andreszumba@mail.fresnostate.edu](mailto:andreszumba@mail.fresnostate.edu)), N5251 Barton Graves 325A, Fresno, CA 93710-8234, and **Khang D Tran** ([khangt@csufresno.edu](mailto:khangt@csufresno.edu)). *Zero distribution of a sequence of polynomials with a recurrence of degree three.*

We study the zero distribution of a sequence of polynomials  $P_n(z)$  defined by a recurrence of degree three

$$P_n(z) = aP_{n-1}(z) + bP_{n-2}(z) + cP_{n-3}(z) + zP_{n-r},$$

where  $1 \leq r \leq 3$  and  $a, b, c$  are real numbers. We show that under certain conditions on  $a, b$ , and  $c$ , the zeros of  $P_n$  will lie on an explicit real interval and are dense there as  $n \rightarrow \infty$ . (Received September 20, 2016)

1125-VN-2718 **Harris Ahmed Mohammed Ismail\*** ([moham189@umn.edu](mailto:moham189@umn.edu)) and **Steven Sperber** ([sperber@umn.edu](mailto:sperber@umn.edu)). *On some applications of a generalized Dwork trace formula to the  $L$ -function associated with exponential sums over Galois rings.* Preliminary report.

We begin with a generalization of the Dwork trace formula which applies to the  $L$ -function associated with certain exponential sums over Galois rings. These are rings of Witt vectors of a finite field having a fixed finite length. The associated  $L$ -function is a generating series encoding information for the corresponding data as one considers finite extensions of the base field. By a character argument, we can apply results to the zeta function which counts solutions to polynomial equations in these Galois rings. In particular, we prove the rationality of the generalized zeta function using the extended trace formula. One can also use Bombieri's argument to obtain bounds on the degree of such  $L$ -functions. There should also be an application to estimating the  $p$ -divisibility of such number counts. (Received September 20, 2016)

1125-VN-2923 **Joseph Gunther\*** ([jgunther@gradcenter.cuny.edu](mailto:jgunther@gradcenter.cuny.edu)), **Daniel Hast** and **Vlad Matei**. *Counting low degree extensions of function fields.*

Recent work of Bhargava, Shankar, and Wang extended results on counting low-degree  $S_n$ -extensions to allow any global field as the base field. Their work uses geometry of numbers for both number fields and function fields. We'll show how, in the function field case, one can instead give algebro-geometric proofs, which shed light

on the geometry present in the number field situation as well. This is joint work with Daniel Hast and Vlad Matei. (Received September 20, 2016)

1125-VN-3017 **Marko Milosevic\*** (marko@uga.edu), **Pete L. Clark** and **Michael Chou**. *Torsion of CM-Elliptic Curves over Abelian Number Fields*.

We give a list of the possible torsion subgroups of elliptic curves with complex multiplication over abelian number fields. (Received September 20, 2016)

## General Session on Outreach

1125-VO-891 **M. Carol Williams\***, Department of Mathematics and Statistics, Texas Tech University, Lubbock, TX 79409, and **G. Brock Williams**. *Quilts, Constructions, and Kids*.

Our department hosts an annual Emmy Noether Day during which over 200 local middle and high school girls attend workshops and compete in math contests. Most participating schools are rural and heavily Hispanic.

We will describe one of the workshops in which students used quilting to explore tilings and geometric constructions. The hands-on activities allowed them to experience how theorems they had seen in their geometry classes can be used to construct quilt patterns.

We also involved undergraduate pre-service teachers who had just completed their required geometry course as tutors for the students, multiplying the impact of the workshop. (Received September 12, 2016)

1125-VO-1687 **Jennifer Nordstrom\*** (jfirkins@linfield.edu). *Mentoring Mathematical Science Fair Projects*.

The Math PLUS program is a partnership between Linfield College and a local middle school, which seeks encourage more mathematics in local and regional science fairs. Linfield College students are paired with middle school students to mentor science fair projects that are required to have a mathematical focus. One goal of the program is to provide access to greater mathematical resources to underserved middle school students who show an interest and aptitude in mathematics. Students are encouraged to research their own questions. Projects have included college level mathematics from number theory to statistics. We will describe the program and discuss the impact on the middle school students, the college student mentors, and the local science fair. The program is funded by a Dolciani Mathematics Enrichment Grant. (Received September 18, 2016)

1125-VO-1840 **Tuwaner Hudson Lamar\*** (tuwaner.lamar@morehouse.edu), 830 Westview Drive, S.W., Atlanta, GA 30314. *Community Outreach: Annual Mathematics Competitions Bootcamp at Morehouse College*. Preliminary report.

Morehouse College Mathematics Department holds a workshop annually devoted to disseminating information about math competitions. Motivated math students, boys and girls, in Grades 5 – 12 are invited to attend a four hour workshop on the campus. The goals of the workshop are to: increase participation in math competitions by members of underrepresented groups; introduce participants to local, state, regional and national math competitions; provide ongoing communication via social media to facilitate and encourage participants throughout the school year as they progress through all levels of math competitions; and provide an opportunity for participants to compete in a Mini-Math Competition. (Received September 19, 2016)

1125-VO-1866 **Gangadhar Acharya\*** (gangadhar.acharya@ttu.edu), Department of Mathematics and Statistics, Broadway and Boston, Lubbock, TX 79409-1042. *A Study of University Mathematics Outreach Programs in the United States*. Preliminary report.

Generally, outreach efforts provide some type of service targeting a population that otherwise might not have access to such services. University mathematics outreach programs often target K-12 students or teachers and have one, or a combination, of the following goals: enhancement of mathematical content knowledge, development of mathematical skills, increased interest in the study of mathematics, and promotion of career choices in mathematically intensive fields. In this descriptive study, we focus on existing university mathematics outreach programs in the US, with particular interest on project goals, project design, and project assessment methods. Analysis of components and characteristics of these outreach programs will be used to provide prototypical models of mathematics outreach programs that might be of use to university mathematics departments interested in developing outreach programs that target K-12 students. (Received September 19, 2016)

1125-VO-1986 **A Morgulis\*** ([amorgulis@bmcc.cuny.edu](mailto:amorgulis@bmcc.cuny.edu)), 199 Chambers St., N599, Math Dept., New York, NY 10007, and **A Han, M Dean and F Prioleau**. *Creating Career Pathways in Mathematics through the Recruitment and Retention of Talented Community College Students*. Preliminary report.

The issue of minority and women participation in STEM has become the crucial question of our time. For decades US has excelled in attracting science and engineering talents from all over the world. Recently women have made significant inroad in the STEM disciplines accounting for 50% of baccalaureate degrees earned nationally in science (including social & behavioral sciences) and engineering. In sharp contrast, African-Americans (8%) & Latinos (8%) continue to comprise a small segment of this population. Just when the US economy requires more scientists, the largest pool of potential workers continues to be isolated from careers in STEM. In this presentation, we will share the preliminary report of the BMCC-CUNY MSEIP Institutional Project – Creating Career Pathways in Mathematics through the Recruitment & Retention of Talented Community College Students. To help improve mathematics performance and encourage more students to follow a career in mathematics, we focus on 3 aspects of success: improving how math courses are taught, recruiting more women & minority students, and providing support and encouragement to students through various programs. Our effective outreach activities and retention programs strategies will be shared during the presentation. (Received September 19, 2016)

1125-VO-2607 **Katelynn D Kochalski\*** ([kdk7rn@virginia.edu](mailto:kdk7rn@virginia.edu)), 141 Cabell Drive, Kerchof Hall, PO Box 400137, Charlottesville, VA 22904. *Mathematicians in the Community: Enriching Middle School Mathematics Education*.

The math topics we typically focus on in middle and high school classrooms aren't necessarily the ones that promote the most excitement or curiosity about mathematics. How can we get young students to recognize that math is much more than they learn in the K-12 education? The UVa Math Ambassadors, an outreach program at the University of Virginia, tries to do just that. By sending graduate students into local middle school classrooms to lead fun math-based activities that use higher level math concepts, we expose students to a range of new mathematics in an age-appropriate way. We'll discuss the program, its primary objectives, and some of our activities. (Received September 20, 2016)

1125-VO-2870 **A. R. Wilzman\*** ([awilzman@gmail.com](mailto:awilzman@gmail.com)), **Brent Lunsford, Jeff Pullen and Chamaree de Silva**. *Outreach Through Fabrication of College-Level Lab Activities for High-school Students*. Preliminary report.

Funded by the Research that Reaches Out Initiative at Mercer University, we continue to conduct an outreach project to aid a local high school by constructing mathematics and physics labs for students. During the summer of 2016, we designed mathematical activities aimed at helping students understand fundamental topics covered in algebra, trigonometry, and statistics. We also constructed several low-cost physics labs – one of which focused on the calculus associated with projectile motion. We plan to spend time in the classroom with the students as we help the instructors implement the activities and labs. (Received September 20, 2016)

1125-VO-2975 **Melinda Lanius and Claire Merriman\*** ([emerrim2@illinois.edu](mailto:emerrim2@illinois.edu)), 1409 W Green St, Urbana, IL 61801, and **Vanessa Rivera Quiñones and Simone Sisneros-Thiry**. *Summer Illinois Math Camp*.

The Summer Illinois Math Camp is a free, week-long math day camp for middle and high school students established and run by graduate students in the Department of Mathematics at the University of Illinois at Urbana-Champaign. In this talk, we will tell the story of how a group of graduate students had an idea - to share the creative, discovery driven side of mathematics with teenagers - and within 6 months created an operational camp. We will share our experience translating college-level mathematics into activities for middle and high school students. (Received September 20, 2016)

1125-VO-3058 **Lisa Rezac\*** ([lmrezac@stthomas.edu](mailto:lmrezac@stthomas.edu)), UST Math Department OSS 201, 2115 Summit Avenue, St. Paul, MN 55105. *Disseminating Mathematical Activities for Outreach Programs*. Preliminary report.

Planning meaningful activities for outreach programs takes time, creativity, and mathematical expertise. We have benefitted greatly from informal sharing of material for different outreach programs, including math camps, enrichment activities and seminars for STEM instructors. We propose a sourcebook to formally disseminate successful outreach activities. This sourcebook will have two parts: first a selection of vetted activities not previously published, and second, a list and review of already published activities. In this talk we will share a proposed format for previously unpublished activities and call for contributions in this format. Outreach directors interested in contributing or using such a sourcebook are encouraged to attend or contact the presenter

and share their ideas. The goal will be to help more outreach program directors have access to great activities and resources that are time-tested and organized according to mathematical content, length of activity, and mathematical maturity level. (Received September 20, 2016)

## General Session on Probability and Statistics

1125-VP-33 **Gina F Reed\*** ([gina.reed@ung.edu](mailto:gina.reed@ung.edu)), Mathematics Department, University of North Georgia, 3820 Mundy Mill Rd., Oakwood, GA 30566. *Analyzing the lead content in drinking water during the Flint water crisis.*

This talk will focus on how to include a data set that will motivate students to apply and test what they are learning in class to a real and relevant problem. The data set is introduced on the first day of class in both an intro class comprised of predominantly non STEM majors and in a required statistics class for math majors. This early and active involvement in a systematic investigation of a real data will hopefully encourage students to complete an optional research project during the semester. (Received June 12, 2016)

1125-VP-35 **Kazeem Adesola Adepoju\*** ([ka.adepoju@ui.edu.ng](mailto:ka.adepoju@ui.edu.ng)), Dept. of Statistics, University of Ibadan, Ibadan, 02. *The beta- fisher snedecor distribution with applications to cancer remission data.*

In this paper a new four-parameter generalized version of the Fisher Snedecor distribution called Beta- F distribution is introduced. The comprehensive account of the statistical properties of the new distributions was considered. Formal expressions for the cumulative density function, moments, moment generating function and maximum likelihood estimation as well as its Fisher information were obtained. The flexibility of this distribution as well as its robustness using cancer remission time data was demonstrated. The new distribution can be used in most applications where the assumption underlying the use of other life time distributions is violated. Keyword: Fisher-Snedecor distribution, Beta-F distribution, Outlier, Maximum likelihood method. (Received June 07, 2016)

1125-VP-115 **Channing S. Parker\*** ([parkercc@dukes.jmu.edu](mailto:parkercc@dukes.jmu.edu)), **Joshua R. Abrams** ([rabrams12@email.arizona.edu](mailto:rabrams12@email.arizona.edu)), **Denise J. Harness** ([harnessd@etsu.edu](mailto:harnessd@etsu.edu)) and **Nina E. Galanter** ([galanter@grinnell.edu](mailto:galanter@grinnell.edu)). *Machine Learning for the Classification of Toxicological Endpoints.* Preliminary report.

The Toxicology Reference Database (ToxRefDB), compiled by the Environmental Protection Agency (EPA) contains data on chemicals and their associated toxicological endpoints. However, this data does not contain all chemicals of interest, and further testing is resource intensive. Here we present machine learning methods used to predict whether substances will have toxic effects on rat test subjects in order to avoid further animal testing. Chemical features associated with each chemical are utilized to generate these predictions. Support Vector Machine and Decision Tree machine learning algorithms are applied to toxicology data sets provided by the Environmental Protection Agency. These methods are tested and improved through cross-validation, parameter optimization, and the committee of machines approach. Feature selection is employed to optimize the models and provide information on which chemical features are potentially relevant to toxicological effects. Methods implemented include PCA, ROC curves, and F-Scores for pre-processing, and sensitivity analysis for post-processing. Long term outcomes of this study are to support further research in reducing the amount of animal testing, as well as in developing mechanistic-based toxicological models. (Received August 01, 2016)

1125-VP-164 **Hasthika S Rupasinghe Arachchige Don\*** ([hasthika@siu.edu](mailto:hasthika@siu.edu)), Department of Mathematics, 1245 Lincoln Dr., Carbondale, IL 62901, and **Lasanthi CR Pelawa Watagoda** ([lasanthi@siu.edu](mailto:lasanthi@siu.edu)), Department of Mathematics, 1245 Lincoln Dr., Carbondale, IL 62901. *Bootstrapping Analogs of the Two Sample Hotelling's  $T^2$  Test.*

Suppose there are two independent random samples from two populations or groups. A common multivariate two sample test of hypotheses is  $H_0 : \mu_1 = \mu_2$  versus  $H_1 : \mu_1 \neq \mu_2$  where  $\mu_i$  is a population location measure of the  $i$ th population for  $i = 1, 2$ . The two sample Hotelling's  $T^2$  test is the classical method, and is a special case of the one way MANOVA model if the two populations are assumed to have the same population covariance matrix. This paper suggests using the Olive (2016ab) bootstrap technique to develop analogs of Hotelling's  $T^2$  test. The new tests can have considerable outlier resistance, and the tests do not need the population covariance matrices to be equal. (Received August 06, 2016)

- 1125-VP-165 **Lasanthi C.R. Pelawa Watagoda\*** ([lasanthi@siu.edu](mailto:lasanthi@siu.edu)), Department of Mathematics, 1245 Lincoln Dr., Carbondale, IL 62901, and **David J Olive** ([dolive@siu.edu](mailto:dolive@siu.edu)), Department of Mathematics, 1245 Lincoln Dr., Carbondale, IL 62901. *Inference After Variable Selection*.

This work presents inference for the multiple linear regression model  $Y = \beta_1 x_1 + \dots + \beta_p x_p + e$  after variable selection, including prediction intervals for a future value of the response variable  $Y_f$  given a  $p \times 1$  vector of predictors  $x_f$ , and testing hypotheses with the bootstrap. If  $n$  is the sample size, most results are for  $n \gg p$ , but prediction intervals are developed that increase in average length slowly as  $p$  increases for fixed  $n$ , if the model is sparse:  $k$  predictors have nonzero coefficients  $\beta_i$  where  $n \gg k$  (Received August 06, 2016)

- 1125-VP-341 **Lina Wu\*** ([lwu@bmcc.cuny.edu](mailto:lwu@bmcc.cuny.edu)), 529 West 42nd Street, Apt. 5K, New York, NY 10036. *A Statistical Approach of Multivariate Data Analysis to Study Effects of Video Games and Online Chat on Mathematics Performance*. Preliminary report.

Regarding heavy video game players for boys and super online chat lovers for girls as a symbolic phrase in the current adolescent culture, this project of data analysis verifies the displacement effect on deteriorating mathematics performance. To evaluate correlation coefficients or regression coefficients between a factor of playing video games or chatting online and mathematics performance compared with other factors, we use multivariate analysis technique and take gender difference into account. We find the most important reason for the negative sign of the displacement effect on mathematics performance due to students' poor academic background. This project was supported by PSC-CUNY Award, jointly funded by The Professional Staff Congress and The City University of New York. (Received August 28, 2016)

- 1125-VP-455 **Min Deng\*** ([mdeng@towson.edu](mailto:mdeng@towson.edu)), Dr. Min Deng, Mathematics Department, Towson University, 8000 York Road, Towson, MD 21252. *Risk Measures for the Mixture of the Popular Models*.

From the data to model is the most important and challenge task in actuarial science field. Because we wish to predict the future risk and uncertainty in order to either avoid the risk, transfer the risk, or reduce the impact of the risk and the mathematical models will enhance the accuracy of the task. There are many popular models (continuous distribution and discrete distributions), such as normal, parato, exponential, binomial, etc. But we realized it is difficult to fit the data to a single popular model etc. Therefore, in this paper we are going to develop the mathematical models by mixture of popular models and discuss the risk connected with those model. We also will give the examples to illustrate our models based on the actual data or artificial data.

Key words: Risk Measurement, Mathematical Models, Probability Distribution, Mixture Distribution, (Received September 02, 2016)

- 1125-VP-647 **Steve Bacinski\*** ([sbacinski@davenport.edu](mailto:sbacinski@davenport.edu)), Davenport University, ATTN: Steve Bacinski, Grand Rapids, MI 49508. *How to Win at Tenzi!*

We will work through a Markov chain analysis of the simple dice game Tenzi to find out the probability of winning in  $k$  rolls, the advantage of rolling a speed  $x$  faster than your opponent, and ultimately how to win at Tenzi. Along the way, we will discover a function with some interesting properties including predictable jumps at every rational number, and continuous at the irrationals. (Received September 08, 2016)

- 1125-VP-685 **Ben Duklewski, Veera Holdai and Barbara Wainwright\*** ([bawainwright@salisbury.edu](mailto:bawainwright@salisbury.edu)). *On the Limitations of Financial Models*. Preliminary report.

