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**Walter Craig\*** ([craig@math.mcmaster.ca](mailto:craig@math.mcmaster.ca)), Department of Mathematics and Statistics,  
McMaster University, Hamilton, Ontario L8S 4K1, Canada. *On the size of the Navier - Stokes  
singular set.*

Consider the hypothetical situation in which a weak solution  $u(t, x)$  of the Navier-Stokes equations in three dimensions develops a singularity at some singular time  $t = T$ . It could do this by a failure of regularity, or more seriously, it could also fail to be continuous in the strong  $L^2$  topology. The famous Caffarelli Kohn Nirenberg theorem on partial regularity gives an upper bound on the Hausdorff dimension of the singular set  $S(T)$ . We study microlocal properties of the Fourier transform of the solution in the cotangent bundle  $T^*(R^3)$  above this set. Our first result is that, if the singular set is nonempty, then there is a lower bound on the size of the wave front set  $WF(u(T, \cdot))$ , namely, singularities can only occur on subsets of  $T^*(R^3)$  which are sufficiently large. Furthermore, if the solution is discontinuous in  $L^2$  we identify a closed subset  $S'(T) \subseteq S(T)$  on which the  $L^2$  norm concentrates at this time  $T$ . We then give a lower bound on the microlocal manifestation of this  $L^2$  concentration set, which is larger than the general one above. An element of the proof of these two bounds is a global estimate on weak solutions of the Navier-Stokes equations which have sufficiently smooth initial data. (Received September 18, 2011)