1077-51-371 **Anton Lukyanenko*** (`Anton@Lukyanenko.net`), 1409 W. Green Street, Mathematics Department, Urbana, IL 61801. *Bi-Lipschitz Extension from Boundaries of Certain Gromov Hypebolic Spaces.*

The compactified Heisenberg group H is the boundary at infinity of complex hyperbolic space $\mathbb{CH}$. A quasi-isometry of $\mathbb{CH}$ extends to a quasi-symmetry of H, and all quasi-symmetries of H arise in this way. Can one say the same of bi-Lipschitz maps of $\mathbb{CH}$?

We define *metric similarity spaces* as spaces X^+ possessing an analogue of the upper half-plane model of hyperbolic space. In particular, $X^+ = X \times \mathbb{R}^+$ for a quasi-homogeneous base metric space X homeomorphic to $\mathbb{R}^n$, and homotheties of X extend to isometries of X^+. Metric similarity spaces include non-compact rank one symmetric spaces such as complex and quaternionic hyperbolic space, as well as warped products of many nilpotent groups with $\mathbb{R}^+$. Metric similarity spaces X^+ are Gromov hyperbolic, and the base X can be identified with the boundary at infinity of X^+ with a horospherical metric. We refer to both as ∂X^+.

For metric similarity spaces X^+ of dimension $n+1 \neq 4$, we show that every quasi-symmetry of ∂X^+ is induced by a bi-Lipschitz map of ∂X^+. In particular, a quasi-symmetry of H is induced by a bi-Lipschitz map of $\mathbb{CH}$, except possibly for the complex hyperbolic plane. (Received August 26, 2011)