Traditional statistical models such as the normal distribution have typically been used to predict the risk related to derivative securities. After the financial meltdown in 2008, some essayists and traders have attempted to explain what went wrong. This paper presents background and some of the perceived problems related to using the normal distribution for predicting rare events. For example, the probability that a single event will exceed the sum of the previous hundred events is a very small probability; so small that one can come to believe that it is zero. However, this probability is not as small when using some "thick-tailed" distributions such as the Pareto and Cauchy distributions. Probabilities of these rare events as well as put option prices will be analyzed and compared using the normal, Pareto, and Cauchy distributions. (Received September 09, 2016)

- 1125-VP-771 **Lakshmi Roychowdhury\*** (lakshmi.roychowdhury@utrgv.edu), School of Mathematical and Stat Sciences, University of Texas Rio Grande Valley, 1201 West University Drive, Edinburg, TX 78539, and **Mrinal K Roychowdhury**. *Optimal quantization for infinite nonhomogeneous distributions*.

Quantization for probability distributions concerns the best approximation of a  $d$ -dimensional probability distribution  $P$  by a discrete probability with a given number  $n$  of supporting points. In this paper, an infinitely generated nonhomogeneous Borel probability measure  $P$  is considered on  $\mathbb{R}$ . For such a probability measure  $P$ , an induction formula to determine the optimal sets of  $n$ -means and the  $n$ th quantization error for every natural number  $n$  is given. In addition, running the induction formula in computer algorithm, some results and observations about the optimal sets of  $n$ -means are obtained. (Received September 11, 2016)

- 1125-VP-823 **John C. Wierman\*** (jwierma1@gmail.com), Dept. of Applied Mathematics and Statistics, 100 Whitehead Hall, Johns Hopkins University, Baltimore, MD 21218. *Bond percolation threshold bounds for Archimedean lattices*. Preliminary report.

In the bond percolation model, a random subgraph is obtained from an infinite connected graph  $G$  by retaining each edge independently with probability  $p$ ,  $0 < p < 1$ . The percolation threshold  $p_c$  is the edge retention probability value above which the random subgraph contains an infinite connected component. The exact percolation threshold is known for only a few graphs, and rigorous bounds for unsolved graphs are generally rather poor. The substitution method uses stochastic ordering, a symmetry reduction, non-crossing partitions, and network flow algorithms to compute improved bounds for several of the eleven Archimedean lattices, which are vertex-transitive tilings of the plane by regular polygons. (Received September 12, 2016)

- 1125-VP-862 **Geoffrey H Schuette\*** (geoffrey.schuette@mavs.uta.edu). *Nonparametric Estimation of Prior Distribution for a Linear Degradation Signal Model*.

Using degradation stochastic models, we study real time conditioning, which monitors complex machines. In engineering, a commonly used model is:

$$S(t_i) = \theta t_i + \epsilon(t_i) \quad \text{for } i = 1, \dots, n$$

where  $S(t_i)$  is the observed signal from the system at a time  $t_i$  which runs from  $[0, \infty)$ ,  $\epsilon(t_i)$  is the error of the model at time  $t_i$ , and  $\theta$  is the prior distribution. In the literature, the Bayesian approach is used to find the posterior distribution, assuming prior distribution is known. We remove the assumption that the prior distribution is known and use a Kernel density estimator to estimate the density of  $\theta$  using a convolution between the unknown prior distribution and the known distribution of the dependent error terms. We derived the rate of convergence of the MISE for this Kernel estimator. (Received September 12, 2016)

- 1125-VP-1072 **Qasim M Al-Shboul\*** (z9889@zu.ac.ae), Zayed University, Dept of Mathematics and Statistics, University College, Dubai, 19282, United Arab Emirates, and **Elies Kouider**. *Best Linear Unbiased Estimators Using Both Double Ranked Set Sampling and Modified Double Ranked Set Sampling Procedures*.

A modified double ranked-set sampling technique is introduced and its efficiency is investigated. The best linear unbiased estimators of the mean, location and scale parameters and generalized variance of the normal, uniform, exponential and Weibull, distributions are obtained by using both the double ranked-set sampling and modified double ranked-set sampling procedures. The efficiencies of these estimators of the mean, location and scale parameters relative to the best linear unbiased estimators using ranked-set sampling for the given distributions are obtained and tabulated using both double ranked-set sampling and modified double ranked-set sampling. Regardless of the distribution or the sample size being used, results show that the efficiency of the best linear unbiased estimators significantly improved when using the modified double ranked-set sampling rather than double ranked-set sampling or ranked-set sampling techniques in estimating the mean, location and scale parameters. (Received September 14, 2016)

- 1125-VP-1097 **Mitra L Devkota\*** (mdevkota@shawnee.edu), 940 Second Street, Mathematical Sciences, Shawnee State University, Portsmouth, OH 45662, and **Gary D Hatfield**. *Study of Autocorrelation of Regression Residuals using Crop Residue Yield Potential*. Preliminary report.

In this study, we studied the autocorrelation of regression residuals using the crop residue yield potential data for North Central region of the USA. Ordinary Least Squares (OLS), Geographically Weighted Regression (GWR), Conditional Autoregressive (CAR), and Simultaneous Autoregressive (SAR) models were fitted for the crop residue yield potential as a function of two climate variables (temperature and precipitation) of crop growing

season. Moran's I and Geary's C were used to test the spatial autocorrelation of the regression residuals. (Received September 14, 2016)

1125-VP-1107 **Sher B. Chhetri\*** (schhetri2013@fau.edu), 777 Glades Rd, Science Building, Room 234, Boca Raton, FL 33431, **Hongwei Long** (hlong@fau.edu), 777 Glades Rd, Science Building, Room 234, Boca Raton, FL 33431, and **Gokarna Raj Aryal** (aryalg@purduecal.edu), 2200 169th Street, Hammond, IN 46323. *The Beta Transmuted Pareto Distribution: Theory and Application*. Preliminary report.

In this work, a new five-parameter beta-transmuted Pareto distribution is introduced and studied. Some important properties of the distribution are discussed and explicit formulas are derived for the mean, mean deviation, entropies, order-statistics and the reliability analysis. The method of maximum likelihood is proposed to estimate the parameters of the distribution. We illustrate the usefulness of the proposed distribution by presenting its application to real-life data. (Received September 14, 2016)

1125-VP-1219 **Jessie M Byrnes\*** (jessie.byrnes@coyotes.usd.edu). *Bayesian Inference on  $P(X < Y)$  Based on Progressive First Failure Censored Samples from Burr Type XII Distributions*. Preliminary report.

Let  $X$  and  $Y$  have two-parameter Burr XII distributions, respectively. The Bayesian inference of stress strength,  $\delta = P(X < Y)$ , is investigated under the progressively first failure-censored samples. Many different loss functions are applied to establish the Bayesian estimators of  $\delta$ . It is due to the complications of computation and no closed forms of estimators, Markov Chain Monte Carlo procedure is proposed for the simulation study. An example is provided for illustration. (Received September 15, 2016)

1125-VP-1260 **Jean-Jacques Kengwong-Keumo\*** (jkengwou@cameron.edu), Department of Mathematical Sciences, Cameron University, Lawton, OK 73505. *Racial and Gender Disparities in Incidence of Lung and Bronchus Cancer in the United States: A Longitudinal Analysis*.

Certain population groups in the United States carry a disproportionate burden of cancer. This work models and analyzes the dynamics of lung and bronchus cancer age-adjusted incidence rates by race (White and Black), gender (male and female), and prevalence of daily smoking in 38 U.S. states, the District of Columbia, and across eight U.S. geographic regions from 1999 to 2012. Between 1999 and 2012, age-adjusted incidence rates in lung cancer have decreased in all states and regions. However, racial and gender disparities remain. Whites continue to have lower age-adjusted incidence rates for this cancer than Blacks in all states and in five of the eight U.S. geographic regions. Disparities in incidence rates between Black and White men are significantly larger than those between Black and White women, with Black men having the highest incidence rate of all subgroups. Assuming that lung cancer incidence rates remain within reasonable range, the model predicts that the gender gap in the incidence rate for Whites would disappear by mid-2018, and for Blacks by 2026. This longitudinal model can help health professionals and policy makers make predictions of age-adjusted incidence rates for lung cancer in the U.S. in the next five to ten years. (Received September 15, 2016)

1125-VP-1332 **Erica Maciejewski\*** (erica.maciejewski@noblis.org). *Predicting Internet Domain Popularity*.

Power-Law distributions are used to represent a functional relationship between two quantities when one varies as a power of another. Several online services offer rankings of internet domain popularity, and charge large sums for regular access to this data. We prove that internet domain popularity follows a power-law distribution, namely the Zipf curve, and provide a new algorithm for generating accurate internet domain popularity rankings free of charge. (Received September 16, 2016)

1125-VP-1408 **Barbara Margolius\*** (b.margolius@csuohio.edu), Department of Mathematics, RT 15th floor, 2121 Euclid Blvd., Cleveland, OH 44115-2214. *Stable Quasi-Birth-Death Processes with Time-varying Periodic Transition Rates are Asymptotically Geometric*. Preliminary report.

Random processes with time-varying periodic transition rates occur in many natural and man-made systems. These include the level of water in the Great Lakes, the number of airplanes arriving to or departing from an airport, the volume of traffic on a city street, internet usage levels, demands for emergency service such as police fire or emergency medical, the volume of calls to a telephone call center, and many more. In this talk we outline an approach for obtaining asymptotic estimates of level probabilities of continuous time Quasi-Birth-Death process with periodic transition rates that can be used to model the aforementioned processes. We illustrate the method with the single server queue and then explore a simple Quasi-Birth-Death process example.

The approach involves finding the roots of the determinant of a matrix related to the generating function for a two-dimensional random walk over a single time period. The level probabilities as the level number tends to infinity tend to periodic functions of the form  $r^n f(t)$  where  $n$  is the level number,  $f(t)$  is a function of time within the period and  $r$  does not depend on time. In other words, the level distribution is asymptotically geometric. The rate,  $r$ , does not depend on time. (Received September 16, 2016)

1125-VP-1413 **Azar Khosravani\*** (akhosravani@colum.edu) and **Constantin Rasinariu**. *n-digit Benford converges to Benford*.

An  $n$ -digit Benford variable behaves as a Benford variable in its first  $n$ -digits, but it is not guaranteed to have a logarithmic digit distribution beyond its  $n$ th digit. The sum invariance property of Benford variables is used to prove that an  $n$ -digit Benford variable converges to Benford as  $n$  approaches infinity. (Received September 19, 2016)

1125-VP-1630 **Khyam Paneru\*** (paneruk@uwv.edu), 800 W Main St, Whitewater, WI 53190, and **Robert N Padgett**. *Pseudo-Likelihood Estimates and Bootstrap Confidence Intervals for the Mean of Zero-Inflated Population*.

Consider an example in an insurance industry where a large number of policyholders do not file a claim (zero claim amount is recorded), and the rest of the policyholders file claims (non-zero claim amount is recorded). A high proportion of zeros in such population, causing the distribution to spike at zero, is known as zero inflation. Zero-inflated population can be viewed as a mixture of zero and non-zero components. Existing methods used to estimate zero-inflated population mean relies on assumptions for asymptotic distribution, and are computationally complex. We extend maximum pseudo-likelihood approach of Chen et al. (2010) to estimate the confidence interval of zero-inflated population mean by applying bootstrapping, a resampling technique. Applying bootstrapping avoids making assumptions for the asymptotic distribution, which makes this method applicable to any distribution type of the non-zeros, and therefore, it becomes mathematically and computationally simpler. We are conducting simulation studies under pseudo-likelihood method to estimate the bootstrap confidence interval of population. We also applied bootstrap technique to estimate the mean of real data with large proportion of zero values. (Received September 18, 2016)

1125-VP-1646 **Shandy Hauk\*** (shauk@wested.org), 400 Seaport Court Ste 222, Redwood City, CA 94063. *Mathematics and Disparate Discipline Cases in the Office for Civil Rights*.

The mission of the U.S. Department of Education, Office for Civil Rights (OCR) is to “ensure equal access to education and to promote educational excellence through vigorous enforcement of civil rights in our nation’s schools.” Part of OCR work is to investigate possible disparities in disciplinary treatment. In-school suspension removes a student from the classroom while still allowing school staff immediate oversight of learning. Out-of-school suspension removes a student from classroom and school, often interrupting learning opportunities. Consider the case of a particular year at Rosa Parks Elementary. In the Civil Rights Data Collection (CRDC) database, the 672 students in the school were identified as 592 Hispanic, 64 Black, 10 White, 4 Asian, and 2 in two or more of these groups. That year, all 7 in-school suspensions were of Hispanic students as were 8 of the 13 out-of-school suspensions. The remaining 5 out-of-school suspensions were of Black students. What does examining these data tell us about the equity or disparity in disciplinary treatment of students? The presentation gives examples from lessons on (a) using basic statistics, (b) proportional reasoning, and (c) framing additional questions to draw conclusions and develop next-steps for examining such scenarios. (Received September 18, 2016)

1125-VP-1651 **Bhikhari P Tharu\*** (btharu@spelman.edu), 350 Spelman Ln SW, Spelman College, Department of Mathematics, Atlanta, GA 30314. *Bayesian Method for Histogram Smoothing*.

We will discuss about Bayesian estimation of probabilities in a histogram. Probabilities are closed to each other with neighboring intervals has been assumed for prior. Multivariate logit transformation has been used with covariance matrix similar to first order autoregressive process. Decomposition of components of covariance matrix has been implemented to update information for posterior. (Received September 18, 2016)

1125-VP-1707 **Netra P Khanal\*** (nkhanal@ut.edu), 401 W Kennedy Blvd, Tampa, FL 33606. *Differential Equation model for carbon dioxide emission*. Preliminary report.

Carbon dioxide (CO<sub>2</sub>) is one of the major contributors in Global Warming. This study focuses on developing a system of differential equations using time series data of significant contributable variables of carbon dioxide in the atmosphere in the continental United States. We define the differential operator as data smoother and use the penalized least square fitting criteria to smooth the data. The proposed model gives an estimate of the rate

of change of carbon dioxide in the atmosphere. The data set is obtained from the Carbon Dioxide Information Analysis Center (CDIAC), the primary climate-change data and information analysis center of the United States Department of Energy. (Received September 18, 2016)

1125-VP-1867 **Andrew Lazowski\***, 5151 Park. Ave. SC220A, Fairfield, CT 06825. *Using Curriculum Infusion to Impact a Probability and Statistics Course.*

Curriculum Infusion (CI) involves exposing students to information on a selected topic by interweaving the content into a course. We will review the evaluation of CI to impact alcohol-related health behaviors and attitudes of college students. We will also review the impact of CI on pedagogy. Course preparation, incorporation of material, and results will be discussed. (Received September 19, 2016)

1125-VP-1982 **Farzana Nasrin\*** ([farzana.nasrin@ttu.edu](mailto:farzana.nasrin@ttu.edu)), Department of Mathematics and Statistics, Texas Tech University, 2500 Broadway, Lubbock, TX 79409, **Ram Iyer** ([ram.iyer@ttu.edu](mailto:ram.iyer@ttu.edu)), Department of Mathematics and Statistics, Texas Tech University, 2500 Broadway, Lubbock, TX 79409, **Eltonzsee See**, Department of Mathematics and Statistics, Texas Tech University, 2500 Broadway, Lubbock, TX 79409, and **Steven Mathews**, West Texas Eye Associates, Lubbock, TX 79424. *Smoothing Splines on Unit Ball Domains with Application to Corneal Topography.*

The increased prevalence of refractive surgery and design of customized contact lens with aspherical back surface necessitates a precise understanding of corneal topography. Slit-scanning elevation topographer (Orbscan II), Scheimpflug imager (Pentacam, Sirius), and optical coherence tomographers (Visante) are non-invasive imaging techniques used to study and understand structures of biological tissues such as corneal topographical elevation. An interesting and important problem is the reconstruction of the shape of the biological tissue from these images. A similar problem arises in Magnetic Resonance Imaging (MRI). We cast the problem as a penalized weighted least squares regression with a penalty on the magnitude of the Laplacian of the surface. We present a novel algorithm to construct the Kimeldorf-Wahba solution for unit ball domains. The solution is the sum of a harmonic function that provides a global fit and a linear combination of asymmetric radial basis functions that provide the local fit. Application of the theory to data from an anterior segment optical coherence tomographer is presented along with computation of simultaneous 95% confidence and prediction bands. A detailed comparison of the reconstructed surface using different approaches is also presented. (Received September 19, 2016)

1125-VP-2174 **Ahmad Alzaghali\***, [ahmad.alzaghali@oswego.edu](mailto:ahmad.alzaghali@oswego.edu), NY. *A New family of continuous distributions.*

A new family of distributions is defined. Some members of the family are defined and studied. Several general mathematical properties of the new family are studied in details. The flexibility of the new family is assessed by applying some members of the family to a real data set and comparing it with other distributions. (Received September 19, 2016)

1125-VP-2271 **Joy M D'Andrea\*** ([jdandrea@usf.edu](mailto:jdandrea@usf.edu)), 8350 N. Tamiami Trail, Sarasota, FL 34243. *Modeling Hurricanes using Exploratory Factor Analysis in conjunction with Non-Response Analysis and Logistic Regression.*

Exploratory factor analysis (EFA) is used to determine the number of latent variables that are needed to explain the correlations among a set of observed variables. In this study, the latent variables are the meteorological measures such as the location of a storm, wind speed and pressure. In this talk, we will demonstrate how EFA can be used to determine the distinct factors that house the terms that explain the variance among the co-dependent variables and how non-response analysis can be applied to model the non-functional relationship that exist in a dynamic system. Further analysis investigates the probability of storms being present in the Atlantic Basin using logistic regression and atmospheric conditions recorded at a fixed location. (Received September 20, 2016)

