1086-51-1147 **Andrei Gabrielov***, agabriel@math.purdue.edu, and **Saugata Basu** and **Nicolai Vorobjov**.
Semi-Monotone Sets, Monotone Maps, and Triangulation of Monotone Families. Preliminary report.

Let $S(t)$, defined for small $t > 0$, be a monotone family (i.e., $S(t) \subset S(u)$ for $t > u > 0$) of compact subsets of a compact set $K \subset R^n$. Both $S(t)$ and K are assumed to be "tame" (e.g., real semialgebraic). The following problem emerges from a conjecture formulated by Gabrielov and Vorobjov (2009) in connection with their work on approximation of a tame set by homotopy equivalent compact sets: Construct a triangulation of K so that restriction of $S(t)$ to each open simplex is equivalent to a "standard" family. The explicit list of $2^n + 1$ standard families is based on lex-monotone Boolean functions in n Boolean variables. A second conjecture claims that K admits a cylindrical cell decomposition into regular cells, such that restriction of $S(t)$ to each open k-cell C is either empty or a regular k-cell, and its boundary in C is either empty or a regular $(k-1)$-cell. To prove these conjectures, Basu, Gabrielov and Vorobjov introduced semi-monotone sets, a generalization of convex sets, and monotone maps, a multivariate generalization of univariate monotone functions. Recent progress towards the proof of the above conjectures will be reported. (Received September 19, 2012)