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Joseph H. Silverman (joseph_silverman@brown.edu) and **Katherine E. Stange***
(stange@math.stanford.edu). *A dynamical system for elliptic divisibility sequences.*

Consider any integer divisibility sequence $(D_n)_{n \geq 1}$ (i.e integer sequence satisfying $n \mid m \implies D_n \mid D_m$) with the property that every prime number eventually appears as a divisor in the sequence. Define a map $\phi : \mathbb{Z}^{>0} \rightarrow \mathbb{Z}^{>0}$ defined by

$$\phi(n) = \min_{k > 0} \{D_k \equiv 0 \pmod{n}\}.$$

Consider the dynamical system given by forward iteration of this map. Cycles of this system are called *aliquot cycles*. We consider the case of elliptic divisibility sequences, a class of divisibility sequences associated to elliptic curves. The study of aliquot cycles leads to a characterisation of the set $S(D)$ of indices n satisfying $n \mid D_n$. In particular, given an index n in $S(D)$, we explain how to construct elements nd in $S(D)$, where d is either a prime divisor of D_n , or d is a product of primes forming an aliquot cycle for D . We draw a connection to the definition of an *amicable pair* for an elliptic curve E/\mathbb{Q} : a pair of primes (p, q) of good reduction for E satisfying $\#E(\mathbb{F}_p) = q$ and $\#E(\mathbb{F}_q) = p$. (Received September 18, 2011)