

1077-03-2897

**Joseph S. Miller\*** (jmillermath@math.wisc.edu). *A small step beyond the Turing degrees.*

The computable real numbers were introduced in Alan Turing's seminal 1936 paper, "On computable numbers, with an application to the Entscheidungsproblem." They were defined to be the reals with computable decimal expansions, but the following year Turing published a correction, "modifying the manner in which computable numbers are associated with computable sequences, the totality of computable numbers being left unaltered." His second representation avoids the problem of non-uniformity at rationals that have finite decimal expansions and is suitable for applying computability theory to functions on the reals.

It turns out that to measure the complexity of continuous function on the reals we need to go beyond the Turing degrees. This is closely related to the reason that Turing rejected his original definition of the computable real numbers: the difficulty of capturing computability on a connected space (the real numbers) using representations in a totally disconnected space (infinite decimal sequences). We discuss the extension of the Turing degrees to a degree structure that can measure the complexity of objects from analysis. We then turn to an interesting class of objects, Levin's neutral measures, that always have non-Turing degree. (Received September 22, 2011)