

1077-11-2192

Douglas Ulmer* (douglas.ulmer@math.gatech.edu), School of Mathematics, Georgia Institute of Technology, Atlanta, GA 30332. *On the arithmetic of the Legendre curve in a tower of function fields*. Preliminary report.

Let p be an odd prime number, f a positive integer, $d = p^f + 1$, $K = \mathbf{F}_p(t)$, and $K_d = \mathbf{F}_{p^{2f}}(u)$ where $u^d = t$. The arithmetic of the Legendre curve

$$y^2 = x(x+1)(x+t)$$

in the tower of fields K_d is very rich. For example, the point

$$P = (u, u(u+1)^{d/2}) \in E(K_d)$$

and its conjugates over K generate a subgroup $V_d \subset E(K_d)$ of finite index and rank $d - 2$. It also turns out that the Tate-Shafarevich group $\text{Sha}(E/K_d)$ is a finite p -group and we have

$$[E(K_d) : V_d]^2 = |\text{Sha}(E/K_d)|.$$

In this talk we will discuss explicit calculations of the quotient $\frac{E(K_d)}{V_d}$ and $\text{Sha}(E/K_d)$ as modules over $\mathbf{Z}_p[\text{Gal}(K_d/K)]$ and explain why they satisfy a Gras-type conjecture. (Received September 21, 2011)