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**C. Joanna Su\*** ([jsu@providence.edu](mailto:jsu@providence.edu)), Dept. of Mathematics and Computer Science,  
Providence College, Providence, RI 02918. *The Third and the Automatic Homotopy Exact  
Sequence of a Fibration in Module Theory.*

The homotopy theory of modules was developed by Peter Hilton in the 1950s, as a natural analog to the existing homotopy theory in algebraic topology. While the concept of a fibration in module theory seemed intuitive and induced a homotopy sequence, the ‘expected’ sequence failed to be discovered. Our search shows that in module theory a fibration induces not just one, but three homotopy sequences - the first, the expected, and the automatic homotopy exact sequences, respectively. Each sequence carries different features. The first and original sequence displays an isomorphism between the relative homotopy groups and the homotopy groups of the ‘fiber’. The expected sequence both displays an analogous appearance to the homotopy sequence of a fibration in topology and inherits the characteristic isomorphism between the relative homotopy groups and the homotopy groups of the induced ‘base space’. Here we discuss the third of the three homotopy sequences - the automatic sequence - which takes place in the projective homotopy theory of modules, whereas the other two sequences took place in the injective homotopy theory of modules. It turns out that the automatic sequence displays an isomorphism between the relative homotopy groups and the ‘strong’ homotopy groups of the fiber. (Received September 18, 2012)