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Stedman M. Wilson* (stedmanw@math.ucla.edu) and **Igor Pak**. *Strengthening Fary's Theorem: Convex and Star-Shaped Realizations of Polyhedral Complexes.*

Fary's Theorem states that every planar graph can be drawn in the plane such that each edge is a straight line segment. We study the problem of strengthening Fary's Theorem in the plane, as well as extending it to higher dimensions. Given a pure d -dimensional topological polyhedral complex (embedded in \mathbb{R}^d), we ask when it can be realized geometrically (that is, rectilinearly embedded in \mathbb{R}^d). We discuss positive and negative results. On the negative side, we exhibit a ($d = 3$) polytopal complex that must have irrational vertex coordinates in any geometric realization. The techniques of this construction provide a novel method for creating topological complexes that cannot be realized geometrically. On the positive side, we show that all ($d = 2$) polyhedral complexes homeomorphic to a ball, as well as a certain class of ($d = 3$) polyhedral balls, may be realized geometrically with convex cells. Furthermore, we show that all ($d = 2$) polyhedral *configurations* (a generalization of a complex) homeomorphic to a ball, as well as a certain class of ($d = 3$) configurations, may be realized geometrically with star-shaped cells. Finally, we mention the analogous yet more restrictive results for $d > 3$. (Received September 14, 2011)