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**Victor D Luo\*** (vd11@williams.edu), 4408 Jensen Place, Fairfax, VA 22032, and **Amanda Bower** and **Steven J Miller**. *Generalized Sum and Difference Sets and  $d$ -dimensional Modular Hyperbolas*.

Many problems in additive number theory, such as Fermat's last theorem and the Twin Prime conjecture, can be understood by examining sums of a set with itself. A finite set  $A \subset \mathbb{Z}$  is considered sum-dominant if  $|A + A| > |A - A|$ . Although it may seem that most sets are difference-dominated since addition is commutative and subtraction is not, by controlling the fringes Martin and O'Bryant in 2007 proved a positive percentage are sum-dominant.

To see interesting behavior, the above results suggest looking at modular sets, as there are no fringes. Good choices arise from modular hyperbolas,  $H_d(1; n) := \{(x_1, \dots, x_d) : x_1 \cdots x_d \equiv 1 \pmod{n}\}$ . In 2009, Eichhorn, Khan, Stein, and Yankov investigated the sizes of  $\{x + x^{-1}\}$  and  $\{x - x^{-1}\}$  where  $x \in H_2(1; n)$ . They proved results about the relative sizes of these restricted sum sets and difference sets, and also proved subsets of  $\mathbb{Z}_n$  were in fact sum-dominated at least 84% of the time. We extend their results to  $d$ -dimensional modular hyperbolas. The key ingredients for the counting are Kloosterman sums, Hensel's lemma, and congruence theory. (Received September 15, 2012)