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There is a strong analogy between the free group G on countably many generators and the purely transcendental field $F = \mathbb{Q}(X_1, X_2, \dots)$. Two notions of basis are relevant. An independent set generating G is known as a *basis* for the group, while an independent set generating the field F is called a *pure transcendence basis*. In fields, the term *transcendence basis* denotes any maximal independent set, whether or not it generates F ; the analogous notion for G is less common, and we simply call it a maximal independent set.

We establish various effectiveness properties of these notions in computable presentations of F and G . For F , the Turing degrees of transcendence bases form an upper cone above the degree of the dependence relation, which is always computably enumerable. In contrast, it is possible for a computable copy of G to have a computable maximal independent set, yet to have noncomputable dependence relation. When one considers independent generating sets, the situation changes: work of McCoy and Wallbaum established that for G , every computable presentation has a Π_2^0 basis, and that this bound is sharp, whereas for fields, many questions about the Turing degrees of pure transcendence bases remain open. (Received September 09, 2012)