

1077-47-558

Michael T Jury* (mjury@uf1.edu), Department of Mathematics, University of Florida, PO Box 118105, Gainesville, FL 32611-8105. “*Noncommutative*” Aleksandrov-Clark measures and function theory in the unit ball. Preliminary report.

Let \mathcal{S} denote the set of functions b , holomorphic in the unit ball of \mathbb{C}^d , such that the kernel $(1 - b(z)\overline{b(w)})(1 - \langle z, w \rangle)^{-1}$ is positive, and write $\mathcal{H}(b)$ for the corresponding reproducing kernel Hilbert space. In one variable these are known as the *de Branges-Rovnyak spaces*. Their theory is well-developed; the central objects are the backward shift operator on $\mathcal{H}(b)$ and the Aleksandrov-Clark (AC) measure μ .

The natural analog of the AC measure in the multivariable setting is a certain positive linear functional on a (non-commutative) operator system. The next difficulty is to understand what should be meant by “backward shift.” We introduce a canonical solution to the Gleason problem in $\mathcal{H}(b)$ which preserves many features of the backward shift in the one-variable setting, and identify a subclass of \mathcal{S} called *quasi-extreme* functions. (In one variable, these are the extreme points of the unit ball of H^∞ .) As an application we obtain a version of Clark’s theorem on rank-one perturbations of the backward shift, and some further function-theoretic results. (For example, $\mathcal{H}(b)$ is z_j -invariant for each $j = 1, \dots, d$ if and only if b is not quasi-extreme.) (Received September 07, 2011)