1125-VP-2420 **Mehdi Razzaghi\*** ([mrazzaghi@bloomu.edu](mailto:mrazzaghi@bloomu.edu)), Department of Mathematics, Bloomsburg University, Bloomsburg, PA 17815. *The Use of Non-Canonical Link Functions in Generalized Linear Models.*

A Generalized Linear Model (GLM) consists of three components. One is the random component which represents the outcome variable. This variable could be binary, multi-class, count or continuous. The second component is the systematic component which consists of a set of explanatory variables. A linear function of these systematic variables is used for prediction of the outcome variable. The third component of a GLM is the link function which, depending on the outcome variable, is defined as a function of the other two components. Now, one general assumption is that the random component has a distribution belonging to the exponential family. In that case,

then there is a natural choice of the link function and is the canonical link. For example, when the outcome is binary, the logistic link defines the canonical link. Similarly, when the outcome is count or continuous, then the Poisson and normal distributions respectively define the canonical links. There is, however, some argument that non-canonical links can also be of value and sometimes even preferred. In this talk, we present the case for the non-canonical link functions and discuss their application. The emphasis will be placed on comparing the logistic and probit links when the outcome is binary or in general categorical. (Received September 20, 2016)

1125-VP-2489 **Eddie Tu\*** (etu@vols.utk.edu), 820 Red Saile Road, Knoxville, TN 37909. *On the Association of Certain Feller Processes.*

We propose methods of characterizing “association,” a strong form of dependence, for certain Lévy-type Feller processes. In particular, our methods utilize the symbol of such processes. We explore the importance of characterizing association of the process by looking at examples, which include solutions of Lévy-driven SDEs. Then we discuss some applications of association for Feller processes. (Received September 20, 2016)

1125-VP-2511 **Josh M Beal\*** (jbeal@iue.edu), Whitewater Hall Room 256, Indiana University East, Chester Blvd., Richmond, IN 47374. *Characterizing the space of distributions of simple stochastic processes.*

In this talk, we present new results regarding the space of distributions for certain discrete time stochastic processes. We begin by discussing the motivation behind our investigation: the classic ‘matching’ or ‘mimicking’ problem. By applying proof techniques used in such ‘matching’ problems, we show that the space of distributions for any simple stochastic process is actually the convex hull for which the process is Markov. (Received September 20, 2016)

1125-VP-2631 **Peter Otto, Benjamin Savoie\*** (bsavoie@umflint.edu), **Ana Wright** and **Renjun Zhu**. *Mixing Times for a Generalization of the Curie-Weiss Model via Aggregate Path Coupling.*

The classical Curie-Weiss (CW) model is the mean-field version of the famous Ising model of statistical mechanics. The model exhibits a second-order, continuous equilibrium phase transition with respect to the magnetization. The mixing time for the Glauber dynamics of the classical CW model has been shown to undergo a transition from rapid to slow mixing at precisely the equilibrium phase transition critical value. In this talk, we present mixing time results for a generalized Curie-Weiss model that exhibits a first-order, discontinuous equilibrium phase transition. We show how the standard path coupling technique fails in this case and how the new method of aggregate path coupling can be applied to determine the mixing time transition value, which is strictly less than the equilibrium phase transition critical value. (Received September 20, 2016)

1125-VP-2666 **Adam Swayze\***, adamrfswayze@gmail.com, and **Mikaela Jordan** and **Joseph Brown**. *If Twitter Could Vote: Predicting Primary Results using Social Media.*

The rapidly growing body of Twitter, Facebook, and other social media data offers novel ways to quantify political affiliation and predict election results. By incorporating such data, we can improve upon traditional prediction methods that rely solely on polling data. The goal of this study is to use natural language processing methods to build models from Twitter to predict outcomes of presidential primary results at the county level. Using word frequencies and order, we construct machine learning models using Random Forests,  $k$ -Nearest Neighbors, Support Vector Machines, and Ensemble Methods. We will compare the effectiveness of such Twitter data driven techniques against more conventional polling methodologies. (Received September 20, 2016)

1125-VP-2706 **Grigory Sokolov\*** (gsokolov@binghamton.edu), 660 Brotzman Rd., Binghamton, NY 13901, and **Georgios Fellouris**. *Decentralized change-point detection in correlated sensor networks.*

Consider a sequential change detection problem, where a number of correlated sensors monitor an environment in real time. At some unknown time there is a change in an unknown subset of sensors; we consider the problem of detecting the time of the change as soon as possible, while controlling the rate of false alarms.

We establish asymptotic optimality of several generalizations of the CUSUM rule under certain communication constraints, and conduct a case study to compare the proposed procedures to the oracle detection rules uninhibited by the aforementioned constraints.

This is joint work with Georgios Fellouris (Department of Statistics, University of Illinois at Urbana-Champaign). (Received September 20, 2016)

1125-VP-2741 **Ramadha D Piyadi Gamage\*** (prdilha@bgsu.edu), **Wei Ning** and **Arjun K Gupta**. *Adjusted Empirical Likelihood for Long-memory Time Series Models*.

Empirical likelihood method has been applied to short-memory time series models by Monti (1997) through the Whittle's estimation method. Yau (2012) extended this idea to long-memory time series models. Asymptotic distributions of the empirical likelihood ratio statistic for short and long-memory time series have been derived to construct confidence regions for the corresponding model parameters. However, computing profile empirical likelihood function involving constrained maximization does not always have a solution which leads to several drawbacks. In this paper, we propose an adjusted empirical likelihood procedure to modify the one proposed by Yau (2012) for autoregressive fractionally integrated moving average (ARFIMA) model. It guarantees the existence of a solution to the required maximization problem as well as maintains same asymptotic properties obtained by Yau (2012). Simulations have been carried out to illustrate that the adjusted empirical likelihood method for different long-time series models provides better confidence regions and coverage probabilities than the unadjusted ones, especially for small sample sizes. (Received September 20, 2016)

1125-VP-2798 **Mintaek Lee\*** (mintaeklee@boisestate.edu) and **Jaechoul Lee** (jaechoullee@boisestate.edu). *A Return Level Analysis of the 2016 Blizzard in New York City*. Preliminary report.

A major winter storm brought up to 36 inches of snow in parts of the Northeastern United States for January 22 – 24, 2016. The 2016 blizzard impacted about 102.8 million people with at least 55 deaths, and caused economic losses in a range of \$500 million and \$3 billion. We apply extreme value methods to extreme snowfall and snow depth data from the New York City area to explicate the blizzard's likelihood in terms of return levels. We use a method suggested by Smith (1990) to correct the underestimated model parameters' standard error due to spatial and temporal dependence in our data. Our preliminary findings indicate that the 2016 blizzard was about a once in a 50-year event when analyzed under appropriate extreme value analysis techniques. We also use several methods to construct better confidence intervals for return levels. (Received September 20, 2016)

1125-VP-2800 **Budhinath Padhy\*** (budhinath@yahoo.com) and **Gemechis Djira**. *A Multivariate Longitudinal Analysis of the effects of Depressive Symptoms, Financial Strain and Self Rated Health on Spiritual Connectedness*.

This talk presents and discusses the results obtained using the longitudinal structural equation modeling methodology to analyze survey data sampled from Center for Medicare and Medicaid Services (CMS) database. In this study two waves of data collected during 2001 and 2004 were used to study the effect of physical, mental and financial strain on spiritual connectedness. Our research objectives are to find which explanatory latent variables have significant effect on spiritual connectedness at Wave 1. Secondly, which latent variables have significant effect on spiritual connectedness at Wave 2. Thirdly, how the spiritual connectedness is affected by race, gender, education and region. Results from our analysis show that the spiritual connectedness at Wave 1 is significantly affected by the financial strain at Wave 1. Secondly, the spiritual connectedness at Wave 2 depends solely on the effect of spiritual connectedness at Wave 1. Thirdly, African Americans are spiritually more connected than white Americans whereas females are spiritually more connected than males. Finally, there is no significant effect of education or region on the spiritual connectedness. (Received September 20, 2016)

1125-VP-2805 **Amanda Lohss\***, agp47@drexel.edu, and **Paweł Hitczenko**. *Corners in tree-like tableaux*.

Tree-like tableaux are combinatorial objects which exhibit a natural tree structure and are connected to the partially asymmetric simple exclusion process (PASEP). There was a conjecture made on the total number of corners in tree-like tableaux and the total number of corners in symmetric tree-like tableaux. We have proven both conjectures based on a bijection with permutation tableaux and type-B permutation tableaux. In addition, we have shown that the number of diagonal boxes in symmetric tree-like tableaux is asymptotically normal and that the number of occupied corners in a random tree-like tableau is asymptotically Poisson. This extends earlier results of Aval, Boussicault, Nadeau, and Laborde Zubieta, respectively. (Received September 20, 2016)

1125-VP-2829 **Gerald Y Agbega\*** (gagbega@ggc.edu), 1000 University Center Lane, School of Science and Technology, Lawrenceville, GA 30043. *Towards Developing Strategies for Winning at Pick-n Lotteries*. Preliminary report.

The state pick-n lotteries include the pick 3, pick 4, and pick 5 games. In place of the "pick" some states use "cash". It is generally believed that there is no way to predict the pick-n lotteries. We use combinatorial techniques such as solution of integer equations, the principle of inclusion-exclusion and permutation of multisets to determine the distribution of link degrees. The distribution of these link degrees point to some winning strategies

for these games. Empirical tests are performed using actual lottery data from some states in the US. (Received September 20, 2016)

1125-VP-2874 **Durga H Kutal\*** ([dkutal2012@fau.edu](mailto:dkutal2012@fau.edu)), 630 NW 13th Street, Apt. 26, Boca Raton, FL 33486. *REML for cure rate model with extra partial information of diagnostic results.* Preliminary report.

In recent years the cure models are widely used to analyses time to event data in the field of medicine and public health researches. We proposed Restricted Maximum Likelihood (REML) estimation of parameter , which is performed based on the expectation maximization (EM) algorithm for the cure model with extra partial information of diagnostic results. We will also look over its simulation studies for the proposed estimation method using R. (Received September 20, 2016)

1125-VP-3039 **Stephanie Thrash\*** ([sthrash@stedwards.edu](mailto:sthrash@stedwards.edu)) and **Nicole Buczkowski.** *Mathematical Analysis of Lottery Voting.*

Standard voting methods rely on deterministic social choice functions to aggregate voter preferences and determine a winner or set of winners. In contrast, lottery voting determines winners by randomly selecting voter ballots. In this presentation, we will investigate lottery voting from a mathematical perspective. In particular, we will provide insight into the effects of varying the size of the set of winners and potential strategies for political parties within the lottery voting system. (Received September 20, 2016)

1125-VP-3067 **Kemal Akoglu\*** ([kakoglu@ncsu.edu](mailto:kakoglu@ncsu.edu)), 3147-B Kings Ct, Raleigh, NC 27606. *The Role of Technology in Overcoming the Common and Resistant Misconceptions about Probability.* Preliminary report.

Probability has an abstract, uncertain nature. Where our conclusions in mathematical problems are and have to be certain, in probability it is only possible to have ‘probabilistic’ conclusions that approximate the theoretical probability in order to make decisions. Because of its uncertain nature, and various other factors, there have been ongoing, resistant misconceptions about probability.

With the help of technology, learners can participate in the process of building probability models and collecting data generated from those models, creating opportunities to improve probabilistic understanding.

Research based suggestions to overcome probabilistic misconceptions will be presented, and the role of technology will be highlighted with examples. (Received September 20, 2016)

1125-VP-3116 **Keshav P. Pokhrel\*** ([kpokhrel@umich.edu](mailto:kpokhrel@umich.edu)), 4901 Evergreen Road, Dearborn, MI 48128. *Regional Discrepancies in Cancer Mortality Rates.*

Incidence and mortality rates are considered as a guideline in developing public health strategies and allocating resources. I will present some applications of functional data analysis techniques to model age-specific cancer mortality rates and forecast entire age-specific functions using exponential smoothing state-space models. The age-specific mortality curves are decomposed using principal component analysis and fit functional time series model with basis functions. Nonpara- metric smoothing methods are used to mitigate the existing randomness in the observed data. Functional time series models are used to model age-specific brain cancer mortality rates and forecast mortality curves with prediction intervals using exponential smoothing state-space model. (Received September 21, 2016)

## *General Session on Teaching and Learning Introductory Mathematics*

1125-VQ-241 **Lisa A Lister\*** ([llister@bloomu.edu](mailto:llister@bloomu.edu)), 224 Ben Franklin Hall, Dept. of MAD Sciences, Bloomsburg University, Bloomsburg, PA 17815, and **Lynne Ipiña** ([ipina@uwyo.edu](mailto:ipina@uwyo.edu)), 320 Ross Hall, Dept. of Mathematics, University of Wyoming, Laramie, WY 82071. *Do Our Calculations Matter if Our Assumptions are Flawed?*

In today’s classroom, it is harder and harder to get our students interested and excited about math. So we thought, why not use what they are interested in, such as movies, television, and video games, to introduce real world problems for them to consider. In this talk we will present one such activity that uses the pop culture phenomenon, Star Trek. Star Trek was a television series in the sixties, with the modern reboot becoming a billion dollar movie franchise. For our activity, we use the episode Trouble with Tribbles, from the original TV series, to motivate our problems. First, the students create models to draw conclusions based on the given assumptions.

More importantly however, the students then look at the assumptions made and see if they themselves make sense; and if they don't, can we believe our conclusions? (Received August 17, 2016)

1125-VQ-489 **Umesh P Nagarkatte\*** ([unagarkatte@yahoo.com](mailto:unagarkatte@yahoo.com)), Department of Mathematics, Office L05H, Medgar Evers College, City University of New York, Brooklyn, NY 11225. *Abstract: Adapting the Singapore Problem Framework to College Level – Performance Report*  
*Presenters: Drs. Umesh Nagarkatte, Joshua Berenbom.*

Singapore Mathematics ranks first in the world in the Trends in International Mathematics and Science (TIMSS) studies. Adapting the Singapore Mathematics Problem Solving Framework to college level, we have revamped Prealgebra, Elementary Algebra and College Algebra & Trigonometry. This effort gave rise to writing three textbooks which involve an innovative set of word problems based on the “Singapore Model Method.” The Framework is represented in a pentagon of interrelated components: Concepts, Skills, Processes, Metacognition, and Attitudes. The “Method” uses rectangular models that analyze word problems to teach students the mechanics involved. We also approach each concept numerically, graphically and algebraically integrating technology. The five components and the approach address diverse learning styles, lead to deeper understanding and prepare students for Calculus. We also considered students’ objectives that address difficulties logically in each topic. We then connect these objectives as a story and solve the problems in the discussion. The adaptation has been tested for three years and is observed to increase passing rates in the three courses. We present the data collected. (Received September 06, 2016)

1125-VQ-511 **Rachel M Bates\*** ([rachel.bates@redlandsc.edu](mailto:rachel.bates@redlandsc.edu)), 1300 S. Country Club Rd, El Reno, OK 73036. *Teacher Education and Quantitative Literacy: Improved Training for Teachers.* Preliminary report.

Quantitative literacy (QL) is a major goal of the National Council of Teachers of Mathematics (NCTM) for teachers of mathematics (NCTM, 2000). Although QL should be an aim of education across all subjects in PK-12, a large part of the responsibility falls to PK-12 educators (Madison, 2008). QL has continued to receive attention because of the growing demands on U.S. residents to understand, utilize, and react to quantitative information in their daily lives. An important component of a stronger quantitative education experience for PK-12 students requires QL educational experiences within teacher education programs. The purpose of this session is to share the results of a study examining pre-service teacher’s experiences completing a Quantitative Literacy course. Participants in this study were pre-service teachers pursuing a Bachelor’s Degree in Education. The education major requires students to complete four undergraduate level mathematics courses. The participants were enrolled in the Quantitative Reasoning course taught by the speaker. Results of surveys, self-reflections and interviews regarding their experiences within the course will be shared. (Received September 04, 2016)

1125-VQ-643 **Judith Quander\*** ([quanderr@uhd.edu](mailto:quanderr@uhd.edu)), University of Houston-Downtown, Department of Mathematics and Statistics, One Main Street, S705, Houston, TX 77002, and **Timothy Redl** ([redlt@uhd.edu](mailto:redlt@uhd.edu)), University of Houston-Downtown, Department of Mathematics and Statistics, One Main Street, S705, Houston, TX 77002. *Building a Community of Practice to Develop and Integrate Innovative Instructional Strategies in College Algebra Classes at the University of Houston-Downtown.* Preliminary report.

In this session, we discuss our work developing, implementing and nurturing a community of both full-time and part-time mathematics faculty who regularly teach College Algebra with a focus on improving their instruction in the course. Thirteen faculty members participated in a summer workshop where we explored the eight mathematical teaching practices described in the NCTM publication Principles to Action and their implications for our teaching freshman-level mathematics. In addition, we developed a collection of rich mathematical tasks to be performed collaboratively by students in the College Algebra classroom. Participants then spent the academic year integrating these activities into their instructional plan and sharing their experiences with all community of practice members. Using an observation tool designed for inquiry-based teaching, participants observed each other’s teaching and provided valuable feedback. This project was supported by the UHD Center for Teaching and Learning Excellence which provided stipends for the participants. We have begun to collect data on faculty members’ experiences and other measures of success. Preliminary findings will be shared. (Received September 08, 2016)

1125-VQ-699 **Alison Reddy\*** ([ared@illinois.edu](mailto:ared@illinois.edu)), 1409 W. Green Street, Department of Mathematics, Urbana, IL 61801. *Corequisite Remediation in a College Algebra Course: Embracing Complete College America.*

At the University of Illinois ALL students scoring below 65 on the placement exam are placed, by default, into College Algebra. Thus students enter the course with very diverse mathematical needs and background

knowledge ranging from beginning algebra to almost precalculus, and the majority of College Algebra students are first generation or underrepresented minorities.

