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Hafedh Herichi and **Michel L. Lapidus*** (lapidus@math.ucr.edu), University of California, Department of Mathematics, Riverside, CA 92521-0135. *The Spectral Operator and Its Applications: Quantized Riemann Zeta Function and Infinitesimal Shift of the Real Line.*

We discuss recent work of H. Herichi and the presenter where we provide a rigorous functional analytic definition of the spectral operator. Earlier, this operator was defined heuristically in a 2006 Springer monograph (2nd ed., 2012) by the presenter and M. van Frankenhuysen as the operator which sends the geometry onto the spectrum of a fractal string. Here, we precisely define it as acting on a suitable scale of Hilbert (or weighted Sobolev) spaces, indexed by the dimension c of the underlying fractal strings, and in terms of suitable boundary conditions. Via the measurable functional calculus for unbounded normal operators, it is equal to the composition of the Riemann zeta function $\zeta(s)$ and D , the underlying differentiation operator or ‘infinitesimal shift’. We plan to discuss several applications of the spectral operator: (1) To a spectral reformulation of the Riemann hypothesis (extending and interpreting from an operator theoretic point of view the corresponding reformulation obtained by the presenter and H. Maier in the 1990s). (2) To different types of ‘phase transitions’ (at $c = 1/2$ and $c = 1$) associated with the Riemann zeta function and its present quantizations. (3) To the study of ‘universality’ (and a new quantum analog) of ζ . (Received September 10, 2012)