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The Riemann hypothesis, conjectured by Bernhard Riemann, claims that all nontrivial zeros of  $\zeta(s)$  lie on the line  $\Re(s) = \frac{1}{2}$ . The density hypothesis is a conjectured estimate  $N(\lambda, T) = O(T^{2(1-\lambda)+\epsilon})$  for any  $\epsilon > 0$ , where  $N(\lambda, T)$  is the number of zeros of  $\zeta(s)$  when  $\Re(s) \geq \lambda$  and  $0 < \Im(s) \leq T$ , with  $\frac{1}{2} \leq \lambda \leq 1$  and  $T > 0$ . The Riemann-von Mangoldt Theorem confirms this estimate when  $\lambda = \frac{1}{2}$ , with  $T^\epsilon$  being replaced by  $\log T$ . In an attempt to transform Backlund's proof of the Riemann-von Mangoldt Theorem to a proof of the density hypothesis by convexity, we discovered a slightly different approach utilizing a *pseudo Gamma function*. This function is devised to be symmetric with respect to  $\Re(s) = \frac{1}{2}$ . It is about the size of the Euler Gamma function on the right side of the line  $\Re(s) = \frac{1}{2}$ . Moreover, it is analytic and does not have any zeros and poles in the concerned open region. Aided by this function, we are able to establish a proof of the density hypothesis. Actually, our result is even stronger, when  $\frac{1}{2} < \lambda < 1$ ,  $N(\lambda, T) = O(\log T)$ . (Received September 20, 2011)