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Maxwell Anselm and **Steven H Weintraub*** (shw2@lehigh.edu), Dept. of Mathematics,
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Let N be an arbitrary positive integer. We consider continued fractions of the form

$$a_0 + \frac{N}{a_1 + \frac{N}{a_2 + \frac{N}{a_3 + \cdots}}},$$

with a_0 a nonnegative integer and a_1, a_2, a_3, \dots positive integers, and refer to them as cf_N expansions.

The $N > 1$ case has both a number of similarities to and some surprising differences from the classical, i.e., $N = 1$, case.

For $N > 1$, every positive real number x_0 has infinitely many cf_N expansions. We develop a natural notion of the best cf_N expansion of x_0 .

We show, for example, that for $N > 1$, every quadratic irrationality has both periodic and nonperiodic cf_N expansions, and that in many cases the best cf_N expansion of a quadratic irrationality is periodic, but, on the grounds of extensive computational results, we conjecture that this is *not* always the case. We establish further results about the form of the best cf_N expansion when it is periodic; this form sometimes but not always more or less resembles the form in the classical case.

In the classical case, continued fractions have a close relationship with Pell's equation, and we investigate the analog for $N > 1$ as well. (Received May 10, 2012)