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**Michael Beeson\*** (beesonpublic@gmail.com). *Tiling a triangle with congruent triangles.*

We investigate the problem of cutting a triangle  $ABC$  into  $N$  congruent triangles (the “tiles”), which may or may not be similar to  $ABC$ . We wish to characterize the numbers  $N$  for which some triangle  $ABC$  can be tiled by  $N$  tiles, or more generally to characterize the triples  $(N, T)$  such that  $ABC$  can be  $N$ -tiled using tile  $T$ . In the first part of the paper we exhibit certain families of tilings which contain all known tilings. We conjecture that the exhibited tilings are the only possible tilings. If that is so, then for there to exist an  $N$ -tiling of any triangle  $ABC$ ,  $N$  must be a square, or 2, 3, or 6 times a square, or a sum of two squares. We have proved the result except for a certain class of exceptional triangles and certain values of  $N$ . For example, we have proved that there are no  $N$ -tilings of any triangle when  $N = 7, 11$ , or  $19$ ; and we have completely solved the case when  $ABC$  is similar to  $T$ . We made use of linear algebra and field theory, and for one case, the algebraic number theory of cyclotomic fields. The simplest unsolved case is  $N = 28$ , with a tile whose sides are 2, 3, and 4, and triangle  $ABC$  has sides 12, 14, and 16. (Received September 04, 2011)