1077-42-115 **Bassam H. Shayya*** (`bshayya@aub.edu.lb`), American University of Beirut, P.O. Box 11-0236 / Mathematics, Riad El-Solh, Beirut, 1107 2020, Lebanon. *Decay of spherical means of Fourier transforms and distance sets of measures.*

Suppose $\mu \in M(\mathbb{R}^n)$ is a measure with $\|\mu\| > 0$, σ is surface measure on the unit sphere $\mathbb{S}^{n-1} \subset \mathbb{R}^n$, and $\phi \in L^2(\mathbb{S}^{n-1})$ is a function with $\|\phi\|_{L^2(\mathbb{S}^{n-1})} > 0$. If

$$\int_0^\infty \int_{\mathbb{S}^{n-1}} |\widehat{\mu}(r\theta)|^2 d\sigma(\theta) r^{n-1} dr = \int_{\mathbb{R}^n} |\widehat{\mu}(\xi)|^2 d\xi < \infty,$$

then, as is well-known, $d\mu \ll dx$, and since $\|\mu\| > 0$, it follows that $|\operatorname{supp} \mu| > 0$. Now by the Cauchy-Schwarz inequality,

$$\Big| \int_{\mathbb{S}^{n-1}} \widehat{\mu}(r\theta)\phi(\theta) d\sigma(\theta) \Big|^2 \leq \|\phi\|^2_{L^2(\mathbb{S}^{n-1})} \int_{\mathbb{S}^{n-1}} |\widehat{\mu}(r\theta)|^2 d\sigma(\theta),$$

so it is natural to ask the question, what can we say about $\operatorname{supp} \mu$ under the weaker assumption

$$\int_0^\infty \Big| \int_{\mathbb{S}^{n-1}} \widehat{\mu}(r\theta)\phi(\theta) d\sigma(\theta) \Big|^2 r^{n-1} dr < \infty?$$

We give an answer to this question in the case $\phi \in C^\infty(\mathbb{S}^{n-1})$. We also give an application of our result to Falconer's distance set problem. (Received July 28, 2011)