The challenge and goal was to embrace Complete College America's call by implementing corequisite remediation to maximize student outcomes within the context of a single course. College Algebra was redesigned to two 50-minute large lectures and one small lab section per week with heavy use of technology for learning and practice outside of class, and course communication.

The redesign has been very successful in meeting the individual needs of all the students and we have seen improved success rates and student satisfaction, and increased pathways to graduation. Collected data will be shared. (Received September 09, 2016)

1125-VQ-1061 **Shumei C Richman\*** ([richmansmc@gmail.com](mailto:richmansmc@gmail.com)). *Formula vs. Concept: A Dual Process for Solving Problems in Beginning Algebra*. Preliminary report.

In my ten plus years teaching remedial algebra courses in colleges, I have searched for root causes of many common mistakes made by my students, and accordingly designed alternative ways of teaching to help them avoid these mistakes. Of the several ways I have designed, one is, when introducing a new formula for solving problems, to add the corresponding concept approach alongside the commonly used formula approach, and discuss the differences. This dual process has been well accepted by most of my students, because of its three main advantages. First, the concept approach, revealing many more math concepts in details, serves as a bridge allowing students not only to learn how to use the formula, but also to understand what it means, and why it works. This deeper level of understanding helps students avoid many mistakes. Secondly, the math concepts revealed help students connect the new formula to what they have learned earlier and so, for the long term, help them build up their mathematical sense. Furthermore, a concept approach often requires students to interpret math symbols and analyze the problem, instead of following steps and plugging in numbers as required by a formula approach, and so helps them build up their abilities to independently tackle problems. (Received September 14, 2016)

1125-VQ-1152 **Christy Sue Langley\*** ([clangley@louisiana.edu](mailto:clangley@louisiana.edu)), Department of Mathematics, P.O. Box 41010, Lafayette, LA 70504-1010. *Using Clickers to Gauge Understanding*.

This talk will give a glimpse into my Mathematics for Finance class; in which I use the University adopted clicker system to gauge understanding of key topics. I will share the ease of integration for both student and instructor. I will include how the anonymity during class helps with student participation. I will also explain how the data could help with encouraging struggling students. (Received September 15, 2016)

1125-VQ-1454 **Frank Vahid** ([vahid@cs.ucr.edu](mailto:vahid@cs.ucr.edu)), Dept of CSE, Univ. of California, Riverside, CA 92521; **Joe Allen\*** ([jalle010@ucr.edu](mailto:jalle010@ucr.edu)), Dept. of CSE, Univ. of California, Riverside, CA 92521, and **Alex Edgcomb** ([aedgcomb@cs.ucr.edu](mailto:aedgcomb@cs.ucr.edu)), Dept. of CSE, Univ. of California, Riverside, CA 92521. *Web-based games to master core skills in introductory college mathematics*.

Numerous games have been proposed to improve mathematics learning outcomes, but with mixed results. A shortcoming is that most games merely plug math problems into a generic game framework, such as into a quiz-based game, a first-person shooter game, or an adventure game. The efficacy of learning through such games is unproven, and some would argue is a bit insulting to the student, especially college students. Meanwhile, education research emphasizes the importance of repetition for skills mastery. Yet homework systems that dryly enforce repetition can lead to disenchanted, anxious, and/or fatigued students. As a partial solution, we are developing engaging web-based math games specifically designed to help students build mastery of core skills. Each game is uniquely created to inherently teach the specific math concept. A solve-for-x game teaches balance by involving a see-saw, yielding more points for quick balance. A tetris-like multiply-terms game involves combining terms to create a goal term, causing the player to automatically recognize that coefficients multiply and exponents add. Similarly for intervals,  $mx + b$ , and more. Trial use by hundreds of students at our university has yielded highly-positive feedback. We intend to make the games freely available on the web. (Received September 16, 2016)

1125-VQ-1563 **Matthew K Voigt\***, 6475 Alvarado Road, Suite 206, San Diego, CA 92120. *Enriching the Flipped Classroom for All Students*. Preliminary report.

There have been several research studies examining the efficacy of flipped classrooms, due in part to its growing popularity in the field of Mathematics. While the results are generally encouraging, they often categorize flipped classrooms using broad strokes, despite the various characteristics of a particular implementation. In addition, the design of such courses often focus on the specifics of flipped classroom pedagogy at the loss of incorporating other critical perspectives. In this research, we designed and implemented a two-week classroom

teaching experiment for calculus students enrolled in a Norwegian University. This unique setting provided the opportunity to include design elements to support students with varying degrees of fluency in English, a challenge that is faced by many educators in the United States. We addressed this issue by providing subtitles and guides to videos, and empowering students to utilize their native language when discussing in the classroom. Furthermore, the course was designed to include aspects of active learning, culturally relevant pedagogy for students in Bodo (Norway) and used mathematical modeling as a tool for social justice to explore the impacts of climate change. (Received September 18, 2016)

1125-VQ-1642 **Michael D. Miner\*** ([michael.miner0@mycampus.apus.edu](mailto:michael.miner0@mycampus.apus.edu)). *Reversing the Feedback: Effective Technique for Assessing Students in an Online College Algebra Course*. Preliminary report.

Online degree programs available to learners seeking degrees outside the traditional college learning environments involve a college algebra course designed to fulfill common general education math requirements. Providing feedback on assessments in an online environment is a meaningful method of informing students on results of assessments. This feedback in many cases often goes ignored or casually read and provides a suboptimal level of interaction with the student. Therefore, a technique of reversing the feedback has proven to add value to students' understanding of the concepts presented in the online College Algebra course and increases the interaction between students and instructor. The discussion presented in this session will outline the technique used in reversing the feedback to get a better insight on students understanding of assessments and increase the interaction between the student and instructor. (Received September 18, 2016)

1125-VQ-1719 **Sheeva Doshireh\*** ([sliddle@gmu.edu](mailto:sliddle@gmu.edu)), 9936 Braddock Road, Fairfax, VA 22032, and **Stephen Liddle**. *Learning Assistants' Roles in Flipping Large Classrooms*.

Over the past few semesters, learning assistants have worked with professors to create flipped classrooms with large class sizes averaging around ninety students per class. Flipped classrooms have a unique structure, generally consisting of the lecture portion of the class being completed outside of the classroom through videos or notes, while in-class time is spent working with students through group work and practice problems. The presentation will discuss the in-class procedures and outside-of-class preparations, examine the professors' and students' views of flipped classrooms, and how the learning assistant program is key to flipping a large classroom. (Received September 19, 2016)

1125-VQ-1906 **Daniel E. Otero\*** ([otero@xavier.edu](mailto:otero@xavier.edu)), Department of Mathematics, Xavier University, Cincinnati, OH 45207-4441. *Preparing Students for Trigonometry with a Primary Source Project*.

In many precalculus courses, there is the obligatory unit on trigonometry. Its goal is to impart to the student as efficiently as possible a long list of useful things to know about the six trigonometric functions: angle measurement, the six functions and their graphs, a dozen or so identities, blah, blah, blah. Students are typically marshaled through these packaged presentations, and must await a (perhaps optional, and typically quite brief) "Applications" section to see where and how these notions are applied before natural questions of why it all matters get addressed. The author is one of seven Principal Investigators for an NSF-funded grant, *Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources* (TRIUMPHS), which is designing curricular materials for teaching standard topics in the university mathematics curriculum via the use of primary historical sources. A one-week-long Primary Source Project for students who are about to study trigonometry will be presented in this talk; it uses excerpts of the writings of Greek, Hindu, Islamic, and European scholars across the long history of the subject to show students that real people interested in natural mathematical problems developed trigonometry to address these problems. (Received September 19, 2016)

1125-VQ-1966 **Bill Zahner, Hayley Milbourne\*** ([hmilbourne@sdsu.edu](mailto:hmilbourne@sdsu.edu)) and **Lynda Wynn**. *Ways Secondary Mathematics Teachers Order Algebra Problems Based on Both Mathematical and Linguistic Complexity: A Case Study*. Preliminary report.

Developing mathematical proficiency among all students (NCTM, 2014) can be a formidable challenge in secondary classrooms with English language learners (ELLs) who are simultaneously developing proficiency in math and fluency in academic English. Prior case studies illustrate that skilled math teachers can create classroom learning environments where ELLs develop robust mathematical proficiency (e.g., Khisty & Chval, 2002; Zahner et al., 2012). Much of secondary math instruction revolves around instructional tasks (Stein et al., 1996). For teachers to create access for ELLs they must understand the interaction of linguistic and mathematical complexity in tasks. In this case study, we describe and summarize interviews with three secondary math teachers of linguistically diverse ninth grade math classes. In the interviews, the teachers analyzed the mathematical and

linguistic complexity of six problems and ordered the problems twice, once focusing on mathematical complexity and once focusing on linguistic complexity. With this information, we can begin to understand the ways in which teachers in linguistically diverse classrooms interpret problems. This may help researchers understand the complex decisions teachers make in the selection and presentation of instructional tasks. (Received September 19, 2016)

1125-VQ-2051 **Abra Brisbin\*** (brisbia@uwec.edu) and **Erica Maranhao do Nascimento**. *Reading vs. Doing: A Comparison of Methods of Teaching Problem-Solving in Introductory Statistics.*

Having students solve practice problems is a well-established technique for teaching problem-solving in math and science. However, some research suggests that having students study worked examples, instead, can be more beneficial to their subsequent performance, possibly due to the reduced cognitive load required. In this study, I compared these two methods of teaching problem-solving in introductory statistics. Six pairs of topics of approximately equal difficulty were chosen from throughout the semester. After an initial demonstration by the instructor, one topic from each pair (chosen randomly) was taught using practice problems; the other was taught by having students read worked examples and answer questions about what they had read. Bayesian and frequentist analyses find weak evidence that student performance is better after reading worked examples. Surprisingly, there is also strong evidence from in-class surveys that students experience greater frustration when reading worked examples. This, combined with the additional class preparation time required to develop the written examples, indicates that worked examples should be a low-priority intervention for instructors wishing to teach problem-solving in statistics. (Received September 19, 2016)

1125-VQ-2219 **Emma Smith Zbarsky\*** (smithzbarsky@wit.edu) and **Joan Giblin** (giblinj@wit.edu). *Testing a Learning Lab Model in First Year Mathematics Courses.*

We implemented a supplemental instruction Learning Lab model in the first year of our mathematics courses—a college algebra/precalculus sequence and a differential and integral calculus sequence. Upper level undergraduates were hired to attend one section of their assigned course, meet with the course coordinator and instructors weekly, and to hold a two hour weekly Learning Lab. In the (optional) Learning Labs, the first hour was devoted to a planned supplemental activity while the second was open for questions and discussion. We will present attendance data, survey results, and an overview of grade outcomes from this project. (Received September 19, 2016)

1125-VQ-2254 **Daniel S. Helman\*** (danielhelmanteaching@yahoo.com). *Using TPR in the pre-calculus class: Math instruction inspired by second-language learning.*

Total Physical Response (TPR) is a second-language learning technique developed from observations of how very young children learn language in a family setting. The point of focus is the act of a parent asking or directing the very young child, e.g. “See that box over there? Go bring it to me” or some similar request that was just at the level of skill and responsibility to give the child a sense of joy and purpose in its completion, even without having mastered all the language production faculties necessary to have made such a request themselves. TPR in the classroom works in a similar fashion, with the teacher often pantomiming the actions that at the same time they are directing students to perform—a grand call and response. The academic literature is clear as to the efficacy of TPR in language-learning, and it has been used as well in science and math contexts. TPR has a track-record of success. Presented herein are examples of using TPR in the pre-calculus class, as well as further explanation of its history and efficacy. Participants should expect to be up and out of their chairs for much of the presentation. A variety of topics will be covered. (Received September 20, 2016)

1125-VQ-2293 **Leah Nicole Hollingsworth\*** (lnh14c@my.fsu.edu), **Kelly Patrick Findley** and **Elizabeth H Jakubowski**. *Do College-Level Mathematics Courses Support Student Success in Introductory Statistics?* Preliminary report.

With statistics education researchers calling for a more conceptually-focused, computationally-lighter statistics 101 course, we ask whether college algebra prerequisites are sensible requirements for statistics at all. We will be sharing our results from our investigation on whether students taking college algebra and other lower-level mathematics classes before introductory statistics are performing better than their peers who take statistics first. Our data was comprised of over 4700 undergraduate students and included their math and statistics course sequence, their grades, and a number of possible mediating variables. The findings from our research may serve to enlighten mathematics departments on sensible math and statistics course sequences for new students and the kind of mathematical content that may serve to strengthen students’ readiness for statistics. (Received September 20, 2016)

1125-VQ-2470 **Anna Mummert\*** ([mummerta@marshall.edu](mailto:mummerta@marshall.edu)). *Reflections on Emporium, Stretch, and Corequisite for Developmental and Gateway Courses*. Preliminary report.

Gateway courses are roadblocks for many college students, resulting in poor retention rates. As state budgets tighten, increasing retention rates becomes more and more important at state funded universities. With a goal of reducing the attrition from college due to the long developmental-gateway course pathway, Marshall University has made a flurry of changes to our developmental and gateway courses over the past few years, including implementing an emporium-style developmental courses and offering corequisite gateway courses. In this talk I will describe Marshall's journey from a two sequence developmental course structure to our planned complete corequisite structure for gateway courses. Hopefully a description of our successes and failures will help other colleges who are working on, or thinking of, similar changes. (Received September 20, 2016)

1125-VQ-2516 **Jayson L Jackson\*** ([jljackson3@una.edu](mailto:jljackson3@una.edu)). *Factoring: Knowing When To Do What You Know How To Do*. Preliminary report.

Students needing math remediation at the college level often enter those classrooms with an appropriate background of knowledge. Even if this is not the case, they usually can learn how to do many of the procedures necessary to pass a remedial college class. Too often, however, even though they know how to work through a problem, they simply do not know when to apply the procedures they already know. This is often the case with basic factoring. Students, when told what category a particular type of factoring problem falls into, are able to complete the factoring process. This talk shows a fun method for helping students to recognize and understand the various types of factoring problems and is appropriate for any algebra class. (Received September 20, 2016)

1125-VQ-2621 **Sarah V. Cook\*** ([sarah.cook@washburn.edu](mailto:sarah.cook@washburn.edu)). *Common Video Resources for Multi-Section Developmental Algebra Courses*. Preliminary report.

At Washburn University, Intermediate Algebra is frequently taught by adjuncts rather than full time instructors. Adjuncts are underpaid and have little time or motivation to develop course resources that could enhance student learning. To help address this, several sample problems with accompanying video solutions were created and made available to all Intermediate Algebra instructors and students. This presentation will discuss motivating factors for creating the videos, technological challenges and solutions, and benefits from having a common resource for the multi-section course. (Received September 20, 2016)

1125-VQ-2644 **Adam J Castillo\*** ([adamj.castillo@austin.utexas.edu](mailto:adamj.castillo@austin.utexas.edu)), 8746 Birmingham Drive, Austin, TX 78748. *Understanding Community College Math Faculty Perceptions and Use of Cooperative Learning*. Preliminary report.

Cooperative learning, students working together in small groups to increase their own and each other's learning, is a well-documented pedagogical approach to promote student learning. However, despite ample research on cooperative learning, there is little research on its use in community college math courses. I conducted a pilot study on community college math faculty at four Texas community colleges involved in major math reform initiatives. These colleges were purposefully selected to identify math faculty familiar with, and with nominal administrative support for, reform teaching strategies such as cooperative learning. The purposive sample was intended to identify participants familiar with cooperative learning, but not necessarily employing it with fidelity, enabling me to characterize: (1) perceptions of community college math faculty regarding cooperative learning and its use and (2) what the implementation of cooperative learning might look like in community college math courses. This talk will focus on pilot study results on faculty perceptions of cooperative learning and how it is used in developmental math courses. I will also discuss the process for testing and modifying research instruments for future research on community college math faculty at different colleges. (Received September 20, 2016)

1125-VQ-2663 **Alexander G. Atwood\*** ([atwooda@sunysuffolk.edu](mailto:atwooda@sunysuffolk.edu)), Department of Mathematics, 533 College Road, Selden, NY 11784. *Introducing Fermi Problems and the Art of Reckoning to Students in an Introductory Statistics Class*.

The art of calculating informed estimates using minimal information, as done by the physicist Enrico Fermi, can be effectively taught to students in an Introductory Statistics Class. Although many students are initially hesitant in making back-of-the-envelope estimates and calculations, they can become progressively much stronger. I will share my experience of what works and what doesn't work in nurturing students' powers of guesstimation, and I will share my Fermi Problems that I use in the classroom. (Received September 20, 2016)

1125-VQ-2707 **Jason Quinley\*** ([jquinley@brookstoneschool.org](mailto:jquinley@brookstoneschool.org)), 9 Spruce St, Columbus, GA 31904.  
*Vector Calculus as a Path to STEM Research*  
*Notes from the Secondary Level.* Preliminary report.

In this talk we give several highlights from teaching a *Vector Calculus* seminar to advanced high-school seniors. Based on the texts from *Herod and Cain* and *Shifrin*, this course blends rigor and intuitive approaches in a manner beyond the typical AP curriculum. This course also gives students both a means for mastering previously seen material and a springboard for encountering more advanced mathematics at the college level. For some students taking BC Calculus concurrently, this course also aids mastery by previewing AP material from a different perspective.

