

1086-11-1974      **Duff G. Campbell\*** ([campbell@hendrix.edu](mailto:campbell@hendrix.edu)), Dept. of Mathematics and Computer Science,  
1600 Washington Ave., Little Rock, AR 72212. *Patterns in continued fractions for*  
 $\sqrt{n}$ . Preliminary report.

In an undergraduate number theory course, I asked my students to find patterns in the continued fraction expansions of  $\sqrt{n}$ , working from a list of expansions for  $n = 1$  to  $n = 100$ . Every time I assign this problem, students find the five “classic” patterns, for  $n = m^2 \pm 1$ ,  $n = m^2 \pm 2$ , and  $n = m^2 + m$ . These patterns occur for every  $m$ . But one year, my students found some other patterns, which only occurred for even  $m$ , or odd  $m$ . They also found two patterns for  $m$  divisible by 3. Inspired by their efforts, I looked at the data myself, and found sixteen other patterns, each restricted to its own congruence class (mod  $M$ ) with  $M$ 's up to 27. In addition, I found other patterns where  $n$  had to satisfy a quadratic condition (such as  $n$  of the form  $4k^2 + 3k + 1$ ). Together, these twenty-odd patterns cover  $\sqrt{n}$  for  $n$  up to 68, and 87 of the first hundred, etc. I have also found patterns in the continued fraction expansions of algebraic integers which are roots of  $x^2 + x - n$ . Here I have five patterns which apply to every  $n$ , fifteen that obey linear congruences, eight that satisfy quadratic congruences and two cubic congruences. Together, these patterns cover all  $n$  up to  $n = 23$ , sixty-eight of the first hundred, etc. (Received September 24, 2012)