

1086-47-853

**Nicholas Young\***, School of Mathematics and Statistics, Newcastle University, Newcastle upon Tyne, NE1 7RU, United Kingdom. *Generalized models and slope functions for the Schur class of the bidisc.*

If a function  $f$  in the Schur class of the bidisc  $\mathbb{D}^2$  satisfies Carathéodory's condition at  $\tau \in \mathbb{T}^2$  then  $f$  has a directional derivative at  $\tau$  in every direction that points into the bidisc. This derivative can be expressed in terms of an analytic function in the Stieltjes class called the slope function of  $f$  at  $\tau$ . We ask: does every function in the Stieltjes class arise as the slope function of some function in the Schur class?

To this end we generalize the notion of a model of  $f$ , that is, a triple  $(H, P, u)$  where  $H$  is a separable Hilbert space,  $P$  is a Hermitian projection on  $H$  and  $u : \mathbb{D}^2 \rightarrow H$  is a map such that, for all  $z, w \in \mathbb{D}^2$ ,

$$1 - \overline{f(w)}f(z) = \langle (1 - w_P^* z_P)u(z), u(w) \rangle_H$$

and

$$z_P = z_1 P + z_2 (1 - P).$$

In a *generalized model* of  $f$  we replace the linear function  $z \mapsto z_P$  by a general contractive analytic operator-valued function on  $\mathbb{D}^2$ . We prove that generalized models with certain regularity properties exist, and with their aid we are able to use an integral representation of functions in the Stieltjes class to show that the answer to the question is yes.

This is joint work with Jim Agler and Ryan Tully-Doyle. (Received September 14, 2012)