This class incorporates research projects using the L<sup>A</sup>T<sub>E</sub>X software, TikZ, visualization programs like *Desmos* and *Wolfram Alpha*, and a 3-D printer. The added benefit occurs from the L<sup>A</sup>T<sub>E</sub>X software as a gentle introduction into programming, as many high school students have yet to take a course in Java or Python. Samples of student work on harmonographs, Lissajous curves, centroids, optimization in multiple dimensions, and differential equations in epidemiology will be presented. (Received September 20, 2016)

1125-VQ-2719 **Galit Eizman\***, po box 1034, Brookline, MA 02446. *Do We Teach the Wrong Thing? The Impact of Mathematical and Scientific Background on Economics Success.*

What Part of this:  $\langle y \rangle = (E(y^2) * \{E(E(U^2)Xun = P(v)/YTZ[e(E)u];$  Or this:  $XyK LbU(x) + Kzy - O(ng)/Kr(y);$  Or this:  $z(x) + g(x^2)/m(y)[k - j(xyz);$  Don't You Understand? For too many economics students and researchers, this question is not a funny joke. Reviewing, reading, understanding and moreover, writing and publishing theoretical models in economics is an extremely high challenge. Do we invest enough time and efforts to simplify and train our students to these highly mathematical requirements? There seems to be too little emphasis on establishing background of advanced mathematics in economics. In this paper, a unique and original data set, collected on individual basis, shows a positive and significant correlation between background in studying math or science as major in college, and being a highly productive, well-known and frequently cited economics researcher. Introducing the Nobel Prize in Economics in 1968, as parallel to Nobel prizes in Physics, Chemistry, Physiology or Medicine, is presented as an exogenous shock which changed the tendency of economics researchers to use more mathematical and scientific terminology. Using it as part of the DID (diff-in-diff) methodology is establishing causality between math studies and economics success. (Received September 20, 2016)

1125-VQ-2812 **Michael P. Doyle\*** ([mdoyle@linsly.org](mailto:mdoyle@linsly.org)). *A Flipped Precalculus Course.* Preliminary report.

Excited to try the “flipped” classroom model, I offer a template from my first implementation in a precalculus class. Included in the discussion is the use of guided reading assignments and notebook checks to motivate students to read the text, randomized group work to keep class time engaged and meaningful, and the mobile app Explain Everything to supplement the more challenging examples from the text. (Received September 20, 2016)

1125-VQ-2815 **Adam Giambrone\*** ([adam.giambrone@uconn.edu](mailto:adam.giambrone@uconn.edu)). *Incorporating Reading-Writing Assignments into a Liberal Arts Mathematics Course.* Preliminary report.

At many institutions, a number of students take a liberal arts math course to satisfy a general education requirement. Consequently, students may come into such a course with the mindset that the course is simply a box to be checked off on a checklist of graduation requirements. In an effort to increase student engagement and help students see how such a course fits into their college education, reading-writing assignments about intelligence, learning, and thinking have been woven into the Elementary Discrete Mathematics courses at the University of Connecticut, starting in the Fall 2015 semester. In this talk, we will discuss the creation and implementation of these reading-writing assignments, using student feedback and responses to highlight some benefits and challenges to incorporating this type of assessment into a liberal arts math course. (Received September 20, 2016)

1125-VQ-3085 **H. Tyrone Washington\*** ([tyrone.washington@millersville.edu](mailto:tyrone.washington@millersville.edu)). *Using iClickers or Plickers and Worked Examples in a College Algebra Course to Foster Discourse.*

In this talk, I will share how I use iClickers and/or Plickers with worked examples in a College Algebra course to foster discourse. Students need a conceptual understanding of fundamental topics in algebra as well as procedural skills. I use the iClickers or Plickers to assess my students' ability to assess the validity of a “fictional” student's work. A worked example problem presents a completed problem and includes questions that focus on specific

steps. Using worked examples allows my students to apply reasoning and sense-making skills as they explain why certain steps were carried out or what misconception is exhibited. (Received September 21, 2016)

1125-VQ-3093 **Wade Ellis\*** (wade25@sbcglobal.net), Wade Ellis, 4562 Alex Drive, San Jose, CA 95130. *Profile of a Quality Collegiate Mathematics Learner.*

The Profile of a Quality Collegiate Mathematics Learner scholarship develops and describes the characteristics acquired by most learners who successfully complete college mathematics requirements for intensive and non-intensive majors. Process Education principles and Learning to Learn practices provides methods for effectively transforming 1st year students into quality collegiate mathematics learners. Since these learner characteristics can be developed, a student's failure because of deficiencies in mathematics becomes an institutional failure and a mathematics department failure. A mathematics department with the strong support of its institution could reduce these failures by aligned its culture and practices with these Process Education principles. Every student: \* Can learn to learn better, regardless of initial learning capacity \* Can become a capable, self-sufficient, life-long learner, even students with limited initial learning capacity \* Can become an empowered learner, one who uses learning processes and self-assessment to improve future performance \* Can and must develop both generic, life-long learning skills but also a specific knowledge base to acquire the necessary expertise to successfully major in a chosen discipline. (Received September 21, 2016)

1125-VQ-3144 **Jalalidin Jaenbai\*** (jalalidin.jaenbai@zu.ac.ae), Zayed University, Department of Mathematics and Statistics, Academic City, Dubai, United Arab Emirates. *Measuring online student's motivation using MyMathLab and fuzzy logic.* Preliminary report.

The process of teaching through an online interactive and educational system such as MyMathLab produces plethora of feedback data from students. Some of these data can be very useful for quickly adjusting teaching strategies to fit specific characteristics of each individual student. In this paper we will discuss how such data can be quickly processed into a measure of a level of motivation in each student. (Received September 21, 2016)

## General Session on Teaching and Learning Calculus

1125-VR-209 **Mary E Pilgrim**, 1874 Campus Delivery, Fort Collins, CO 80523, and **Jessica R. Gehrtz\*** (jrgehrzt@gmail.com), 1874 Campus Delivery, Fort Collins, CO 80523. *Optimization Problems: Understanding Students' Struggles.*

Students struggle with solving problems that are within the context of a real-world situation. We explored student thinking through various levels of Calculus I optimization problems. We will share what was learned from student responses, curriculum and instructional impacts, and how we plan to connect findings to GTA training. (Received August 12, 2016)

1125-VR-383 **N Bradley Fox\*** (foxb@apsu.edu) and **Ramanjit Sahi.** *Revitalizing Calculus to Connect the Dots.*

Calculus is a vital component in the mathematical study of change. Due to the reduction of course credit hours at our university, our challenge was to redesign the Calculus course so that the same course content is offered to students without sacrificing student engagement and while improving student success. In this presentation, we will look at how interactive curricular material, focused on conceptual and application problems, has improved student learning. (Received August 30, 2016)

1125-VR-1529 **William Freed\*** (william.freed@concordia.ab.ca), Concordia University, 7128 Ada Blvd., Edmonton, Alberta T5B 4E4, Canada. *The Joys of Teaching Infinitesimal Calculus.* Preliminary report.

Infinitesimal calculus, aka non-standard analysis, greatly increases student enjoyment of and proficiency in the first calculus course. Calculus terms are self-explanatory,  $dy/dx$  vs  $f'(x)$ ; the definite integral symbol is not just a name but live mathematics. The differential is basic, not just a gimmick to be converted into an approximation formula or to make the change the change of variable method of integration seem intuitive. Derivation of formulas in general are more straight forward. Proofs of the Extreme Value Theorem or the Riemann Integrability of a Continuous Function over a Closed Interval are intuitive and short. Infinitesimal methods were used by the discoverers of the calcul. The absence of proof of the existence of infinitesimals was always a blight on the otherwise very productive subject of the calculus. In the mid-twentieth century Abraham Robinson showed infinitesimals could be introduced in a consistent mathematical way. Robinson's work is inaccessible to beginning

calculus students and just postulating the existence of infinitesimals is unsatisfying. The presenter gives a quasi-rigorous demonstration of the existence of infinitesimals. Some of the highlights of elementary infinitesimal calculus are shown. (Received September 17, 2016)

1125-VR-1916 **J.C. Price\*** (jprice12@ggc.edu). *A Canned Flipped Calculus Experience*. Preliminary report.

In this talk we will discuss our three years of experience with flipping calculus. We will briefly review the technologies used to screencast and distribute lectures, which are available at [www.youtube.com/user/drprice765](http://www.youtube.com/user/drprice765). Our focus will be on the instructional design and the materials created to run the course. The main result is an active learning environment that encourages students to openly discuss mathematics, compare and contrast ideas, and work together to solve problems. Our goal is to provide the basic knowledge and resources necessary to run a flipped calculus class with minimal effort. (Received September 19, 2016)

1125-VR-2222 **Amanda J Mangum\*** (amangum@niagara.edu), 5795 Lewiston Road, Niagara University, NY 14109. *Fostering Comprehensive Learning Through Concept Worksheets and Mastery-Based Testing*. Preliminary report.

This talk will discuss two methods utilized to encourage students to engage with the material from a Calculus I course in such a way that they leave the course fully understanding the key material and the concepts. Mastery-based testing was implemented in this course to reduce test anxiety and to encourage students to revisit material until they master it. (Students may re-take tests over concepts until they master them.) The second method implemented was assigning worksheets tailored to common misconceptions and mistakes seen in student work. These worksheets, completed either in class or as homework, should also help students process the material before they are tested over concepts. By actively working together to understand and to complete the worksheets, students can help each other comprehend the material at a deeper level. This talk will also discuss the shortcomings and challenges encountered with these approaches. (Received September 19, 2016)

1125-VR-2268 **Juliana V. Belding\*** (juliana.belding@bc.edu) and **Peter M. Garfield** (garfield@math.ucsb.edu). *Calculus Applied! An Online Resource for Students and Teachers of Calculus to Explore Calculus' Connections to Other Fields Through the Lens of Practitioners*.

Millions of students take calculus each year, many as a prerequisite for another field of study or future career. But the exposure students get to applications of calculus is often limited to short problems on homework or in a textbook.

In this talk, we'll introduce "Calculus Applied!", a massive open online course (MOOC) we will launch in Summer 2017, which features practitioners from various fields discussing actual problems in their fields. The course is designed for participants to actively engage in topics from mathematical biology, economics, physics, psychometrics, and medical imaging, among others, using short videos of experts discussing the role of calculus in modeling and making sense of problems, interspersed with problems to try and dynamic graphing exercises.

This MOOC is aimed not only at students in the first year of calculus, but also at high school and college instructors who want to supplement their single-variable calculus course.

Time-permitting, we'll discuss the potential impact of these types of activities on student attitudes about the relevance of mathematics to their future fields of study. (Received September 20, 2016)

1125-VR-2356 **Jonathan P Keiter\*** (jkeiter@esu.edu), East Stroudsburg University, Mathematics Department, 200 Prospect St, East Stroudsburg, PA 18301. *Teaching Contour Diagrams using 3D Models*.

We describe several projects that help students understand contour diagrams. Using laser cut sheets that form cones, paraboloids, and other shapes, students trace out the edges to create the contour diagram of the surface. Students also build their own simple 3D models based on a contour diagram or a function. These projects are useful at the start of a multivariate Calculus course, helping the students see the connections between the contour diagram and the surface. (Received September 20, 2016)

1125-VR-2384 **Matthew Leingang\*** (leingang@nyu.edu), 251 Mercer St, New York, NY 10012, and **Selin Kalaycioglu** and **Drew Youngren**. *Side-by-side comparison of a single instructor's flipped and traditional sections*. Preliminary report.

The authors have developed an extensive set of materials for a flipped Calculus I course, and have taught sections with those materials. In prior semesters, exam scores and learning outcomes have been compared between students in traditional sections and those in the flipped sections. In general, students in the flipped sections have done better, but that could have been an effect of the instructor rather than of the model.

In the Fall 2016 term, the first author taught two sections of Calculus I, one each in the traditional and flipped models. We will compare outcomes between the two, to control for any possible instructor effect. (Received September 20, 2016)

1125-VR-2409 **Ellen R. Swanson and Lesley W. Wigglesworth\*** ([lesley.wigglesworth@centre.edu](mailto:lesley.wigglesworth@centre.edu)), 600 W. Walnut Street, Danville, KY 40422. *Improving Student Success in Calculus Using an Algebra Supplement Course.*

Weak algebra skills can be a stumbling block for students in many college courses, especially Calculus. A supplemental algebra course was offered for students who were taking Calculus I at Centre College, during a reorganization of course offerings. The students were simultaneously enrolled in the traditional Calculus course and took an additional hour course in algebra. Using the results of a pre- and post-test, we analyze the improvement of the students in comparison to those not enrolled in the additional course. We found, in general, that the scores on the test improved as did the confidence in the students in the supplemental course. (Received September 20, 2016)

1125-VR-2574 **Austin Mohr\*** ([amohr@nebrwesleyan.edu](mailto:amohr@nebrwesleyan.edu)). *Using Points-Free Grading to Promote Perseverance in Calculus.*

In a points-free grading scheme, one sets criteria for acceptable and unacceptable work as opposed to issuing a numerical score. Notably, the concept of "partial credit" does not exist in a points-free course. By setting a clear, discrete level at which credit is obtained, students are required to persevere through challenging work in instances where they might have otherwise accepted partial credit. The safety net of partial credit is replaced with the opportunity for revision of unacceptable work, allowing students to respond to instructor feedback where it is most useful. I will discuss the framework I used for my first points-free course (Vector Calculus), offer reflections on its effectiveness, and share student perspectives. (Received September 20, 2016)

1125-VR-2597 **Steven M. Hetzler\*** ([smhetzler@salisbury.edu](mailto:smhetzler@salisbury.edu)), Department of Math & C.S., Salisbury University, 1101 Camden Ave, Salisbury, MD 21801. *Introducing Picard's Theorem in Integral Calculus: an Interesting Example.*

A non-standard method of estimating continuously compounded interest will be presented. This method is accessible to students with sufficient integral calculus experience to find the average value of an integrable function on a closed, bounded interval. The method produces Taylor Polynomials for the exponential function by considering the interest due on a principal, the interest due on the interest due on the principal, and so forth. By approaching the Taylor Polynomials from a new perspective, students can be led to see a generalization which shows that a unique solution exists for certain differential equations (Picard's Theorem). (Received September 20, 2016)

1125-VR-2651 **Martha H Byrne\*** ([byrnema@sonoma.edu](mailto:byrnema@sonoma.edu)), Mathematics and Statistics Department, 1801 E Cotati Ave, Rohnert Park, CA 94928. *Cooperative Curve Sketching: An Activity for Classes.*

The 2014 Freeman report calls for mathematics faculty to abandon traditional lecturing in favor of active learning. This paper presents an active learning, cooperative activity for a derivative calculus class. Curve sketching is an important part of any derivative calculus curriculum as it requires students to synthesize many concepts and pieces of information about functions, their graphs, and their derivatives; however, commonly used textbooks present the problems with a procedural framework that allows students to complete the activities without engaging in the deep, critical thinking we hope for. The activity presented is designed to get students talking to each other and working collaboratively to synthesize given information, seek missing information, and sketch the curve of an unspecified function. While engaged in the activity, students need to talk about the relationships between concavity and asymptotes, continuity and differentiability, first and second derivatives, and extrema and roots. The presenter will discuss variations on the activity and provide the audience with a template to take back to their classrooms. (Received September 20, 2016)

1125-VR-2672 **Mary Vlastnik Armon\*** ([marmon@knox.edu](mailto:marmon@knox.edu)), Knox College Box 1, 2 E. South St., Galesburg, IL 61401. *An Oral Final Exam in a Distance Applied Calculus Course.*

The Associated Colleges of the Midwest (ACM) have, over the past few summers, offered a distance course in applied calculus for students at its member schools. The course has from its inception included virtual small-group tutorial sessions; the 2016 implementation added a virtual oral exam as a portion of the final exam. We will discuss the course as a whole; the decision-making process that led to the adoption of an oral final; and the construction, implementation, and assessment of the final exam. (Received September 20, 2016)

- 1125-VR-2704 **Terry L. Barron\*** (tbarron@ggc.edu). *A Guide for Understanding and Achievement: Using Developmental Counseling as a Tool to Provide Effective Communication for Calculus Students Learning in a Hybrid Format.*

Teaching calculus in a hybrid format is challenging. Time constraints, the pace of content delivery and the expectation of providing timely and effective feedback are all stressors in the hybrid design. Informative and effective communication is paramount to provide students with the critical feedback necessary to be successful in the hybrid environment. Developmental counseling combines feedback based on interdependent pedagogical benchmarks such as mastery of the content, overall progress and inclusive performance. This study compared hybrid calculus classes that used development counseling with hybrid classes that did not use developmental counseling nor specifically any of its components. Results indicate that developmental counseling significantly improves the D-F-W rate for hybrid calculus classes. Conclusions indicate that further research is necessary to study the positive impact of developmental counseling across all mathematics courses and throughout all instructional delivery methods. (Received September 20, 2016)

- 1125-VR-2712 **Houssein El Turkey** and **Salam Turki\*** (sturki@eric.edu). *Implementation of Pre and Post Class Readings in Calculus.* Preliminary report.

Active learning practices highly depend on students' preparation for class in advance. However, reading Calculus can be a challenging task to students. We address this concern by assigning targeted pre-class readings and reading quizzes in two Calculus II classes. To study the effectiveness of these, we also provided them as post-class readings in two other classes. We report on our implementation and we discuss students' feedback about the readings and quizzes. (Received September 20, 2016)

- 1125-VR-2723 **Amanda M. Akin\*** (aakin000@leeu.edu), LU Campus Box #1137, 1120 N Ocoee Street, Cleveland, TN 37311, and **Allison B. Bernhard** (abernh00@leeu.edu), LU Campus Box #0441, 1120 N Ocoee Street, Cleveland, TN 37311. *How Do First Year Calculus Students' Proof Schemes Change Over the Course of a Semester?*

Learning mathematics involves a great development of the mind- a shift from memorization and simple computation to deductive and analytical reasoning. Many students will encounter a calculus course before or during their college career, and it is seen as a fundamental course in their development within the field of mathematics. It would be appropriate to view Calculus I as a course in which students develop intellectually as well as analytically. The purpose of this research is to study this supposed evolution of intellectual need and the development of analytical reasoning in mathematics students, namely first-year calculus students. Through questionnaires and interviews throughout the learning process, we map this growth of intellectual need and mathematical reasoning in order to observe and understand the development that is vital in this foundational course. (Received September 20, 2016)

- 1125-VR-2724 **Allison M. Wolf\*** (awolf9@utk.edu). *Using Low-Stakes Writing to Promote Engaged Learning.* Preliminary report.

