1077-65-635 **Runchang Lin***, Department of Mathematics, TAMIU, 5201 University Blvd, Laredo, TX 78041, and **Martin Stynes**, Department of Mathematics, National University of Ireland, Cork, Ireland.
A balanced finite element method for singularly perturbed reaction-diffusion problems.

Consider the singularly perturbed linear reaction-diffusion problem $-\varepsilon^2\Delta u + bu = f$ in $\Omega \subset \mathbb{R}^d$, $u = 0$ on $\partial\Omega$, where $d \geq 1$, the domain boundary $\partial\Omega$ is (when $d \geq 2$) Lipschitz-continuous, and $0 < \varepsilon \ll 1$. It is argued that for this type of problem, the standard energy norm is too weak a norm to measure adequately the errors in solutions computed by finite element methods: the multiplier ε^2 gives an unbalanced norm whose different components have different orders of magnitude. A balanced and stronger norm is introduced, then for $d \geq 2$ a mixed finite element method is constructed whose solution is quasi-optimal in this new norm. By a duality argument it is shown that this solution attains a higher order of convergence in the L_2 norm. Error bounds derived from these analyses are presented for the cases $d = 2, 3$. For a problem posed on the unit square in $\mathbb{R}^2$, an error bound that is uniform in ε is proved when the new method is implemented on a Shishkin mesh. Numerical results are presented to show the superiority of the new method over the standard mixed finite element method on the same mesh for this singularly perturbed problem. (Received September 09, 2011)