1077-46-1227 **Igor Klep*** (igor.klep@fmf.uni-lj.si), Univerza v Ljubljani, Fakulteta za matematiko in fiziko, Jadranska 19, 1111 Ljubljana, Slovenia, and **J. William Helton**, **Scott McCullough** and **Markus Schweighofer**. *Convex Positivstellensatz, linear matrix inequalities and complete positivity.* Preliminary report.

Given polynomials p and q, it is natural to ask: does one dominate the other? That is,

$$\text{does} \quad q(x) \geq 0 \quad \text{imply} \quad p(x) \geq 0 \text{ ?} \tag{Q}$$

In this talk we focus on *free noncommutative* polynomials p, q and substitute matrices for the variables x_j. In case the positivity domain $\mathcal{D} = \{X \mid q(X) \succeq 0\}$ is *convex*, the domination question (Q) has an elegant answer. First of all, $\mathcal{D}$ then has a linear matrix inequality (LMI) representation, i.e., $\mathcal{D} = \{X \mid L(X) \succeq 0\}$ for a linear pencil L. Furthermore, the following "perfect" *Positivstellensatz* holds: p is *positive semidefinite* on the LMI domain $\mathcal{D}$ if and only if it has a weighted sum of squares representation with optimal degree bounds:

$$p(x) = s(x)^T s(x) + \sum_j f_j(x)^T \, L(x) \, f_j(x), \tag{A}$$

where $s(x), f_j(x)$ are vectors of polynomials of degree no greater than $\deg(p)/2$.

We shall also discuss the linear variant of (Q) and show how *LMI domination* is essentially equivalent to *complete positivity*. (Received September 18, 2011)