Teaching calculus often demands finding a delicate balance between supporting students who have gaps in their background skills and encouraging interaction with the material. Full integration of semi-structured, low-stakes writing assignments can help you shape your course, allowing an environment where students feel supported, start taking intellectual risks, and build the skills expected of more mature students while learning the material. We present here an easy-to-implement vehicle for incorporating writing to support student learning in the mathematics classroom, along with preliminary results. (Received September 20, 2016)

- 1125-VR-2999 **Nela Lakos\*** (lakos.1@osu.edu), 231 West 18 th Ave, Dept of Math OSU, Columbus, OH 43210. *Challenges and Benefits of Tight Coordination of Calculus 1 at OSU.*

Every year more than 4000 students take Calculus 1 at OSU. Coordination was introduced in order to provide a degree of uniformity to this operation that involves more than 40 Lecturers and Recitation Instructors. Several pilots have been introduced (Flip&Flexible, Active Learning, Ximera- using OSU's open-source text book) and common assessments, together with surveys, provide rich source of data for measurement and comparison of effectiveness of different styles of teaching and setups. Common assessments include in-class quizzes, written homework, online homework, and common exams that are group- graded using the common grading guide. Recitation worksheets have been developed and used in all sections for students to work on either in groups or individually during the Recitation class and to complete at home. Email is the main mode of communication between the Coordinator, Lecturers and Recitation Instructors. In addition, biweekly Lecturers - Coordinator meetings are held where important issues are discussed and decisions made. Most Recitation Instructors are first

year graduate students with no prior teaching experience, and this tightly coordinated, structured class serves as valuable first teaching experience. (Received September 20, 2016)

## *General Session on Teaching and Learning Advanced Mathematics*

1125-VS-797 **Samer S. Habre\*** ([shabre@lau.edu.lb](mailto:shabre@lau.edu.lb)), Lebanese American University, P.O. Box 13-5053, Beirut, Chouran 1102 2801, Lebanon. *A Writing Assignment in a Complex Analysis Course.*

The use of writing as a learning activity in mathematics has witnessed popularity among educators lately. While introductory courses (such as calculus and differential equations to name a few) are benefiting the most from the spread of this active learning strategy, writing may be promoted in advanced and abstract mathematics as well. While such courses are often taught using the lecture format, writing assignments be it short essays, journal or expository writing, provide an avenue for students to develop their own thoughts and perspectives on the mathematics they are studying. Teachers also benefit from such activities as they receive a wealth of information about their students and about the course enabling them to improve their teaching. In this presentation, I report on a writing assignment in a first course in complex analysis. The goal of this project was to highlight the similarities and differences between key concepts that are explored in real and complex analysis. (Received September 12, 2016)

1125-VS-884 **Javad Namazi\*** ([namazi@fd.edu](mailto:namazi@fd.edu)), FDU, 285 Madison avenue, Madison, NJ 07930. *Essential statistics for mathematics majors.* Preliminary report.

Designing a one-semester statistics course for mathematics and computer science majors who have not been exposed to probability before can be a challenge since the instructor has to sift through the high volume of pre-requisite probability theory to pick the most essential items in order to most rapidly get to the main goal of covering enough of basic statistics. We discuss some strategies and challenges. (Received September 12, 2016)

1125-VS-2160 **Blain Anthony Patterson\*** ([bapatte3@ncsu.edu](mailto:bapatte3@ncsu.edu)). *Cryptography: Decoding Student Learning.*

Cryptography is typically used as a means of keep information safe. However, cryptography can also be used as a powerful teaching tool, as it puts mathematics in a dramatic and realistic setting. It also allows for a natural way to introduce topics such as modular arithmetic, matrix operations, and elementary group theory. The purpose of this presentation is to analyze student work on problems from the domain of cryptography using the SOLO taxonomy via tasked based interviews. Students interviewed were assessed on how they made connections to various areas of mathematics through solving cryptography problems. Analyzing these interviews showed that students have a strong foundation in number theoretic concepts such as divisibility and modular arithmetic. Although cryptography should not be a required course by mathematics majors, concepts from this field could be introduced in courses such as linear algebra, abstract algebra, number theory, and discrete mathematics. (Received September 19, 2016)

1125-VS-2167 **Toke L Knudsen\*** ([toke.knudsen@oneonta.edu](mailto:toke.knudsen@oneonta.edu)), Dept. of Mathematics, Comp. Sci., and Stat, SUNY Oneonta, 108 Ravine Parkway, Oneonta, NY 13820, and **Keith M Jones** ([keith.jones@oneonta.edu](mailto:keith.jones@oneonta.edu)). *Medieval India's Solution to the Pell Equation as a Classroom Project.*

As part of the successful NSF grant TRIUMPHS (Transforming Instruction in Undergraduate Mathematics via Primary Historical Sources), we contributed a module as project writers. The module, entitled, *The Pell equation in Indian Mathematics*, focuses on how to solve quadratic indeterminate equations in integers by examining original sources on how this was done in medieval India. The talk will outline the work on building the module as well as a report on how the module was tested in a Number Theory at SUNY Oneonta in Fall 2016. (Received September 19, 2016)

1125-VS-2248 **Hwa Young Lee\***, [hylee00@uga.edu](mailto:hylee00@uga.edu), and **Hamilton Lee Hardison.** *Foregrounding the Background: Two Uses of Coordinate Systems.*

Coordinate systems, such as the conventional Cartesian coordinate plane, are used as representational tools in various domains of mathematics such as algebra, geometry, and statistics. Perhaps because of their prevalence in the presentation, learning, and teaching of mathematics, often times coordinate systems are taken for granted. In this report we will present a distinction between two uses of coordinate systems—spatial coordination and

quantitative coordination—and discuss the educational implications that follow from this distinction. We will illustrate this distinction with examples from an analysis of algebra, geometry, and statistics textbooks. By foregrounding differences in the ways coordinate systems are used in mathematics, we provide a tool for examining students' difficulties in understanding representations of quantitative relationships or geometrical objects in coordinate systems. Additionally, we encourage mathematics educators to attend to these different uses of coordinate systems in order to support students' balanced understanding of both uses. (Received September 20, 2016)

1125-VS-2438     **Josip Derado\*** ([jderado@kennesaw.edu](mailto:jderado@kennesaw.edu)), Department of Mathematics, Kennesaw State University, 1100 South Marietta Pkwy, Marietta, GA 30060, and **Mary L Garner** ([mgarner@kennesaw.edu](mailto:mgarner@kennesaw.edu)). *Point Reward System (PRS) - A New (R)evolutionary Learning Assessment Method*. Preliminary report.

The Point Reward System (PRS) is a new learning assessment method designed to deal with a diversity of students' abilities and interest. PRS is an assessment method that facilitates learning by flexibly accommodating students' individual needs and yet it is easy and practical to implement in the classroom. It has been shown that PRS increases retention rates, retains students' interest in the material throughout the semester and build students' confidence in their abilities to deal with complex topics. In this talk, we will present the results of a comparison between PRS and a commonly used assessment method. In particular, we will discuss differences in anxiety levels (we compare exam stress levels as well as overall stress levels caused by grade pressure), students involvement in course (measured by their class participation and class attendance), and retention of learned material (analyzed using longitudinal study of students individual learning progress). We also discuss the effect PRS has on instructors and instructors' ability to tailor the course to better serve the students. (Received September 20, 2016)

1125-VS-2466     **Tessa F Weinstein\*** ([weinsteint@morningside.edu](mailto:weinsteint@morningside.edu)), Department of Mathematical Sciences, 1501 Morningside Ave, Sioux City, IA 51106. *Promoting Metacognition in an Over-easy Geometry Classroom*.

A precise definition of mathematical maturity is elusive at best, and yet, as mathematics educators, a quality we all wish to inculcate in our students. While mathematical maturity is difficult to quantify and possibly impossible to teach, we can develop the habits of a mathematically mature mind by explicitly promoting metacognition. When presented with a novel problem, knowledge of the prerequisite skills is necessary but not sufficient for success. An individual must be able to orchestrate various cognitive components to be a successful problem solver. This talk examines an effort to promote healthy metacognition, bolster academic challenge, and increase time-on-task in an over-easy style Geometry classroom at a small liberal arts college. (Received September 20, 2016)

1125-VS-2519     **Elizabeth Niese\*** ([niese@marshall.edu](mailto:niese@marshall.edu)), Department of Mathematics, Marshall University, One John Marshall Drive, Huntington, WV 25755. *Teaching research skills in undergraduate mathematics courses*.

In mathematics it is not uncommon for students to have their first research experience while completing their senior capstone course. In this presentation, I discuss some of the research skills that students need and ways to encourage the development of research skills in earlier coursework. These include: inquiry, problem-solving, written communication, oral communication, reading mathematics, and proof-writing. Specific assignment/project types for courses ranging from College Algebra to Abstract Algebra will be discussed. Most of the assignments can be implemented in a traditional classroom without significant course redesign. This project was initiated as part of the Faculty Learning Community on Pedagogy of Undergraduate Research at Marshall University. (Received September 20, 2016)

1125-VS-2760     **Randy Combs\***, Box 60787, WT Station, Canyon, TX 79015. *Inverting the Advanced Calculus and Abstract Algebra Classrooms*.

In this paper we discuss one instructor's experience in using the inverted classroom structure to teach proof based, upper level Advanced Calculus and Abstract Algebra courses. The structure of the inverted classroom model allows students to begin learning the new mathematics prior to the class meeting. By front-loading the learning of a new concept, students can use valuable class time for exploring and solving complex problems with the instructor available for guidance and assistance. This paper compares student performance in the inverted classroom with student performance in the direct instruction classroom. The researchers also share student perceptions of the inverted classroom experience.

(Received September 20, 2016)

1125-VS-3087 **Michael Obiero Oyengo\*** ([mchlyng2@illinois.edu](mailto:mchlyng2@illinois.edu)), 1409 W Green St, Urbana, IL 61801, and **David A. Stern** ([d.a.stern@reading.ac.uk](mailto:d.a.stern@reading.ac.uk)). *Closing a cycle by helping develop the next generation of African problem solvers.*

Like many bright students graduating with excellent degrees from African universities I lacked the skills needed to think independently, to solve problems, be creative and innovative. This is partly due to the culture of learning in our schools and universities. During my masters', I was lucky to work under the guidance of a visiting international professor who helped me develop these skills. The same cannot be said of most young African students who aspire to advance their studies and do meaningful research. African Institute for Mathematical Sciences (AIMS) provides a cultural shift by bringing together a diverse group of Africa's brightest students in a structured environment where they can learn from the best professors around the world, and from themselves. Mathematical Problem Solving is one of the skills courses taught at AIMS, and is tailored to meet the deficiencies that most young African students have. By design, the course requires the students to find their own solutions to problems, develop their own writing and presentation skills. We will discuss our experiences in teaching this course at AIMS Tanzania in the fall of 2016. We will also discuss the broader vision of AIMS in nurturing the next generation of African mathematical scientists. (Received September 21, 2016)

## General Session on Topology

1125-VU-323 **Nicolas Petit\*** ([nicolas.petit@oberlin.edu](mailto:nicolas.petit@oberlin.edu)), 10 N Professor St, King Building 205, Oberlin, OH 44074. *Finite-type invariants for virtual knots.*

We will go over some past and recent results on some finite-type invariants of order one (according to the Kauffman definition) for virtual knots, with special focus on the categories of framed virtual knots and long virtual knots, both framed and unframed. (Received August 26, 2016)

1125-VU-595 **Lyll Reid\*** ([lyall.reid@ucf.edu](mailto:lyall.reid@ucf.edu)). *Lattice-Valued Convergence Spaces.* Preliminary report.

Let  $L$  be a complete lattice satisfying an infinite distributive property (Frame) with top element  $\top$ . Then the  $\top$ -filters on a set  $X$  can be used to define a  $\top$ -convergence space. Properties of this category are investigated and its relationship to the category whose objects consist of all stratified  $L$ -convergence spaces is given. (Received September 07, 2016)

1125-VU-946 **Adu A Nathaniel\*** ([nadu@knights.ucf.edu](mailto:nadu@knights.ucf.edu)), Department of Mathematics, University of Central Florida, Orlando, FL 32816. *Strongly Symmetric Compactifications.* Preliminary report.

Convergence approach spaces possess both quantitative and topological properties. These spaces are equipped with a structure which provides information as to whether or not a sequence or filter approximately converges. Compactification of objects in a certain subcategory are studied here. (Received September 13, 2016)

1125-VU-1298 **Irma E. Stevens\*** ([istevens@uga.edu](mailto:istevens@uga.edu)), 110 Carlton St., University of Georgia, Athens, GA 30602. *Topological Data Analysis of Students' Responses to MAA Surveys on College Calculus.*

Topological data analysis (TDA) is a powerful method that can be less sensitive to noise than its competitors (e.g., PCA or HCA) and can handle large data sets. The method has been used to study a broad range of data sets taken from FDG-PET scans to fingerprints. This study focuses on the data set from the MAA national study of college calculus starting in 2012. A subset of the data was chosen in order to focus on the 797 participants in the study who completed both the pre- and post-course Likert scale surveys which included topics such as high school preparation for calculus, attitudes towards mathematics, and students' thoughts on the structure and instruction of their calculus course. The data was explored in three different ways: pre-/post-variables, two variables and a normalized frequency variable, and various variables of interest based on previously reported statistical analysis of the data set. Using the Rips method of creating simplicial complexes on a combination of the aforementioned variables, the persistent homologies of up to the second homology group were constructed using persistence diagrams. I will discuss the persistence diagrams and interpretations of them based on the 3D plots of combinations of three variables. (Received September 16, 2016)

1125-VU-1484 **Jocelyn R Bell\***, [bell@hws.edu](mailto:bell@hws.edu). *The proximal infinite game.*

The proximal infinite game is a two player infinite game played in a uniform space. The first player chooses elements of the uniformity while the second player selects points in the space. In this talk we will illustrate the

rules of this game as well as discuss some topological results resulting from the existence of a winning strategy for the first player in this game. (Received September 17, 2016)

1125-VU-1704 **Allison Henrich, Elsa Magness and Kayla Perez\*** ([perkay05@evergreen.edu](mailto:perkay05@evergreen.edu)), The Evergreen State College, QuaSR Center, 2700 Evergreen Parkway NW, Olympia, WA 98505, and **Briana Zimmer**. *Knot Fertility and Lineage*. Preliminary report.

Is your favorite knot fertile? We define a knot  $K$  to be a parent knot of a knot  $H$  if some number of crossings in a minimal crossing projection of  $K$  can be resolved to produce a diagram of  $H$ . We say that  $K$  is fertile if it is a parent knot of every knot with a smaller crossing number than itself. In this talk, we will explore families of knots and their relative fertility. We also explore ways to find the trefoil in every knot. (Received September 18, 2016)

1125-VU-1726 **Safia Chettih\***, [safia@reed.edu](mailto:safia@reed.edu). *Topology of Non-k-Equal Configurations on Graphs*.

Configuration spaces of  $n$  points on a graph, where no two points are equal, have homology that is well-known in the unordered case, while they elude a general combinatorial description. Recently, there has been new interest in configurations of  $n$  points where no  $k$  points are equal, otherwise known as non- $k$ -equal configurations. In this talk, we will present new results which give a discretized model for non- $k$ -equal configuration spaces on graphs, and explain the implications for the combinatorial and geometric structures interrelating configurations on graphs. (Received September 19, 2016)

1125-VU-1797 **Mehmet Emin Aktas\*** ([maktas@math.fsu.edu](mailto:maktas@math.fsu.edu)), 208 Love Building, 1017 Academic Way, Tallahassee, FL 32306. *Deformations in Dessin D'enfants of Trigonal Curves*.

Dessin D'enfants, which are bipartite ribbon graphs, have been used for studying the topology of curve complements in recent years. In this paper, we define a new equivalence relation for Dessin D'enfants and classify them up to total degree 5. We also show the deformation spaces of Dessin D'enfants for each class. Both the classification and the deformation have an important role in understanding topology of corresponding curves. (Received September 19, 2016)

1125-VU-1877 **Shiva Shankar Rai\***, Department of Mathematics and Statistics, Broadway and Boston, Lubbock, TX 79409-1042. *Pseudo-Endpoints of a nondegenerate Chainable Continua*. Preliminary report.

A point  $x$  in a nondegenerate chainable continuum  $X$  is said to be a *Pseudo-endpoint* if for each neighborhood  $U$  of  $x$  and  $\forall \epsilon > 0$  there is an  $\epsilon$ -chain  $\mathcal{C}$  covering  $X$ , such that  $\mathcal{C}_1 \subset U$ . This property was originally pointed out by R. H. Bing(1951). We construct nondegenerate chainable continua with  $m$ , and  $n$  number of endpoints and pseudo-endpoints respectively, where  $m < n$  and  $m$  and  $n$  are nonnegative integers. We further study several interconnections between pseudo-endpoint(s) and endpoint(s) of a nondegenerate chainable continua. (Received September 19, 2016)

1125-VU-2227 **David Sumner Lipham\*** ([ds10003@auburn.edu](mailto:ds10003@auburn.edu)), David Lipham, Department of Mathematics and Statistics, Auburn University, Auburn, AL 36849. *Widely-connected sets in the bucket-handle continuum*.

A connected space  $X$  is said to be widely-connected if every non-degenerate connected subset of  $X$  is dense in  $X$ . The object of this talk is the construction of widely-connected subsets of the Knaster bucket-handle continuum. We answer a question of Paul Erdős and Howard Cook by constructing a completely metrizable example. Then, we discuss some open problems which are motivated by the question: "How closely related are widely-connected sets and indecomposable continua?" (Received September 21, 2016)

1125-VU-2563 **Marian Anton and Landon Renzullo\*** ([landonrenzullo@gmail.com](mailto:landonrenzullo@gmail.com)), Department of Mathematical Sciences, CCSU, New Britain, CT 06050. *On an Algorithm in Data Homology*. Preliminary report.

