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William W. Johnston* (bwjohnst@butler.edu), Butler University, 4600 Sunset Avenue, Indianapolis, IN 46208. *An Alternate Proof that the Hermite Functions Form a Basis for $L^2(\mathbb{R})$* . Preliminary report.

The standard elegant but advanced proof that the Hermite functions $\{H_n(x)e^{-x^2/2}\}_{n=0}^{\infty}$ form an orthonormal basis for $L^2(\mathbb{R})$ traditionally proceeds by forming an associated complex function, proving it entire, and using its complex function theoretic properties (such as those following from an application of Liouville's theorem and the mean value theorem) to show the corresponding Fourier transform is zero almost everywhere. This presentation describes an alternative proof, which is an expansion of a method of Richard Wheeden and Antoni Zygmund that they applied to $L^2(-\pi, \pi)$. This more basic technique uses real function descriptions of the Schwarz inequality and the fundamental theorem of calculus (on an infinite range of integration) as its most sophisticated tools. The proof is then accessible to undergraduate majors. (Received September 25, 2012)