The study of data clouds using distance matrices and the homology of their associated Rips complexes may reveal relevant information about various projections of these complexes in lower dimensions. In this context, we present work in progress on an algorithm based on graph theory dealing efficiently with these lower dimensional projections that may reveal new topological insights into highly interconnected networks. (Received September 20, 2016)

1125-VU-2567 **E. T. Brown\*** ([brownet@jmu.edu](mailto:brownet@jmu.edu)), James Madison University, Department of Mathematics and Statistics, MSC 1911, Harrisonburg, VA 22807. *Partial Metrics and Pathological Topologies*.

Partial metrics are a generalization of metrics which allow for non-zero self-distances. They arise naturally in computing, making them a comparatively intuitive topic for undergraduates. Topologies generated by partial metrics are  $T_0$  but do not as a rule satisfy other separation axioms.

In this talk we will discuss some pathological topologies which arise naturally in the context of partial metrics, and comment on the opportunities these provide for undergraduate exploration and/or research. (Received September 20, 2016)

1125-VU-2616 **Luke Morgan Williams\*** ([lukewilliams@math.ksu.edu](mailto:lukewilliams@math.ksu.edu)). *Infinite Families of Non-Stein Rational Balls*. Preliminary report.

A standard obstruction to a 4-manifold being realized as a Stein domain relies on the adjunction inequality. This obstruction fails to apply directly in the case when the manifold in question is a rational homology ball. In spite of this, I will demonstrate how this obstruction can be modified to produce infinite families of irreducible rational balls that do not support a Stein structure (in either orientation). (Received September 20, 2016)

1125-VU-2617 **Ryan J Jensen\*** ([jensen@math.utk.edu](mailto:jensen@math.utk.edu)), University of Tennessee, Knoxville, TN 37996. *Localization of Coarse Structures*. Preliminary report.

A localization of a coarse structure is a new method of creating a coarse structure from an existing one. We investigate which coarse properties are preserved through localization. (Received September 20, 2016)

1125-VU-2640 **John E Mosley\*** ([jmosley@exeter.edu](mailto:jmosley@exeter.edu)). *In Search of Class Representatives for  $SU$ -Cobordism*.

Interest in the question of preferred class representation of cobordism classes has existed since the study of cobordism began in the 1950's. A classic result of John Milnor states that every stably almost complex cobordism class contains a nonsingular algebraic variety. That is, if we're interested in a complex cobordism class, we can find a nonsingular algebraic variety to represent it. In this talk we will consider the ring of special unitary cobordism, and show that the analogous result does not hold. (Received September 20, 2016)

1125-VU-2660 **Cary Malkiewich\*** ([cmalkiew@illinois.edu](mailto:cmalkiew@illinois.edu)). *The Reidemeister trace in pictures*.

We give a brief pictorial exposition of joint work with John Lind, in which we build a geometric model for the "refined Reidemeister trace." When  $E \rightarrow B$  is a smooth fiber bundle, this gives a stable map between the spaces of free loops  $LB \rightarrow LE$ . If time permits, we will sketch the generalizations and their applications, which range from dynamics on compact manifolds to algebraic K-theory. (Received September 20, 2016)

1125-VU-2776 **Qayum Khan and Gerrit Smith\*** ([gsmith35@slu.edu](mailto:gsmith35@slu.edu)), 220 N. Grand Blvd., St. Louis, MO 63103. *Realizing Incompressible 3-Manifolds in Stable 4-Manifolds*. Preliminary report.

Consider a separating 3-dimensional submanifold of a smooth 4-dimensional manifold whose fundamental group is mapped injectively into the fundamental group of the 4-manifold. The Seifert-van Kampen theorem implies that the fundamental group of the 4-manifold has an injective amalgamated product structure. This work attempts to characterize when the converse holds, allowing for stabilization of the 4-manifold by forming the connected sum with  $S^2 \times S^2$  factors.

In the case that the universal cover of the 4-manifold is not spin, there is a single algebraic obstruction in the 3rd homology group of the fundamental group of the 3-manifold. In the case that the 4-manifold is spin, there are potentially two additional obstructions. (Received September 20, 2016)

1125-VU-2947 **R. M. Argus\*** ([rargus@masonlive.gmu.edu](mailto:rargus@masonlive.gmu.edu)), 8256 Vernon Street, Manassas, VA 20109. *Generalized Erdos-Type Spaces*. Preliminary report.

Erdos space is defined as the set  $E$  of all points in the Hilbert space  $\ell^2$  of square summable sequences which have rational coordinates. and is a canonical example of a totally disconnected topological space which is not zero-dimensional. In this talk, we discuss these and other properties of  $E$ , including those of  $E$  as a subspace of  $\ell^p$  for  $p > 2$ , and find minimal Erdos-type spaces. (Received September 21, 2016)

1125-VU-2958 **Ashley Berger\*** ([berger@ou.edu](mailto:berger@ou.edu)), 601 Elm Ave, Norman, OK 73019, and **sepideh Stewart** ([ssewart@math.ou.edu](mailto:ssewart@math.ou.edu)), 601 Elm Ave, Norman, OK 73019. *Schema as a theoretical framework*. Preliminary report.

In this talk we present a theoretical framework based on Skemp's idea of schema, in order to investigate undergraduate students' understanding of Topology. According to Skemp (1971), concepts are embedded in a

hierarchical structure of other concepts, these levels in the structure being classifications of concepts. As the concepts are paired together, relations between them as well as classifications are also possible. The complexity of this hierarchical structure comes from the fact that these classifications of concepts and relations are not unique, giving way to multiple hierarchical structures, which can be interrelated. When components of these conceptual structures come together to make a structure that would not be realized by only looking at the individual components, the resulting structure is called a schema. A schema integrates existing knowledge, serves as a tool for future learning, and allows for understanding to take place. Without a suitable schema in position, students will have difficulty in understanding or making sense of new concepts. The proposed framework will promote schematic learning and seek to identify whether the presence or absence of a certain schema will have an effect in understanding new knowledge. (Received September 20, 2016)

1125-VU-2985 **Ik Jae Lee\***, Department of Mathematics, Rowan University, 201 Mullica Hill Rd., Glassboro, NJ 08028, and **Hieu D Nguyen**, Department of Mathematics, Rowan University, 201 Mullica Hill Rd., Glassboro, NJ 08028. *Alternating Minimum Braids and Caterpillar Graphs*. Preliminary report.

In this talk, we prove a correspondence between alternating minimum braids and caterpillar graphs. (Received September 20, 2016)

1125-VU-3051 **Colin C Adams** ([cadams@williams.edu](mailto:cadams@williams.edu)) and **Gregory Kehne\*** ([gregorykehne@gmail.com](mailto:gregorykehne@gmail.com)). *Decompositions of multi-crossing link complements into bipyramids*.

Knots and links are typically studied in 2-crossing projections, and given such a projection of a link  $L$  a dual pair of decompositions of the complement manifold  $S^3 \setminus L$ —into octahedra at the crossings and into bipyramids in the faces—are known. Generalizing these constructions, we present a dual pair of decompositions of  $S^3 \setminus L$  into bipyramids, given any multi-crossing projection of  $L$ . When  $L$  is a hyperbolic link, these decompositions give new upper bounds on  $\text{vol}(S^3 \setminus L)$  given its multi-crossing projection. These bounds are in fact realized by three closely related infinite planar tiling weaves: the square weave, the triple weave, and the right triangle weave. (Received September 20, 2016)

1125-VU-3099 **Juan S. Villeta-Garcia\*** ([villeta2@illinois.edu](mailto:villeta2@illinois.edu)). *The Hungarian Horntail (THH) and Other Mathematical Beasts*. Preliminary report.

Algebraic K-Theory is often thought of as “the” universal additive invariant of rings. This magical functor has proven very difficult to compute (we don’t even know it for the integers!). One crack at this golden egg has been to study its “first derivative”, Topological Hochschild Homology (THH), and reconstruct invariants from it (Witt vectors, Topological Cyclic Homology, etc.). We describe a procedure, inspired by Waldhausen’s additivity theorem, that recovers some of these mathematical monstrosities, and ties in to work of Lindenstrauss and McCarthy on the Taylor tower of Algebraic K-Theory. (Received September 21, 2016)

1125-VU-3117 **Christine Caples\*** ([christine-caples@uiowa.edu](mailto:christine-caples@uiowa.edu)). *Classifying Tangles Using Invariants*. Preliminary report.

A knot can be thought of as a knotted piece of string with the ends glued together. A tangle is formed by intersecting a knot with a 3-dimensional ball. The portion of the knot in the interior of the ball along with the fixed intersection points on the surface of the ball form the tangle. Tangles can be used to model protein-DNA binding, so another way to think of a tangle is in terms of segments of DNA (the strings) bounded by the protein complex (the 3-dimensional ball). Like knots, the same tangle can be represented by multiple diagrams which are equivalent under deformations (no cutting or gluing allowed). A tangle invariant is a value that is the same for equivalent tangles, thus an invariant can be used to tell us when two tangles are decidedly different. Tangles can be classified into families which allows one to compute invariants more quickly as well as study properties of tangles that may be useful for solving tangle equations. (Received September 21, 2016)

## General Session on Assorted Topics

1125-VW-171 **Randall E. Cone\*** ([recone@salisbury.edu](mailto:recone@salisbury.edu)), 128 Henson Hall, Salisbury University, Salisbury, MD 21801. *Fight the Powers that Be: A Reflection on the Future of Our Professional Societies*.

Memberships in professional associations around the world are in decline; mathematics societies are no exception. Given the ease of communication and collaboration in today’s world, professionals are forming their own networks,

independent of formalized structures associated with such societies. Yet, instead of moving to dynamic shared-governance models, some society administrations are retreating deeper into exclusive oligarchical structures.

Only recently have some of these societies begun to understand the need for wider inclusiveness, diversity, and social sensitivity, and only after enough noise was made by the general membership. Cultural walls and prejudicial barriers continue to exist between mathematics societies, and they are carefully maintained – if only for some old and exhausted form of posterity, or as a self-imposed apartheid. Is this socially just? Do we want to encourage our students to join such societies? If our professional societies are to survive the digital age, they must join in common and open purpose, or rightfully perish in the dust of this new age of globalization, social interaction, and entrepreneurial professionalism. In this talk, we provide further analysis of these subjects, and how they relate to the classroom experience. (Received August 07, 2016)

1125-VW-187     **Caroline Maher Maher-Boulis\*** (cmaherboulis@leeuniversity.edu), NASCM Department, Lee University, 1120 N. Ocoee St., Cleveland, TN 37320, and **Lauren Jeneva Clark** (dr.jenevaclark@utk.edu), 7238 Austin Park Lane, Knoxville, TN 37920. *Career Contexts: How PD Can Prompt Connections in Secondary Classrooms.*

In an effort to help local teachers implement the Common Core State Standards for Mathematics in an innovative and fun way, we have offered a professional development for high school teachers in summer 2016. The workshop also included preparation of students for post-secondary education and for success in nonacademic mathematical careers. The project targeted mathematical content in Algebra, Functions and Geometry present in the integrated pathway approved by the Tennessee State Board of Education. During this talk we will give an overview of the ideas incorporated in the workshop and present the results of the pre- and post-tests conducted in the workshop. (Received August 10, 2016)

1125-VW-271     **Chris D. Lynd\*** (clynd@bloomu.edu), 229 Ben Franklin Building, 400 E. 2nd St, Bloomsburg, PA 17815. *Undergraduate Research Projects in Discrete Dynamical Systems.*

There are many open problems in this area of mathematics that are accessible to undergraduate students. This talk will focus on the behavior of sequences that are generated by recurrence relations.

We will present an undergraduate project that led to a paper on left nested radicals that was recently accepted for publication in Mathematics Magazine. We will also present some open problems and give examples of how undergraduates can contribute to research in this area.

With little mathematical background, students can analyze the sequences generated by a recurrence relation by performing computer simulations, discovering patterns, and making conjectures about the behavior of the sequences. With a background in real analysis and dynamical systems, students can prove some of their conjectures. (Received August 22, 2016)

1125-VW-422     **Cynthia J. Huffman\*** (cjhuffman@pittstate.edu), Math Dept., Pittsburg State University, 1701 S. Broadway, Pittsburg, 66762. *Digital Storytelling in a History of Mathematics Class.*

Storytelling is a fun way to grab the attention of students and to help them gain an understanding of concepts. We will discuss benefits of using storytelling in the mathematics classroom, both in general and specifically in a history of mathematics course, plus take a look at one of the free apps available for educators (Adobe Spark Video) for creating a narrated video. Spark allows instructors and students to create stunning digital stories in a matter of minutes. Also, a report on the results of using digital storytelling assignments in a History of Mathematics course will be included in the presentation. (Received September 01, 2016)

1125-VW-472     **Kale Oyedeji\*** (kale.oyedeji@morehouse.edu), Department of Physics, Morehouse College, Atlanta, GA 30314-3773, and **Ronald E. Mickens** (rmickens@cua.edu), Department of Physics, Clark Atlanta University, Atlanta, GA 30314. *Mickens Law of Cooling.*

Key Words: Newton's law of cooling, nonlinear differential equations, heat transfer

An object placed in an environment such that the two systems have different temperatures will either increase or decrease its temperature according to whether  $T_0 < T_e$  or  $T_0 > T_e$ . The widely used Newton's law of cooling can be used to determine the temperature of the object at time  $t$ , i.e.,

$$\frac{dT(t)}{dt} = -\lambda [T(t) - T_e], \quad T(0) = T_0, \quad t > 0, \quad (1)$$

where  $\lambda$  is a positive parameter. A major unwanted feature of Eq. (1) is that its solution takes an unlimited amount of time to reach the equilibrium temperature in contrast to experimental evidence that  $T_e$  is achieved

in a finite time. We discuss and generalized a resolution to this enigma proposed by Mickens (Georgia Journal of Science, Vol. 67 (2009), pp55). His formulation replaces Eq. (1) by the expression

$$\frac{d\Delta}{dt} = -\lambda [\text{sign}(\Delta)] |\Delta|^p, \quad \Delta(0) = \Delta_0.$$

where  $\Delta(t) = T(t) - T_e$  and  $0 < p < 1$ . (Received September 03, 2016)

1125-VW-486 **Torina Lewis\*** (tlewis@cau.edu), Department of Mathematical Sciences, Clark Atlanta University, Atlanta, GA 30314, and **Ronald E. Mickens** (rmickens@cua.edu), Department of Physics, Clark Atlanta University, Atlanta, GA 30314. *The Geometric Triangular Periodic Functions.*

Key Words: periodic functions

The (trigonometric) circular and (Jacobi) elliptic functions are two well-known classes of periodic functions. They are based, respectively, on geometric properties of the circle and ellipse; see, W. Schwalm, "Elliptic functions sn, cn, dn as trigonometry". We demonstrate that a similar type of analysis can be done for a triangle where vertices lie on the unit circle. The associated periodic functions are defined and labeled  $tc(\theta)$ ,  $ts(\theta)$  and  $td(\theta)$ , i.e., the triangular cosine, sine, and dine functions. We calculate their explicit representations in terms of  $\theta$ , and derive several of their fundamental properties. Finally, plots of these functions are given for one period in the variable  $\theta$ . (Received September 03, 2016)

1125-VW-679 **Ralph P Grimaldi\*** (grimaldi@rose-hulman.edu), Rose-Hulman Institute of Technology, 5500 Wabash Avenue, Terre Haute, IN 47803. *Extraordinary Subsets: A Generalization.*

For  $n$  a positive integer, a subset  $S$  of  $[n]$  is called extraordinary if  $|S|$  equals the smallest element of  $S$ . The number of such extraordinary subsets, for a given  $n$ , is counted by  $F_n$ , the  $n$ th Fibonacci number. For  $1 \leq k \leq n$ , we call a subset  $S$  of  $[n]$   $k$ -extraordinary if  $|S|$  equals the  $k$ th smallest element of  $S$ . When  $k = 1$  such a subset  $S$  is 1-extraordinary (or, simply extraordinary). If we let  $a_{n,k}$  count the number of  $k$ -extraordinary subsets of  $[n]$ , we examine how  $a_{n,k}$  is related to  $a_{n-1,k}$  and  $a_{n-2,k}$ . Further, we find that  $\sum_{k=1}^n a_{n,k} = 2^{n-1}$  and that  $\sum_{i=1}^n a_{i,k} = a_{n+2,k} - a_{n+1,k-1}$ . (Received September 09, 2016)

1125-VW-821 **Jianjun Huang\*** (jhuang@wpi.edu), Department of Mathematical Sciences, 100 Institute Rd, Worcester, MA 01609, and **Sarah D Olson** (sdo@wpi.edu), Department of Mathematical Sciences, 100 Institute Rd, Worcester, MA 01609. *Sperm movement under the effect of a wall in Stokes flow.*

We are interested in studying wall interactions of elastic rods and microorganisms such as sperm in Stokes flow. In the presence of a wall, we employ the regularized method of images that satisfies the zero flow boundary condition. A Kirchhoff rod model is used with a preferred curvature and twist. We determine that an elastic rod is still able to reach its preferred equilibrium configuration even when bounded by a wall. However, the time to reach the equilibrium and location (tilt) may change. To study sperm motility, the tail is assumed to propagate with either a planar (sinusoidal) or helical wave. We find that the swimming speed and angle of the swimmer near the wall varies based on the material properties of the swimmer, preferred flagellar bending, as well as the plane that the swimmer is initialized in relative to the wall. (Received September 12, 2016)

1125-VW-851 **John Tyler Risher\*** (rishtj@email.sc.edu), 807 Hampton St., Walterboro, SC 29488. *Two Inequalities Involving AM, GM, and HM.*

Let  $A_{a_j}$ ,  $G_{a_j}$ , and  $H_{a_j}$  indicate the arithmetic mean, geometric mean, and harmonic mean of sequence  $\{(a_j)_i\}_{i=1}^n$  respectively. We first prove that  $G_{a_1} + \dots + G_{a_m} \leq G_{a_1+\dots+a_m}$ , and  $H_{a_1} + \dots + H_{a_m} \leq H_{a_1+\dots+a_m}$ , where  $G_{a_1+\dots+a_m}$  and  $H_{a_1+\dots+a_m}$  are the geometric mean and the harmonic mean of the sequence  $\{(a_1)_i + \dots + (a_m)_i\}_{i=1}^n$ . Applying these two inequalities, we then generalize two results introduced by Lai and Kim, involving the reciprocals of  $\sum A_{a_j}$ ,  $\sum G_{a_j}$ , and  $\sum H_{a_j}$ . (Received September 12, 2016)

1125-VW-1190 **Ricardo E Rojas\*** (ricardo.rojas@northern.edu), 1200 South Jay Street, Aberdeen, SD 57401. *Factoring Quadratics: The Bijection That Lies Beneath.*

We can factor a quadratic polynomial with integer coefficients if and only if it has a discriminant that is a perfect square; furthermore, if such a factorization exists, then it is, essentially, unique. This leads us to a bijection from ordered pairs of linear polynomials with integer coefficients to quadratic polynomials with integer coefficients that we can factor. This bijection is multiplication; its inverse is factorization. To my knowledge, this inverse bijection of factorization has never been explicitly written down and presented as the function that it is; this presentation, aimed at both math education majors and 7-12 math teachers, shall do just that. (Received September 15, 2016)

- 1125-VW-1195 **David Richeson\*** (richesod@dickinson.edu), Department of Mathematics and Comp Science, Dickinson College, Carlisle, PA 17103. *Sugihara's Impossible Cylinder Illusion*. Last spring Kokichi Sugihara created a YouTube video called *Ambiguous Optical Illusion* that shows a variety of three-dimensional objects that look like one shape when viewed from the front but like a different shape in a mirror propped up behind it. The video went viral and was viewed 5 million times in the first week. In this talk we show the mathematics behind this illusion—a straightforward application of vectors, lines, and parametric curves in three-dimensional space. We also created a paper template that you can use to make your own impossible cylinder and wow your friends. (Received September 15, 2016)
- 1125-VW-1427 **Stephanie Loewen** (loewens@mail.gvsu.edu) and **Tristan Wells\*** (trwells22@ksu.edu). *Outer Billiards, Fuchsian Groups, and Fundamental Regions*. Outer (Dual) polygonal billiards is a simple plane based dynamical system on a convex polygon. A discrete subgroup of isometries of the hyperbolic plane is called a Fuchsian group if it consists of orientation-preserving transformations. Any Fuchsian group possesses connected, convex fundamental regions. In this project we would like to explore fundamental regions of the outer billiard map defined with regular tilings and possibly quasiregular tilings in the hyperbolic plane. This research was conducted as part of the 2016 REU program at GVSU. (Received September 16, 2016)
- 1125-VW-1440 **Steven Gottlieb\*** (sgottlieb60@hotmail.com), PO Box 133, Mt Marion, NY 12456, and **Chris McCarthy** (mccarthybmcc@yahoo.com), BMCC/CUNY, Department of Mathematics, 199 Chambers St, New York, NY 10007. *When do we get erroneous roots?* Preliminary report. Given the equation  $\sqrt{x+a} = bx+c$ , precisely when do we expect to get two, one or zero solutions and when do we get erroneous solutions? (Received September 16, 2016)
- 1125-VW-1496 **Leandro Junes\*** (junes@calu.edu), 1053 Pennsylvania Avenue, California, PA 15419, and **Rigoberto Florez** and **Eva Czabarka**. *Enumerations on Non-decreasing Dyck Paths*. A non-decreasing Dyck path of length  $2n$  is a diagonal lattice path from  $(0, 0)$  to  $(2n, 0)$ , consisting of  $n$  up-steps (along the vector  $(1, 1)$ ) and  $n$  down-steps (along the vector  $(1, -1)$ ), such that the path never goes below the  $x$ -axis and the sequence of its local minima forms a non-decreasing sequence. We provide several statistics on non-decreasing Dyck paths related with peaks, valleys, and pyramids. In particular, we provide close formulas for peaks, pyramid weights, and indexed sums of pyramid weights for all non-decreasing Dyck paths of length  $2n$ . We also show that an indexed sum on pyramid weights depends only on the size and maximum element of the indexing set. (Received September 17, 2016)
- 1125-VW-1531 **Emelie A Kenney\*** (kenney@siena.edu). *Mathematics Education Research Abroad: Observations from ICME-13*. Learning mathematics at the college level assumes students possess a modicum of preparation before they arrive on campus, and were the beneficiaries of well-schooled, sophisticated educators who sought to implement best practices. Nevertheless, in many colleges, we find that student preparation is not optimal. Further, despite the emphasis in the United States on standardized testing, the U.S. still does not appear at the top of any worldwide lists for excellence in mathematics scores. In the most recent Programme for International Student Assessment (PISA) study, the United States is listed in 36th place among 65 participating entities and 27th among 34 Organisation for Economic Cooperation and Development (OECD) countries. What are other nations doing differently? Does culture play a role in the outstanding performance of Asian students? How is the European didactic tradition helping students of European nations learn mathematics? Here, I report on some results of comparative country studies presented at the 13th International Congress on Mathematical Education (ICME-13), and describe what aspects of international mathematical education research can be transported to the United States for the benefit of collegiate mathematics students and prospective teachers of mathematics. (Received September 17, 2016)
- 1125-VW-1697 **Susanna Molitoris-Miller\***, Kennesaw State University, Department of Mathematics, Clendenin Building Rm 3039 (MD1102), Kennesaw, GA 30144, and **Brian Kronenthal** and **Jathan Austin**. *The Settlers of "Catanbinatorics"*. One of the distinguishing features of The Settlers of Catan board game is the dynamic game board which is reconstructed for a new board every time you play. With nineteen resource tiles, eighteen number chips, and nine ports to arrange the possibilities seem endless. In this session we use combinatorial techniques to count the number of possible boards as well as explore other interesting combinatorial problems inspired by the board assembly (Received September 18, 2016)

1125-VW-1763 **Colm Mulcahy\*** (colm@spelman.edu). *Irish mathematicians in American mathematics—a historical perspective*. Preliminary report.

From Canada to Costa Rica and Honolulu to Houston, and stretching back in over two centuries in time, Irish mathematicians have steadily shown up in various parts of the Americas to study and to work. Some have had real impact, in publishing, academia, and elsewhere.

We'll survey several of the more notable, including Robert Adrain, who beat Gauss to the punch with the method of least squares, and department builders and shapers such as James McMahan (Cornell), Frank Murnahan (Johns Hopkins) and John Synge (Univ of Toronto). We'll also shine a light on people who played other significant roles, such as one who secured the preservation of Oberwolfach and another whose statistical analysis contributed to a US Supreme Court decision. (Received September 19, 2016)

1125-VW-1892 **Mohamed Allali\*** (allali@chapman.edu). *Principal Component Analysis in Image Processing*.

Digital image processing is becoming a strong application area of modern mathematics. In this talk, I will show, through practical examples, how principal component analysis applied to digital images can be incorporated into many mathematics courses. This approach makes the courses more visual and interesting for instructors and students. (Received September 19, 2016)

1125-VW-1931 **Xueyi Lei\*** (leix@jay.washjeff.edu), 50 South Lincoln Street, Washington, PA 15301. *Constructing a matroid from a finite group*.

One can take two separate paths from a finite group to a matroid. We first describe the direct construction of a matroid from a finite group. We then discuss the cycle matroid of a Cayley graph of a group. Using small examples, we answer the obvious question: Are the two resulting matroids always the same? (Received September 19, 2016)

1125-VW-1988 **R. Daniel Hurwitz\*** (dhurwitz@skidmore.edu), Department of Mathematics, Skidmore College, 815 North Broadway, Saratoga Springs, NY 12866-1632. *Subtraction Squares*. Preliminary report.

The title refers to the following iterated process: taking four numbers and regarding them as labels of a square, let the absolute values of the differences between adjacent labels create a new set of labels for a square. Repeat the procedure. Using positive integers, this is sometimes used as a mechanism to practice subtraction in elementary schools. But there are numerous interesting mathematical questions which arise from the process. Among them are: Do the squares "stabilize"? If so, after how many iterations? If not, for which type of initial labels? And what about "subtraction n-gons" for  $n$  greater than four? We will consider these and perhaps other issues. (Received September 19, 2016)

1125-VW-2008 **Annie Han\*** (yhan@bmcc.cuny.edu), 199 Chambers Street., N599M, New York, NY 10007, and **Fangyang Shen**. *Four-Movement Classical Symphony: Mentoring Pre-Service Teachers Through IBL Model*. Preliminary report.

For over thirty years, our nation has expressed concern about the lack of competitiveness of U.S. students in the areas of science, technology, engineering, and mathematics (STEM). Although national and local policies have emphasized STEM education, progress has been slow. Without a qualified pool of teachers who have degrees in STEM fields, we continue the cycle of unprepared STEM students taught by underprepared teachers. STEM teachers with weak backgrounds simply do not promote passion and commitment in students to pursue STEM careers. A major component of increasing students' STEM achievement is raising the quantity and quality of STEM teachers. The New York City Tech and BMCC Noyce NEST Project focuses on these concerns by uniting the two colleges together to mentor pre-service teachers through Inquiry-Based Learning. Our Four-Movement Classical Symphony model fosters students' interest early on to spark a lasting desire to pursue a career as a STEM teacher. Many success stories involve an inspiring mentor. The City Tech and BMCC NEST Project aspires to provide that opportunity for future teachers. The premise of this presentation is to share our success stories of the Four-Movement Classical Symphony: Mentoring Pre-Service Teachers Through Inquiry-Based Learning Model. (Received September 20, 2016)

1125-VW-2241 **Dan L. Seth\*** (dseth@wtamu.edu), Department of Mathematics, WTAMU, Canyon, TX 79015. *Enhanced Student Learning with Bi-weekly MINITAB Labs in Statistics*. Preliminary report.

The MINITAB program has been integrated into Mathematical and Engineering Statistics at WTAMU the past five years. This past year the course was restructured to include bi-weekly MINITAB labs in a hybrid format. Labs were posted on the classroom management system with class time replaced with time in a computer

laboratory. Students are introduced to MINITAB tools and explore or visualize theories. The presentation will illustrate tools developed to visualize Continuous PDF's and the Central Limit Theorem. Most explorations incorporated fresh data that the students collect. Examples presented will include Hypothesis Testing for the Mean, ANOVA and Real Estate Data, and a Chi-squared test for Vehicle Types. Incorporation of the labs has enhanced learning and engendered an interest in statistics as a meaningful subject of study. Assessment results will be presented (Received September 20, 2016)

1125-VW-2525 **Ryan Luke\*** ([ryan.luke@louisville.edu](mailto:ryan.luke@louisville.edu)). *Behavior of Residuated Maps with respect to the Way-Below Relation*. Preliminary report.

We will examine residuated mappings on a function lattice and how they behave with respect to the way-below relation. In particular, which residuated  $\phi$  has the property that  $F$  is way-below  $\phi(F)$  for  $F$  in appropriate sets. We will also consider an application to contractions and probabilistic metric spaces. (Received September 20, 2016)

1125-VW-2629 **Kimberly Muller\*** ([kmuller@lssu.edu](mailto:kmuller@lssu.edu)), 650. W. Easterday Ave., School of Mathematics and Computer Science, Sault Ste. Marie, MI 49783. *Connecting the Math and Science Practices*. Preliminary report.

The Standards of Mathematical Practice and the Standards of Science and Engineering Practice in the NGSS have many commonalities. In fact, with a well-designed activity, one can provide opportunities for K-12 students to use a large majority of both sets of Standards in a single lesson. The speaker will discuss ways in which collaboration both inside and outside the classroom can help aid the students in developing a deeper understanding of mathematics. (Received September 20, 2016)

1125-VW-2765 **Susan Licwinko\*** ([slicwinko@bmcc.cuny.edu](mailto:slicwinko@bmcc.cuny.edu)), Department of Mathematics, BMCC, The City University of New York, 199 Chambers Street, New York, NY 10007, and **Jean W. Richards, Daniela Bardac-Vlada and Lucio M-G Prado**. *New Directions for Developmental Mathematics in Community Colleges*.

Faced with an increasing failure and withdrawal rates in remedial mathematics in community colleges for non-STEM students, we present alternative pathways for developmental algebra, which is critical for student advancement towards graduation. Often, non-stem students need a credit-bearing mathematics course to satisfy requirements for their degree. This talk will argue that while elementary algebra is important for certain disciplines, alternative pathways must be made available to students who struggle through it several semesters; who then often end up dropping out from college. There are several alternatives which integrate algebraic concepts into credit-bearing classes including statistics, fundamentals of mathematics (sometimes referred to as Liberal Arts mathematics), and quantitative literacy. These innovative pathways represent a cornerstone in the evolution of developmental mathematics (Received September 20, 2016)

1125-VW-2789 **Jennifer K Ulrich\*** ([jkm154@psu.edu](mailto:jkm154@psu.edu)), School of Science, 4701 College Drive, Erie, PA 16563. *The Discrete Sheffer Sequences and Schrodinger Form*.

In this talk, I develop a discrete analogue of the one-dimensional time-independent Schrodinger equation through the theory of classical discrete orthogonal polynomial sequences. The difference equation is first established in a general context and then specific solutions are obtained involving each of the discrete Sheffer sequences (the Meixner, Charlier and Krawtchouk polynomials). In turn, I also develop a first-order difference equation for each of these polynomials; the derivation of the latter two does not appear in the literature. Finally, I supplement my analysis by graphing several solutions to the Schrodinger equation and conclude my talk with some future directions. (Received September 20, 2016)

1125-VW-2799 **John Robert Botzum\*** ([botzum@kutztown.edu](mailto:botzum@kutztown.edu)), 5528 Heather lane, Orefield, PA. *"It Does Matter How You Slice It: The Combinatorics of Pizza-Slicing"*.

Mathematics students are urged to recognize patterns and form general conclusions. However, students of elementary mathematics are cautioned against assuming that conjectures formed inductively are necessarily true in the general case. A classic example of a reasonable conjecture that is false arises in the solution of the following problem : What is the maximum number of regions formed by pairwise connections of  $n$  points on the circle? We will employ elementary counting methods to solve this problem and more general problems. (Received September 20, 2016)

1125-VW-2827 **Tucker L. Dowell\*** ([tucker.dowell@pop.belmont.edu](mailto:tucker.dowell@pop.belmont.edu)), **Daniel Biles** and **Glenn Acree**. *Stochastic Social Network Model for the Dissemination of Ideas.*

A lot of work has been done using graph theory to represent social networks. We use some of this knowledge to drive an idea for a model to map the dissemination of ideas within social networks. Each vertex of this graph represents a person, and weighted edges are used to represent connections between people while vertices are weighted based on network centrality measures. Associated with each vertex (person) is a set of Stochastic Differential Equations representing the disposition of the person toward a certain aspect of an idea. For example, if we were modeling a presidential election, there would be a certain probability a person would lean toward the Republican, Democratic, Libertarian, or Green Parties at any particular time. The goal of this research is to test the validity of the proposed model, seeing, in particular, if we can use the model to make predictions about how the disposition of the entire network will change over time. (Received September 20, 2016)

1125-VW-2880 **Brandon Samples\*** ([brandon.samples@gcsu.edu](mailto:brandon.samples@gcsu.edu)), Georgia College, Department of Mathematics CBX 017, Milledgeville, GA 31061, and **Emily Baum** ([emily.baum@uga.edu](mailto:emily.baum@uga.edu)), University of Georgia, College of Education, Athens, GA 30602. *Supplemental Instruction Shaping Student Success.*

Academic support programs are essential when it comes to increasing student success. Such programs are particularly vital for the mathematical sciences, which have historically lower success rates. Building a program that provides effective academic enhancement opportunities for all students requires a consideration of many factors. One must identify the areas of greatest need and provide high impact instructional support while considering a myriad of resource limitations. In this talk, we will examine a supplemental instruction program and highlight the key factors that contribute to quality academic enhancement. We will provide both statistical evidence and qualitative feedback about the effectiveness of such a program. We will also highlight the mentorship and professional development opportunities afforded our Supplemental Instruction (SI) Leaders that truly make learning a vertically integrated, dynamic collaborative experience. (Received September 20, 2016)

1125-VW-2914 **Sheida Riahi\*** ([sr1315@msstate.edu](mailto:sr1315@msstate.edu)), Department of Mathematics and Statistics, Mississippi State, MS 39762, and **Prakash Patil**. *Measuring and Testing Central Symmetry in Bivariate Settings.* Preliminary report.

A concept of symmetry plays an important role in statistics. For example detection of abnormalities as asymmetrical patterns in the thermographic images is linked to the concept of symmetric regression functions or the null distributions of  $t$  and  $F$  statistics in the univariate general linear model depend on spherical symmetry of the error distribution. For a multivariate distribution one can define different kinds of symmetry, e.g. central, spherical, elliptical symmetry etc. Our interest here is to study these symmetries in bivariate set-up with the intention of measuring (quantifying) and testing for different kinds of asymmetries. For that here as a first step we extend the recently proposed quantification of asymmetry and tests for symmetry in Partlett and Patil (2015) to the central symmetry in bivariate settings. (Received September 20, 2016)

1125-VW-3018 **Erin E. Williams\*** ([ewilliams50@uco.edu](mailto:ewilliams50@uco.edu)). *Categorization of all Newton maps of rational functions conjugate to quadratic polynomials.*

Conjugation can be used to simplify many questions which arise when iterating complex valued functions. We say that two functions,  $f$  and  $g$ , are conjugate if there exists a Möbius transformation,  $M$ , such that  $M \circ f \circ M^{-1} = g$ . In this talk we will consider the iteration of Newton maps ( $R(z) = z - \frac{r(z)}{r'(z)}$ ) where  $r(z)$  is a rational function, and categorize all such maps which are conjugate to  $z^2 + c$ . (Received September 20, 2016)

1125-VW-3140 **Paul R. Bialek\*** ([pbialek@tiu.edu](mailto:pbialek@tiu.edu)), Dept of Mathematics, Trinity International University, 2065 Half Day Rd, Deerfield, IL 60015. *How to Choose a Graduate School in Mathematics.*

How do you choose which graduate school in Mathematics to attend? The author has researched the topic and will share the results with you. (Received September 21, 2016)